

# Long Run Effective Demand: Introducing Residential Investment in a Sraffian Supermultiplier Stock-Flow Consistent Model

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Eastern Economic Association

46 th Annual Conference - 2020

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The model employed here is a work in progress.

## Abstract

In this paper, we build a fully specified parsimonious Sraffian supermultiplier stock-flow consistent model (SSM-SFC) with residential investment. This means we present the flow of funds tables and balance sheets only for the strictly necessary institutional sectors (households, firms and banks); growth is led by “non-capacity creating” autonomous expenditure (in this case, residential investment); and non-residential investment is a induced expenditure. The introduction of residential investment implies that our SSM-SFC model has two real assets: firms’ productive capital and households’ real estate. The numerical simulation experiments report the main standard results of Sraffian Supermultiplier growth models: (i) changes in income distribution affect growth only during the traverse; (ii) the rate of growth of autonomous expenditure (residential investment) alone explains growth in steady state; (iii) the rate of capacity utilization converges towards the normal one. As a particular result, an increase of the rate of growth of residential investment causes a reduction of the share of real estate in total real assets. Therefore, this model introduces housing on Sraffian Supermultiplier agenda and extends the range of autonomous expenditures alternatives.

**Keywords:** Residential investment; Sraffian Supermultiplier; Stock-Flow Consistent approach.

## 1 Introduction

The Sraffian Supermultiplier (SSM) growth model defines a key role to non-capacity creating expenditures to the understanding of economic growth and capital accumulation. Serrano (1995b) original contribution — and also more recent papers (FREITAS; SERRANO, 2015) — presents the SSM model in a rather parsimonious way. The main reason to this approach is to present it as an alternative closure within the demand-led growth theory tradition (SERRANO; FREITAS, 2017).

In working with SSM we must deal with some issues: which expenditures are autonomous, how are they determined, how are they financed, and its consequences. Pariboni (2016) and Fagundes and Freitas (2017), for instance, emphasize debt-financed consumption. Brochier and Macedo e Silva (2018) introduces SSM in a more financially complex economic framework — the Stock-Flow Consistent (SFC) approach — in which the autonomous consumption depends on household financial wealth. Nevertheless, residential investment has been systematically neglected. Despite its absence in the theoretical literature, there is a growing empirical literature drawing attention for its

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role to macroeconomics dynamics (LEAMER, 2007; JORDÀ et al., 2014; FIEBIGER, 2018; FIEBIGER; LAVOIE, 2018).

The aim of the paper is to include residential investment into the the Sraffian Supermultiplier model within a SFC framework. In the next section we will introduce some stylized facts on the relation between dwelling investments and macroeconomic dynamics. Section 3 will briefly present the SSM as an alternative closure to demand-led growth theory and will also assess how different authors deal with alternative non-creating capacity autonomous expenditures components. In section 4 we present our SSM-SFC with residential investment financed by mortgage loans. In contrast with the other models, our SSM-SFC has two real assets: firms' capital and household' real estate. We do some numerical simulations to evaluate the existence of steady-state and the effects of changes on: residential investment rate of growth; functional income distribution; rate of interest. Section five concludes the paper.

## 2 Empirical motivation

A current trend among empirical research on demand-led growth is about the role of non-capacity creating autonomous expenditures. Freitas and Dweck (2013) present a growth accounting decomposition for the Brazilian economy. Their work show the importance of those expenditures in explaining Brazilian GDP growth between 1970 and 2005. Braga (2018) shows evidence that economic growth and firms investment are explained by unproductive expenditures in Brazilian economy from 1962 to 2015. For the USA, Girardi and Pariboni (2016) show that autonomous expenditures do cause long run effects on the growth rate. Girardi and Pariboni (2018) bring evidence that autonomous expenditures determine the investment share on GDP for twenty OECD countries.

Nevertheless, there still is a lack of studies on the role of residential investments specifically. With the exception of Leamer (2007), most of those studies were published after the Great Recession 2008-2009 — which made it clear how important this particular expenditure is to USA economic dynamics. Leamer (2007, p. 2) shows the central role in explaining US business cycles in the post-war period. Accordingly to Leamer, US business cycles have the pattern “*[f]irst homes, then cars, and last business equipment*” (LEAMER, 2007, p. 8).

