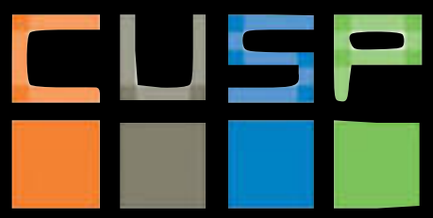
Urban Informatics

Fall 2015

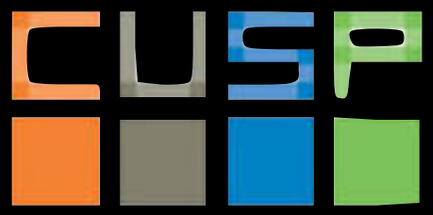
dr. federica bianco fb55@nyu.edu

 @fedhere 



Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: *p*-value, statistical significance
- Systematic errors



V: Likelihood and Regression Models

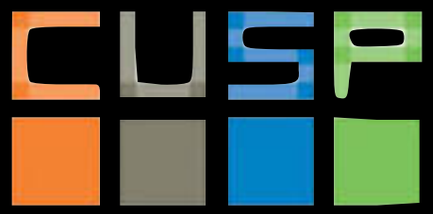
Recap:

- Good practices with data: falsifiability, reproducibility
- Basic data retrieving and munging: APIs, Data formats
- Basic statistics: distributions and their moments
- Hypothesis testing: *p*-value, statistical significance
- Systematic errors

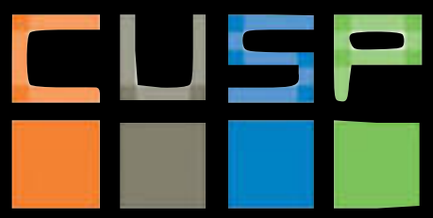
Today:

- Statistical error

- Goodness of fit tests
- Likelihood
- Linear Regression
- Predictive models



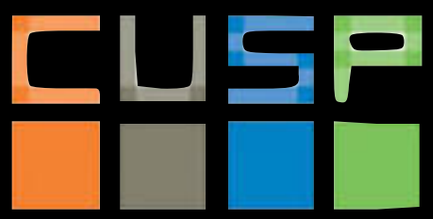
False Positives and False Negatives



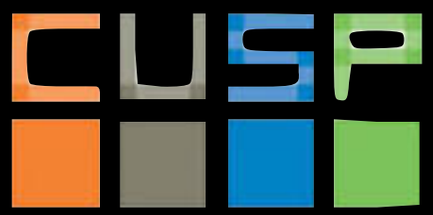
V: Likelihood and Regression Models

False Positives and False Negatives

	H_0 is True	H_0 is False
H_0 is falsified	Type I error False Positive important message gets spammed	True Positive
H_0 is not falsified	True Negative	Type II error False negative Spam in your Inbox

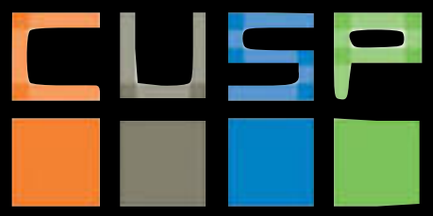


Errors and uncertainties.



V: Likelihood and Regression Models

systematic errors



V: Likelihood and Regression Models

Errors and uncertainties.

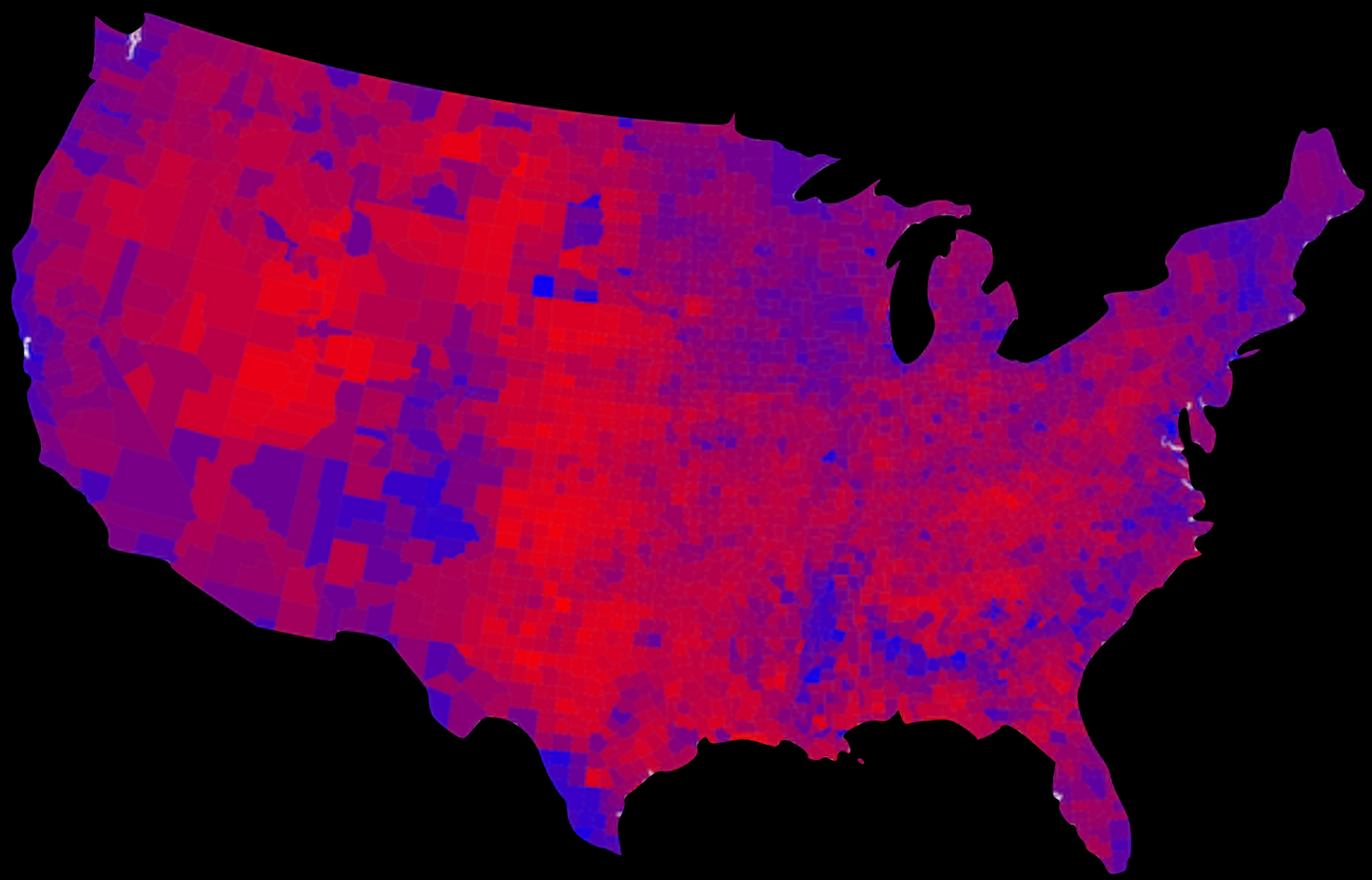
- Systematic error

  tendency to *systematically*
  underestimate/overestimate the average
  difference between the *population* and the subset you test or *sample* because the sample is intrinsically different or the measurements are consistently off

Errors and uncertainties.

IV: Statistical analysis

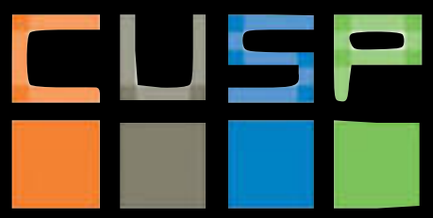
Where are the Real Errors in Political Polls?



http://blogs.scientificamerican.com/guest-blog/where-are-the-real-errors-in-political-polls/

CUSP

Rasmussen Reports mostly finds its sample group through landline phones, which many people no longer use. For those who do not have landline phones, Rasmussen uses an online survey. There are a couple of problems with this methodology. First, *this means the company can only reach people who have landline phones or Internet access.* Yet, according to Nate Silver, the founder and editor of FiveThirtyEight, 23 percent of adults do not have a landline, 4 percent don't answer their landline and 2 percent don't have a phone at all. So Rasmussen's method could definitely bias the poll towards the wealthier and older segments of the population that still uses landlines, both of which tend to vote Republican.



Bias in measurements: know your data

- Systematic error: SURVEY BIAS

UNBIASED survey:
average from *all* samples
equals *population* average

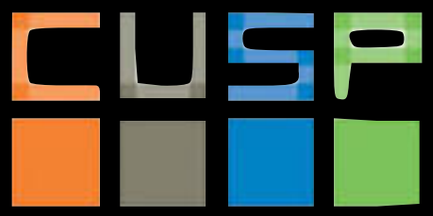
Undercoverage bias
Self selection bias
Social desirability bias

Small number statistic
Publication Bias
Data Dredging

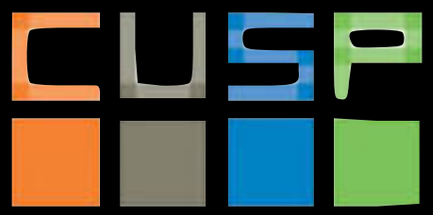
Errors and uncertainties.

- Systematic error
  tendency to *systematically*
  underestimate/overestimate the average
  difference between the *population* and the subset
  you test or *sample* because the sample is intrinsically
  different or the measurements are consistently off

Solution: Good experimental design
Calibration (to assess systematics induced by your measurements)
Simulations (to assess the systematics induced by your analysis)



statistical errors



Errors and uncertainties.

- Systematic error

- Stochastic & Random error

  unpredictable uncertainty in a measurement
  due to lack of sensitivity in the measurement or
  to stochasticity in a process

Errors and uncertainties.

- Stochastic & Random error
  unpredictable uncertainty in a measurement
  due to lack of sensitivity in the measurement
  or
  to stochasticity (inherent randomeness) in a process

4.0 +/- 0.1 cm

Errors and uncertainties.

- Stochastic & Random error

unpredictable uncertainty in a measurement due to lack of sensitivity in the measurement or to stochasticity in a process

2.0 +/- ε cm, ε > 0.1 cm

Repeated measurements give ε (*bootstrap* data…)

Errors and uncertainties.

- Stochastic & Random error

Deterministic systems have no randomness in their evolution. *Chaos* is deterministic….

Stochastic processes can be *completely random*: the probability of any event is disjoint from that of the previous one
These are Poisson processes:

Errors and uncertainties.

- Stochastic & Random error

Deterministic systems have no randomness in their evolution. *Chaos* is deterministic….

Stochastic processes can be
completely random: the probability of any event is disjoint from that of the previous one
These are Poisson processes:
they are described by a Poisson distribution.

A discrete distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate and independently of the time since the last event.

Errors and uncertainties.

- Stochastic & Random error

  Poisson processes :
  https://github.com/fedhere/UInotebooks/blob/master/poisson%20vs%20gaussian.ipynb

Errors and uncertainties.

- Stochastic & Random error

  Poisson processes :
  https://github.com/fedhere/UInotebooks/blob/master/poisson%20vs%20gaussian.ipynb

Binomial

discrete bivariate

2 parameters: n,p

support: [0 , ∞]

moments: np, $\sqrt{npq}$,>0

Poisson

discrete univariate

1 parameters: λ

support: [0 , ∞]

moments: λ, $\sqrt{\lambda}$,>0

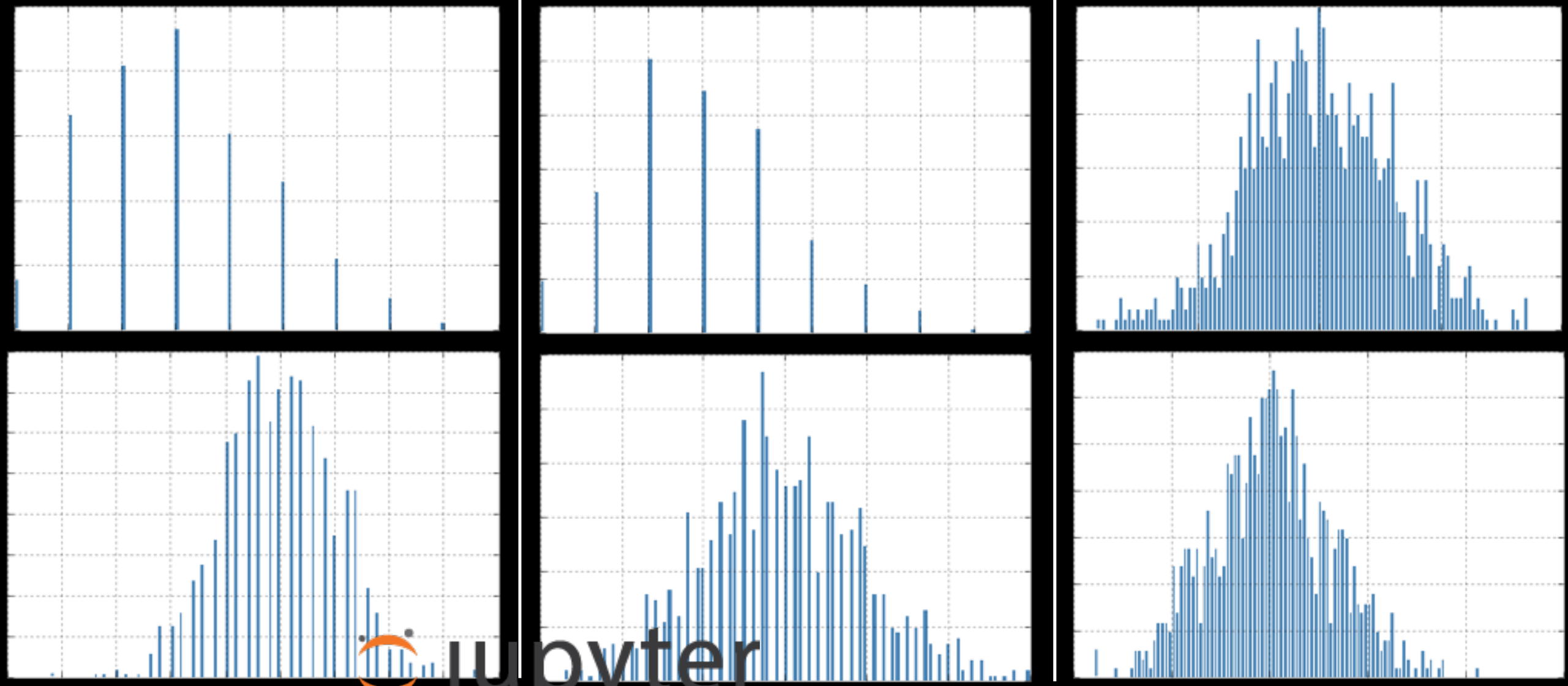
Gaussian

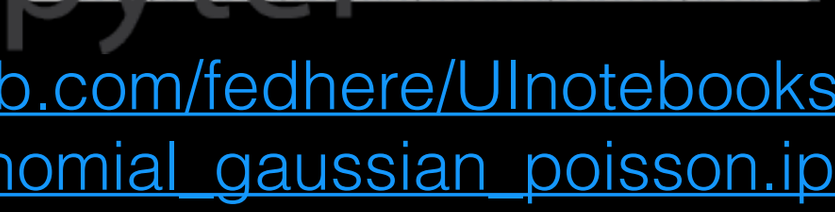
continuous bivariate

2 parameters: μ, σ

support:[- ∞ , ∞]

moments: μ, σ, 0

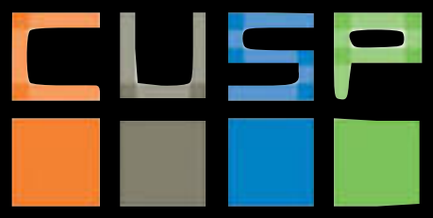




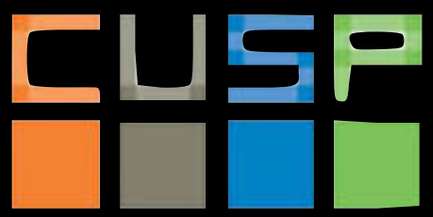
https://github.com/fedhere/UInotebooks/blob/master/binomial_gaussian_poisson.ipynb

for large enough λ
a Poisson distribution $P(\lambda)$
parametrized by λ
tends to a
Gaussian distribution $N(\mu, \sigma)$
of mean $\mu = \lambda$ and standard deviation $\sigma = \sqrt{\lambda}$

