$$F(x) \approx f(a)\frac{(x-x_m)(x-b)}{(a-x_m)(a-b)} + f(x_m)\frac{(x-a)(x-b)}{(x_m-a)(x_m-b)} + f(b)\frac{(x-a)(x-x_m)}{(b-a)(b-x_m)} = P(x)$$

Sea $h=\frac{b-a}{2} = x_m - a = b - x_m$

$$P(x) = \frac{F(a)}{+2h^2}(x-x_m)(x-b) + \frac{F(x_m)}{-h^2}(x-a)(x-b) + F(b)\frac{(x-a)(x-x_m)}{2h^2}$$

Para resolver $\int P(x)\,dx$:

Se resuelve la primera integral:

$$\int \frac{F(a)}{2h^2}(x-x_m)(x-b)\,dx = \frac{F(a)}{2h^2}\int(x-x_m)(x-b)\,dx$$

La integral

$$\int (x-a)(x-b)\,dx = (x-a)\frac{(x-b)^2}{2} - \frac{(x-b)^3}{6}$$

Entonces

$$\int \frac{F(a)}{2h^2}(x-x_m)(x-b)\,dx = \frac{F(a)}{2h^2}\left[(x-a)\frac{(x-b)^2}{2} - \frac{(x-b)^3}{6}\right]$$

$$\int_a^b P(x) = \int_a^b \frac{F(a)}{2h^2}(x-x_m)(x-b) + \frac{F(x_m)}{-h^2}(x-x_m)(x-b) + \frac{F(b)}{2h^2}(x-x_m)(x-b)\,dx$$

$$= \frac{F(a)}{2h^2}\left[(x-a)\frac{(x-b)^2}{2} - \frac{(x-b)^3}{6}\right] + \frac{F(x_m)}{-h^2}\left[(x-a)\frac{(x-b)^2}{2} - \frac{(x-b)^3}{6}\right] + \frac{F(b)}{2h^2}\left[(x-a)\frac{(x-x_m)^2}{2} - \frac{(x-x_m)^3}{6}\right]\Bigg|_a^b$$

$$= \frac{F(a)}{2h^2}\left[(x_m-a)\frac{(a-b)^2}{2} + \frac{(a-b)^3}{6}\right] + \frac{F(x_m)}{-h^2}\left[\frac{(a-b)^3}{6}\right] + \frac{F(b)}{2h^2}\left[(b-a)\frac{(b-x_m)^2}{2} - \frac{(b-x_m)^3}{6} + \frac{(a-x_m)^3}{6}\right]$$

$$= \frac{F(a)}{2h^2}\cdot\frac{2}{3}h^3 + \frac{F(x_m)}{-h^2}\cdot -\frac{4}{3}h^3 + \frac{F(b)}{2h^2}\frac{2h^3}{3}$$

$$= \frac{h^3}{3}\left[\frac{F(a)}{h^2} + \frac{4F(x_m)}{h^2} + \frac{F(b)}{h^2}\right] = \frac{h}{3}\left[F(a)+4F(x_m)+F(b)\right]$$