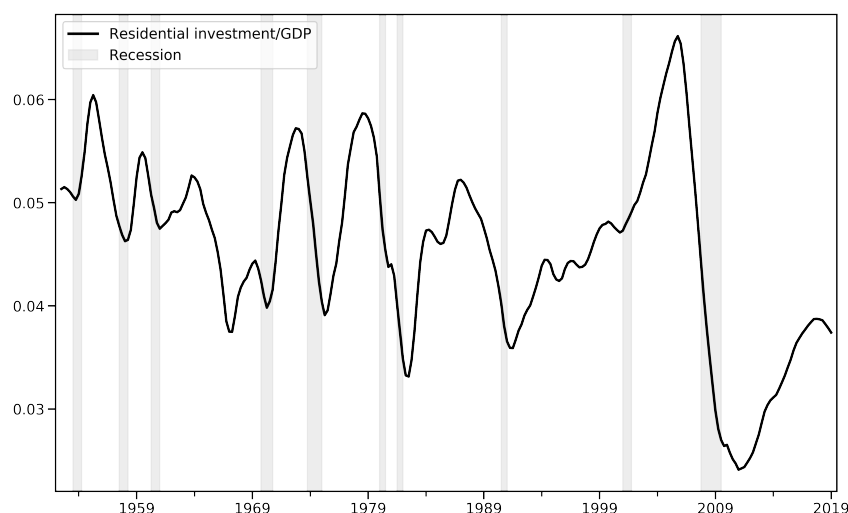
Figure 1 shows how the behavior of residential dynamics can help to predict recessions. Recessions are anticipated by a reduction of residential investment share of GDP, while the expansion of those expenditures precedes economic recovery. The fall of dwellings expenditures in 1966-67 are an exception because the increase of military expenditures because of Vietnan War offset an eventual economic downturn (LEAMER, 2007, p. 20). Another exception is the dot-com bubble 2000 crisis that was not caused by residential investment. The Great Recession 2008-2009 is the one in which this pattern is the most evident.

Figure 2 depicts the relevance of business cycles in an alternative way<sup>3</sup>. Each cycle is represented in a different panel. The vertical axis represents residential investment-GDP ratio and the horizontal axis represents the rate of capacity utilization as a proxy for business cycle. Economic recovery is

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<sup>3</sup>A similar depiction of business cycles can be find in Fiebiger (2018).

Figure 1 – Residential Investments as share of GDP  
quarterly moving average



**Source:** Federal Reserve Bank of St. Louis, authors' elaboration

generally characterized by a growth rate of residential investment greater than GDP rate of growth — with the 1991-2000 period being a particular case. The result is a bigger residential investment-GDP ratio and an also bigger rate of capacity utilization. As firms investment follows the capital stock adjustment principle the accumulation rate increase with the goal to adjust the effective rate of capacity utilization to the normal/planned one. The increase of firms investment growth rate causes the GDP to grow faster than residential investment, therefore reducing the latter share on GDP and also reducing the rate of capacity utilization<sup>4</sup>. Therefore its is possible to see the stylized fact about the relation among residential investment and economic cycle.

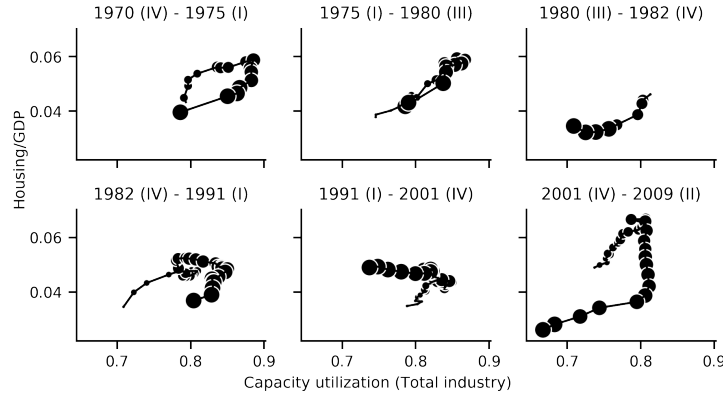
There is also an indirect relation between housing and aggregate demand. Accordingly to Teixeira (2012), real estate is one of the most commons means of wealth to US households and it serves as collateral to borrowing. Zezza (2008) and Barba and Pivetti (2009) show that credit-financed consumption was one of the main drivers of economic growth before 2008 subprime crises. Households would increase their indebtedness as houses prices went up as a way to “realize” capital gains without selling their homes during the house bubble of the 2000s (TEIXEIRA, 2015).

Understood how important is residential investment to economic dynamics (specially to the US economy), in the next section we will present and compare different heterodox models of economic growth with special focus on how they incorporate non-capacity creating autonomous expenditures.

<sup>4</sup>The works of Fiebiger (2018) e Fiebiger and Lavoie (2018) also report residential investment as an important determinant of economic cycles. Those works associate economic instability to the behavior of (at least some) autonomous expenditures in spite of the behavior firms investment — as it follows capital stock adjustment principle. Dejuán (2017) and Teixeira (2015) find similar results.

Figure 2 – Share of residential investment and capacity utilization during business cycles

(Dots size grow in time)



**Source:** Federal Reserve Bank of St. Louis, authors' elaboration.

### 3 Heterodox growth models and alternative closures

Harrod (1939) opens the research agenda of modern economic growth theory. He does that by extrapolating Keynes (1936) principle of effective demand to a growing economy. Accordingly to Harrod, Keynes had not took in consideration the dynamic implications of the new productive capacity created by net investment greater than zero. He proposes to connect the multiplier effect (which encapsulates investment as a source of demand) to the capital stock adjustment principle (which represents investment as source of new productive capacity). His main goal in proceeding this way is to analyze the conditions for balanced growth between supply and demand.

