

Unit 2 exercises

1. Read the wiki at Sharma et al (2022) and then attempt the exercises below:

i. For each clause (a) - (f) below, create truth tables for each to answer the question of when each statement is false.

- a. $\sim P$
- b. $P \wedge Q$
- c. $P \vee Q$
- d. $P \rightarrow Q$
- e. $P \leftrightarrow Q$
- f. $P \rightarrow (\sim Q)$

ii. Consider the statement $(\sim Q) \rightarrow (\sim P)$.

- a. When is it false?
- b. Now consider $P \rightarrow Q$. When is it false?
- c. Do you believe these two compound statements mean the same thing?
- d. Construct the truth table for the statement $(\sim Q) \rightarrow (\sim P)$. Then revisit your answer to (c).

iii. Construct the truth table for $P \text{ XOR } Q$.

iv. Construct truth tables for the following statements.

- a. $\sim (P \wedge Q)$
- b. $P \vee (Q \wedge R)$
- c. $P \vee (Q \vee R)$
- d. $(P \vee Q) \vee R$ (Compare to the previous statement.)
- e. $(P \rightarrow Q) \wedge (Q \rightarrow P)$

Exercise 1

P	Q	a. $\sim P$	b. $P \wedge Q$	c. $P \vee Q$	d. $P \rightarrow Q$	e. $P \leftrightarrow Q$	f. $P \rightarrow (\sim Q)$
T	T	F	T	T	T	T	F
F	T	T	F	T	F	F	T
T	F		F	T	T	F	T
F	F		F	F	T	T	T

Exercise 2

a. $(\sim Q) \rightarrow (\sim P)$ is false when $\sim Q$ is true and $\sim P$ is false, which corresponds to the case where Q is false and P is true.

b. $P \rightarrow Q$ is false when P is true and Q is false

c. No – inverse relation

d.

Q	P	$\sim Q$	$\sim P$	a. $(\sim Q) \rightarrow (\sim P)$	b. $P \rightarrow Q$
T	T	F	F	T	T
F	T	T	F	T	F
T	F	F	T	F	T
F	F	T	T	T	T

Exercise 3

P	Q	$P \text{ XOR } Q$
T	T	F
F	T	T
T	F	T
F	F	F

Exercise 4

a. $\sim(P \wedge Q)$

P	Q	$P \wedge Q$	$\sim(P \wedge Q)$
T	T	T	F
F	T	F	T
T	F	F	T
F	F	F	T

b. $P \vee (Q \wedge R)$

P	Q	R	$Q \wedge R$	$P \vee (Q \wedge R)$
T	T	T	T	T
T	T	F	F	T
T	F	T	F	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	F
F	F	T	F	F
F	F	F	F	F

c. $P \vee (Q \vee R)$

P	Q	R	$Q \vee R$	$P \vee (Q \vee R)$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	T	T
F	F	T	T	T
F	F	F	F	F

d. $(P \vee Q) \vee R$

P	Q	R	$P \vee Q$	$(P \vee Q) \vee R$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	F

Both $P \vee (Q \vee R)$ and $(P \vee Q) \vee R$ result in the same truth table as expected due to the associative property of OR.

e. $(P \rightarrow Q) \wedge (Q \rightarrow P)$

P	Q	$P \rightarrow Q$	$Q \rightarrow P$	$(P \rightarrow Q) \wedge (Q \rightarrow P)$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T