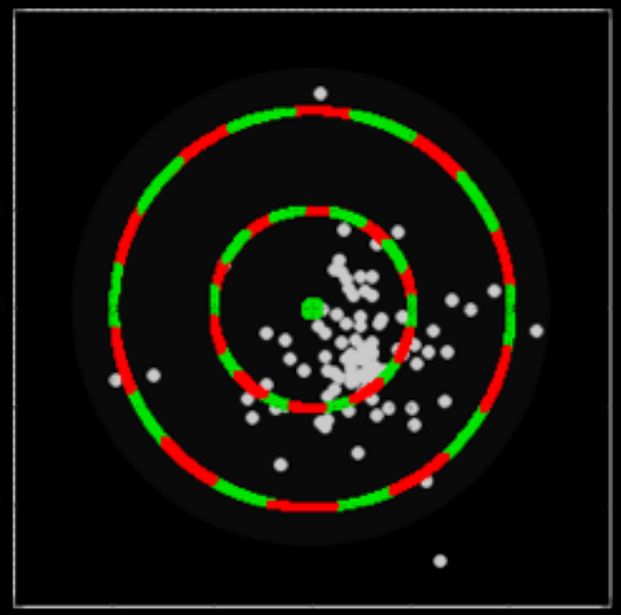
$$P(\lambda) \xrightarrow[\lambda->\infty]{} N(\lambda, \sqrt{\lambda})$$

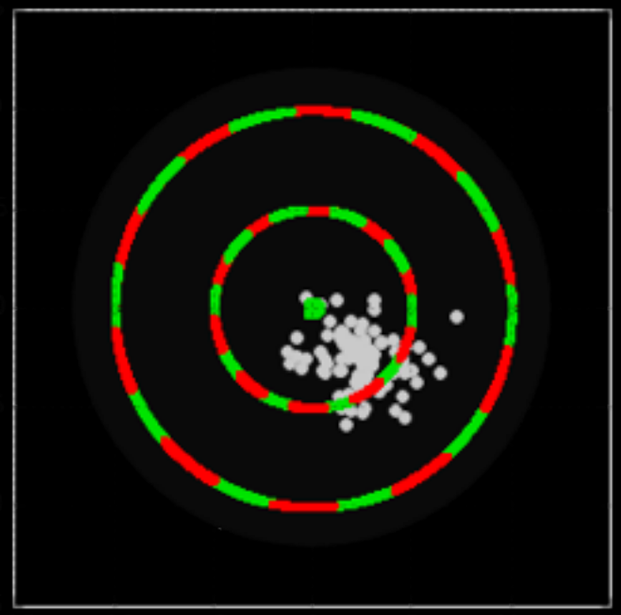


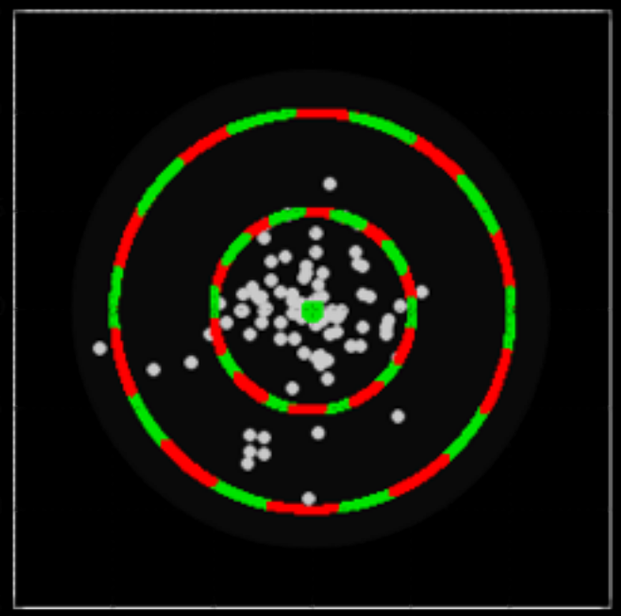
Systematic	Statistical
Biases the measurement in one direction	No preferred direction
Affects the sample regardless of the size	Shrinks with the sample size (typically as $\sqrt{N}$)
Any distribution (usually we use Gaussian though)	Gaussian or Poisson

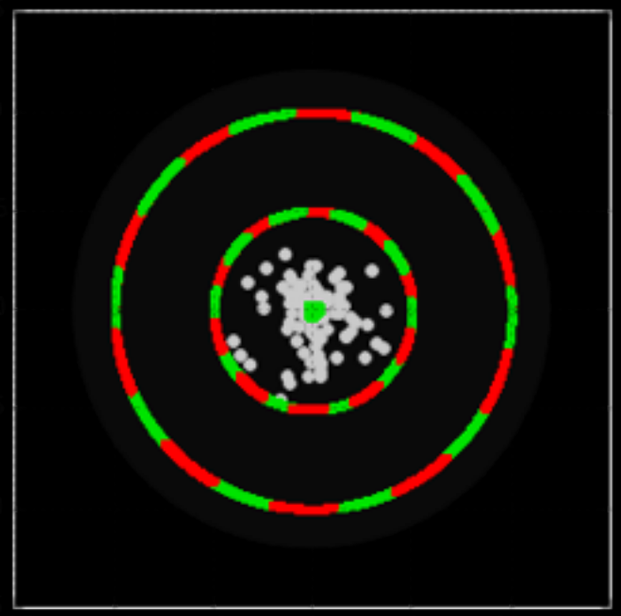


PRECISION









ACCURACY

CUSP

IV: Statistical analysis

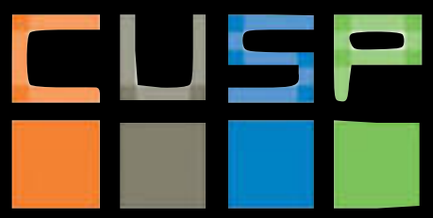
Error propagation

IID: Independent identically distributed:
add in quadrature for linear data operations

$$x_1 \pm \mathcal{E}(x_1)$$
$$x_2 \pm \mathcal{E}(x_2)$$

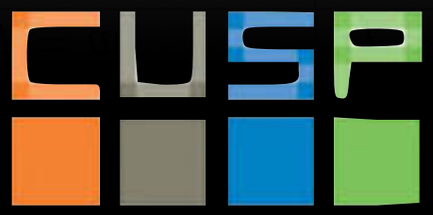
$$\overline{x} = \frac{x_1 + x_2}{2}$$

$$\mathcal{E}(\overline{x}) = \sqrt{\mathcal{E}(x_1)^2 + \mathcal{E}(x_2)^2}$$



Function	Variance	Standard Deviation
$f = aA$	$\sigma_f^2 = a^2\sigma_A^2$	$\sigma_f = a\sigma_A$
$f = aA + bB$	$\sigma_f^2 = a^2\sigma_A^2 + b^2\sigma_B^2 + 2ab\,\sigma_{AB}$	$\sigma_f = \sqrt{a^2\sigma_A^2 + b^2\sigma_B^2 + 2ab\,\sigma_{AB}}$
$f = aA - bB$	$\sigma_f^2 = a^2\sigma_A^2 + b^2\sigma_B^2 - 2ab\,\sigma_{AB}$	$\sigma_f = \sqrt{a^2\sigma_A^2 + b^2\sigma_B^2 - 2ab\,\sigma_{AB}}$
$f = AB$	$\sigma_f^2 \approx f^2\left[\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 + 2\frac{\sigma_{AB}}{AB}\right]$	$\sigma_f \approx \lvert f\rvert\sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 + 2\frac{\sigma_{AB}}{AB}}$
$f = \frac{A}{B}$	$\sigma_f^2 \approx f^2\left[\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 - 2\frac{\sigma_{AB}}{AB}\right]$ [11]	$\sigma_f \approx \lvert f\rvert\sqrt{\left(\frac{\sigma_A}{A}\right)^2 + \left(\frac{\sigma_B}{B}\right)^2 - 2\frac{\sigma_{AB}}{AB}}$
$f = aA^b$	$\sigma_f^2 \approx \left(abA^{b-1}\sigma_A\right)^2 = \left(\frac{fb\sigma_A}{A}\right)^2$	$\sigma_f \approx \left\lvert abA^{b-1}\sigma_A\right\rvert = \left\lvert\frac{fb\sigma_A}{A}\right\rvert$
$f = a\ln(bA)$	$\sigma_f^2 \approx \left(a\frac{\sigma_A}{A}\right)^2$ [12]	$\sigma_f \approx \left\lvert a\frac{\sigma_A}{A}\right\rvert$
$f = a\log_{10}(A)$	$\sigma_f^2 \approx \left(a\frac{\sigma_A}{A\ln(10)}\right)^2$ [12]	$\sigma_f \approx \left\lvert a\frac{\sigma_A}{A\ln(10)}\right\rvert$
$f = ae^{bA}$	$\sigma_f^2 \approx f^2\left(b\sigma_A\right)^2$ [13]	$\sigma_f \approx \lvert f\left(b\sigma_A\right)\rvert$
$f = a^{bA}$	$\sigma_f^2 \approx f^2\left(b\ln(a)\sigma_A\right)^2$	$\sigma_f \approx \lvert f\left(b\ln(a)\sigma_A\right)\rvert$
$f = A^B$	$\sigma_f^2 \approx f^2\left[\left(\frac{B}{A}\sigma_A\right)^2 + \left(\ln(A)\sigma_B\right)^2 + 2\frac{B\ln(A)}{A}\sigma_{AB}\right]$	$\sigma_f \approx \lvert f\rvert\sqrt{\left(\frac{B}{A}\sigma_A\right)^2 + \left(\ln(A)\sigma_B\right)^2 + 2\frac{B\ln(A)}{A}\sigma_{AB}}$

https://en.wikipedia.org/wiki/Propagation_of_uncertainty#Linear_combinations



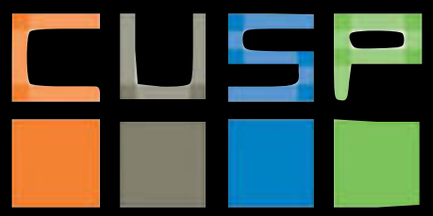
Covariance matrix

$$\vec{x} = \{x_1, x_2, x_3, \ldots, x_n\}$$

$$f_k = \sum_i^n A_{ki} x_i$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \ldots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \ldots & \sigma_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_{m1} & \sigma_{m2} & \ldots & \sigma_m^2 \end{pmatrix}$$

$$\Sigma(f) = A\Sigma^x A^\top$$



Covariance matrix

$$\vec{x} = \{x_1, x_2, x_3, \ldots, x_n\}$$

$$f_k = \sum_i^n A_{ki} x_i$$

IF Independent variables

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \ldots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \ldots & \sigma_{2n} \\ \ldots & \ldots & \ldots & \ldots \\ \sigma_{m1} & \sigma_{m2} & \ldots & \sigma_m^2 \end{pmatrix}$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & 0 & \ldots & 0 \\ 0 & \sigma_2^2 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ 0 & 0 & \ldots & \sigma_m^2 \end{pmatrix}$$

$$\Sigma(f) = A \Sigma^x A^\top$$

$$\Sigma(f)_{ij} = \sum_k^n A_{ik} \Sigma_k A_{jk}$$

Reporting Your Results

It is essential that the systematic error be reported separately from the imprecision part of the reported value

Statistical Concepts and Procedures
by United States. National Bureau of Standards 1969

Keep statistical, systematic errors separate. Report results as something like:

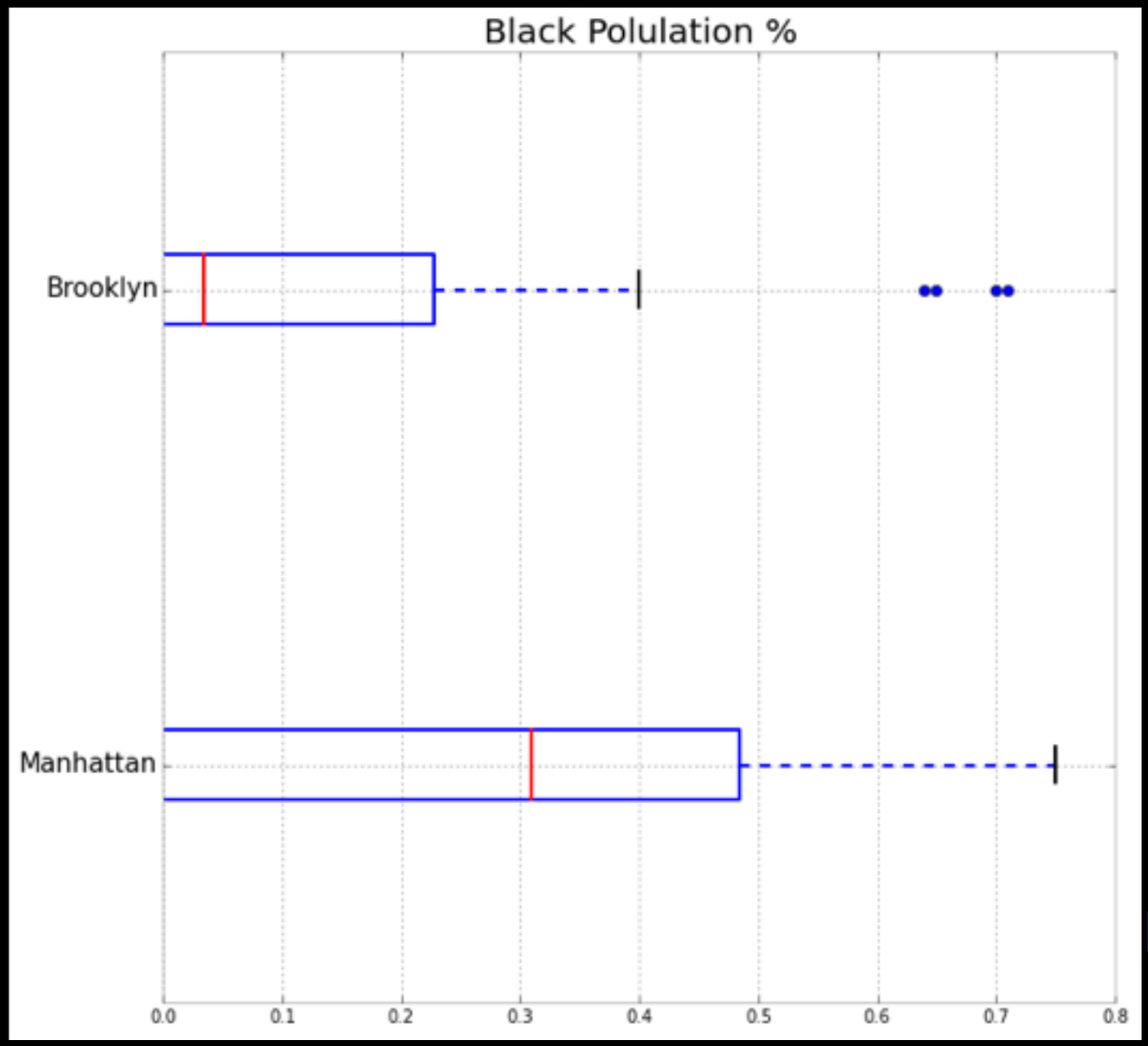
$x = [965 \pm 30(\text{stat}) \pm 12(\text{sys})]$ number of car accidents

Add in quadrature (note that this assumes Gaussian distribution) compare with known values $32 = \sqrt{30^2 + 12^2}$:

$x = [965 \pm 32(\text{total})]$ number of car accidents

Reporting Your Results

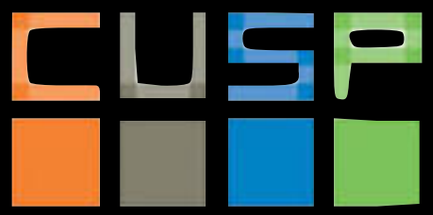
Percentage of Black population by Borrow (Manhattan vs Brooklyn)



jupyter

https://github.com/fedhere/UInotebooks/blob/master/black_percentage.ipynb

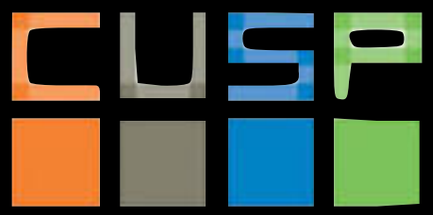
Goodness of fit



You have some data, and an idea of how it should look: a *model*

Is it a good model?