Following Keynes (1936), Harrod takes all consumption as induced by income — implying that all non-capacity creating autonomous expenditures ( $Z$ ) are null. Therefore, investment ( $I$ )<sup>5</sup> is the key variable in determining both output ( $Y$ ) and capacity levels ( $K$ ).

$$Y = \frac{I}{s} \quad (1)$$

$$\Delta K = I \quad (2)$$

In the above equation  $s$  represents marginal propensity to save which is identical to average propensity to save due to the absence of autonomous consumption. From equation 2 it is possible to deduct that capital stock rate of growth ( $g_K$ ) is determined by and is equal to investment growth rate ( $g_I$ )<sup>6</sup>. From the identity between savings and investment<sup>7</sup> we can show the equation to capital

<sup>5</sup>For simplicity we will always consider investment net of depreciation in the remainder of this paper

<sup>6</sup>This determination is not instantaneous, demonstrating the role time lags play in investment dual effect: first it generates income, then it generates productive capacity

<sup>7</sup>We can rewrite this identity as

$$\frac{I}{K} = \frac{S}{K} \frac{Y}{Y_{fc}} \frac{Y_{fc}}{Y}$$

accumulation is:

$$g_I = g_K = \frac{s}{v}u \quad (3)$$

where  $u$  is the degree of capacity utilization ( $u = Y/Y_{fc}$ ). From equation 1, we can realize that output growth rate is determined by investment growth rate. Hence, equation 3 represents the actual output growth rate for any degree of capacity utilization, for a given marginal propensity to save and a given capital-output ratio. Firms are in equilibrium when they operate with the planned/normal degree of capacity utilization ( $u_N$ , for simplicity we can consider equal to one) and so there is no need to adjust the rate of capital accumulation. Harrod (1939) names this the warranted rate of growth ( $g_w$ ).

$$g = \frac{s}{v}u$$

$$g_w = \frac{s}{v} \quad (4)$$

Balanced growth occurs when actual and warranted growth rates are equal. If the actual growth rate is greater (smaller) than the warranted rate of growth, the degree of capacity utilization is greater (smaller) than the planned/normal one. In this case, firms seek to increase (reduce) their output capacity aiming to adjust the effective degree of capacity utilization to the normal/planned one. Increasing (reducing) investment rate of growth has an immediate effect on output rate of growth and after some lag on capital stock rate of growth. The result is an even greater divergence between normal/planned degree of capacity utilization and the effective one<sup>8</sup>. The supposed adjustment mechanism in Harrod's model in fact unstabilizing. Harrod (1939) calls this process fundamental instability<sup>9</sup>.

Therefore modern growth theory first appear with the challenge to build a stable demand-led growth model<sup>10</sup>. The authors that came after Harrod had tried to deal with this task — and it is their effort that we will analyze and verify at what cost they succeeded.

Cambridge growth model was one of the first attempts to tackle this issue (KALDOR, 1955-56; 1957; ROBINSON, 1962; PASINETTI, 1962). In this model investment is autonomous eliminating the source of instability in Harrod's model. Cambridge growth model has a “kaleckian” economic structure (KALECKI, 1954), dealing explicitly with the income distribution between workers and capitalists. There is no savings out of wages — capitalists do all savings ( $S = s_p \cdot FT$ , where  $s_p$  is the marginal propensity to save out of profits,  $FT$  is total profit). Modifying equation 3, and following Serrano, Freitas and Behring (2018) exposition, we have

$$\frac{I}{K} = \frac{S}{K} \frac{Y}{Y_{fc}} = s_p \frac{FT}{K} \frac{Y}{Y_{fc}}$$

$$g = s_p \cdot (1 - \omega) \cdot u \cdot R \quad (5)$$

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$Y_{fc}$  is capacity output.

<sup>8</sup>The recurring effort to the adjust the degree of capital utilization by means of increasing (decreasing) the rate of growth of investment will make this divergence grows even more.

<sup>9</sup>For a mathematical demonstration of this instability see, for instance, Serrano, Freitas and Behring (2018).

<sup>10</sup>Neoclassical authors also had tried to avoid the fundamental instability by giving up the idea of demand-led growth (SOLOW, 1956)

$$r = (1 - \omega) \cdot u \cdot R$$

where  $R$  the maximum rate of profit (the reciprocal of capital-output ratio  $\nu$ ),  $\omega$  is the wage share ( $\omega = W/Y$ ) and  $r$  is the actual rate of profit.

Kaldor (1955-56) affirms that Keynesian multiplier determines output level only in the short-run, when prices and wages are rigid. On the long-run output level is equal to the potential output ( $Y = Y_{fc}$ ) and prices and wages would be flexible. In this scenario, changes in rate of growth of autonomous investment would change functional income distribution by means of change in prices. From equation 5, making output equal to capacity output ( $u = 1$ ), we have:

$$(1 - \omega) = \frac{g}{s_p \cdot R}$$

Where growth rate determines the profit share ( $1 - \omega$ ) — and also the profit rate —, for a given marginal propensity to save out of profits and a given maximum rate of profit. Changes in functional distribution of income are the adjustment mechanism of the warranted rate of growth towards the actual one, ensuring the model stability.

