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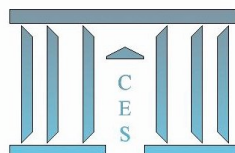
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**Macroeconomic effects of consumer debt:  
three theoretical essays**

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# Macroeconomic effects of consumer debt: three theoretical essays

Olivier Allain<sup>1</sup>

Abstract. Post-Keynesian economists have quite recently begun to draw attention to the consumer debt. However, as they omit the principal payment, they implicitly assimilate this debt as perpetual loans. The goal of this article is mainly methodological. We first develop a ‘Keynesian’ overlapping generations framework assuming that people borrow when they are young and service their debt (interests and principal) in the following periods. Defaults on the principal are also taken into account. We then analyze the theoretical properties of the equilibriums (multiplier effect, stability conditions) resulting from the introduction of this framework in three types of models that differ in regard of who are the debtors and who are the creditors: workers can borrow from capitalists (essay 1) or from their peer (essay 2); capitalists can borrow from their peer (essay 3).

Key words: Consumer debt, Keynesian models, Equilibrium instability, Overlapping generations models.

JEL codes: E12, E2, E21

## 1. Introduction

Post-Keynesian economists have quite recently begun to draw attention to the consumer debt. Their analyses put the stress on the causes of the surge in consumer debt ratio as well as on its economic consequences. About the first issue, the supply-side factors, mainly the financial

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deregulation and increased competition between financial institutions, appear to have played a leading role. However, many authors point the role of the demand-side factors: the evolution of social norms and consumers' needs (Cynamon and Fazzari, 2008, 2013), the conspicuous consumption and Veblen's effect in a context of growing inequalities (Barba and Pivetti, 2009; Palley, 2010; Wisman, 2013; Kappeler and Schütz, 2014; Kim *et al.*, 2014a, 2014b), or a wealth effect resting on notional wealth (Bhaduri *et al.*, 2006; Bhaduri, 2011).

About the consequences of this phenomenon, several authors have explored the properties of the equilibrium resulting from the introduction of consumer debt in a demand-led framework. In such framework, the level of economic activity partly rests upon the propensity to consume. Actually, it is commonly admitted that consumer lending involves a rise in the propensity to consume for the concerned households. However, the payment of interests induces an income distribution from high-consumption agents (the debtors) to low-consumption agents (the creditors), therefore the uncertainty of the final impact of borrowing on the overall propensity to consume and, consequently, on the level of economic activity and growth (Palley, 1994, 2002; Dutt, 2006; Hein, 2012; Charpe and Flaschel, 2013).

Another macroeconomic consequence relates to the consumer debt (un)sustainability and the (in)stability of the equilibrium. First, the debt accumulation might be uncontrolled. According to Barba and Pivetti (2009) who make a parallel with the snowball effect of the sovereign debt, this happens if the rate of interest is higher than the rate of growth of the household income. However, unlike for governments, the payment of interests may act as a discipline device on consumption, thus preventing the consumer debt explosion, a point which has been clarified by Dutt (2006) or Charpe and Flaschel (2013) among other. Charpe and Flaschel (2013, p.55) suggest another destabilizing mechanism based on a positive loop between consumption and the banks' credit supply: more consumption implies better performance for banks which supply more credit and support consumption, *etc.*

Second, the surge in consumer debt may involve a rise in default rates, therefore more financial fragility and a rise in systemic risk (Cynamon and Fazzary, 2008, 2013). In particular, banks may react by credit rationing, which can generate the vicious circle at work during financial crises: the tightening of credit, the deterioration of economic activities and the accumulation of non-performing loans in the creditors' balance sheets (Palley, 1994; Charpe and Flaschel, 2013).

From a methodological point of view, the theoretical models available in the exiting literature may be sorted in three classes according who are the debtors and who are the creditors in the system. This is a core question since the macroeconomic effects of debt depend on who spends, who saves and how borrowing and debt servicing affect the income distribution and then the overall propensity to consume. In most models including for instance Dutt (2006), Hein (2012), and Charpe and Flaschel (2013), workers are supposed to borrow from capitalists. For their part, Kim *et al.* (2014a) assume that workers partly borrow from their peer. Eventually, capitalists can borrow from capitalists, as in Bhaduri *et al.* (2006) in which the households' debt refers to a wealth effect.

The starting point of this article is the observation that, while all these models include the payment of interests as a constraint on consumption, they omit the payment of the principal. Such omission may be relevant for a sovereign debt: Treasury can take out a new loan to repay the old one, a process which assimilates public bonds to perpetual loans. But a household has a finite life expectancy. He can't transform his liabilities into a perpetual debt; he must repay the principal one time or another; or, if he can't, he makes default.

The theoretical challenge is to take into account the household finite life expectancy in a macroeconomic model in which "households" taken as an aggregate have an indefinite life expectancy. This difficulty can be resolved through a 'Keynesian' overlapping generations framework, assuming that people borrow when they are young and service their debt in the following periods. The default possibility can also be introduced.

As a result, everybody commits to service his debt, interest and principal. The increase of the propensity to consume (at the time of the debt issuing) is then followed by several periods during which the household must reduce his consumption. This means firstly that a household cannot enter into perpetual debt, and secondly that the net global effect of debt on the propensity to consume is *a priori* undetermined.

This framework is labelled 'Keynesian' because, contrary to the orthodox approach, it neither depends upon the hypothesis that economic agents predict future states of the world, nor upon that of intertemporal utility maximization. The only assumptions are that households borrow as they are young (whatever the reason: impatience, conspicuous consumption, *etc.*) but have to service their debt afterwards.

This Keynesian overlapping generations framework is then successively introduced in each of the three classes of models according to who are the debtors and who are the creditors. Note

that the aim of the present article is mainly methodological: it is to focus on the theoretical properties of the equilibriums resulting from this innovation.

Because of this methodological goal, the other hypotheses included in the models will remain quite unsophisticated: the explanations of the borrowing behavior are leaved aside so that the ‘propensity to borrow’ is assumed to be exogenously given, the rate of accumulation is also assumed to be given, etc. As a result, some of the models properties will appear to be counterintuitive or at odds with empirical facts. The interest of these counterintuitive properties is to underscore the lacking hypotheses which should be introduced in the modelling. For example, we will show that, under reasonable assumptions, a rise in consumer debt isn’t destabilizing by itself, therefore the conclusion that an hypothesis must be changed in order to generate some instability (the propensity to borrow must be made endogenous). Another example: as debt defaults have by themselves a positive impact on economic activity, specific assumptions (such as the degradation in the state of confidence) must be added in order to account for financial fragility.

Section 2 is devoted to the model in which workers borrowing is financed by capitalists (essay 1). In section 3, it is assumed that workers partly borrow from their peer (essay 2). In section 4, capitalists finance themselves (essay 3). The main results are summarized in the concluding section.

For sake of place and simplicity, it isn’t possible to deal with each essay in depth. The aim is rather to emphasize the converging outcomes resulting from the introduction of principal payment in an overlapping generations framework.

## **2. Essay 1: Capitalists finance the workers’ debt**

### *2.1. Model structure and stock-flow consistency*

We suppose an economy with four agents: workers, capitalists, banks and firms. The ex post accounting are reported in Table 1 (Balance-sheet matrix) and Table 2 (Transactions flow matrix).<sup>2</sup>

[Table 1 around here]

[Table 2 around here]

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<sup>2</sup> Symbols with plus signs describe sources of funds whereas negative signs indicate uses of funds.

In accordance with the overlapping generations hypothesis, it is assumed that young workers borrow because their consumption ( $C_{yw}$ ) is higher than their wage ( $W_y$ ). Their new loans ( $NL$ ) are then:

$$NL = C_{yw} - W_y \quad (1)$$

In addition of the interests on their loans ( $i_l L$  where  $i_l$  is the rate of interest on loans and  $L$  the amount of loans), the older workers have to pay a part of the principal to the banks: the *principal payment* ( $PP$ ) which is preceded by a negative sign indicating that it is a use of funds for these workers. The  $\lambda_p$  parameter is a binary parameter (the  $p$  subscript standing for *principal*) whose value is 1 if this payment is taken into account in the analysis and 0 if it is omitted.

When  $\lambda_p = 1$ , workers can default (see also Charpe and Flaschel, 2013). For convenience, default is not specified as a fraction of loans but as the fraction  $\theta$  of workers' principal payment. The idea is that older workers make default when they aren't able to pay back this principal. Default is supposed to be definitive without any rescheduling opportunity. The effective amount of principal payment is then  $(1 - \theta)\lambda_p PP$  which is assumed to be the only worker's saving (workers don't make any deposit):

$$(1 - \theta)\lambda_p PP = W_o - C_{ow} - i_l L \quad (2)$$

where  $W_o$  and  $C_{ow}$  represent the wage and consumption of older workers. Eventually, the variation of loans corresponds to the new loans minus the effective principal payment minus default, that is:

$$\dot{L} = NL - (1 - \theta)\lambda_p PP - \theta\lambda_p PP \quad (3)$$

Of course,  $\lambda_p = 0$  means that workers never pay back the principal so that they have contracted a perpetual debt.