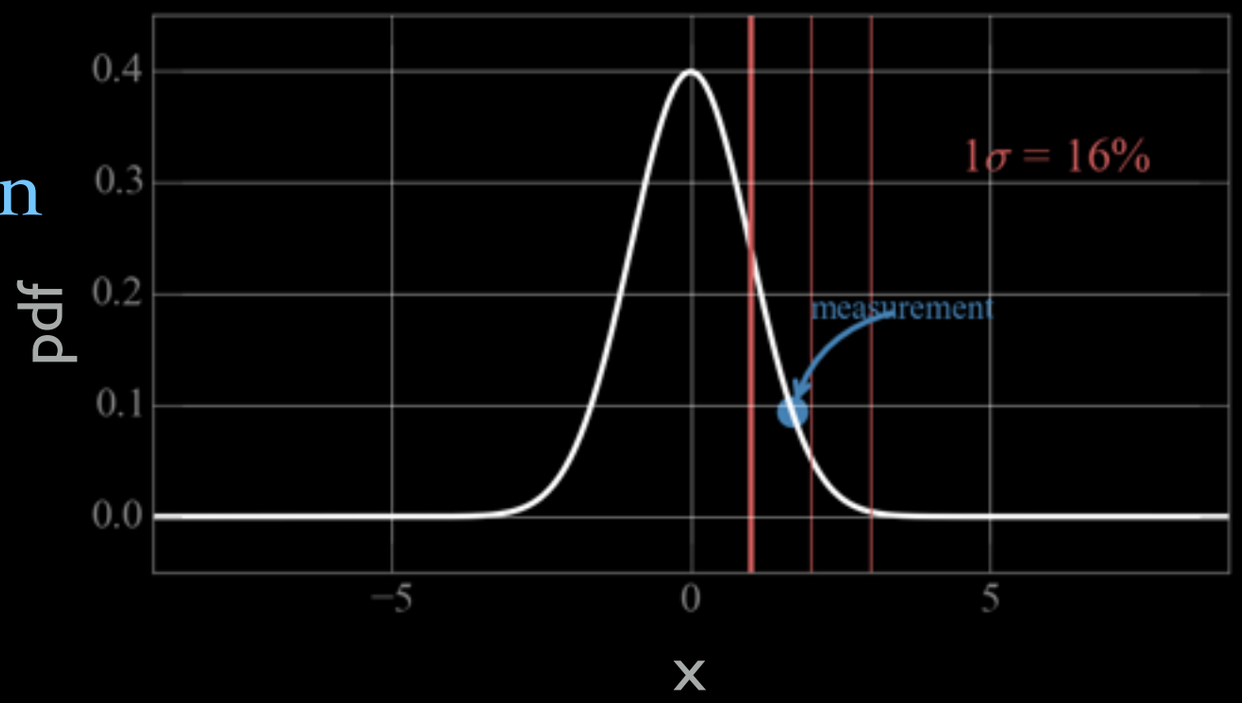
Goodness of fit



Probability Distribution Function

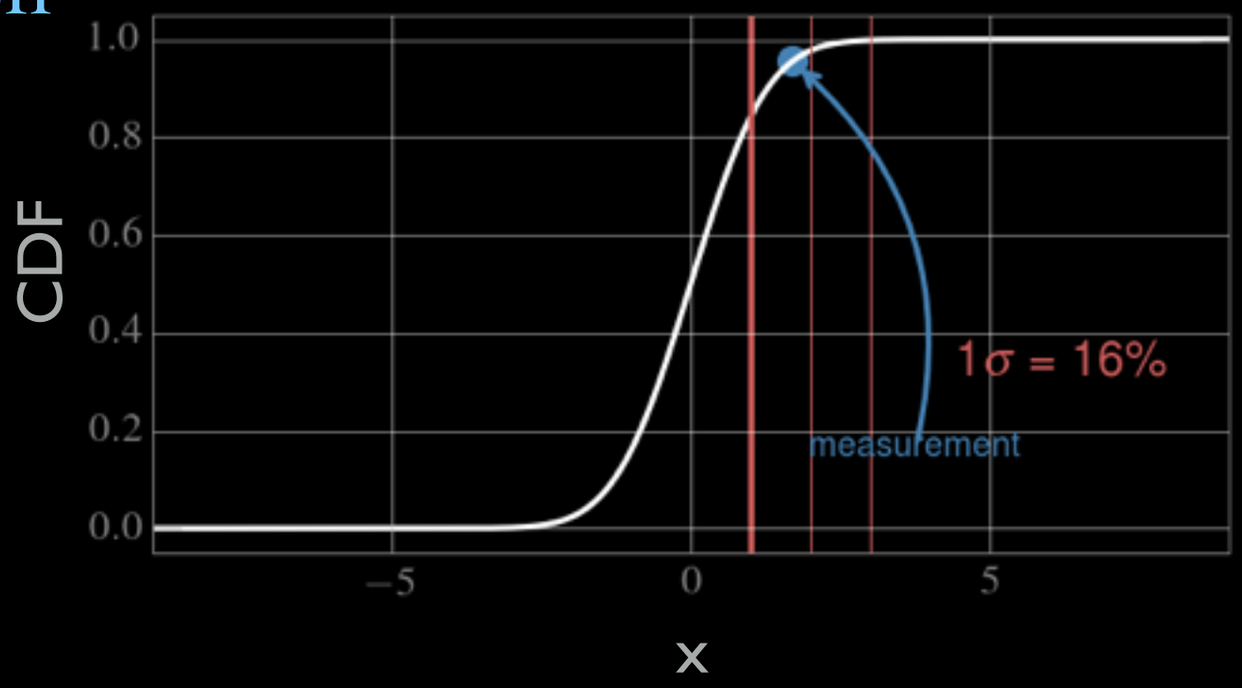
$$f_{x_0}(x) \sim p(x=x_0)$$

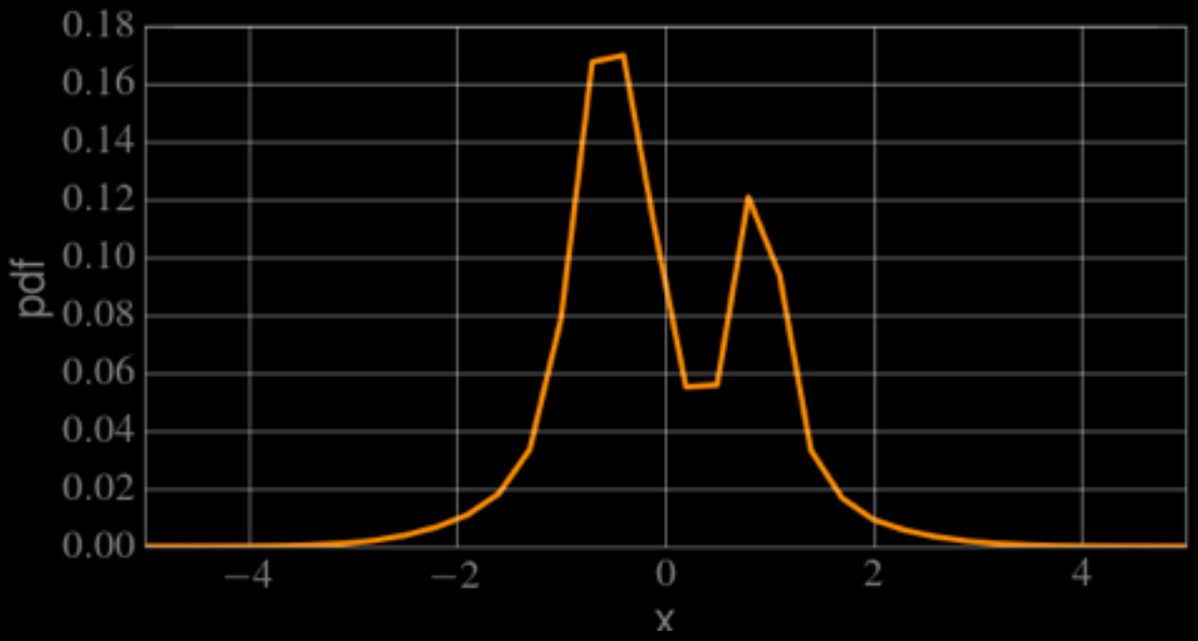
$$f_{x_0}(x) \sim p(x>x_0-dx) \bigcap p(x<x_0+dx)$$