An alternative to Cambridge model is the Kaleckian growth model based on the works of Kalecki and Steindl (KALECKI, 1954; 1971; STEINDL, 1952; 1979)<sup>11</sup>. Steindl (1979) share some Cambridge features in his model, as autonomous investment and fully induced consumption, but disagree on the stabilizing mechanism. In economies where prevail market structures with oligopolies, prices would be rigid even in the long run because of rigid mark-ups. Therefore, income distribution could not bear to be the adjustable variable to close the model<sup>12</sup>. Steindl also asserts that there is no reason to suppose that output would always be equal to capacity output. In his model, changes of the growth rate would be accommodated by changes of the degree of capacity utilization — that could be permanently different from the normal/planned one. From equation 5, we can derive the Kaleckian closure, as presented by Serrano, Freitas and Behring (2018)<sup>13 14</sup>:

$$u = \frac{g}{s_p \cdot R \cdot (1 - \omega)} \quad (6)$$

Up to this point it is possible to say that demand-led growth theory faced a “trilemma”. It did not seem possible to reconcile in the same model exogenous distribution, normal/planned degree of capacity utilization and stability. Each one of the three closure give up one of those features — as it is represented at figure 3<sup>15</sup>.

The Sraffian Supermultiplier growth model, developed by Serrano (1995a) and Bortis (1997), shows that this is a false trilemma. This model presents exogenous distribution (as Harrod and Kaleckian models), normal/planned degree of capacity utilization (as Cambridge<sup>16</sup>) and stability. The new

<sup>11</sup>The work of those authors are at the origin of neo-kaleckian growth models (traditional and Marglin-Bhaduri types), as formulated by Rowthorn (1981), Dutt (1984) e Bhaduri and Marglin (1990) among others.

<sup>12</sup>For a more detailed assessment of Cambridge model, see Serrano (1988), Serrano, Freitas and Behring (2018) and Serrano and Freitas (2017).

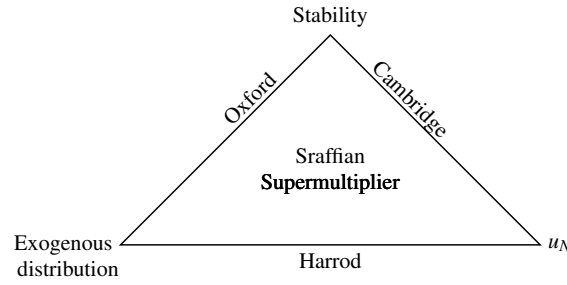
<sup>13</sup>Serrano (1995a) call those models “Oxford models” because both economists worked at that university

<sup>14</sup>For a critical assessment of this particular closure see Serrano (1988), Serrano, Freitas and Behring (2018) and Serrano and Freitas (2017)

<sup>15</sup>These figure is inspired by the “trilemma” presented by Cesaratto (2015).

<sup>16</sup>Despite Harrod’s model does not present an effective tendency towards the normal/planned degree of capacity utilization, it does have the capital stock adjustment principle that aims to do that.

Figure 3 – “Impossible” trilemma



**Source:** Author's elaboration

feature of the model is the introduction of non-capacity creating autonomous expenditures. The existence of this kind of expenditure implies a difference between marginal and average propensity to save. Other feature of this model is a fully induced firms investment function and a gradual adjustment of the marginal propensity to invest. Those features together ensure the convergence of actual degree of capacity utilization to the normal/planned one ( $u \rightarrow \bar{u}_N$ ) (FREITAS; SERRANO, 2015)<sup>17</sup>. Therefore, output growth is explained by the rate of growth of non-capacity creating autonomous expenditures ( $g_z$ ). Changing one more time equation 3, we have:

$$g = \frac{S}{Y} u \cdot R$$

$$\frac{I}{Y} = \frac{S}{Y} = \frac{g_z}{u_N \cdot R} \quad (7)$$

The equation 7 shows that a greater (smaller) rate of growth of non-capacity creating autonomous expenditures will cause a greater (smaller) investment share in output<sup>18</sup>.

At this point, it is worth noting a recent trend in the Kaleckian tradition. In face of the criticism of the non adjustment of the degree of capacity utilization towards the normal one, Allain (2015) introduces some modifications into the neo-Kaleckian baseline model and replicate some of the main results of the Sraffian Supermultiplier model.

Following the path opened by Allain (2015), neo-Kaleckian authors started to explore the consequences of introducing different non-creating capacity autonomous expenditures. Nah and Lavoie (2017) introduce exports as the main driver of growth. Despite achieving the main results of the Sraffian Supermultiplier model as well, their model can present different accumulation regimes (profit-led or wage-led) depending on how the real exchange rate reacts to changes in income distribution.