It is important to underline that defaults don't directly affect the variation of loans ( $\dot{L}$ ). Indeed, assuming  $\lambda_p = 1$ , two cases must be distinguished: either older workers pay  $PP$  to their creditors so the debt diminishes by  $PP$ ; or they make a default of  $\theta PP$ , pay  $(1 - \theta)PP$  so that debt diminishes by  $PP$  once again.

However default negatively affects banks profitability:

$$F_b = i_l L - i_m M - \theta\lambda_p PP \quad (4)$$

where  $i_m$  is the rate of interest on deposits and  $M$  the amount of deposits. We assume that banks belong to capitalists who get the whole ( $F_b$ ) as dividends.<sup>3</sup> The banks and capitalists accountings can then be merged together, hence a simplification of the model specification. Another simplification will be introduced as we assume that  $i_m = 0$ .

The capitalists financing capacity is the difference between their income (composed by banks  $F_b$  and firms  $F_f$  dividends) and their consumption spending ( $C_c$ ). This capacity is used to buy firms equities ( $\dot{E}$ ) and to increase their deposits ( $\dot{M}$ ) so that:

$$F_f + (i_l L - i_m M - \theta \lambda_p PP) - C_c = \dot{M} + \dot{E} \quad (5)$$

As it clearly appears in Table 2, the variation of deposits finances (*via* the banks) the variation of loans ( $\dot{M} = \dot{L}$ ).

The firms' accounts are very simple. Firms produce consumption ( $C$ ) and capital ( $I$ ) goods. Note that  $I$  stands for *gross* investment which includes capital depreciation ( $\delta K$ ). This depreciation is financed by a fraction of profits, the remaining part being distributed through dividends  $F_f$ . Eventually, equity issues finance the *net* investment.

## 2.2. Workers' consumption: a Keynesian overlapping generations framework

We built a model with  $V$  generations and no demographic growth so that  $1/V$  is the weight of each generation. In period  $t$ , all workers are supposed to have the same productivity, regardless of how their age. Noting  $W_t$  the global wage bill, each vintage receives  $W_t/V$ .

According to the overlapping generations model, it is assumed that workers borrow when they are young (whatever the reason) and service their debt in the following periods. The consumption behavior thus evolves over time for each generation. Suppose that young workers of vintage  $v$  enter the labor market at time  $t$  and consume more than their wages in that period:

$$C_{v,t} = (1 + c_{yw}) \frac{W_t}{V} \quad (6)$$

where  $c_{yw}$  stands for the young workers' 'propensity to borrow'. This propensity is assumed to be exogenous, as in Dutt (2006) who specifies that the "desired level of borrowing (...) can be interpreted as being determined by lenders, by borrowers or by both" (p.347). But the function can of course be made more explicit. For instance, the propensity to borrow is

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<sup>3</sup> On the contrary, Charpe and Flaschel (2013) assume that banks retain a fraction of their income.



endogenous in Charpe and Flaschel (2013) where it depends on bank performances. In Hein (2012), workers' borrowing rests on the amount of saving that rentiers decide to lend. Other authors introduce income inequalities (Barba and Pivetti, 2009) or consumption inequalities (Setterfield and Kim, 2013) to take the relative income hypothesis into account. In other words, there is a wide variety of specifications. We keep the simplest hypothesis of an exogenous propensity to borrow in order to focus on the consequences of the principal payment whatever the motivation of debt.

The debt for vintage  $v$  is:

$$L_{v,t} = NL_t = c_{yw} \frac{W_t}{V} \quad (7)$$

The principal payment is assumed to be uniform in every period until the end of the life span of the generation and the fulfillment of the intertemporal budget constraint:

$$PP_{v,t+n} = \frac{L_{v,t}}{V-1} \quad (n = 1, \dots, V-1) \quad (8)$$

The remaining debt for the vintage  $v$  at time  $t+n$  is then:

$$L_{v,t+n} = L_{v,t+n-1} - PP_{v,t+n} \quad (n = 1, \dots, V-1) \quad (9)$$

As workers don't borrow anymore after their first period and as they save no more than their principal payment, the consumption behavior for the next periods ( $t+n$ ) is given by:

$$C_{v,t+n} = \frac{W_{t+n}}{V} - (1-\theta)PP_{v,t+n} - i_l L_{v,t+n-1} \quad (n = 1, \dots, V-1) \quad (10)$$

where, contrary to the assumption made by Charpe and Flaschel (2013), the direct effect of defaults ( $\theta PP$ ) on workers' consumption is positive:<sup>4</sup> while the principal payment reduces the possibility to consume, any default on this payment restores this possibility.<sup>5</sup>

We now consider the aggregation of the  $V$  workers vintage which are present at time  $t$ , each of which having borrowed:

$$L_{v,t-v} = \frac{c_{yw}W_{t-v}}{V} \quad (v = 0, \dots, V-1) \quad (11)$$

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<sup>4</sup> There may be a negative indirect effect *via* a credit rationing or a rise in the rate of interest. We will discuss this points later.

<sup>5</sup> An implicit assumption is that (at a microeconomic level, represented by small case letters) every defaulting worker get a higher income ( $w - i_l l$ ) than his principal payment ( $pp$ ). Otherwise, if  $w - i_l l < pp$ , the possibility to consume preserved by the default is only restricted to the income (that is,  $c = w - i_l l < pp$ ). The remaining part of the default ( $pp - w + i_l l$ ) doesn't enable any consumption. However, this microeconomic feature is left aside because of the difficulty to take it into account in a macroeconomic model.

Except for the youth, each vintage has to pay:

$$PP_{v,t} = \frac{L_{v,t-v}}{V-1} \quad (v = 1, \dots, V-1) \quad (12)$$

The aggregate amount of principal payment at time  $t$  is then:

$$PP_t = \sum_{v=1}^{V-1} PP_{v,t} = \frac{c_{yw}\Omega W_t}{V} \quad (13)$$

with:

$$\Omega = \frac{1}{g^{(V-1)}} [1 - (1+g)^{1-V}] \quad (14)$$

where  $g$  denotes the growth rate of the wage bill.<sup>6</sup> It can be shown that  $\Omega$  decreases as  $V$  and/or  $g$  increases. More precisely:  $g > 0$  implies that  $0 < \Omega < 1$ ;  $g \rightarrow 0$  implies that  $\Omega \rightarrow 1$ ; and,  $g < 0$  implies that  $\Omega > 1$ .

Finally, the aggregate consumption for the  $V$  workers vintage at time  $t$  is:

$$C_{wt} = \left(1 + \frac{c_{yw}}{V}\right) W_t - (1 - \theta)PP_t - i_t L_{t-1} \quad (15)$$

Substituting  $PP_t$ , adopting continuous time and rearranging in order to offer greater generality, this function can be rewritten:

$$C_w = \left\{1 + \left[1 - (1 - \theta)\lambda_p\Omega\right] \frac{c_{yw}}{V}\right\} W - \lambda_i i_t L \quad (16)$$

where the term in braces represents the propensity to consume out of wages (the level of loans being given). The parameter  $\lambda_p$  is introduced here in order to distinguish the overlapping generations model where the principal payment is included ( $\lambda_p = 1$ ) with others models where this payment is omitted (that is,  $\lambda_p = 0$ ). In the latter case, it will be assumed to that  $\theta = 0$  for the sake that workers can't default on the principal of their debt if they have no principal to pay to their creditors.

Another parameter,  $\lambda_i \in [0,1]$  (the  $i$  subscript standing for *interests*) is added in order to take into account the impact of the payment of interests on consumption. Note that  $\lambda_p = \lambda_i = 1$  are the only conditions for the intertemporal budget constraint to be satisfied and the model to remain consistent as an overlapping generations model. In that case, debt servicing plays as a perfect discipline device on the consumption behavior. This is our reference model.

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<sup>6</sup> Because there is no demographic growth,  $g$  is the rate of growth of individual wages as well as of the wage bill. As it will be made explicit below, the increase in the wage bill results from the increase in the capital stock which involves a rise in labor productivity.

This reference model will be compared to a model where the debt servicing doesn't act as a perfect discipline device on consumption, that is  $\lambda_p = 0$  and/or  $\lambda_i < 1$ .<sup>7</sup> Such hypotheses are inconsistent in an overlapping generations model, but they can be made consistent in a model with no generation and perpetual debt.

Note that the two hypotheses,  $\lambda_p = 0$  and  $\lambda_i < 1$ , have different implications on the workers behavior:  $\lambda_i < 1$  implies new borrowing for workers to pay their interests whereas  $\lambda_p = 0$  doesn't imply new borrowing (it just means that workers don't pay their principal, or that the model omits this payment).