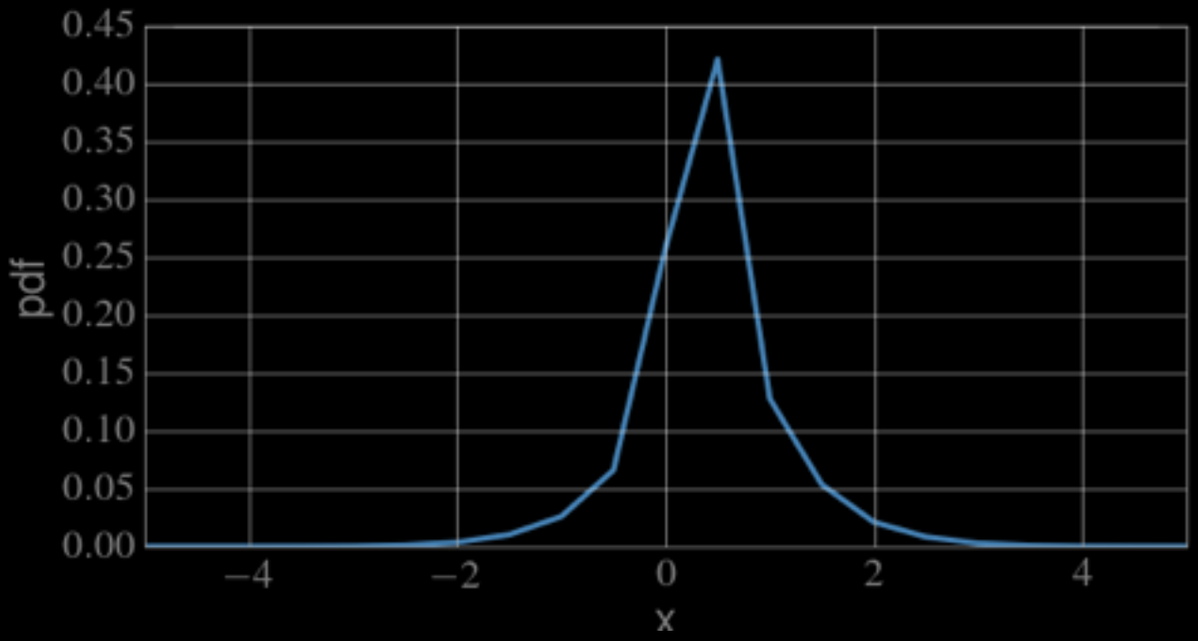


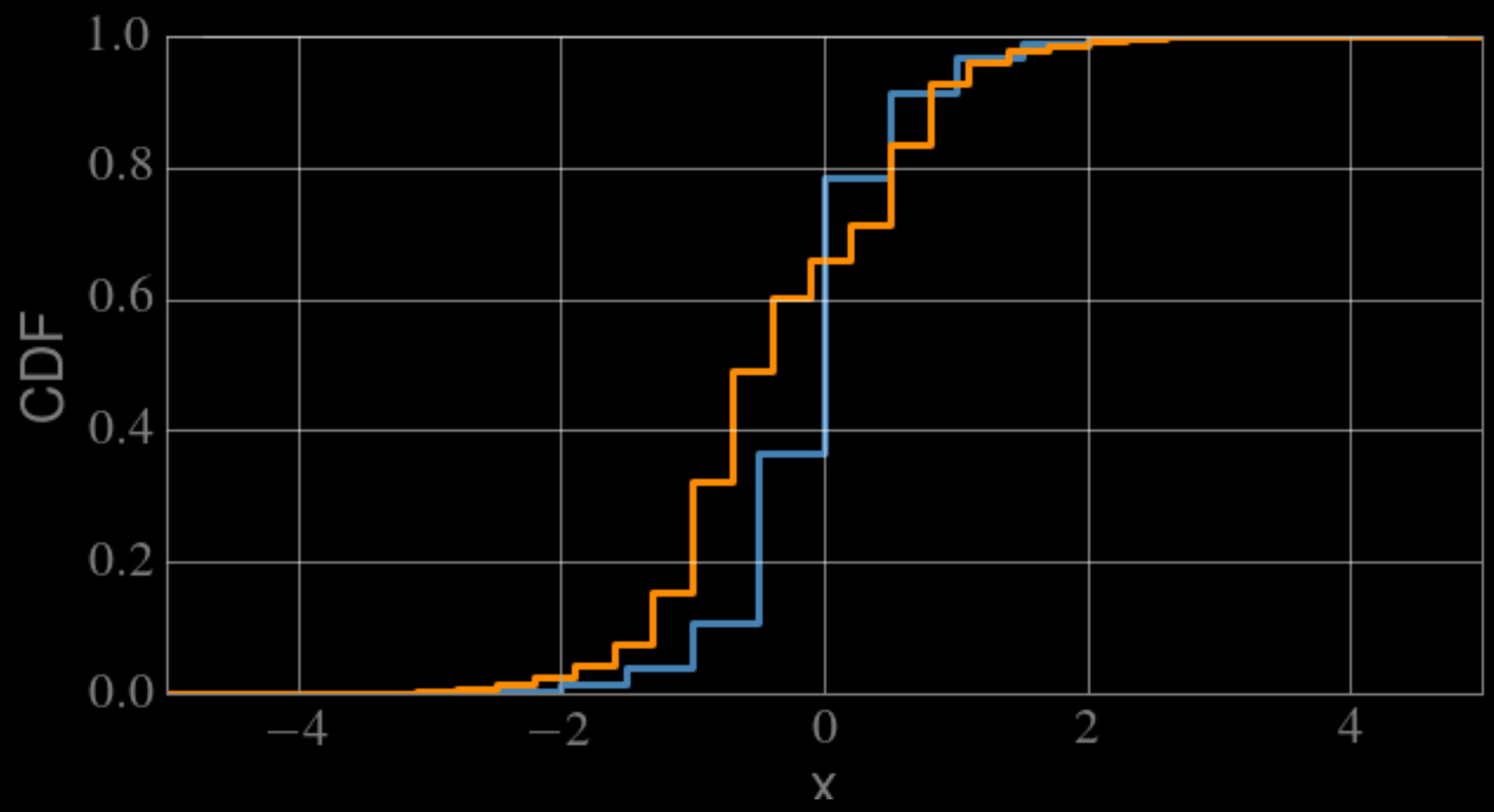
Cumulative Distribution Function

$$F_{x_0}(x) = P(x<x_0)$$









CUSP

IV: Statistical analysis

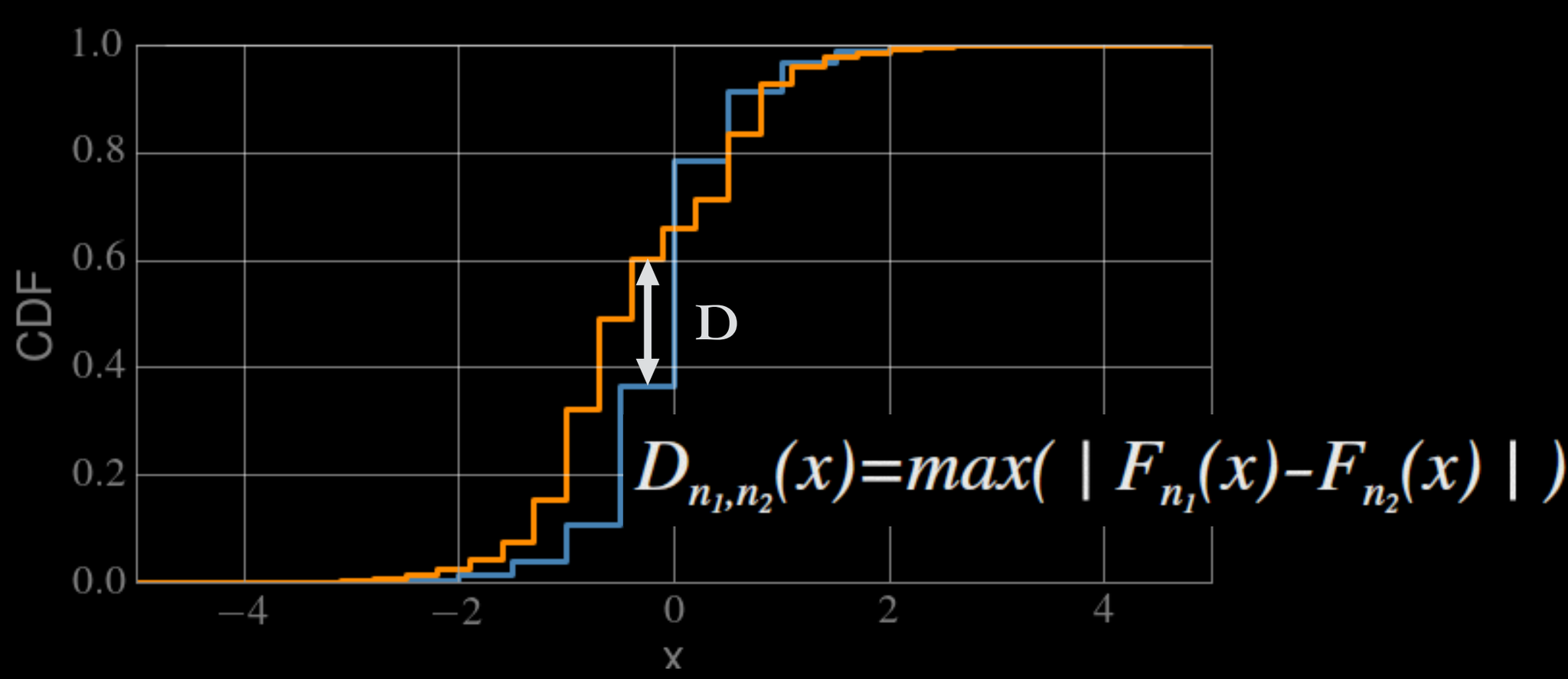
Two sample Kolmogorov Smirnoff test:

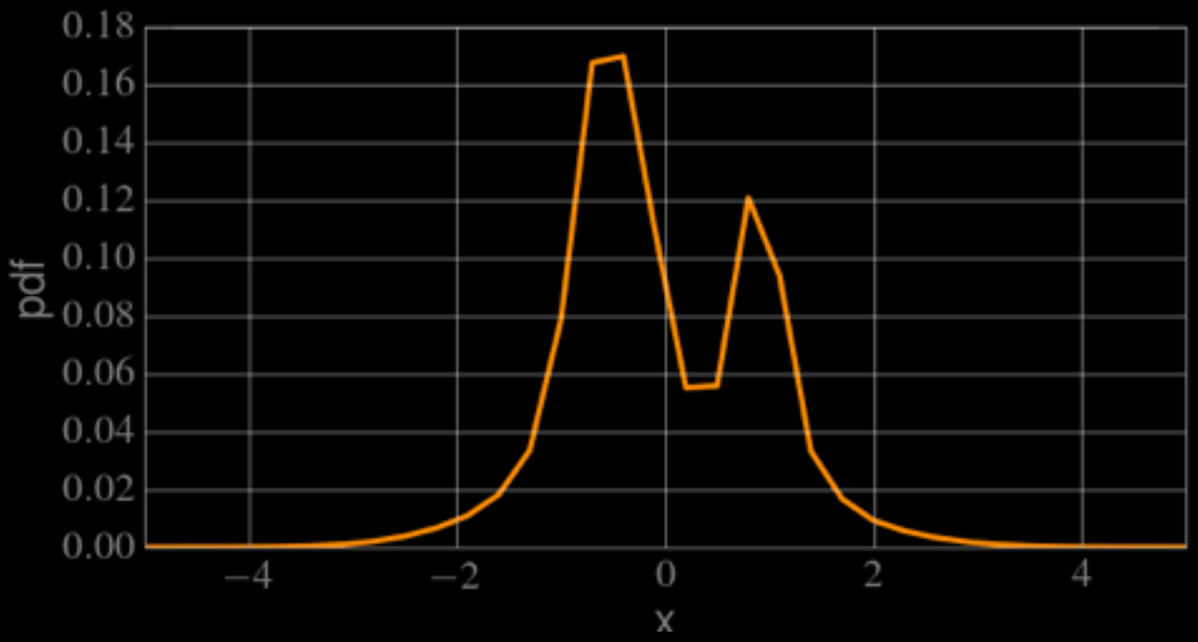
null hypothesis H_0: the samples come from the same parent distribution

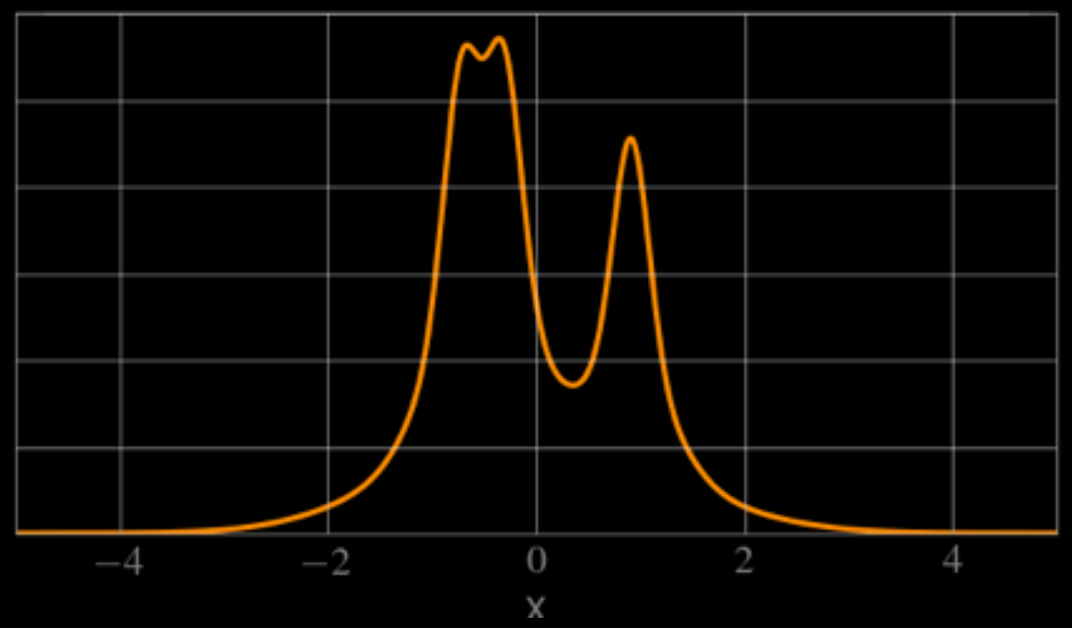
H_0 is rejected at level α if $D(n_1,n_2) > c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$

with $c(\alpha)$ given by a table

NOTE: it ONLY woks in 2D where the Euclidian distance is uniquely defined!




0.18
0.16
0.14
0.12
0.10
0.08
0.06
0.04
0.02
0.00
pdf
−4
−2
0
2
4
x

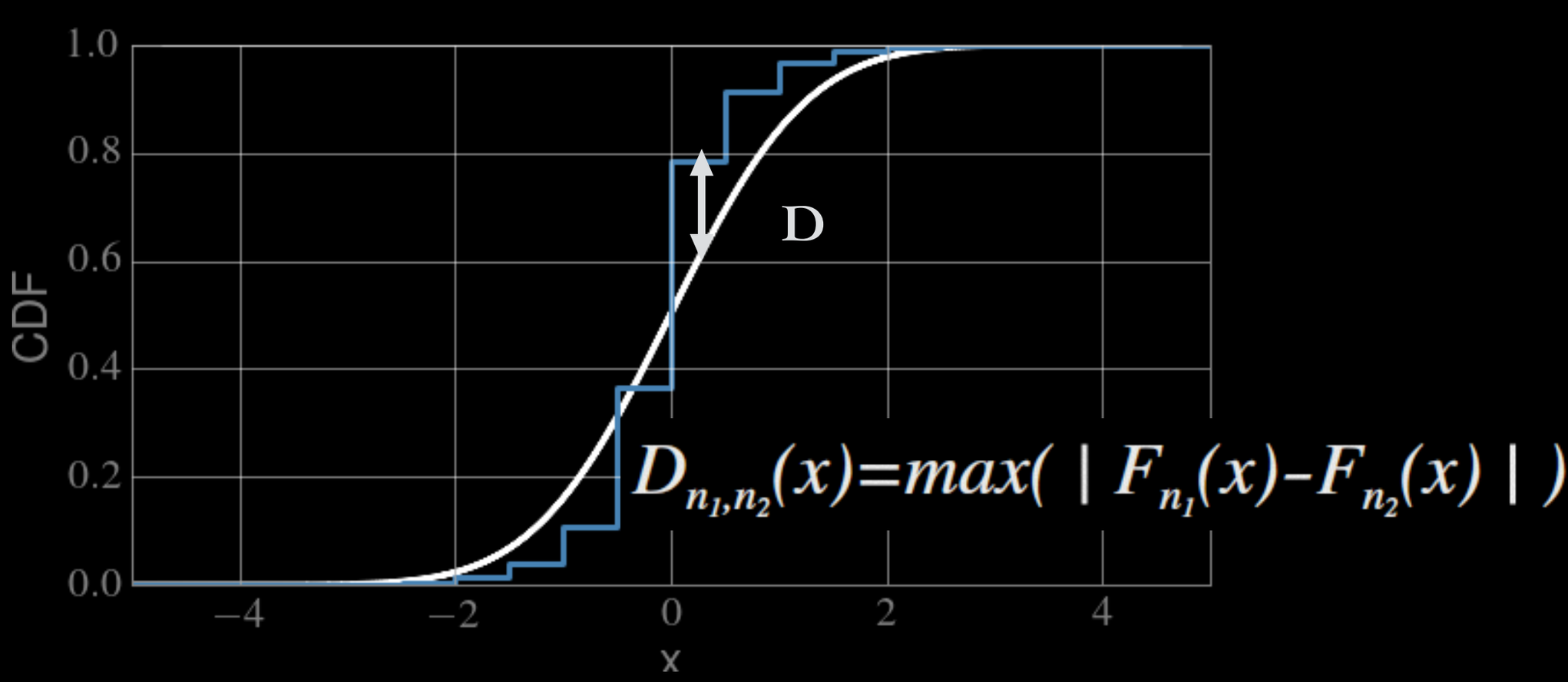

−4
−2
0
2
4
x

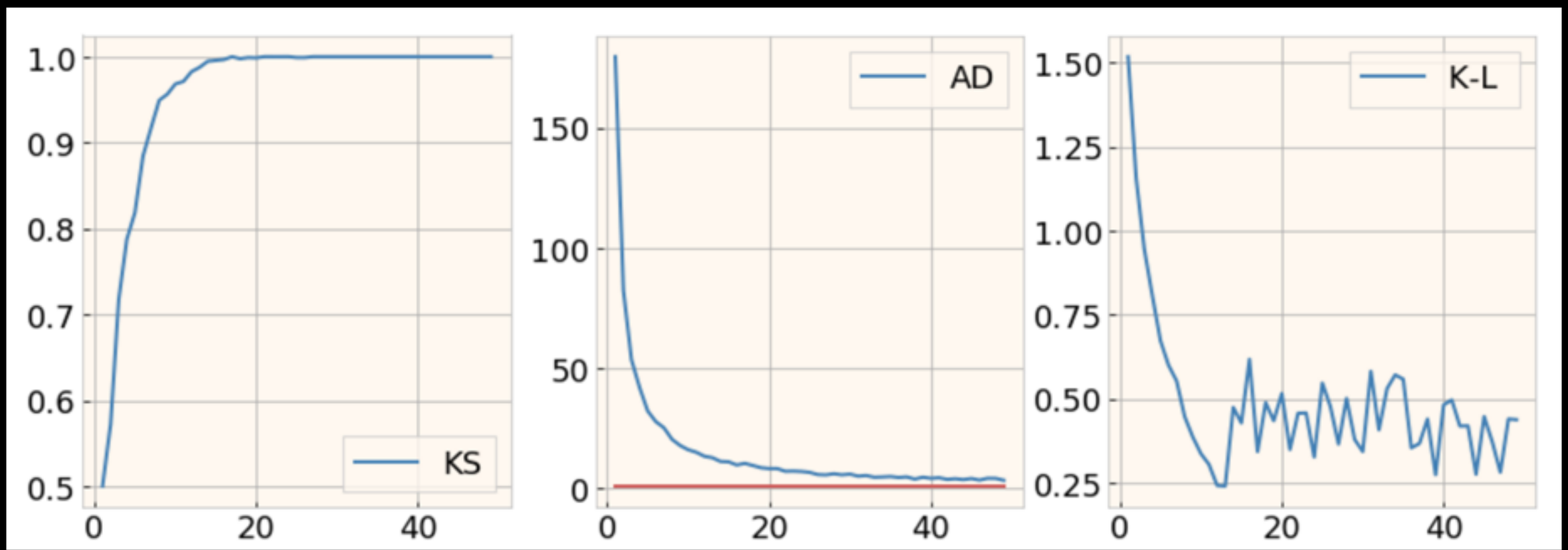
1.0
0.8
0.6
0.4
0.2
0.0
CDF
−4
−2
0
2
4
x

Goodness-of-fit Kolmogorov Smirnoff test:

null hypothesis H_0: the sample does comes from the *model* distribution

H_0 is rejected at level α if $\sqrt{n}\, D_n > K_\alpha$ where $P(K \leq K_\alpha) = 1-\alpha$





https://github.com/fedhere/UInotebooks/blob/master/oh_my_goodness_of_fit.ipynb

IV: Statistical analysis

Compare Tests for Correlation and Goodness of fit:

The following are 3 tests that assess correlation between 2 samples:

- Pearson's test
- Spearman's test
- K-S test

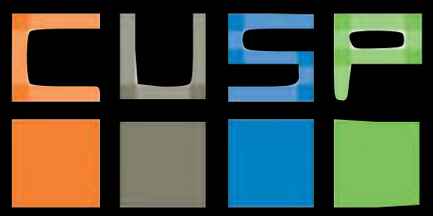
e.g. Estimate number of MTA Bus passengers at different hours (morning, afternoon, or in time chunks as 7:30-10:30, 10:30-1:30, 1:30-3, 3:6, 6:9, you can do it per bus line, per origin or destination neighborhood…)

The following are 5 tests that can be used to assess the goodness of fit of a model

- K-S
- Pearson's Chi squared
- Anderson-Darling
- K-L Divergence
- Likelihood ratio

e.g. Estimate number of MTA Bus passengers per bus line within an interval of time: are the passengers randomly distributing on busses.