Dutt (2006), Palley (2010) and Hein (2012) present a model with debt-financed consumption. Nevertheless, as those models were built under standard neo-Kaleckian assumptions, the stability is reached only if consumption grows at the same rate of capital accumulation. This implies that in

<sup>17</sup>The next section will present a detailed exposition of this model. At this point we are only interested in showing it as an alternative closure to demand-led growth theory

<sup>18</sup>Freitas and Serrano (2015) shows a complete stability proof of this model. Serrano and Freitas (2017) compares all alternative closures presented in this section.

those models debt-financed consumption is not really autonomous. In this sense Pariboni (2016) works on an alternative based on the Sraffian Supermultiplier. In his work, the causality is reversed and it is the rate of accumulation that gradually converges towards the rate of growth of debt-financed consumption

Brochier and Macedo e Silva (2018) was the first effort to introduce the Sraffian Supermultiplier model in a fully specified stock-flow consistent framework. They present a non-parsimonious model with four institutional sectors: households, non-financial firms, banks and government. The results are at odds with the standard Sraffian Supermultiplier model. Besides exhibiting in a tendency of the degree of capacity utilization to converge to the normal/planned one, their model produces a wage-led accumulation regime. An increase of the wage share causes faster growth and accumulation at the fully adjusted position.

From the literature review, it is possible to realize an absence of residential investment-led growth models, despite its relevance to the macroeconomic dynamics. The next section will present a first attempt to fill this gap: a fully specified parsimonious Sraffian Supermultiplier stock-flow consistent model where residential investment financed by mortgage lending is the the source of growth.

## 4 The Sraffian Supermultiplier Stock-Flow Consistent with residential investment model

### General equations<sup>19</sup>

Our model is the most parsimonious as possible: a closed economy without government sector. Output ( $Y$ ) is determined by aggregate demand and it is the sum of consumption ( $C$ ) and ( $I_t$ ).

$$Y = C + I_t$$

From the institutional sectors perspective, household expenditures have two components (consumption and residential investment) and firms just one (firms' investment. From the demand side perspective, investment expenditures have two components and consumption just one. Firms are the responsible for the investment that creates productive capacity to the private business sector of the economy ( $I_f$ ). The novelty of the model is to introduce a second component to investment: residential investment ( $I_h$ ). Our simplifying hypothesis asserts that it is all made by household sector. The real assets created by this investment (housing) is household property and does not create productive capacity for the business sector. Therefore, this economy produces two types of real assets: firms productive capital ( $K_f$ ) and households housing ( $K_h$ ).

$$Y = [C + I_h] + [I_f] \tag{8}$$

$$K = K_f + K_h \tag{9}$$

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<sup>19</sup>The model was built in Python with pysolve3 package. The programming codes are available upon request.



Denoting the share of firms capital in total real assets as  $k$ , we can rewrite equation 9 as:

$$k = \frac{K_f}{K} \quad (10)$$

$$K = k \cdot K + (1 - k) \cdot K$$

Assuming a Leontief production function and infinity elasticity of labor supply, capacity output is determined by capital stock accumulated by firms.

$$Y_{FC} = \frac{1}{v} K_{f-1} \quad (11)$$

$$u = \frac{Y}{K_f} \cdot v \quad (12)$$

In this model, as in Sraffian Supermultiplier and Kaleckian tradition, income distribution is exogenous:

$$\omega = \bar{\omega} \quad (13)$$

$$\omega = \frac{W}{Y}$$

$$W = \omega \cdot Y \quad (14)$$

Table 1 presents the balance sheet matrix for all institutional sectors. Households hold financial wealth as money deposits at banks ( $M$ ), while finance their residential investment by mortgages ( $MO$ ). Their total net wealth is the sum of their net financial wealth ( $V_h$ ) and their real assets, *i.e.* housing ( $K_h$ ). Firms finance their investment primarily by undistributed profits and the residual by loans from the banking sector — thus they do not have deposits. Banks first create credit *ex nihilo* and then get the deposits, paying the same interest rate that they charge.

Table 1 – Balance sheet matrix

	Households	Firms	Banks	$\Sigma$
Deposits	$+M$		$-M$	0
Loans		$-L$	$+L$	0
Mortgages	$-MO$		$+MO$	0
$\Sigma$ Net financial Wealth	$V_h$	$V_f$	$V_b$	0
Capital		$+K_f$		$+K_f$
Houses	$+K_h$			$+K_h$
$\Sigma$ Net Wealth	$NW_h$	$NW_f$	$NW_b$	$+K$

**Source:** Author's elaboration

The next table (2) presents the transactions flows matrix and the flow of funds. This table accounts for all for all economic relations between the institutional sectors ensuring the lack of “black holes” and making all relations between financial and real sides explicit (DOS SANTOS; MACEDO E SILVA, 2010).