Finally, the variation of loans is given by:

$$\dot{L} = C_{yw} - W_y - \lambda_p PP + (1 - \lambda_i)i_l L \quad (17)$$

After substituting, it comes that:

$$\dot{L} = (1 - \lambda_i)i_l L + (1 - \lambda_p\Omega) \frac{c_{yw}}{V} W \quad (18)$$

which is a positive function of  $i_l$ ,  $L$ ,  $\gamma$  (via a decrease in  $\Omega$ ),  $c_{yw}$  and  $W$ , while it is a negative function of  $\lambda_i$ ,  $\lambda_p$  and  $V$ . The introduction of the principal payment parameter ( $\lambda_p = 1$ ) then entails conflicting changes in consumption: a negative direct effect on the propensity to consume but a positive indirect effect *via* the reduction in the amount of loans and then the payment of interests. In the same way, an increase in  $\lambda_i$  has a negative direct effect on consumption (which is more disciplined by the payments of interests) but a positive indirect effect *via* the reduction of the amount of loans.

### 2.3. *Equilibrium and dynamics analyses*

The behavior of the other agents must be specified to complete the model. First, a very simple investment function is retained in order to keep the focus on household debt, independently of the vivid debate on the investment specification amongst the different Post-Keynesians approaches. It is thus assumed that investment maintains the growth of the capital stock ( $K$ ) at an exogenous, positive rate ( $\gamma > 0$ ). However, as it is interesting to occasionally highlight the

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<sup>7</sup> Although the models in the existing literature generally assume that  $\lambda_i = 1$ , it will be instructive to look at the consequences of the assumption  $\lambda_i = 0$  on the model properties.

models properties under the assumption of a zero (or even negative)  $\gamma$ , capital depreciation ( $\delta$ ) is also introduced in the model.<sup>8</sup> We then have:

$$I = (\gamma + \delta)K \quad (19)$$

Second, capitalist are supposed to spend the propensity  $c_c$  of their net income in consumption, that is:<sup>9</sup>

$$C_c = c_c(F_f - \delta K + i_l L - \theta \lambda_p PP) \quad (20)$$

Finally, the production function being the well-known function *à la* Leontief and assuming that outcome is not labor restricted result in:

$$Y = uK \quad (21)$$

where  $u$  represents the rate of capacity utilization. Under these hypotheses, the goods market equilibrium is given by:

$$Y = C_w + C_c + I \quad (22)$$

Substituting each function, noting  $\pi$  the profit share<sup>10</sup> and  $\lambda = L/K$  the debt ratio, and assuming (temporarily) that this ratio is exogenous, the *short-run* goods market equilibrium is given by:<sup>11</sup>

$$u^* = \Phi[\gamma + (1 - c_c)\delta + (c_c - \lambda_i)i_l\lambda] \quad (23)$$

where the term in brackets is assumed to be positive and where  $\Phi = \left\{ (1 - c_c)\pi - \left\{ 1 - [1 - (1 - c_c)\theta]\lambda_p\Omega \right\} (1 - \pi) \frac{c_{yw}}{v} \right\}^{-1}$  is also assumed to be positive to satisfy the Keynesian stability assumption.

[Table 3 around here]

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<sup>8</sup> This is made necessary by the fact that, investment being the only autonomous component of the aggregate demand in our framework, the model converges to a zero solution if  $\gamma = 0$ . Introducing capital depreciation allows a positive solution even if  $\gamma = 0$ .

Of course, the rate of accumulation can't be lastingly null or negative in a capitalist economy. However, it may be the case in a short or medium period of time.

<sup>9</sup> The capitalists' net income is assumed to be positive, involving some restrictions on the capital depreciation and/or default ( $F_f + i_l L > \delta K + \theta \lambda_p PP$ ). Besides, it is assumed that principal payment affects the financing capacity of capitalists, not their consumption.

<sup>10</sup> The profit share is here assumed to be exogenous. See Charpe and Flaschel (2013) for a model with an endogenous determining of income distribution.

<sup>11</sup> Note that the rate of growth of the wage bill,  $W = (1 - \pi)Y$ , adjusts to the rate of growth of income,  $Y = uK$ , which is equal to the rate of accumulation, hence  $g = \gamma$ .

The comparative statistics are reported in Table 3. As the issue about defaults will be analyzed in a subsequent section, we suppose here that  $\theta = 0$ . It must be noted that most Keynesian results are preserved whatever the value of  $\lambda_p$  and  $\lambda_i$ : increases in both the rate of accumulation ( $\gamma$ ), the rate of depreciation ( $\delta$ ) or the propensities to consume ( $c_{yw}$  and  $c_c$ ) as well as a decrease in the profit share ( $\pi$ ) entail a rise in the rate of capacity utilization.

Turning to considerations about households' debt, several points deserve attention.<sup>12</sup> First, the greater the value of  $\lambda_p$  or  $\lambda_i$ , the smaller the rate of capacity utilization because debt servicing acts as a more restrictive discipline device on the older workers consumption.

Second, an increase in  $i_l$  or in  $\lambda$  induces a rise in the amount of interests on loans. The resulting effect in  $u^*$  depends on the value of  $\lambda_i$ . If  $\lambda_i = 1$ , more interests on loans entail a shift in income from older workers (who fully consume their disposable income) to capitalists (who save a fraction of their income), therefore a fall in consumption and then a fall in  $u^*$ . On the contrary, assuming  $\lambda_i = 0$  means that older workers don't adjust their consumption and borrow to pay their interests; the aggregate workers consumption remains unaffected while the capitalists consumption is feed by the interests they receive; it results a rise in  $u^*$ .

Third, it is worth examining what happens if debt servicing plays as a perfect discipline device ( $\lambda_p = \lambda_i = 1$ ) when the economic growth is very weak ( $\gamma \rightarrow 0$ ). In this case, remembering that  $\gamma \rightarrow 0$  implies  $\Omega \rightarrow 1$ , the goods market equilibrium can be rewritten:

$$u^* = \frac{\delta - i_l \lambda}{\pi} \quad (24)$$

Interestingly, the equilibrium is no longer dependent on the young workers' propensity to consume ( $c_{yw}$ ) because the positive effect of an increase in  $c_{yw}$  on youth consumption is completely offset by the negative effect on the consumption of their elders.

In the short run, the debt ratio is assumed to be exogenously given. But of course it isn't. In the *long run*, it can be shown that:

$$\dot{\lambda} = (1 - \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi) u - [\gamma - (1 - \lambda_i) i_l] \lambda \quad (25)$$

where the dot denotes the rate of change ( $\dot{\lambda} = d\lambda/dt$ ). The dynamics of  $\lambda$  is given by:

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<sup>12</sup> We don't insist on the negative sign of  $du^*/dV$  which principally results from the structure of the model: as  $V$  increases, the same wage bill is divided among more vintage which implies a cut in the young workers' income and then a cut in the rate of capacity utilization. This outcome is closely related to the hypothesis that the workers' borrowing is restricted to the only first period of their span of life.

$$\frac{d\lambda}{d\lambda} = (1 - \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi) \frac{du^*}{d\lambda} + (1 - \lambda_i) i_l - \gamma \quad (26)$$

with:

$$\frac{du^*}{d\lambda} = \Phi(c_c - \lambda_i) i \quad (27)$$

so that:

$$\frac{d\lambda}{d\lambda} = \left[ 1 - \lambda_i + (1 - \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi) \Phi(c_c - \lambda_i) \right] i_l - \gamma \quad (28)$$

whose sign depends on the value of several parameters including  $\lambda_i$ . In addition, the condition for the debt ratio to remain constant is:

$$\lambda^* = \frac{(1 - \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi)}{\gamma - (1 - \lambda_i) i_l} u \quad (29)$$

or, in the plane  $(\lambda, u)$ :

$$\dot{\lambda} = 0 \quad \Leftrightarrow \quad u = \frac{\gamma - (1 - \lambda_i) i_l}{(1 - \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi)} \lambda^* \quad (30)$$

This is a linear function (the *constant debt ratio curve*) whose slope can be positive or negative depending on the sign of the numerator.

The goods market equilibrium given by  $u^*$  can also be represented in the plane  $(\lambda, u)$  by a straight line (the *goods market equilibrium curve*) whose intercept,  $\Phi[\gamma + (1 - c_c)\delta]$ , is positive and whose slope,  $\frac{du^*}{d\lambda} = \Phi(c_c - \lambda_i) i_l$ , takes the sign of the term in parentheses. Finally, the long-run equilibrium is given by the intersection of the two curves.

It must be stressed that  $\lambda_i$  has a greater influence than  $\lambda_p$  in the long-run analysis because it affects the sign of both  $d\dot{\lambda}/d\lambda$  and  $du^*/d\lambda$ . Consequently, we suppose in the next section that workers reduce their consumption to pay their interests on loans ( $\lambda_i = 1$ ). We leave the issues about borrowing the interests ( $\lambda_i = 0$ ) or defaulting workers ( $\theta > 0$ ) for further sections.