In HW3 you used 2 out of these tests to assess if 2 samples are related (measure their correlation, or decide if they come from the same parent distributions)
Now in HW4 use 2 of the goodness of fit tests to see if a dataset comes from a normal distribution, or from another distribution (where possible) of your choice.



Compare Tests for Correlation and Goodness of fit:

The following are 3 tests that assess correlation between 2 samples:

- Pearson's test
- Spearman's test
- K-S test

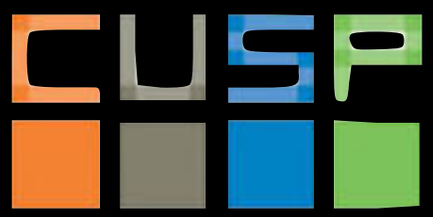
e.g. Age distribution of male vs female Citibikes riders. Age distribution in different seasons. Age distribution for long/short trips

The following are 5 tests that can be used to assess the goodness of fit of a model

- K-S
- Pearson's Chi squared
- Anderson-Darling
- K-L Divergence
- Likelihood ratio

e.g. Estimate Age of riders: could be Gaussian, could be lognormal, power law, some bimodal distribution…

In HW3 you used 2 out of these tests to assess if 2 samples are related (measure their correlation, or decide if they come from the same parent distributions)
Now in HW4 use 2 of the goodness of fit tests to see if a dataset comes from a normal distribution, or from another distribution (where possible) of your choice.

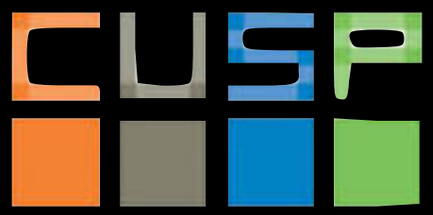


Homework: 1. Compare Tests for Goodness of fit

(real data)

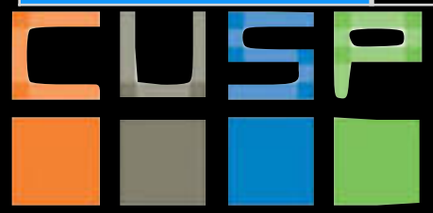
Test whether a gaussian model for the age distribution of citibike drivers is a sensible model, or if you can find a better fit with another distribution. Use 2 tests: KS, AD, KL, chisq to do this. Test at least 2 distributions.

Optional (extra credit): Divide your sample geographically: by Borrow + split Manhattan in an Uptown and a Downtown sample (use your discretion to do so) and see if you notice any differences in how the age distribution can be modeled.



Tests Cheat Sheet:
2 (+) samples comparison

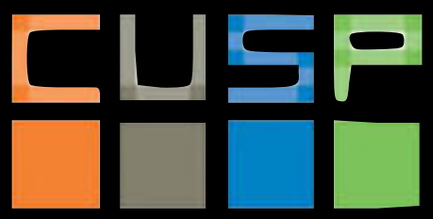
	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\mid F_{n_1}(x)-F_{n_2}(x)\mid)$	$c(\alpha)\sqrt{\frac{n_1+n_2}{n_1 n_2}}$	*Non parametric 2 samples only*
K-sample Anderson-Darling	$ADK=\frac{n-1}{n^2(k-1)}\sum_{i=1}^{k}\frac{1}{n(i)}\left(\sum_{j=1}^{L}h_j\frac{(nF_{ij}-n_iH_j)^2}{H_j(n-H_j)-nh_j/4}\right)$	•AK table	*Non parametric, N samples*
Pearson's	$r_{xy}=\frac{1}{n-1}\sum_{i=1}^{n}\left(\frac{x_i-\overline{x}}{s_x}\right)\left(\frac{y_i-\overline{y}}{s_y}\right)$	The interpretation of a correlation coefficient depends on the context and purpose	-1 anticorrelated 0 uncorrelated 1 correlated .
Spearman's	$\rho=1-\frac{6\sum(x_i-y_i)^2}{n(n^2-1)}$	t test $t=r\sqrt{\frac{n-2}{1-r^2}}$	*ranked data only* p-value from t-test, Fisher's transformation +z score, permutation test



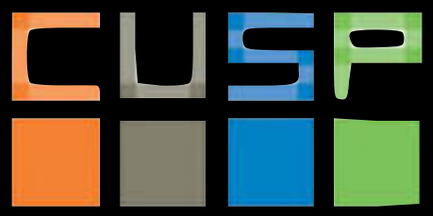
Tests Cheat Sheet:

goodness of fit

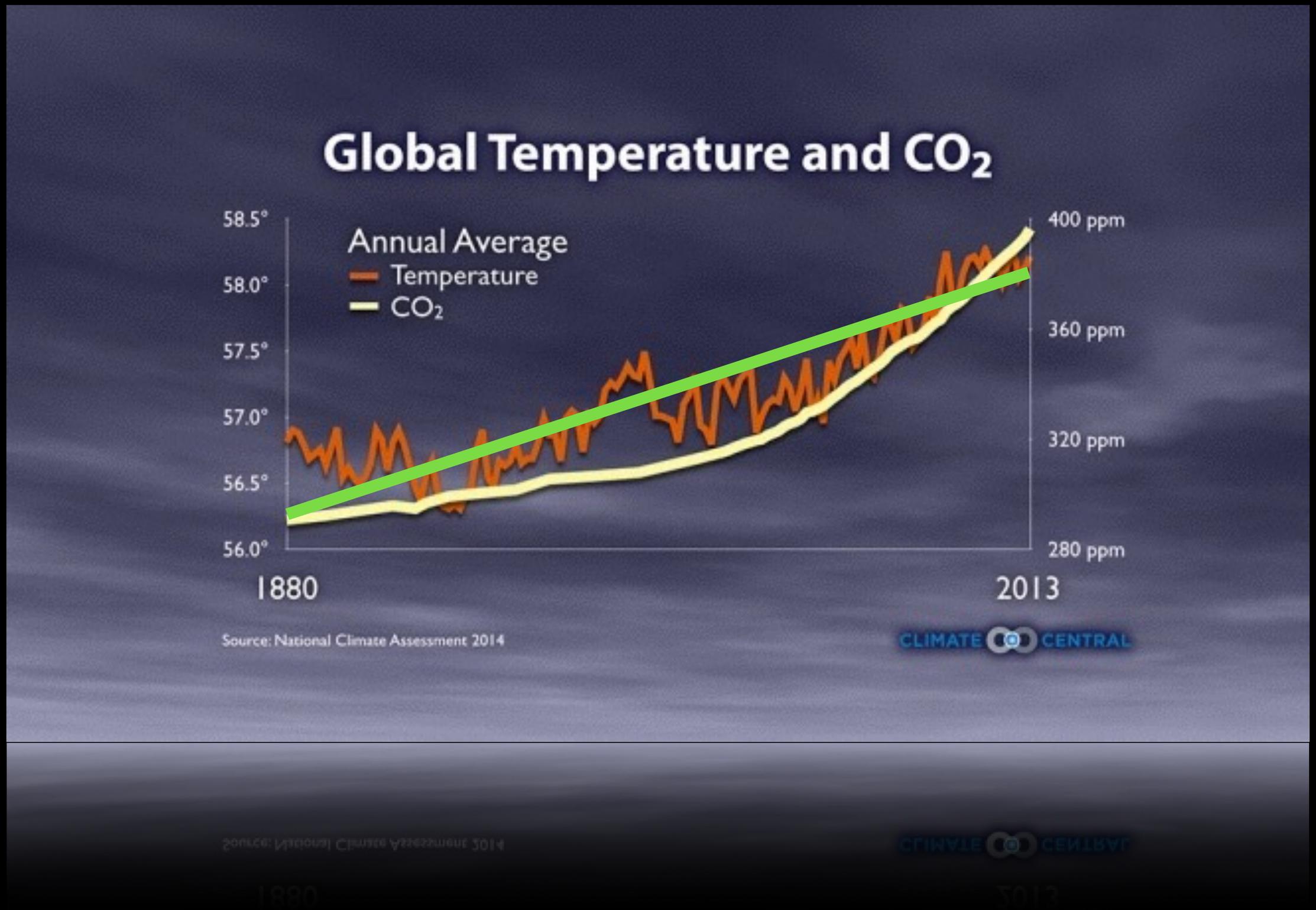
	metric (statistic)	compare to	
KS	$D_{n_1,n_2}(x)=max(\mid F_n(x)-F\ (x)\mid)$	$\frac{K_\alpha}{\sqrt{n}}$	power in the core only
Pearson's chi square	$\chi^2_{red}=\frac{\chi^2}{df}=\frac{1}{df}\sum\frac{(O-E)^2}{\sigma^2}$	scipy.stats.chisquare(f_obs, f_exp=None, ddof=0, axis=0)[	
Anderson-Darling	$A=n\int_{-\infty}^{\infty}\frac{(F_n(x)-F(x))^2}{F(x)\ (1-F(x))}\,dF(x)$	scipy.stats.anderson(x, dist='norm')	power in the tails
K-L divergence	$D_{KL}\ =\ -\int_x p(x)\ log(q(x))\ +\ p(x)\ log(p(x))$	scipy.stats.entropy(pk. qk=<not None>)	relates to information entropy
Likelihood ratio	$\frac{L\ (model\ \ 1\mid\ data)}{L\ (model\ \ 2\mid\ data)}$		suitable to bayesian analysis