Table 2 – Transactions flow matrix and flow of funds

	Households		Firms		Banks	Total $\Sigma$
	Current	Capital	Current	Capital		
Consumption	$-C$		$+C$			0
Non-Residential investment			$+I_f$	$-I_f$		0
Residential investment		$-I_h$	$+I_h$			0
<b>[Product]</b>			$[Y]$			$[Y]$
Wages	$+W$		$-W$			0
Profits	$+FD$		$-FT$	$+FU$		0
Interest (deposits)	$+r_m \cdot M_{-1}$				$-r_m \cdot M_{-1}$	0
Interest (loans)			$-r_l \cdot L_{-1}$		$+r_l \cdot L_{-1}$	0
Interest (mortgages)	$-r_{mo} \cdot MO_{-1}$				$+r_{mo} \cdot MO_{-1}$	0
<b>Subtotal</b>	$+S_h$	$-I_h$		$+NFW_f$	$+NFW_b$	0
Change in deposits	$-\Delta M$				$+\Delta M$	0
Change in mortgages		$+\Delta MO$			$-\Delta MO$	0
Change in Loans				$+\Delta L$	$-\Delta L$	0
<b>Total</b>	0	0	0	0	0	0

Source: authors' elaboration

## Firms

In order to produce, firms purchase capital goods ( $-I_f$  in capital account) and hire works, whom total remuneration is the economy wage bill ( $w$ ). Their total profits ( $FT$ ) are a residual between their sales ( $Y$ ) and total wages ( $W$ ). Firms retain part ( $\varphi_F$ ) of profits net of interest payments ( $FU$ ) — to reinvest — and distribute the rest to households ( $FD$ ).

$$FT = Y - W \quad (15)$$

$$FU = \varphi_F \cdot (FT - r_l \cdot L_{-1}) \quad (16)$$

$$FD = (1 - \varphi_F) \cdot (FT - r_l \cdot L_{-1}) \quad (17)$$

Firms investment is fully induced by the level of effective demand (FREITAS; SERRANO, 2015), and its growth rate changes accordingly to the capital stock adjustment principle. This implies that firms changes their investment plans when the actual degree of capacity utilization is different from the normal/planned one. Firms investment determines the change in productive capital stock.

$$I_f = hY \quad (18)$$

$$\Delta h = h_{-1} \cdot \gamma_u \cdot (u - \bar{u}_N) \quad (19)$$

$$\Delta K_f = I_f \quad (20)$$

Where  $h$  is the marginal propensity to invest and  $\gamma_u$  must be sufficiently small in order to the adjustment be gradual<sup>20</sup>.

Firms finance the part of investment that exceeds undistributed profits by bank loans, paying the interest rate  $r_l$  for it. We assume an elastic supply of credit for investment. Moreover, tables 1 and 2 makes firms net financial wealth ( $NW_f$ ) and net financial balance ( $NFW_f$ ) explicits.

$$NW_f = K_f - L \quad (21)$$

$$\Delta L = I_f - FU \quad (22)$$

$$NFW_f = FU - I_f \quad (23)$$

## Banks

Banks do not have an active role in our model — as in most part of SFC literature. They create money as credit is demanded and just after they collect deposits. Firms finance part of their investment with credit ( $L$ ) and households finance all their residential investment with mortgages ( $MO$ ), as already mentioned. Each operation has its own interest rate defined by a spread over deposits interest rate ( $r_m$ ) autonomously determined by banks.

$$r_l = r_m + \text{spread}_l \quad (24)$$

$$r_{mo} = r_m + \text{spread}_{mo} \quad (25)$$

$$r_m = \bar{r}_m \quad (26)$$

For simplicity, we assume all the spreads to be null. The interest rate on mortgages and on firms loans are the same as on deposits.

Banks net balances ( $NFW_b$ ) are defined by interests received net of interests payments. As those interests are the very same, the net balance is necessarily zero. Deposits can be determined as a residuum and from table 1 we determine banks net wealth (zero as well):

$$NFW_b = r_{mo} \cdot MO_{-1} + r_l \cdot L_{-1} - r_m \cdot M_{-1} \quad (27)$$

$$\Delta M = \Delta L + \Delta MO \quad (28)$$

$$NW_b = V_b \equiv 0 \quad (29)$$

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<sup>20</sup>The size of this parameter guards a fundamental relation to the stability of the model, Freitas and Serrano (2015).

## Households

This is the most complex institutional sector of our model. We assume consumption ( $C$ ) is entirely induced by wages and households do not have access to consumption loans. Disposable income ( $YD$ ) is the sum of wages, distributed profits and received interests on deposits, net of interests payments on mortgages<sup>21</sup>.

$$C = \alpha \cdot W \quad (30)$$

$$YD = W + FD + \bar{r}_m \cdot M_{-1} - r_{mo} \cdot MO_{-1} \quad (31)$$

where  $\alpha$  is the marginal propensity to consume and it is equal to one.