#### 2.4. Long-run properties if the payment of interests plays as a discipline device on consumption

It is assumed here that  $\lambda_i = 1$  and  $\theta = 0$ . Note that the denominator of equation (29) simplifies so that:

$$\lambda^* = \frac{(1-\lambda_p\Omega)\frac{c_{yw}}{V}(1-\pi)}{\gamma}u \quad (31)$$

The convergence condition depends on the sign of the derivative:

$$\frac{d\lambda}{d\lambda} = - \left[ (1 - \lambda_p\Omega) \frac{c_{yw}}{V} (1 - \pi) \Phi(1 - c_c) i_l + \gamma \right] \quad (32)$$

A sufficient condition for this derivative to be negative is that the rate of accumulation ( $\gamma$ ) is positive. However, even if  $\gamma < 0$ , the system remains stable as long as:

$$(1 - \lambda_p\Omega) \frac{c_{yw}}{V} (1 - \pi) \Phi(1 - c_c) i_l > -\gamma \quad (33)$$

We suppose that this condition holds. The graphical solution corresponds to Figure 1.

[Figure 1 around here]

The numerator of the *constant debt ratio curve* (equation 31) simplifies so that its slope is positive and independent from the interest rate.

On the other hand, the slope of the *goods market equilibrium curve* (equation 23) is negative. Note that there is no dynamics here as the rate of capacity utilization is fixed instantly to its equilibrium level  $u^*$ .

The main outcome is that the long-run equilibrium  $(\lambda^*, u^*)$  is both positive and stable. In other words, there's no risk of households debt unsustainability as it is the case for the public debt as soon as consumption depends on income, *i.e.* as soon as the payment of interests disciplines the workers consumption. As pointed by Charpe and Flaschel (2013), “the recessionary effect of higher debt stabilizes the accumulation of debt because it reduces the disposable income of workers as well as their level of consumption” (p.53). This result confirms that of Dutt (2006, p.652). It also confirms Charpe and Flaschel (2013) as well as Hein (2012) in the sense that, in their models, instability stems from an endogenous shift in some parameters: the increase in the propensity to borrow, the rise in the profit share, *etc.* It seems thus useful to distinguish the issue of the stability *in itself* (that is for a given value of the parameters) with another issue that questions the stability of the economic system if some parameters are subject to endogenous changes (see below).

[Table 4 around here]

The comparative statistics are reported in Table 4. About the impacts on the rate of capacity utilization, the short-run results for several parameters ( $c_{yw}$ ,  $\pi$ ,  $\lambda_p$  and  $V$ ) are confirmed provided that the constraint  $\gamma > (1 - c_c)i_l$  is satisfied. Otherwise, the short-run effect is more

than offset by an opposite effect in the workers' consumption resulting from the change in the payment of interests.

Assume for instance a rise in young workers' borrowing ( $c_{yw}$  increases). The two curves on Figure 1 make a clockwise rotation around their own points of intersection with the horizontal axis. This results in an unambiguous rise in the debt ratio. Besides, if  $\gamma > (1 - c_c)i_l$ , the dominant rotation is that of the *goods market equilibrium curve* that brings about an increase in the rate of capacity utilization ( $u^*$ ) as the rise of wages resulting from a relatively high rate of economic growth ( $\gamma$ ) makes it possible for young borrowing workers to support a higher level of consumption. Conversely, if  $\gamma < (1 - c_c)i_l$ , the dominant rotation is that of the *constant debt ratio curve*: the previous effect is dominated by the debt burden impact which involves a shift in income from borrowers to creditors who have a lower propensity to consume; then a drop both in consumption and economic activity. Such outcome has already been underlined in several contributions including Dutt (2006, p.355), Hein (2012, p.25), and Charpe and Flaschel (2013, p.53).

Now assume a shift in  $\lambda_p$  (from 0 to 1). The two curves on Figure 1 make a counter-clockwise rotation around their point of intersection with the horizontal axis. This implies an unambiguous drop in the debt ratio resulting from older workers paying back the principal to their creditors. If  $\gamma > (1 - c_c)i_l$ , the rate of capacity utilization decreases too because, as in the short run, older workers have to worsen their consumption in order to service the principal payment. Conversely, if  $\gamma < (1 - c_c)i_l$ : the lower level of indebtedness involves low interests paid to capitalists, therefore a higher older workers' consumption that enhances the rate of capacity utilization.

The only ambiguous effect on the equilibrium debt ratio ( $\lambda^*$ ) results from a change in  $\gamma$ : on the first hand, an increase in  $\gamma$  boosts economic activity, wages and then youth indebtedness; in the other hand, it reduces the share of the previously contracted debt in the national income. Besides, an increase in the rate of interest entails a decline in the older workers' consumption (as  $\lambda_i = 1$ ), hence a decline in economic activity, wages and, again, youth indebtedness.

Eventually, it must be stressed that a very weak economic growth ( $\gamma \rightarrow 0$  implying  $\Omega \rightarrow 1$ ) implies that principal payment offsets the new contracted loans. There is no variation of loans and the long-run debt ratio remains close to zero. In that case, the long-run rate of capacity utilization simply becomes  $u^* = \delta/\pi$ .



## 2.5. More comments about workers' consumption

The workers consumption functions proposed in the existing literature generally take the following form:<sup>13</sup>

$$C_w = W - i_t L + \dot{L} \quad (34)$$

At a first glance, the consistency of such ‘without microeconomic foundation’ specification is questioning. First, what is the relevance of the introduction of workers borrowing when their consumption is lower than their wages ( $C_w < W$ )? Second, this formulation can suggest that workers enter in perpetual debt as soon as they borrow ( $\dot{L} > 0$ ): how can they pay the principal if they consume more than their income in every period ( $C_w > W - i_t L$ )?

An important contribution of our overlapping generations model is to provide ‘microeconomic foundations’ to the above specification. First, it confirms that borrowing is fully consistent with the case where workers consume less than their whole wages. Second, the proof is made that the ‘without microeconomic foundation’ specification remains consistent if the principal payment is taken into account (provided that  $\gamma > 0$  so that  $\Omega < 1$ ). Indeed, assuming  $\lambda_p = 1$ , it can be shown that:

$$C_w = W - i_t L + (1 - \Omega) \frac{c_{yw}}{\gamma} W \quad (35)$$

where the last term corresponds to the variation in borrowing ( $\dot{L}$ ). Thus, the *constraint* that workers don’t consume more than their income during their lifetime (the intertemporal budget constraint) is fully consistent with the *result* that workers (taken as an aggregate) consume more than their income at time  $t$ . The reason lies in economic growth that makes it possible for young workers to consume today a fraction of their further, growing wages while older workers pay back the principal which is proportional with their previous, lower wages.<sup>14</sup>

In other words, the ‘without microeconomic foundation’ specifications where principal payment is omitted and where it is *assumed* that borrowing enables workers to consume more than their income are fully consistent with the *results* of our overlapping generations model including the principal payment.

It is also worth to compare workers consumption with their wages, that is:

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<sup>13</sup> See for instance Dutt (2006), Hein (2012), or Charpe and Flaschel (2013).

<sup>14</sup> On the contrary, workers consume no more than their income if  $\gamma \rightarrow 0$  (*i.e.*  $\Omega \rightarrow 1$ ). They consume less than their income if  $\gamma < 0$  (*i.e.*  $\Omega > 1$ ).

$$\frac{c_w - W}{K} = \frac{(1 - \lambda_p \Omega)(1 - \pi)c_{yw}}{V} \left(1 - \frac{i_l}{\gamma}\right) u^* \quad (36)$$

If  $\gamma$  is positive (hence  $\Omega < 1$ ), the sign is given by the last term in parentheses. Therefore borrowing enables the aggregate consumption of workers to be greater than their wage provided that  $\gamma > i_l$ : the amount of borrowing is higher than the amount of debt servicing since the high rate of growth (which reduces the debt ratio) goes together with a cheap rate of interest. Conversely, workers consume less than their wage when  $\gamma < i_l$ : the amount of borrowing is now lower than the amount of debt servicing because of a low rate of growth and a high rate of interest.

Another, central implication of the model is that an increase in the young workers' consumption implies more sacrifices for their elders. A way to show this is to calculate the effect of a shift in the 'propensity to borrow' ( $c_{yw}$ ) on the ratio  $C_{ow}/C_{yw}$  where the young workers' consumption is given by equation (6) while their elders consumption function is:

$$C_{ow} = C_w - C_{yw} \quad (37)$$

After substitution and manipulation, it comes that:

$$\frac{dC_{ow}}{dC_{yw}} = - \frac{V - 1 + \left[ \frac{i_l}{\gamma} (1 - \lambda_p \Omega) + \lambda_p \Omega \right]}{(1 + c_{yw})^2} < 0 \quad (38)$$

Thus an increase  $c_{yw}$  induces a decrease in the consumption of the older workers relative to that of the youth generation.