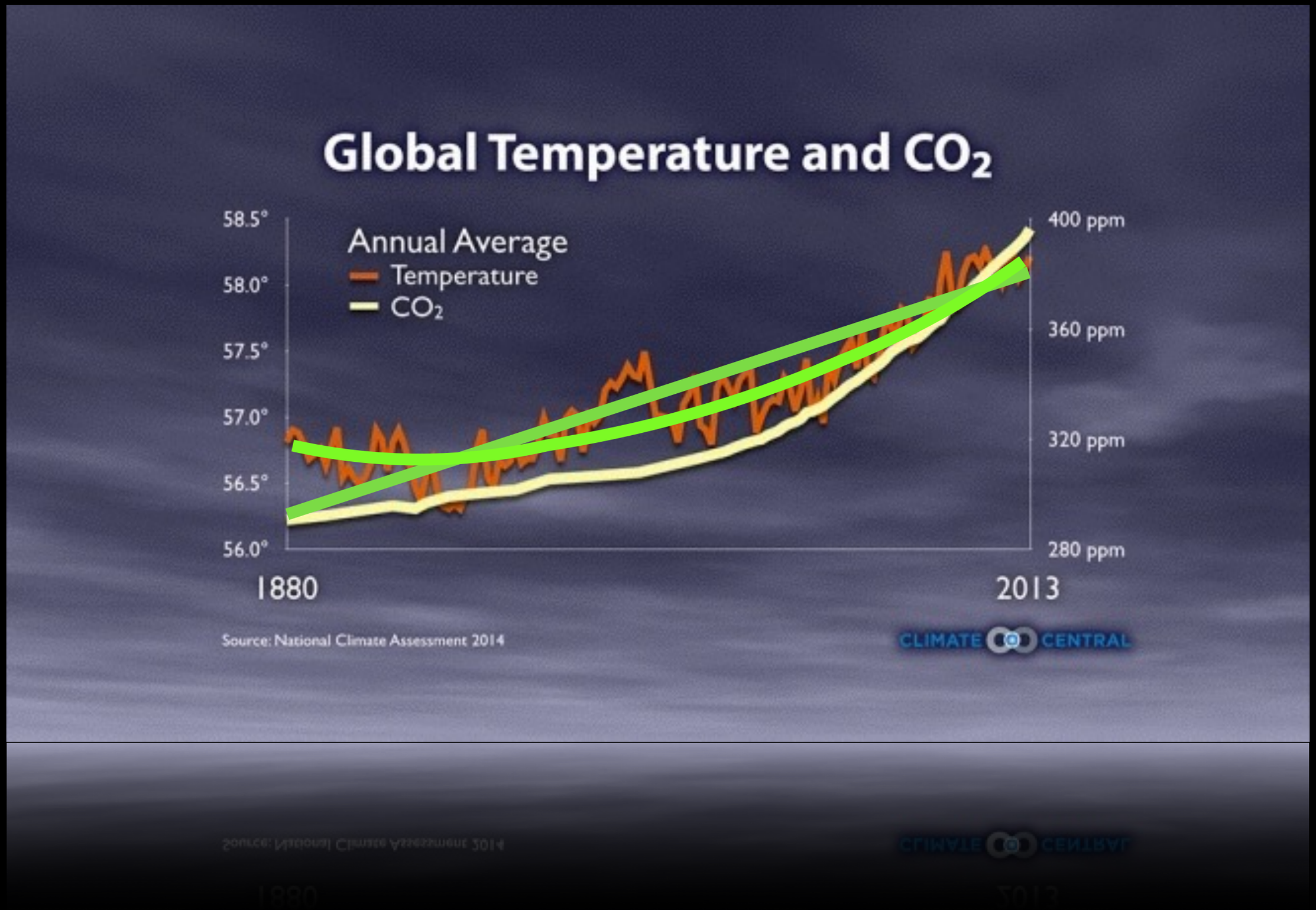
Predictive



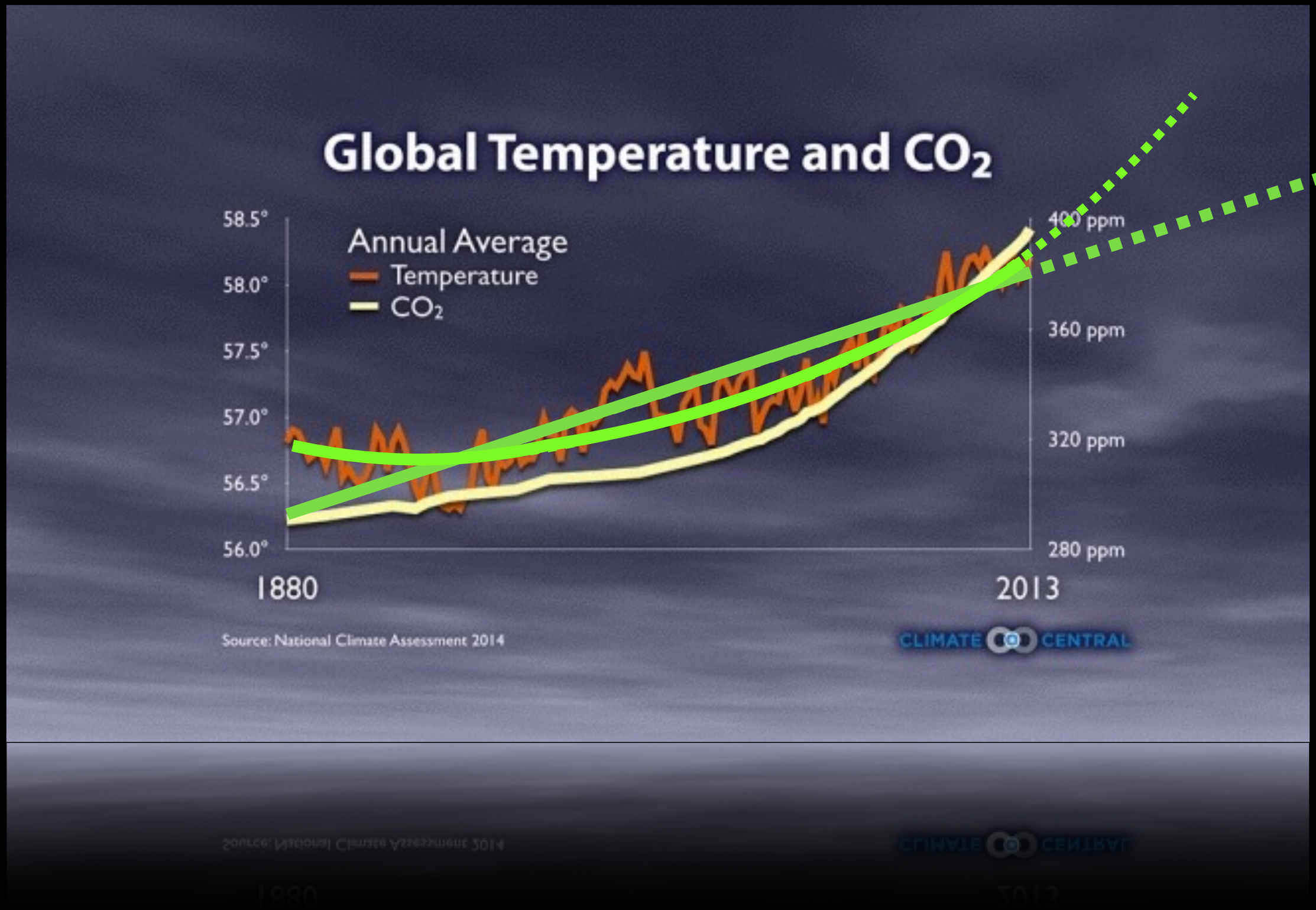
V: Likelihood and Regression Models



V: Likelihood and Regression Models



V: Likelihood and Regression Models

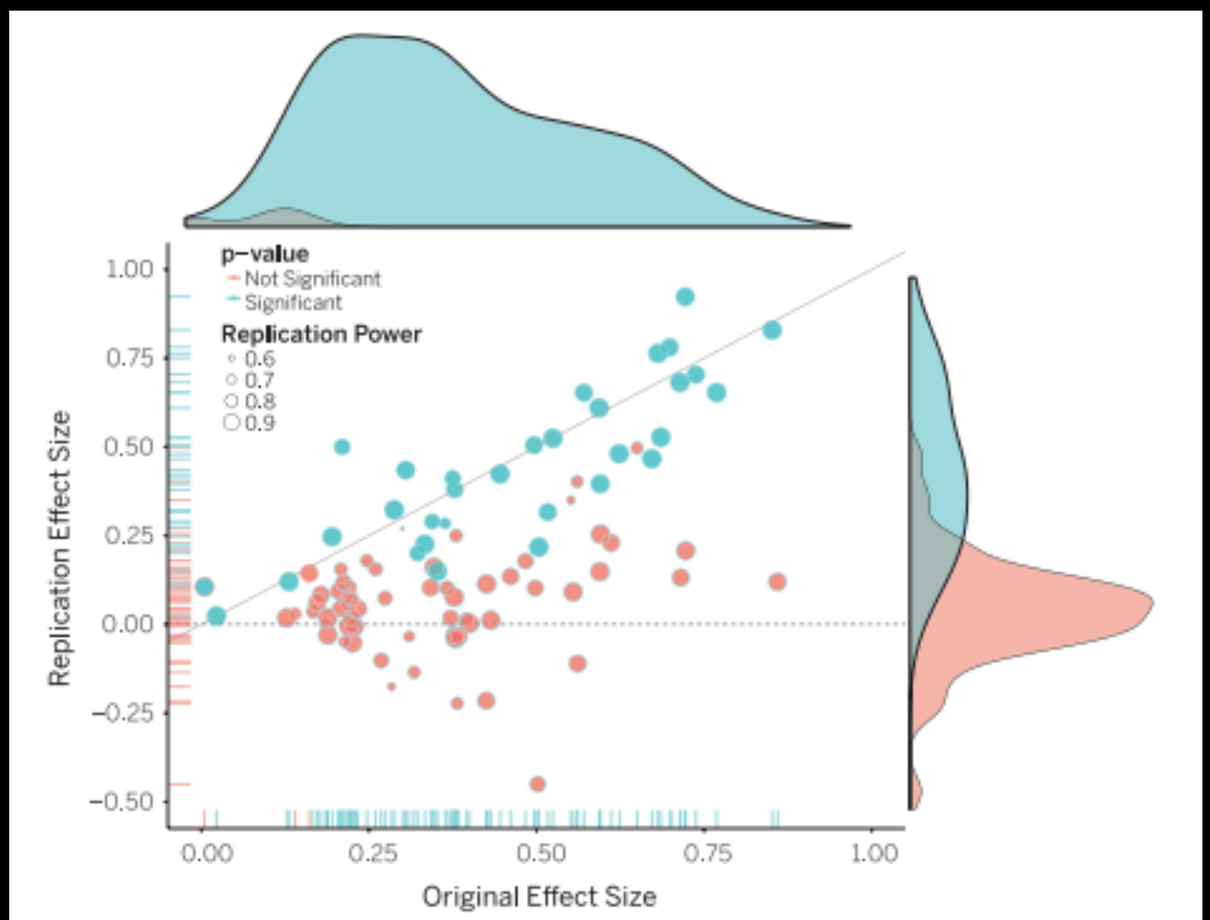

Global Temperature and CO_2
Annual Average
Temperature
CO_2
58.5°
58.0°
57.5°
57.0°
56.5°
56.0°
400 ppm
360 ppm
320 ppm
280 ppm
1880
2013
Source: National Climate Assessment 2014
CLIMATE CENTRAL

RESEARCH ARTICLE SUMMARY

PSYCHOLOGY

Estimating the reproducibility of psychological science

Open Science Collaboration*



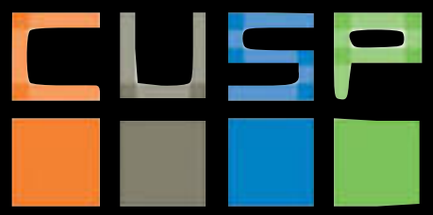
Original study effect size versus replication effect size (correlation coefficients). Diagonal line represents replication effect size equal to original effect size. Dotted line represents replication effect size of 0. Points below the dotted line were effects in the opposite direction of the original. Density plots are separated by significant (blue) and nonsignificant (red) effects.

http://www.sciencemag.org/content/349/6251/aac4716.full.pdf

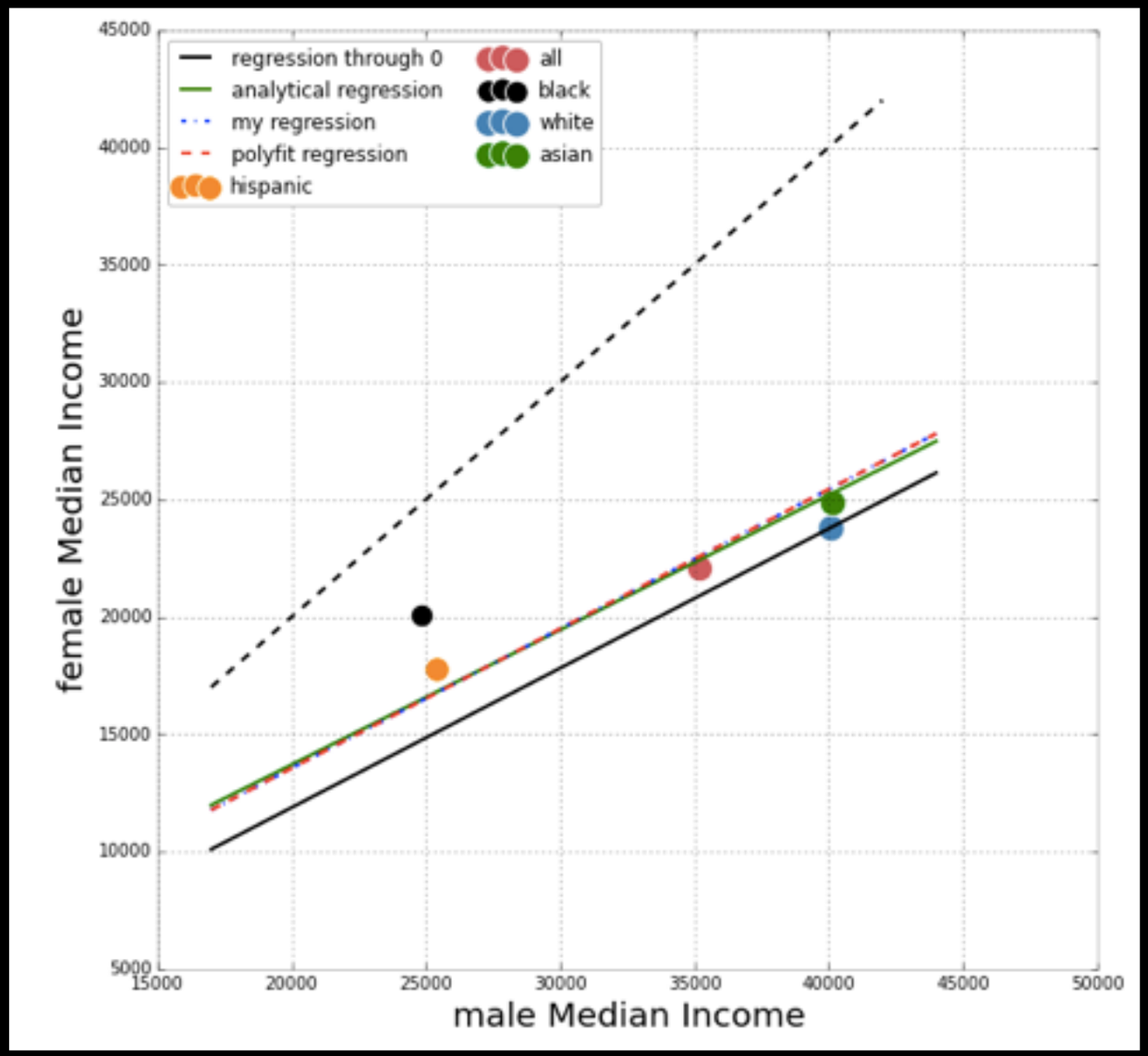
CUSP

V: Likelihood and Regression Models

Why?



V: Likelihood and Regression Models

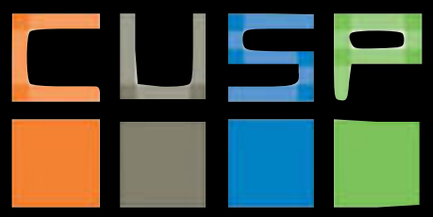


jupyter

CUSP

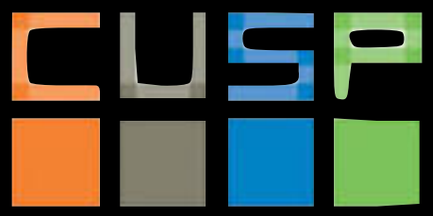
V: Likelihood and Regression Models

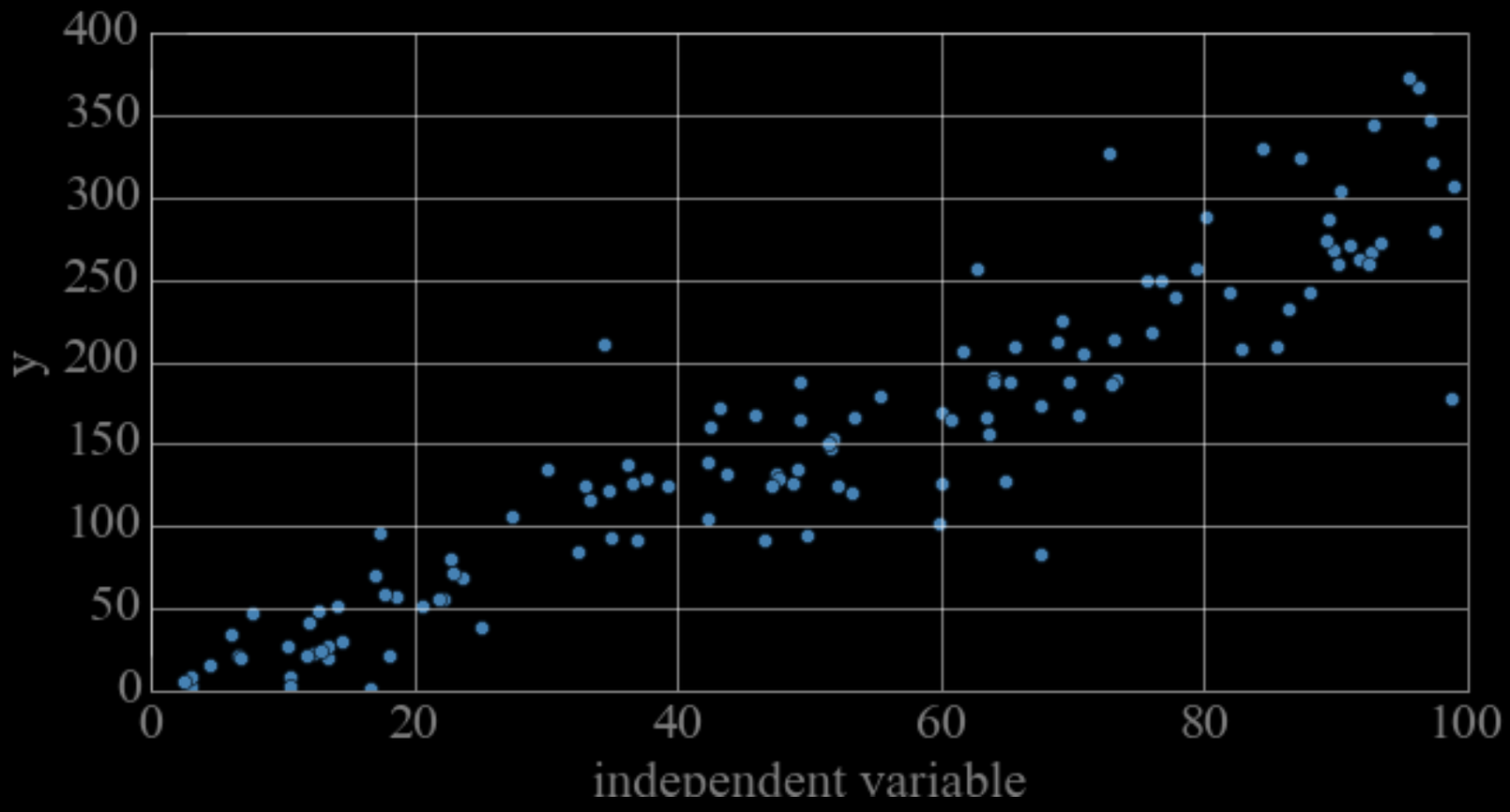
https://github.com/fedhere/UInotebooks/blob/master/OLS/line_fit_and_residuals.ipynb