Household savings ( $S_h$ )<sup>22</sup> are disposable income net of consumption (equal to wages, accordingly to our hypothesis). At odds with SFC literature, savings are not the same thing as the net balance ( $NFW_h$ )<sup>23</sup>. The cause of this difference is the inclusion of residential investment.

$$S_h = YD - C \quad (32)$$

$$NFW_h = S_f - I_h \quad (33)$$

Residential investment is the only autonomous expenditure ( $Z$ ) of the model, with a rate of growth ( $g_z$ ) exogenously given. As households are the only institutional sector realizing it, the supply ( $I_{hs}$ ) and demand ( $I_h$ ) for housing is the very same thing.

$$Z = I_h \quad (34)$$

$$I_h = (1 + \bar{g}_z) \cdot I_{h-1} \quad (35)$$

$$I_{hs} = I_h \quad (36)$$

As already mentioned, we assume that households finance their investment by mortgage loans. This implies that residential investment determines households indebtedness.

$$\Delta MO = I_h \quad (37)$$

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<sup>21</sup>For simplicity, we do not take in consideration the amortization of mortgage debt.

<sup>22</sup>As residential investment is fully funded by mortgages, savings are equal to change on household deposits:

$$\Delta M = S_h$$

However, this is a redundant equation and its is not necessary to be specified in the model.

<sup>23</sup>All net balances must add up to zero for the model to be consistent. If households have a surplus (deficit), firms must have a deficit (surplus), as banks net balance is zero.

## 4.1 Numerical simulations and experiments

After presenting the framework of the model, we can now show its main results. Output level is determined by the level of residential investment and by the size of the supermultiplier:

$$Y = \left( \frac{1}{1 - \omega - h} \right) I_h \quad (38)$$

The expression between brackets is the supermultiplier. Output growth rate — out of a steady state position — is determined by the following equation:

$$g = \frac{h_{-1} \cdot \gamma_u \cdot (u - \bar{u}_N)}{1 - \omega - h(t)} + \bar{g}_Z \quad (39)$$

In a fully adjusted position, when the actual degree of capacity utilization is equal to the normal one, output growth is fully determined by residential investment rate of growth:

$$g = \bar{g}_Z \quad (40)$$

in this sense, the investment share of output is:

$$h = \bar{g}_Z \frac{u_N}{v} \quad (41)$$

The ratio between the stock of firms capital and total capital is:

$$k = \frac{K_f}{K} = \frac{h}{(1 - \omega)} \quad (42)$$

This ratio is a positive function of the investment share in output (from equation 41 also a positive function of residential investment rate of growth) and of the wage share.

The remainder of this section presents the results of the following experiments: (i) increase of residential investment growth rate; (ii) wage share increase; (iii) increase of the interest rate. The results are summarized in appendix 1.

### Increase of residential investment growth rate

An increase of residential investment growth rate implies a greater growth rate of aggregate demand, therefore an increase of the degree of capacity utilization. Firms revise their investment plans accordingly to capital stock adjustment principle and gradually increase their marginal propensity to invest, causing output to temporarily grow faster than residential investment. The new full adjusted position will be characterized by: a greater rate of growth of output, equal to the rate of growth of residential investment; a greater investment share of output; and convergence towards the normal/planned degree of capacity utilization.

Those results (figure 4) are in line with Sraffian Supermultiplier literature. The main distinctive characteristic of our model is the existence of two real assets. A result that can at first be considered counter intuitive is that a increase of residential investment growth rate causes a reduction of housing share in total capital (*i.e.* an increase of  $k$ ).

## Increase of the wage share

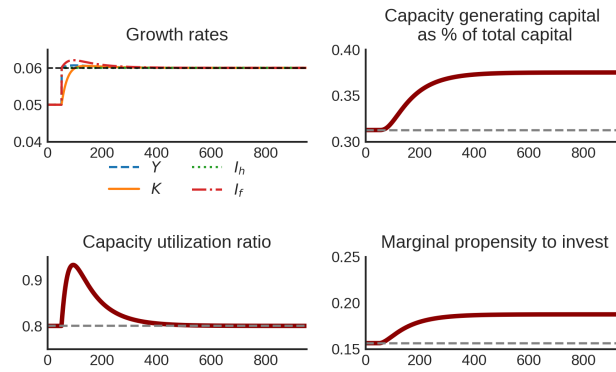
The increase of the wage share increases the rate of growth of output and the degree of capacity utilization. But those effects are just temporarily, because the rate of growth of residential investment does not change (figure 5). The main results are: marginal propensity to invest increases temporarily, and then returns to its baseline value; degree of capacity utilization quickly converges to the normal/planned one.

Despite the effect on output growth rate being only temporary, this change has a permanent effect on firms capital share of total capital. The increase of the wage has a level effect on output and on output capacity, without affecting residential investment. The result is an increase of the share of firms capital in total real assets.