Even if this article doesn't put the stress on the explanations of the workers behavior, this outcome raises questions about the conspicuous consumption and the relative income hypotheses which are frequently put forward in recent literature.<sup>15</sup> Actually, let us suppose an increase in the profit share. This entails a drop in economic activity and consumption for each agent, but the *aggregate* consumption for workers decreases relative to that of capitalists. According to the relative income hypothesis, assume that workers attempt to preserve their relative standard of life by increasing their 'propensity to borrow' ( $c_{yw}$ ).<sup>16</sup> However, this rise is good for young workers to the detriment of their elders. So, borrowing doesn't make it

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<sup>15</sup> See for instance Barba and Pivetti (2009), Palley (2010), Wisman (2013), Kappeler and Schütz (2014), Kim *et al.* (2014a, 2014b).

<sup>16</sup> Actually, two alternative specifications may be considered. First, the value of the propensity to borrow may depend from the profit share, for instance:  $c_{yw} = c_y + \zeta\pi$ . Second, the propensity to borrow  $c_{yw}$  may apply to the discrepancy between profits and wages as in  $C_w = W + c_{yw} \frac{\pi}{1 - \pi} - i_l L$ .

possible to improve the standards of life of workers in a homogeneous way. That questions the relevance of the models where the relative income hypothesis is based on a borrowing behavior. The reason is that if a household enhances his standard of life by borrowing, he has to make sacrifices in the following periods to service his debt.<sup>17</sup>

Such behavior leads to another interesting outcome: as the rise in  $c_{yw}$  boosts both the rate of capacity utilization and the debt ratio, the income of capitalists is enhanced as well as their consumption. In our framework, it can be shown that the workers attempt fails so that  $C_w/C_c$  suffers from a new decline. What happens if young workers insist? They raise their propensity to borrow once again, that involves the same consequences, then another rise in  $c_{yw}$ , etc. Finally, the system which is stable in itself (that is, for a given value of the parameters) can become unstable as soon as one of its parameters is subject to an endogenous change. That is the kind of instability that is highlighted by some economists: the system could reach a stable equilibrium but an agent modifies his behavior because he isn't satisfied by some properties of this equilibrium (here, the inequality in consumptions).<sup>18</sup>

## 2.6. Long-run properties if the payment of interests doesn't play as a discipline device on consumption

We now assume that the payment of interests doesn't play as a discipline device on consumption, *i.e.*  $\lambda_i = 0$ . As workers don't save, they have to borrow to pay their interests. Of course, such assumption is inconsistent with the intertemporal budget constraint. As it had already been underlined, our goal here is mainly methodological. We attempt to answer to the question: what happens if the interests payment is omitted in the consumption function?

That may be a cause of households over-indebtedness. Under the *ceteris paribus* hypothesis, it results in a higher rate of capacity utilization ( $du^*/d\lambda_i < 0$ ) because of the rise in the older workers' consumptions (equation 23 and Table 3). Conversely, the necessity to borrow for paying the interests leads to a greater equilibrium debt ratio (equation 29).

If  $\lambda_i = 0$ , the debt ratio dynamics becomes:

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<sup>17</sup> At this stage, a nuance should be introduced: if the borrowing behavior isn't fully consistent with the relative income hypothesis, it would be easier to combine with the conspicuous consumption hypothesis. Actually, young workers can buy the same big car as capitalists', but they have to eat potatoes in the following periods to service their debt.

<sup>18</sup> On the contrary, the well-known problem of public debt unsustainability stems from the instability of the system in itself (for a given value of the parameters).

$$\frac{d\lambda}{d\lambda} = \Psi i_l - \gamma \leq 0 \quad (39)$$

where  $\Psi = 1 + (1 - \lambda_r \Omega) \frac{c_{yw}}{V} (1 - \pi) \Phi c_c$  is higher than unity. As a result, a sufficient condition for the derivative to be positive is that  $i_l > \gamma$ . In addition, the slope of the equilibrium rate of capacity utilization in the plane  $(\lambda, u)$  is now positive as:

$$\frac{du^*}{d\lambda} = \Phi c_c i_l \quad (40)$$

It can be shown that three cases must be distinguished:

- (a)  $i_l > \gamma$  (cf. Figure 2);
- (b)  $i_l < \gamma$  but  $\Psi i_l > \gamma$  since  $\frac{du^*}{d\lambda} \Big|_{u=u^*} > \frac{du}{d\lambda} \Big|_{\lambda=0}$  (cf. Figure 3);
- (c)  $i_l < \gamma$  and  $\Psi i_l < \gamma$  since  $\frac{du^*}{d\lambda} \Big|_{u=u^*} < \frac{du}{d\lambda} \Big|_{\lambda=0}$  (cf. Figure 4).

[Figure 2 around here]

[Figure 3 around here]

[Figure 4 around here]

The two first cases produce instability and households' debt unsustainability. In particular, in case (b) the convergence toward the equilibrium debt ratio  $\lambda^*$  systematically raises  $\lambda$ , inducing an increase in  $u^*$ , then an increase in  $\lambda$ ... Only the third case (c) produces stability. Note that the condition to be in (c) rather than in (b) is:

$$\frac{(1-c_c)\pi}{(1-\lambda_p\Omega)(1-\pi)\frac{c_{yw}}{V}} > \frac{c_c i_l}{\gamma - i_l} + 1 \quad (41)$$

Stability thus results from high  $\gamma$ ,  $V$ ,  $\lambda_p$  and  $\pi$ , while  $i_l$ ,  $c_c$  and  $c_{yw}$  must be small. Here again, the  $\lambda_p$  parameter doesn't play a crucial role in the analysis except that  $\lambda_p = 1$  makes it more likely to reach system stability and debt sustainability.<sup>19</sup>

## 2.7. Workers default on principal payment

We now assume that workers service their debt ( $\lambda_i = \lambda_p = 1$ ) but that some of them make default ( $\theta > 0$ ). As it has been pointed above, our main goal is methodological, that is we

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<sup>19</sup> As noted earlier, there is no symmetry between the two hypotheses  $\lambda_p = 0$  and  $\lambda_i = 0$ : the absence of principal payment ( $\lambda_p = 0$ ) doesn't induce any increase in borrowing; on the contrary, it is the new borrowing resulting from  $\lambda_i = 0$  which can cause system instability.

first analyze the consequences of defaults on the properties of our model. As a result, it will be shown that, everything else being equal, defaults have a positive impact on economic activity as well as on the global income of creditors. In other words, specific assumptions should be introduced in the model, such as state of confidence, to illustrate the financial fragility ensuing from defaults.

Let us remind first that, according to our definition of defaults as a fraction of workers' principal payment, defaults don't have any direct impact on the loans level (Table 2): if workers owe  $PP$  to their creditors, the cut in this loans level is  $PP$  if they pay  $PP$  as well as if they pay  $(1 - \theta)PP$  and make default on  $\theta PP$ .

Debt dynamics and stability conditions thus remain unchanged. The system converges toward its long-run equilibrium (equations 23 and 31).<sup>20</sup> In other words, defaults don't involve any households' debt unsustainability by itself.

[Table 5 around here]

The other main result is that the only effects of defaults on the model go through the multiplier effect: consumption of older workers increases since a default makes it possible for them to keep more income for consumption; this effect is partly offset by the decrease of capitalists' consumption resulting from default altering their income. Defaults then support the economic activity, so the wage bill and the young workers' indebtedness. In other words, defaults indirectly imply a rise in the rate of capacity utilization and, consequently, in the debt ratio (Table 5). In addition, it can be shown that an increase in the 'propensity to borrow' ( $c_{yw}$ ) whose effect on the rate of capacity utilization were negative if  $\gamma < (1 - c_c)i_l$  (cf. Table 4) can now have a positive impact.

Several extensions would be explored to deal with the analysis of defaults and their consequences in a more realistic way. First, the rate of defaults would be made endogenous, assuming for instance that it increases with a fall in the workers income (Charpe and Flaschel, 2013).