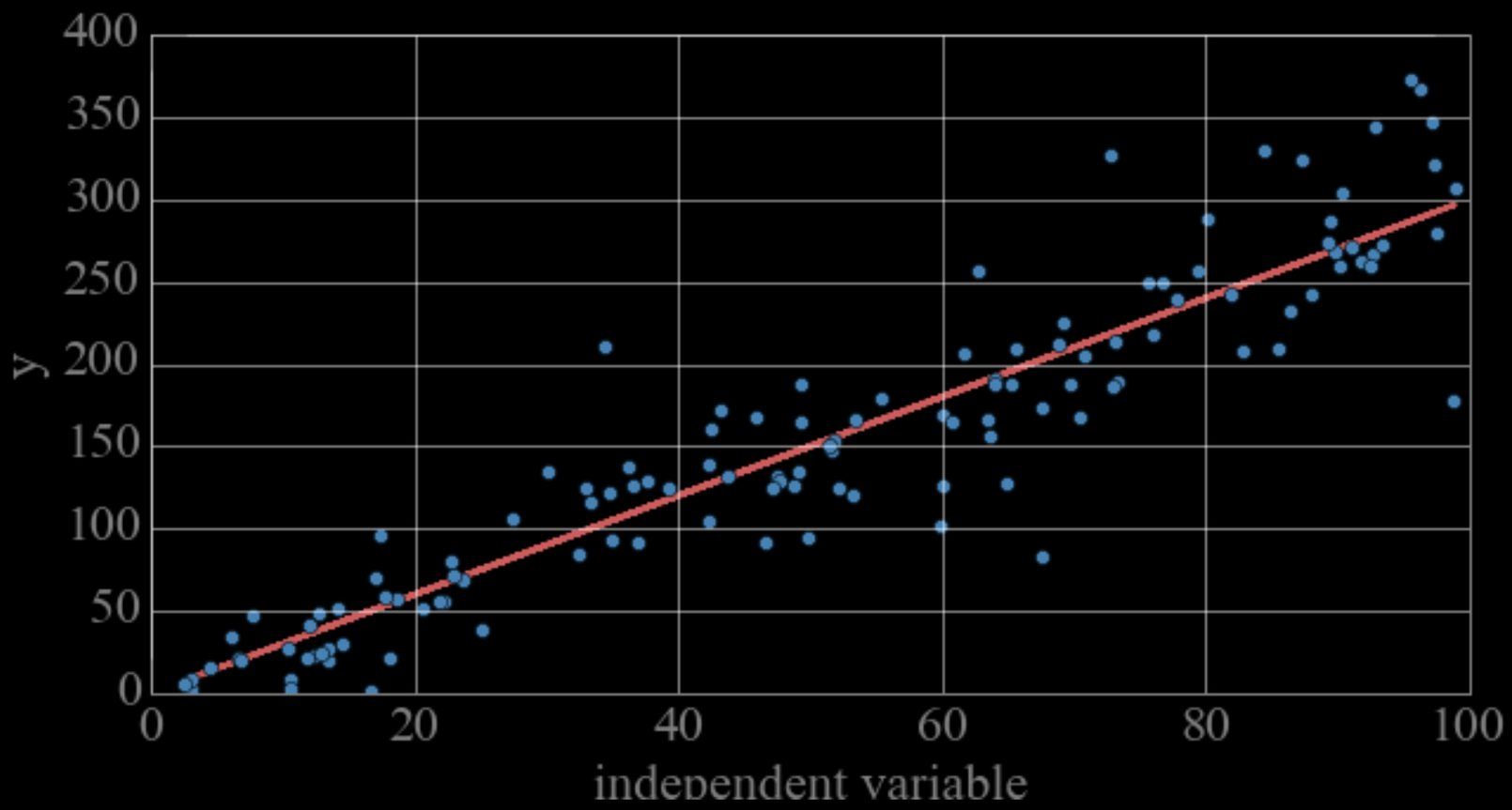
V: Likelihood and Regression Models

How?

