①

Sean x_0, x_1, x_2 puntos a interpolar

Sean $F(x_0)=y_0$, $F(x_1)=y_1$, $F(x_2)=y_2$

$a = F[x_0, x_1, x_2]$

$$\rightarrow a = \frac{\left(\frac{y_2-y_1}{x_2-x_1}\right)-\left(\frac{y_1-y_0}{x_1-x_0}\right)}{x_2-x_0}$$

$$b = \frac{y_1-y_0}{x_1-x_0} - \left(x_0 + x_1 \cdot \frac{\left(\frac{y_2-y_1}{x_2-x_1}\right)-\left(\frac{y_1-y_0}{x_1-x_0}\right)}{x_2-x_0}\right)$$

$$c = y_0 - \left(x_0 \cdot \frac{y_1-y_0}{x_1-x_0}\right) + x_0 x_1 \cdot \left(\frac{\frac{y_2-y_1}{x_2-x_1} - \frac{y_1-y_0}{x_1-x_0}}{x_2-x_0}\right)$$

(2)

$h_1 = x_1 - x_0$

$h_2 = x_2 - x_1$

$h_2 + h_1 = x_2 - x_0$

$f(x_2) = a(x_2 - x_2)^2 + b(x_2 - x_2) + c$

$f(x_2) = c$

$f(x_1) = a(x_1 - x_2)^2 + b(x_1 - x_2) + f(x_2)$

Despejando b;

$$b = \frac{f(x_1) - a(x_1 - x_2)^2 - f(x_2)}{x_1 - x_2} = \frac{f(x_1) - f(x_2)}{x_1 - x_2} - \frac{a(x_1 - x_2)^2}{x_1 - x_2}$$

$$b = f[x_1, x_2] - a(x_1 - x_2) = f[x_1, x_2] + a(x_2 - x_1) = f[x_1, x_2] + ah_2$$

$$f(x_0) = a(x_0 - x_2)^2 + b(x_0 - x_2) + f(x_2)$$

$$a = \frac{f(x_0) - (f[x_1, x_2] + ah_2)\cdot(x_0 - x_2) - f(x_2)}{x_0 - x_2}$$

$$a = f(x_0) - \left(\left(\frac{f(x_1) - f(x_2)}{x_1 - x_2} + a(x_2 - x_1)\right)(x_0 - x_2) - f(x_2)\right) \Big/ x_0 - x_2$$

$$a - \frac{ah_2(x_0 - x_2)}{x_0 - x_2} = \frac{f(x_0) - f[x_1, x_2]x_0 \cdot (x_0 - x_2) - f(x_2)}{x_0 - x_2}$$

$$a(1 - h_2) = \frac{f(x_0)}{x_0 - x_2} - \frac{(f(x_1) - f(x_2))(x_0 - x_2)}{(x_1 - x_2)(x_0 - x_2)} - \frac{f(x_2)}{x_0 - x_2}$$

$$a(1 - h_2) = \frac{f(x_0)}{x_0 - x_2} - f[x_1, x_2] - f(x_2) \Rightarrow a = \frac{f(x_0)}{(x_0 - x_2)(1 - h_2)} - \frac{f[x_1, x_2]}{1 - h_2} - \frac{f(x_2)}{(x_0 - x_2)(1 - h_2)}$$

$$a = \frac{1}{x_0 - x_2}(f(x_0) - f(x_2)) - \frac{f[x_1, x_2]}{1 - h_2} = \frac{f[x_0, x_2]}{1 - h_2} - \frac{f[x_1, x_2]}{1 - h_2}$$

③ $x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2-4ac}}$ $\quad \epsilon = \left|\frac{x_3 - x_2}{x_3}\right| < 1 \cdot 10^{-10}$

$= \left|1 - \frac{x_2}{x_3}\right| < 1 \cdot 10^{-10}$

$\rightarrow x_2 \approx x_3$

Sea $b > 0$:

$x_3 = x_2 + \frac{-2c}{b \pm \sqrt{b^2-4ac}}$

$\rightarrow \left|1 - \frac{x_2}{x_3}\right| = \left|1 - \frac{x_2}{x_2 + \frac{-2c}{b \pm \sqrt{\quad}}}\right.$

Si $b \geq 0$: $\frac{-2c}{b \pm \sqrt{b^2-4ac}}$ donde $b + \sqrt{b^2-4ac} > 0$ y $\frac{-2c}{b+\sqrt{4ac}} < 0$, a menos que $c < 0$.

Solo si $c < 0$, $\left|1 - \frac{x_2}{x_3}\right| < 1$