In addition, it would be interesting to introduce the reactions of capitalists (or banks) when they face increasing defaults. Intuitively, defaults should cause a decline in the consumption of capitalists. However, the impact remains ambiguous as the direct negative impact on capitalists' income goes together with the indirect positive impact via the rate of capacity

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<sup>20</sup> It also can be shown that, in the long run, the amount of capitalists' deposits grow at the same rate ( $\gamma$ ) than both the stock of capital and the amount of debt.

utilization. So it isn't sure that capitalists (or banks) taken as a whole suffer from a rise in default. Nevertheless, on microeconomics grounds, one can expect that some of them suffer a loss (the ones who had lent to the defaulting workers). The likelihood to suffer a loss may involve a degradation in the state of confidence. Capitalists can therefore try to protect themselves, firstly by increasing the interest rate of loans ( $i_l$ ). According to Table 4, it implies a decline in the debt ratio. However, this decline doesn't result from a decrease in the 'propensity to borrow' but from a fall in the rate of capacity utilization. In other words, the model specification isn't appropriate here since the young workers behavior isn't affected by the rate of interests.<sup>21</sup>

Another policy for capitalists is credit rationing. This can be formally introduced by assuming that only a fraction of the 'propensity to borrow' *desired* by the workers ( $c_{yw}$ ) is granted by the creditors, this fraction being inversely related to the (previously made endogenous) default ratio ( $\theta$ ). For sake of space, we don't develop this specification here. However, Tables 4 and 5 give an idea of the outcomes that may be expected. Assume for instance an increase in the profit share ( $\pi$ ) implying a decline in both the rate of capacity utilization and the debt ratio.<sup>22</sup> Because their wages fall, older workers face difficulties in servicing their debt, hence an increase in the default ratio ( $\theta$ ). As some capitalists suffer a loss, the credit is rationed. That entails a decrease in the young workers consumption which implies, through the multiplier effect, a new fall in economic activity and wages, therefore a new rise in the default ratio, *etc.*<sup>23</sup> Such scenario doesn't involve downward instability by itself as the fall in the young workers propensity to consume stops as soon as  $c_{yw}$  reaches zero: if capitalists refuse to lend, consumer debt vanishes and we go back to the usual Keynesian model without borrowing.

Note that there can't be upward instability in such scenario as the young workers propensity to consume is upward bounded by  $(1 + c_{yw})$ . It is not the case in Charpe and Flaschel (2013) where the propensity to borrow depends partly on workers' desire, and partly on banks performance. According to the authors, this specification captures the supply-side explanation of consumer debt: the credit expansion can result from a rise in the banks supply of loans which could itself be related to banks performance. This initially causes upward instability as the "proactive bank behaviour generates a destabilizing feedback channel in which debt and

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<sup>21</sup> See Charpe and Flaschel (2013) for another specification including such a policy.

<sup>22</sup> It is also assumed that  $\gamma > (1 - c_c)i_l$ , otherwise an increase in  $\pi$  would have an undetermined effect on  $u^*$ .

<sup>23</sup> See also Dutt (2006, p.359) and Charpe and Flaschel (2013, p.56-57).

consumption feed each other, leading to over-indebtedness” (p.55). However, this lately causes downward instability as “debt default reduces bank net equity and produces a tightening of credit” (p.57).<sup>24</sup>

### **3. Essay 2: Workers’ saving at least partially finances their own debt**

Following Kim *et al.* (2014a), we now assume that workers’ saving is positive and enables a partial financing of the workers’ debt. Of course, we keep our overlapping generations framework and put the stress on the principal payment issue.

For sake of both space and simplicity, it is assumed that workers don’t make default ( $\theta = 0$ ) and don’t borrow for paying the interests on loans ( $\lambda_i = 1$ ). It is also assumed that the rate of accumulation is non-negative ( $\gamma \geq 0$ ).

#### *3.1. Model structure and stock-flow consistency*

The ex post accounting are reported in Table 6 (Balance-sheet matrix) and Table 7 (Transactions flow matrix).

[Table 6 around here]

[Table 7 around here]

An important implication of the overlapping generation framework is that each new generation must inherit the deposits of the older generation of the previous period. Both young and older workers so receive interests on their deposits. Young workers borrow  $NL$  to consume more than their income. Of course, it should be surprising to suppose that young workers borrow whereas they have inherited their parent deposits. The most convincing answer could be to assimilate monetary deposits to some precautionary saving and to assume that young workers prefer to borrow to finance some (conspicuous) consumption rather than empty their deposits.

For sake of simplicity and without any consequence on the results, it is also assumed that they make deposits ( $\dot{M}_{yw}$ ) in the same way as their elders. Hence the following loans contracted by the young workers:

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<sup>24</sup> Somehow, Hein (2012) proposes a *pure* model of credit rationing in which “credit going to workers does not depend on workers’ net income but on rentiers’ income and saving” (p.8), the “rentiers’ saving [being] split in fixed proportion between additional lending to workers and buying additional equity issued by the firms” (p.7).

$$NL = C_{yw} - (W_y + i_m M_{yw}) + \dot{M}_{yw} \quad (42)$$

Older workers for their part use their income to consume, to service the debt, principal and interests, and to make deposits ( $\dot{M}_{ow}$ ):

$$W_o + i_m M_{ow} = C_{ow} + \lambda_p PP + i_l L + \dot{M}_{ow} \quad (43)$$

By construction, the variation of loans ( $\dot{L} = NL - \lambda_p PP$ ) is equal to the variation of deposits made both by workers ( $\dot{M}_w = \dot{M}_{yw} + \dot{M}_{ow}$ ) and capitalists ( $\dot{M}_c$ ), so that:

$$NL - \lambda_p PP = \dot{L} = \dot{M}_w + \dot{M}_c \quad (44)$$

Assume that both  $\dot{L}$  and  $\dot{M}_w$  are positive. It must be stressed that the accounting framework allows for two opposite streams of funding. On the one hand, if  $\dot{L} > \dot{M}_w$ , workers deposits are too low to finance their debt and must be completed by the capitalists deposits ( $\dot{M}_c > 0$ ). On the other hand, if  $\dot{L} < \dot{M}_w$ , workers deposits exceed the debt financing. As a result, the variation of deposits made by capitalists is negative ( $\dot{M}_c < 0$ ) which means that the excess of workers deposits is captured by capitalists and contributes to the investment financing.<sup>25</sup>

Banks receive the interests on loans ( $i_l L$ ) and distribute the interests on deposits ( $i_m M$ ). The difference (if any) corresponds to the banks profit ( $F_b$ ). We take again the core assumption that banks belong to capitalists. Capitalists' income is thus the sum of the interests on their deposits, banks' profit and firms' net profit ( $F = F_b + F_f$ ). The saving of capitalists finances the other agents through bank deposits and purchase of equities.

$$F + i_m M_c - C_c = \dot{M}_c + \dot{E} \quad (45)$$

The accounting of firms is the same than in the first essay.

The main differences with the first essay are that both young and older workers are assumed to make deposits in each period and that the income now includes the interests on deposits. Drawing on Kim *et al.* (2014a), two scenarios are distinguished according to the workers' behavior. In a *scenario a*, older workers first use their wage to meet debt servicing obligations, and then consume a conventional fraction of their disposable wage. Latter, in a *scenario b*, it will be supposed that older workers first spend a part of their wage in consumption, and then distribute their saving between debt servicing and monetary deposits.

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<sup>25</sup> Of course, the latter case inconsistent with a stationary equilibrium in a growing economy. However, it can occur in a short or medium period of time, depending on both the agents' behavior and the value of the parameters.



### 3.2. Workers' consumption (scenario a)

In *scenario a*, it is assumed that young as well as older workers have a lower than unity propensity to consume out of their disposable income ( $c_w$ ). However, young workers borrow to finance a part of their consumption spending.<sup>26</sup> For the younger vintage  $v$  at time  $t$ , we thus have:<sup>27</sup>

$$C_{v,t} = (c_w + c_{yw}) \frac{W_t + i_m M_{w,t}}{V} \quad (46)$$

and

$$L_{v,t} = NL_t = c_{yw} \frac{W_t + i_m M_{w,t}}{V} \quad (47)$$

According to the overlapping generations framework, the intertemporal budget restriction is satisfied if, in the following periods, these workers first service their debt and then distribute their disposable income between consumption and deposits. The consumption behavior for the same vintage  $v$  as it grows older is then:

$$C_{v,t+n} = c_w \left( \frac{W_{t+n} + i_m M_{w,t+n}}{V} - PP_{v,t+n} - i_l L_{v,t+n-1} \right) \quad (n = 1, \dots, V-1) \quad (48)$$

where

$$PP_{v,t+n} = \frac{L_{v,t}}{V-1} \quad (n = 1, \dots, V-1) \quad (49)$$

and

$$L_{v,t+n-1} = L_{v,t+n-2} - PP_{v,t+n-1} \quad (n = 1, \dots, V-1) \quad (50)$$

The aggregate amount of principal payment at time  $t$  is then:<sup>28</sup>

$$PP_t = \sum_{v=1}^{V-1} PP_{v,t} = c_{yw} \frac{\Omega(W_t + i_m M_{w,t})}{V} \quad (51)$$

<sup>26</sup> In Kim *et al.* (2014a), borrowing depends on a consumption target that explicitly relates to the emulation effect of the relative income hypothesis which is developed in Setterfield and Kim (2013).

<sup>27</sup> As it has been pointed, this is a simplifying hypothesis that has no significant implication on the model outcomes. If young workers don't save and if  $c_w$  stands for the propensity to consume of the only older workers, it can be shown that  $c_w$  has to be replaced by  $(V-1)c_w/V$  in the aggregate consumption function and further in the model resolution.