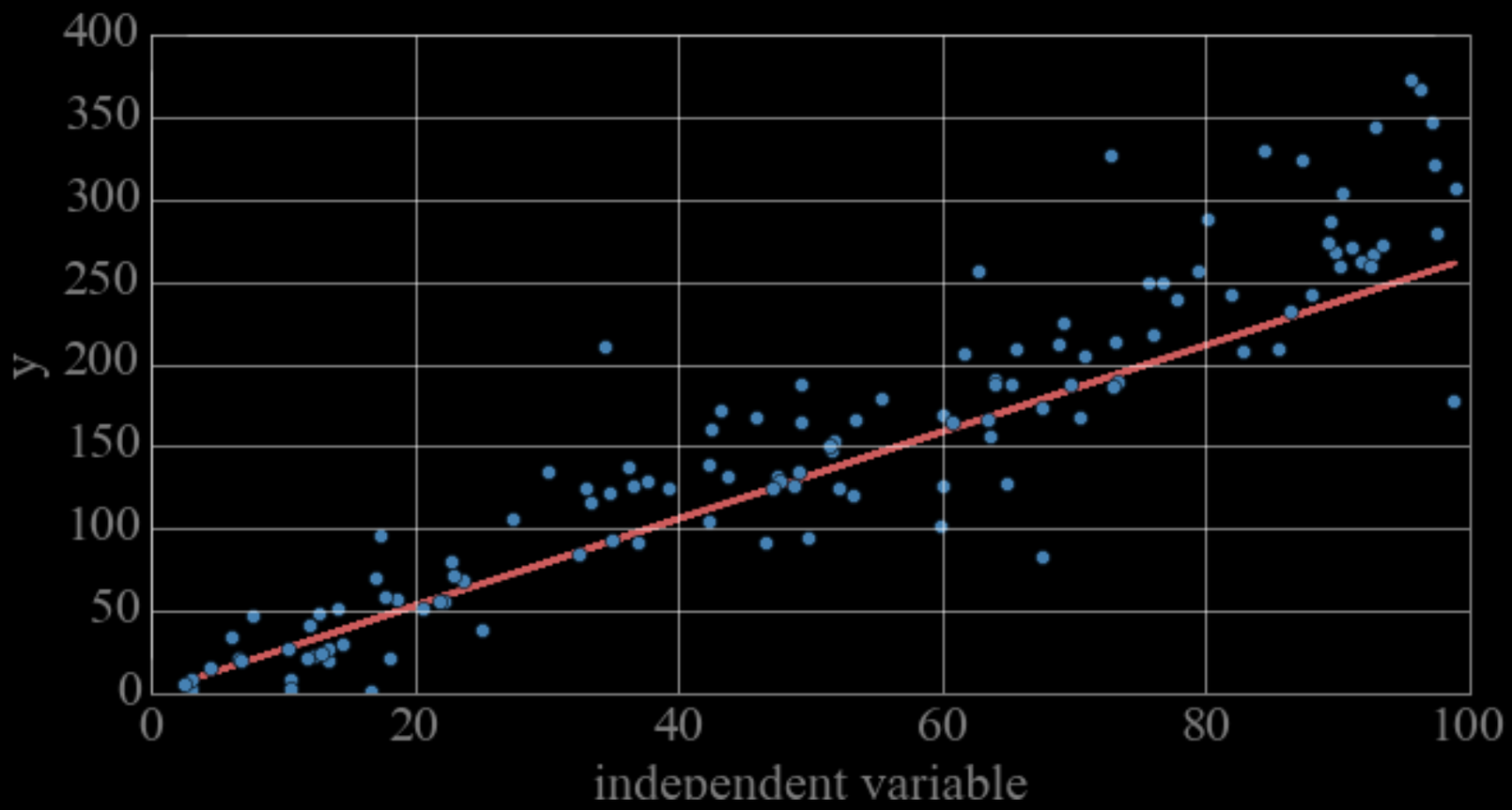




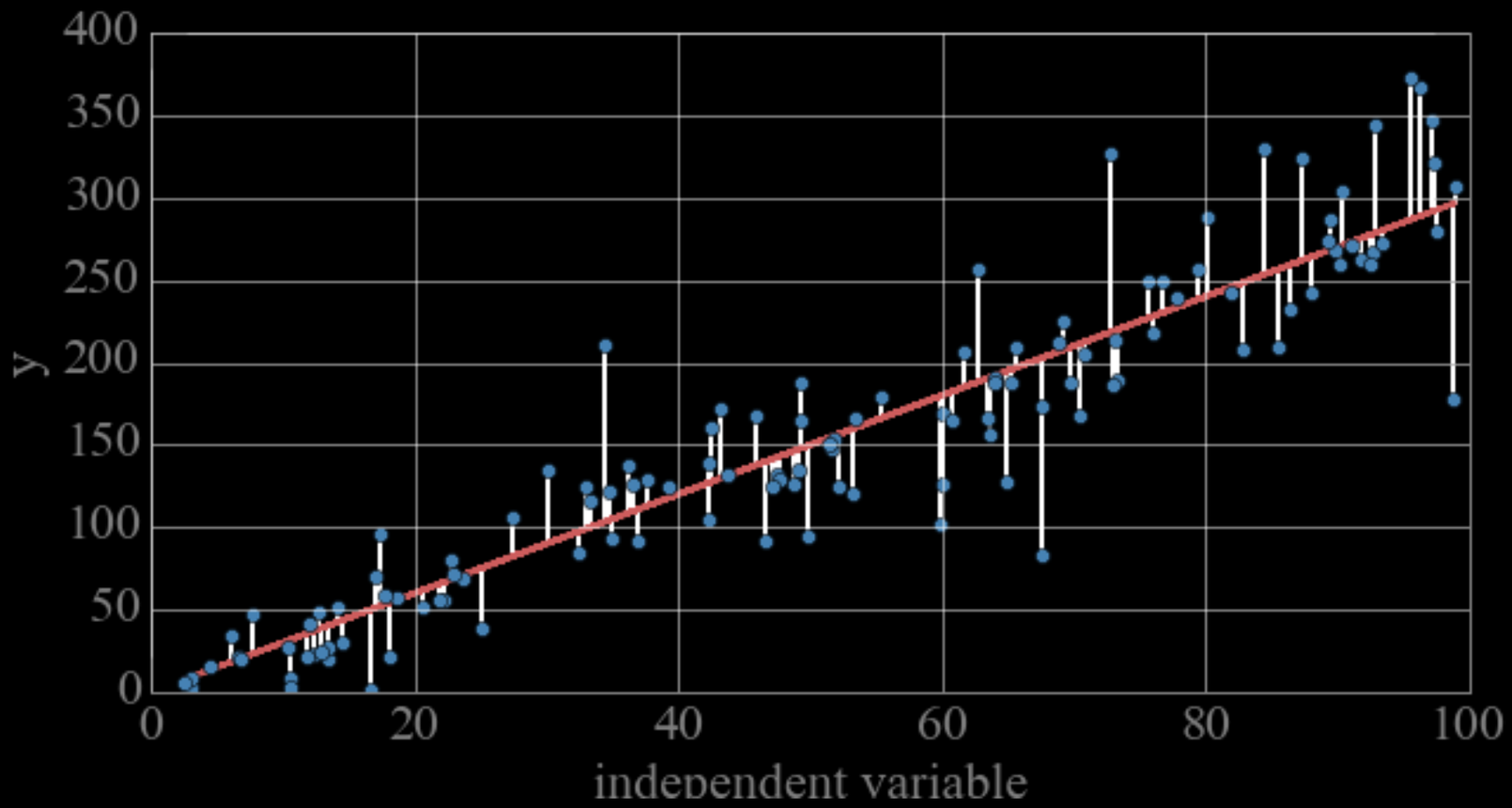
V: Likelihood and Regression Models



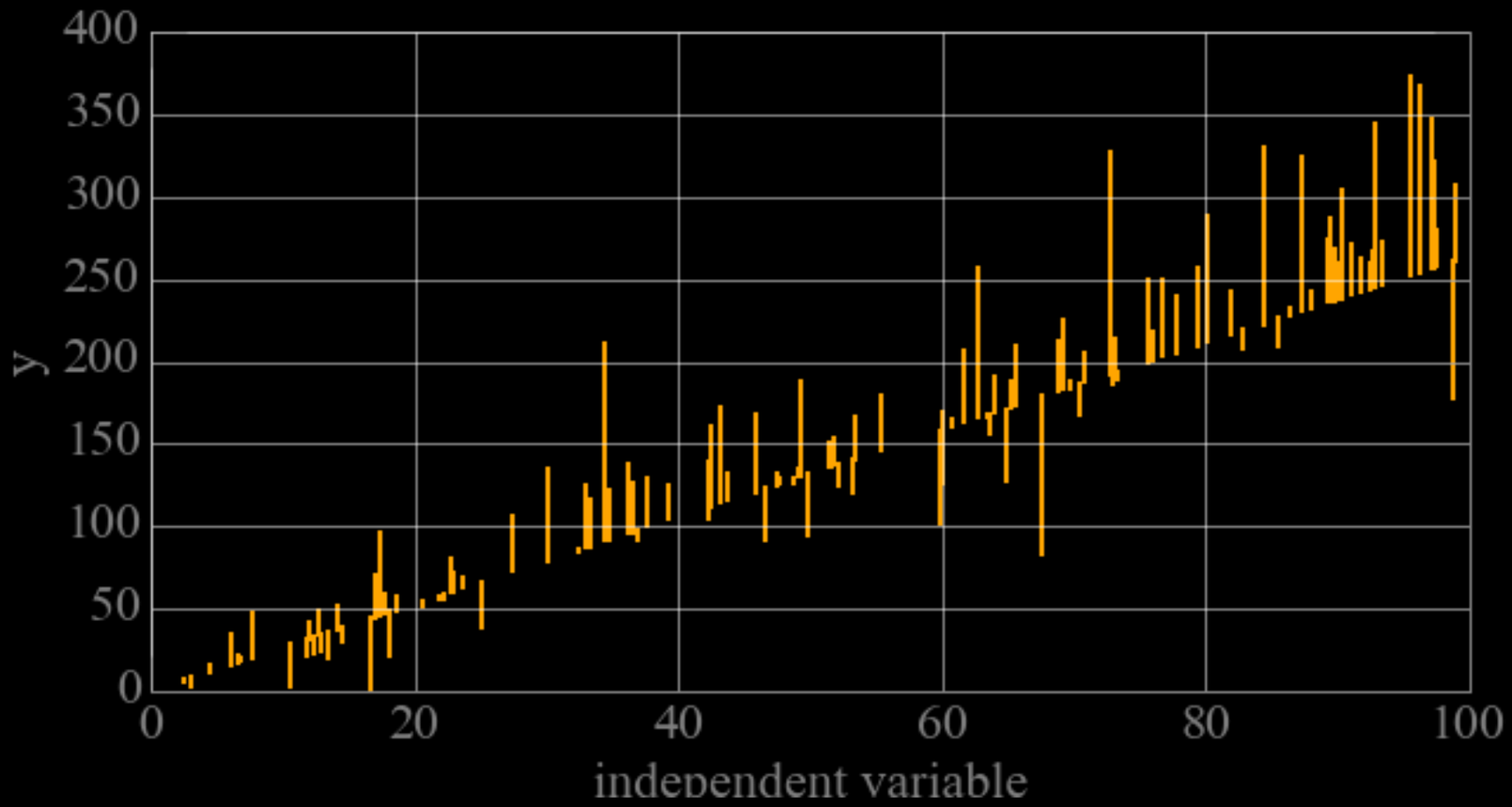
V: Likelihood and Regression Models



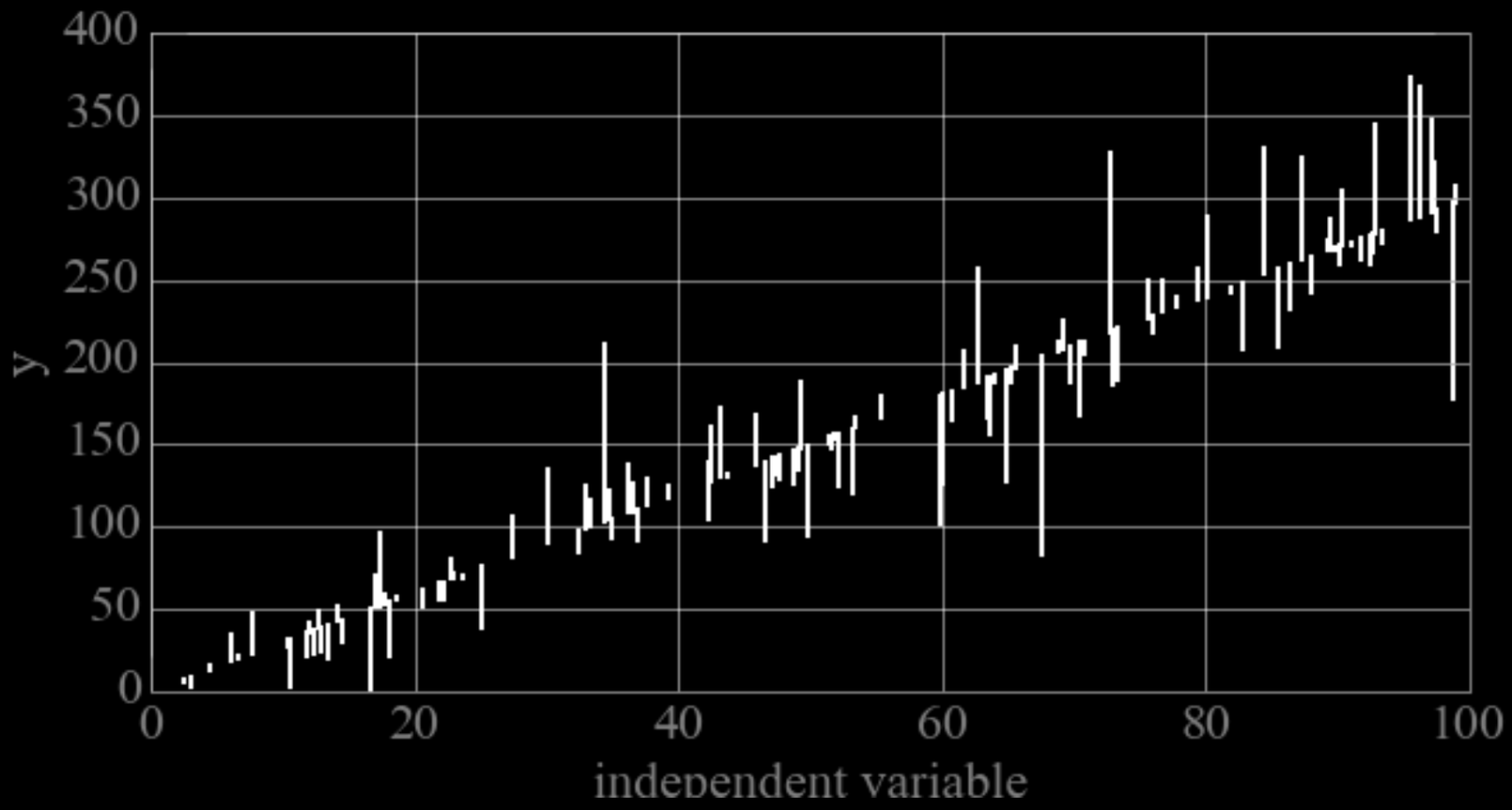
V: Likelihood and Regression Models



V: Likelihood and Regression Models



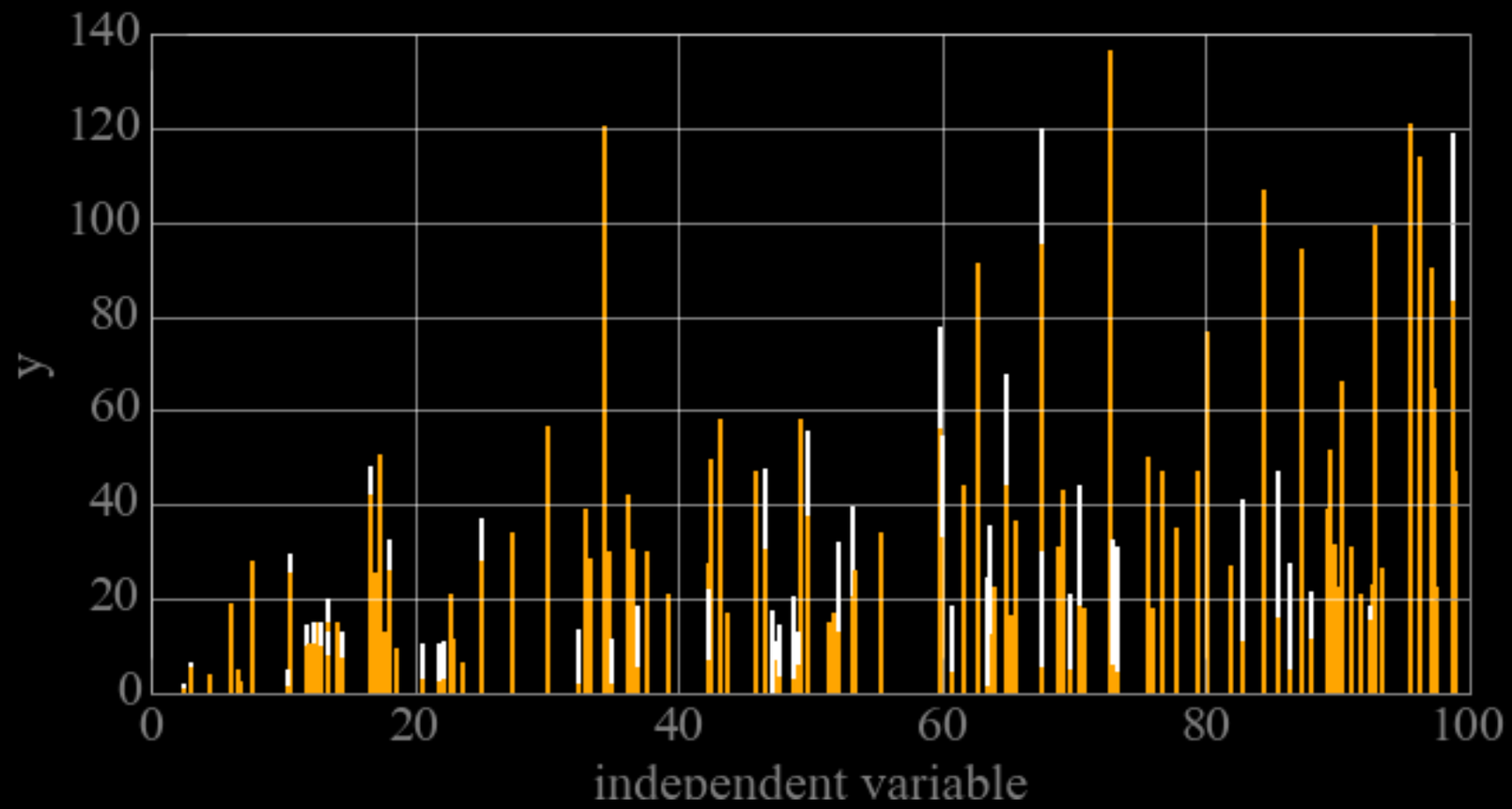
V: Likelihood and Regression Models



V: Likelihood and Regression Models

Sum of residuals squared $\sum_i (|y_i - (mx_i + b)|)$

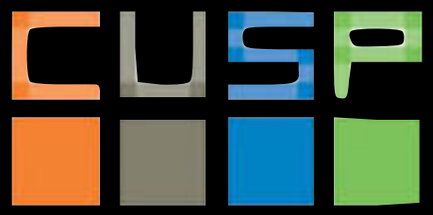
11655.34 12155.24



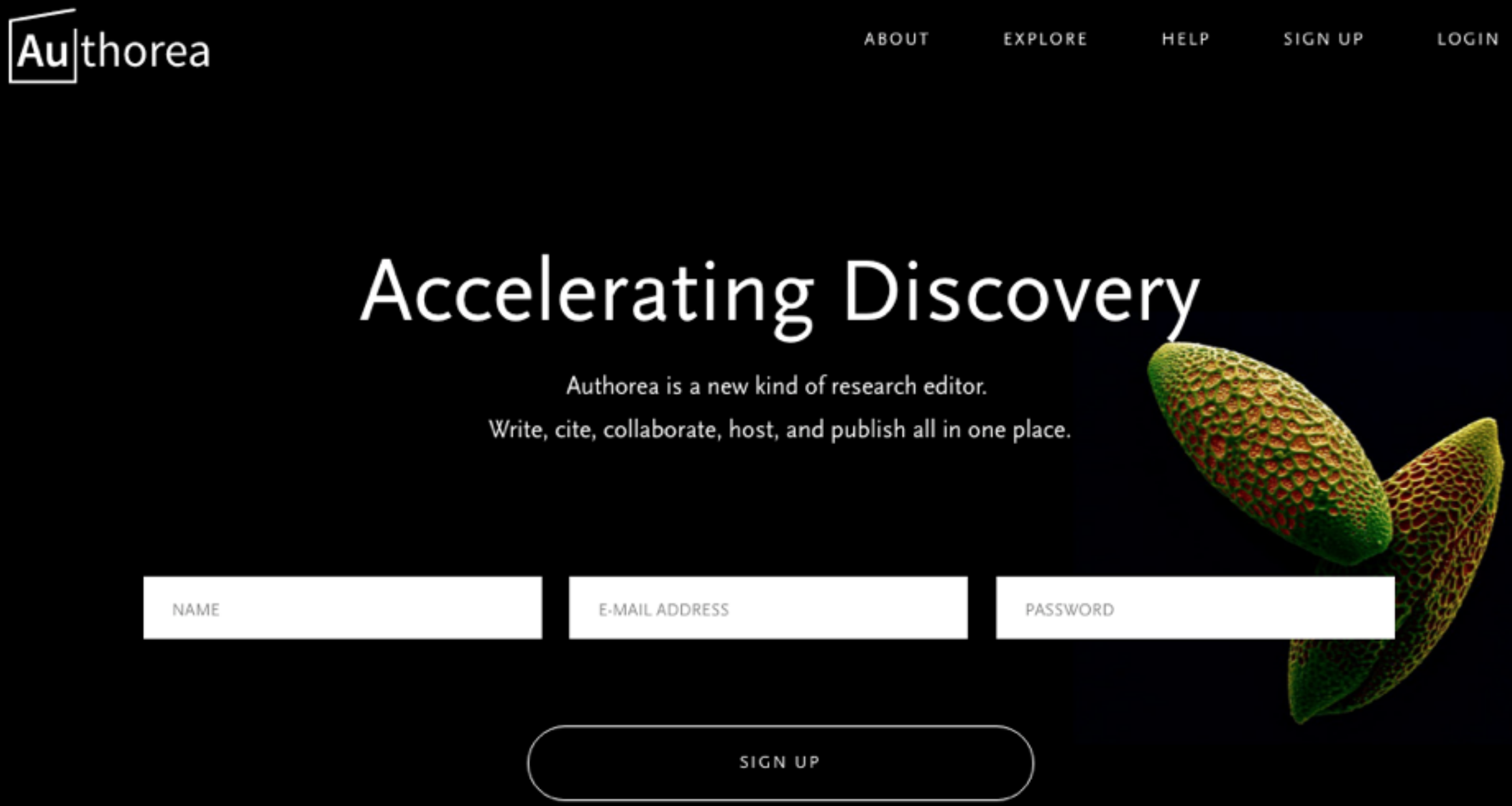
Homework: 2. linear regression and prediction

GENDER INCOME GAP

you may know that it is estimated that women earn about 78% of men in the same job position. Can we test that on NYC income data? Can we turn that into a prediction: if you get hired at a certain stipend as a men, what should you expect to make as a woman? (or from the point of view of a job employer, perhaps not one with a very strong moral compass, what should I offer to a woman job candidate, given what I would offer a man for the same job?)



Assigned reading



10 Simple Rules for the Care and Feeding of Scientific Data

https://www.authorea.com/users/3/articles/3410/_show_article

CUSP

V: Likelihood and Regression Models

MUST KNOWS:

- Systematic and Statistical Errors
- Precision vs Accuracy
- Errors are generally added in quadrature
- Goodness of fit testing (why, how, few tests)
- Least square fits (OLS)

Resources:

Sarah Boslaugh, Dr. Paul Andrew Watters, 2008
Introduction to General Linear Regression (chap 12)
https://books.google.com/books/about/Statistics_in_a_Nutshell.html?id=ZnhgO65Pyl4C

David M. Lane et al.
Introduction to Statistics (XVIII)
regression : Chapter 14
http://onlinestatbook.com/Online_Statistics_Education.epub
http://onlinestatbook.com/2/index.html

Error Analysis from UPenn physics labs
these are prepared for the phyiscs undergrads labs, which I taught while in gradschool. The examples are really physics, so more relevant to remote sensing than social science type projects, but the error propagation etc works the same way
http://virgo-physics.sas.upenn.edu/uglabs/lab_manual/Error_Analysis.pdf