## Increase of the rate of interest

An increase of the rate of interest does not have any effect on output growth, nor on the degree of capacity utilization. The share of firms capital on total capital does not change, because there is no change on distribution or the marginal propensity to invest. The only effect is a reduction of households savings due to payments of interest on mortgages increasing more than interests on deposits received. The result is a greater compromise of disposable income with interest payments<sup>24</sup>. There is no changes of the rest of long-term results.

Figure 4 – Effects of an increase of residential investment rate of growth



Source: Authors' elaboration

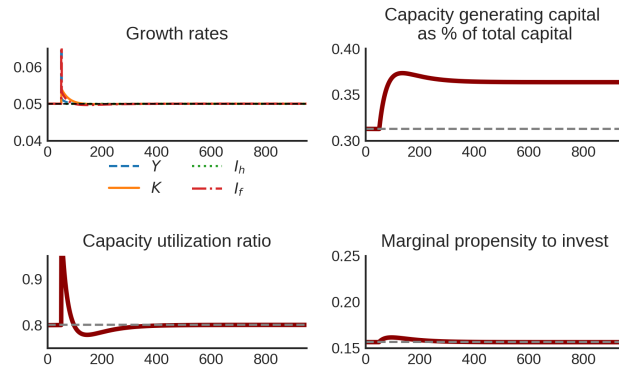
## 5 Final remarks

In this paper we have built a model aiming to contribute to the Sraffian Supermultiplier growth model, taking in consideration recent efforts of embedding it in a SFC framework. The main distinc-

<sup>24</sup>This effects can be seen in the following equation:

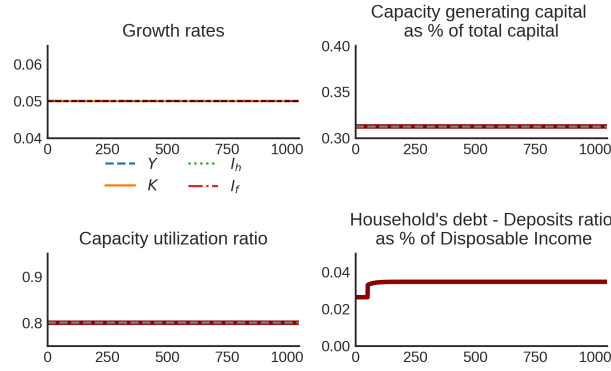
$$\frac{r_m \cdot M_{-1} - r_{mo} \cdot MO_{-1}}{YD}$$

Figure 5 – Effects of an increase of wage share



Source: Authors' elaboration

Figure 6 – Effects of an increase of of interest rate



Source: Authors' elaboration

tive characteristic of our model is putting residential investment as the driver of growth. The choice of this specific expenditure is due to recent empirical works showing the relevance of dwelling expenditures for macroeconomic dynamics, as seen in section 2. Section 3 shows that we could not find any work that incorporated this feature in a demand-led growth model.

As expected, our model presents the main results of Sraffian Supermultiplier growth models: convergence towards normal/planned degree of capacity utilization is achieved by means of changes of marginal propensity to invest; and steady state output growth rate is determined by the rate of growth of autonomous expenditures — in this case, residential investment. The main distinction of our model is that there are two different kinds of real assets: firms capital and housing.

The experiments show that an increase of residential investment growth rate results in a smaller housing share in total capital. Although seemingly counter intuitive, this result is a consequence of capital stock adjustment principle. Firms investment must temporarily grows faster than residential investment in order for the actual degree of capacity utilization to converge towards the normal/planned one. This is the cause of changes in the share of firms capital in total capital.

The other two experiments reports the results expected for Sraffian Supermultiplier growth mod-

els. An increase of wage share only has level effect on output and on capacity output. But precisely because of this level effect on capacity output, firms capital share in total capital increases. An increase of interest rate has only the effect of increasing firms debt-disposable income ratio.

It is important highlighting that this is just the first step of a wider research agenda on the role of residential investment for economic growth and business cycles. Future research should increase the degree of complexity of this model. Some possible improvements of this model include: exploring the determinants of residential investment to connect housing bubbles with aggregate demand and the inclusion of debt-financed consumption.

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# 1 Appendix

Table 3 – Experiments summary

	Base scenario	$\Delta g_Z$	$\Delta \omega$	$\Delta r_m$
$\alpha$	1,000	1,000	1,000	1,000
$g_{I_h}$	0,050	0,060	0,050	0,050
$\varphi_F$	0,400	0,400	0,400	0,400
$\gamma_u$	0,010	0,010	0,010	0,010
$g_k$	0,050	0,060	0,050	0,050
$g_Z$	0,050	0,060	0,050	0,050
$h$	0,156	0,187	0,156	0,156
$\omega$	0,500	0,500	0,510	0,500
$r_m$	0,020	0,020	0,020	0,025
$Ik$	0,313	0,375	0,319	0,312
$u$	0,800	0,800	0,800	0,800
$u_N$	0,800	0,800	0,800	0,800
$v$	2,500	2,500	2,500	2,500

**Source:** Authors' elaboration