It is also assumed that every vintage has the same amount of deposits at the beginning of every period which shouldn't be the case because the variation in deposits differs for the young and the older workers. However, every vintage is consecutively young and older and a young generation inherits from his predecessors. We consequently suppose that the differences in the amount of deposits are small in the long period.

<sup>28</sup> In this computing, the wage bill  $W_t$  and the monetary deposits  $M_{w,t}$  are supposed to grow at the same rate  $g$ .

where, as in the first essay:

$$\Omega = \frac{1}{g^{(V-1)}} [1 - (1 + g)^{1-V}] \quad (52)$$

Note that workers saving can't be legitimated here by a postponement of consumption because the propensity to consume out of income is the same at every age. It can't no longer be explained by debt servicing because this is extra-saving, after the workers have service their debt. Here once again, the better reason seems to invoke some precautionary saving.

Finally, aggregating for the  $V$  workers vintage at time  $t$ , substituting, introducing  $\lambda_p$  ( $\lambda_p = 1$  if principal paiement is taken into account;  $\lambda_p = 0$  if it is omitted) and adopting continuous time, it comes that:

$$C_w = \left[ c_w + (1 - c_w \lambda_p \Omega) \frac{c_{yw}}{V} \right] (W + i_m M_w) - c_w i_l L \quad (53)$$

$$\dot{L} = (1 - \lambda_p \Omega) c_{yw} \frac{W + i_m M_w}{V} \quad (54)$$

and

$$\dot{M}_w = (1 - c_w) \left[ \left( 1 - \lambda_p \Omega \frac{c_{yw}}{V} \right) (W + i_m M_w) - i_l L \right] \quad (55)$$

where the last equation stands for the variation in workers deposits. The term in brackets (the disposable income after debt servicing, principal and interests) is supposed to be positive.

### 3.3. Equilibrium and dynamics analyses (scenario a)

The only innovation is that capitalists no longer receive the whole interests on loans ( $i_l L$ ). This amount must be reduced by the interests on the deposits served to the workers ( $i_m M_w$ ) so:

$$C_c = c_c (\Pi - \delta K + i_l L - i_m M_w) \quad (56)$$

where it is assumed that  $c_c < c_w$ . Substituting in the goods market equilibrium (equation 22), noting  $\mu_w = M_w/K$  the ratio of workers deposits and rearranging leads to:

$$u^* = \frac{\left[ c_w - c_c + (1 - c_w \lambda_p \Omega) \frac{c_{yw}}{V} \right] i_m \mu_w + \gamma + (1 - c_c) \delta - (c_w - c_c) i_l \lambda}{1 - c_w + (c_w - c_c) \pi - (1 - c_w \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi)} \quad (57)$$

In this equation,  $\mu_w$  causes great difficulties because of its own dynamics. The simplest solution is to assume that the rate of interest on deposits is zero ( $i_m = 0$ ). It means that, as in the first essay, the whole interests on loans are paid to capitalists as banks profits. Note in

addition that assuming  $i_m = 0$  is not inconsistent with the hypothesis that workers save for precautionary purposes. Introducing this assumption, the equilibrium rate of capacity utilization becomes:

$$u^* = \frac{\gamma + (1 - c_c)\delta - (c_w - c_c)i_l\lambda}{1 - c_w + (c_w - c_c)\pi - (1 - c_w\lambda_p\Omega)\frac{c_{yw}}{v}(1 - \pi)} \quad (58)$$

One can check that the results of the comparative statistics are unchanged (see Table 4). In particular, the multiplier effect is lower if principal payment is included ( $\lambda_p = 1$ ) than if it is omitted ( $\lambda_p = 0$ ). The only innovation is the presence of the workers propensity to consume ( $c_w$ ):

$$\frac{du^*}{dc_w} > 0 \quad \Leftrightarrow \quad \left(1 - \lambda_p\Omega\frac{c_{yw}}{v}\right)(1 - \pi)u - i_l\lambda > 0 \quad (59)$$

This condition simply corresponds to the assumption that the disposable income after debt servicing is positive so workers can both consume and make deposits. This condition being satisfied, the higher the workers propensity to consume, the greater the economic activity. If  $c_w = 1$ , the model goes back to the configuration of the first essay where borrowing is completely financed by capitalists. In other words, everything else being equal, economy activity is higher if the workers' debt is financed by capitalists because funds are transferred toward agents with a higher propensity to consume.

The change in the debt ratio is the same as in the first essay (assuming  $\lambda_i = 1$ ):

$$\dot{\lambda} = \left(1 - \lambda_p\Omega\right)\frac{c_{yw}}{v}(1 - \pi)u - \gamma\lambda \quad (60)$$

Here again, the derivative  $\frac{du^*}{d\lambda}$  being negative, we assume that  $d\dot{\lambda}/d\lambda < 0$  so that the system converges towards its long-run equilibrium.<sup>29</sup> The debt ratio therefore converges toward its equilibrium level:

$$\lambda^* = \frac{(1 - \lambda_p\Omega)\frac{c_{yw}}{v}(1 - \pi)}{\gamma}u \quad (61)$$

As before, the debt ratio is smaller if the older workers pay the principal to their creditors ( $\lambda_p = 1$ ). In this case once again, a very weak economic growth ( $\gamma \rightarrow 0$  implying  $\Omega \rightarrow 1$ ) results in a zero long-run debt ratio. Workers' saving then is completely captured by capitalists (*via* banks) and contributes to the investment financing without any retribution.

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<sup>29</sup> Actually, the convergence condition is that  $(1 - \lambda_p\Omega)\frac{c_{yw}}{v}(1 - \pi)\frac{du^*}{d\lambda} < \gamma$ . The derivative  $\frac{du^*}{d\lambda}$  being negative, a sufficient condition for  $d\dot{\lambda}/d\lambda$  to be negative is  $\gamma > 0$ .

More generally, the direction of the streams of funding is a matter of parameter since at equilibrium:

$$\frac{I - \dot{M}_w}{K} = (1 - c_w)(1 - \pi)u^* \left\{ \left[ \frac{1 - c_w \lambda_p \Omega}{1 - c_w} + \frac{i_l}{\gamma} (1 - \lambda_p \Omega) \right] \frac{c_{yw}}{V} - 1 \right\} \quad (62)$$

Hence:

$$\frac{I - \dot{M}_w}{K} > 0 \quad \Leftrightarrow \quad \left[ \frac{1 - c_w \lambda_p \Omega}{1 - c_w} + \frac{i_l}{\gamma} (1 - \lambda_p \Omega) \right] \frac{c_{yw}}{V} - 1 > 0 \quad (63)$$

It results that a workers contribution to the investment financing is more likely to occur if  $c_w$  and  $c_{yw}$  are low and if workers pay the principal ( $\lambda_p = 1$ ).<sup>30</sup>

### 3.4. Workers' consumption (scenario b)

In the previous scenario, as pointed by Kim *et al.* (2014a), “any increase in debt servicing will (*ceteris paribus*) reduce both consumption and saving out of current income” (p. 42). However, these authors refer to the data analysis proposed by Lusardi *at al.* (2011) to consider “a second scenario in which workers are assumed to consume a conventional fraction of their gross wage income, using the residual to fund either debt servicing or current saving, as the demands of the former allow. In this second scenario, then, working households regard saving as a luxury that is foregone first (before consumption out of current income is affected) in the event that they confront higher debt-servicing obligations” (p. 42). In our overlapping framework, the contracted loans by the younger vintage  $v$  at time  $t$  amounts to:

$$L_{v,t} = NL_t = c_{yw} \frac{W_t}{V} \quad (64)$$

and their consumptions to:

$$C_{v,t} = (c_w + c_{yw}) \frac{W_t}{V} \quad (65)$$

The consumption of the same vintage in the following periods becomes:

$$C_{v,t+n} = c_w \frac{W_{t+n}}{V} \quad (n = 1, \dots, V - 1) \quad (66)$$

The aggregate consumption function in continuous time for the  $V$  workers vintage at time  $t$  is simply:

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<sup>30</sup> Note that a rise in the rate of accumulation  $\gamma$  increases the eventuality that workers finance investment if  $\lambda_p = 0$  but it has an opposite effect if  $\lambda_p = 1$  as it implies a decrease in  $\Omega$  whose effect is dominant.

$$C_w = \left(c_w + \frac{c_{yw}}{V}\right) W \quad (67)$$

The variation of loans can be rewritten:

$$\dot{L} = (1 - \lambda_p \Omega) c_{yw} \frac{W}{V} \quad (68)$$

Finally, the variation of workers' deposits corresponds to saving on the gross wages augmented by the interests on deposits ( $i_m M_w$ ) and diminished by the principal ( $\lambda_p \Omega \frac{c_{yw}}{V} W$ ) and interests payments ( $i_l L$ ):

$$\dot{M}_w = \left(1 - c_w - \lambda_p \Omega \frac{c_{yw}}{V}\right) W + i_m M_w - i_l L \quad (69)$$

### 3.5. Equilibrium and dynamics analyses (scenario b)

With the same consumption function for capitalists as in the previous scenario and considering once again that  $i_m = 0$ , the goods market equilibrium is now given by:

$$u^* = \frac{\gamma + (1 - c_c)\delta + c_c i_l \lambda}{1 - c_c \pi - \left(c_w + \frac{c_{yw}}{V}\right)(1 - \pi)} \quad (70)$$

The rate of capacity utilization is higher (*ceteris paribus*) than in the previous scenario<sup>31</sup> for the sake that borrowing boosts the young workers' consumption spending while debt servicing doesn't lessen the workers consumption (on the contrary, the interests on loans increase the capitalists' consumption). As a consequence, the  $\lambda_p$  parameter no longer appears in this equilibrium: economic activity isn't impeded by the principal payment when  $\lambda_p = 1$ .

Note also that the derivative  $du^*/d\lambda$  is now positive. As a consequence, the debt ratio doesn't necessarily converge towards its equilibrium (equation 61) since the sign of the derivative

$$\frac{d\lambda}{d\lambda} = \frac{(1 - \lambda_p \Omega) \frac{c_{yw}}{V} (1 - \pi) c_c}{1 - c_c \pi - \left(c_w + \frac{c_{yw}}{V}\right)(1 - \pi)} i_l - \gamma \quad (71)$$

can be either positive or negative depending on the value of several parameters. The resulting properties are the same than in the first essay, while it was assumed that workers borrow to pay their interests ( $\lambda_i = 0$ ). Actually, if  $d\lambda/d\lambda > 0$ , the system can be represented by Figure 3 where economic activity and debt ratio strengthen each other in a classical spiral: an increase in  $\lambda$  implies an increase in  $u$  because capitalists consume the interests on loans; reciprocally, the increase in  $u$  entails a rise in wages, in the young workers' borrowing, and

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<sup>31</sup> Indeed, one can check that the numerator of  $u^*$  is higher in *scenario a* than in *scenario b*, and that conversely its denominator is lower in *scenario a* than in *scenario b*.

then in  $\lambda$ . Otherwise, if  $d\dot{\lambda}/d\lambda < 0$ , the equilibrium is stable and is represented by Figure 4. Note that a higher  $c_c$  leads to greater instability ( $d\dot{\lambda}/d\lambda = 0$  if  $c_c = 0$ ) whereas a greater  $\lambda_p$  makes it more likely to obtain a stable equilibrium.

Besides, as in *scenario a*, the direction of the stream of funding is a matter of parameter: capitalists may contribute to the financing of the workers debt or, conversely, the workers saving may contribute to the financing of investment. Moreover, as pointed by Kim *et al.* (2014a), the *scenario b* holds only if the older workers' saving is enough to service their debt. Formally, the deposits made by these workers ( $\dot{M}_{ow} = \dot{M}_w - \dot{M}_{yw}$ ) must be positive. After substituting and rearranging, it comes that:<sup>32</sup>

$$\dot{M}_{ow} > 0 \quad \Leftrightarrow \quad (1 - c_w)(V - 1) > \left[ \frac{i_l}{\gamma} (1 - \lambda_p \Omega) + \lambda_p \Omega \right] c_{yw} \quad (72)$$

This condition is more likely to occur if the rate of accumulation ( $\gamma$ ) is high and if the propensity to consume ( $c_w$ ), the 'propensity to borrow' ( $c_{yw}$ ) and the interest rate ( $i_l$ ) are low. Note however that  $\lambda_p$  has an ambiguous impact on this condition since:

$$\frac{d \left[ \frac{i_l}{\gamma} (1 - \lambda_p \Omega) + \lambda_p \Omega \right]}{d\lambda_p} = \left( 1 - \frac{i_l}{\gamma} \right) \Omega \leq 0 \quad (73)$$

In other words, when  $\gamma > i_l$  the condition is more easily satisfied (that is, the right member of the inequality is smaller) if  $\lambda_p = 0$ : the debt is high both because the rate of growth ( $\gamma$ ) is high and workers don't pay the principal, but the cheap rate of interest  $i_l$  keeps the amount of interests at a low level which enables to make deposits. Conversely, when  $\gamma < i_l$  the condition is more easily satisfied if  $\lambda_p = 1$ : the debt is weak both because of a low rate of growth and workers pay their principal; hence the amount of interests remains at a low level despite a high rate of interest so workers can make some deposits.

On the other hand, if the formal condition isn't satisfied, not only the capitalists must contribute to the workers' debt financing but the older workers also are unable to service their whole debt. In that case, at least three options are open to the workers. First, they may choose to reduce  $c_w$  to increase their saving. However, this will cause a fall in aggregate consumption and thus in economic activity. This behavior also means that workers consumption is disciplined by debt servicing, hence the necessity to go back to the *scenario a*.

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<sup>32</sup> The youngest generation is excluded since it doesn't face any debt servicing.

A second option for the older workers is to borrow a part of their interests from the capitalists. Such a solution would nonetheless require a reformulation of the model in order to reintroduce the  $\lambda_i$  parameter (with  $\lambda_i < 1$ ) whose effects were analyzed in the first essay. Such amendment would probably result in an unstable equilibrium and an unsustainable debt when the rate of interests is higher relative to the rate of growth.

Third, some older workers can make default on their debt. Here again, the model would be modified in order to reintroduce the  $\theta$  parameter and it is likely that the outcomes will meet those of the first essay.

In summary, the *scenario b* where debt servicing affects saving but not consumption can suffer from instability for two different, not exclusive reasons. First, because economic activity and debt ratio can feed each other in the usual destabilizing spiral. Second, because the lack of saving for debt servicing can generate an increase in workers' borrowing.

### 3.6. Further comments about workers' consumption

One can check that, here as in the first essay, the aggregate consumption of workers can be greater than their wage. Now, it is a matter of several parameters but it is more likely to occur if both propensities ( $c_{yw}$  and  $c_w$ ) are high and, in *scenario a*, if  $i_l$  is lower than  $\gamma$ .

Furthermore, once again, it can be shown that an increase in the propensity to borrow implies an increase in the consumption for young workers to the detriment of their elders in both scenarios (the ratio  $C_{ow}/C_{yw}$  declines). The relative income hypothesis is thus faced with the same limits than in the first essay: it isn't fully relevant if it rests on a borrowing behavior.

However the story changes if the relative income hypothesis directly applies on the propensity to consume ( $c_w$ ). If the workers react to a rise in some inequalities (for instance a rise in  $\pi$ ) by increasing  $c_w$ , they will all enhance their consumption with only small distributive effects among them.<sup>33</sup> This outcome suggests that the relative income hypothesis may be less consistent with a rise in the propensity to borrow than with a rise in the lower than unity propensity to consume out of wages (or out of income).

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<sup>33</sup> These distributive effects result from the decrease in the debt ratio and thus in the amount of debt servicing which entail a small rise in  $C_{ow}/C_{yw}$ : the increase in  $C_{ow}$  is greater than that of  $C_{yw}$ , but  $C_{yw}$  increases nevertheless and young workers don't have to make any sacrifice.

However, it is worth to note that the 'first disappointing consequence' presented in the first essay may hold here: the rise in  $c_w$  may induce a new rise in  $C_c/C_w$ .

#### **4. Essay 3: Capitalists finance their own debt**

In the third essay, we draw on Bhaduri *et al.* (2006) who suggest that consumption includes a wealth effect that is financed by borrowing for the sake that most capital gains (or losses) are only virtual. As the authors remind us, “the market normally works with only a relatively small fraction of the total number of stocks actually transacted at the margin. This sets notionally the price for these assets, at which their owners might calculate their notional capital gains and losses, but without realizing them. And, this must necessarily be so; because, if everyone or even a substantial number of the owners of this virtual wealth were to try to realize their notional gain through actual trading, stock prices would crash with much of the perceived wealth vanishing immediately. Thus, virtual wealth has to remain largely virtual by the very logic of its existence!” (Bhaduri *et al.*, 2006, p.413).

Drawing on Bhaduri *et al.* (2006), we propose a model where capitalists are supposed to borrow in order to finance the wealth effect on their consumption. A core hypothesis here is that workers consume their whole wages so that only capitalists have a financing capacity: they therefore must finance their own debt.

However, our model rests upon an important simplification of Bhaduri *et al.* (2006): the wealth evaluation is made with historical rather than notional prices. As for the two previous essays, we concentrate on the consequences of the principal payment introduction in a model where capitalists’ saving finances capitalists’ debt. The wealth specification with notional prices is then a sophistication which is not addressed right now.

##### *4.1. Model structure and stock-flow consistency*

As before, we suppose a model with four agents: workers, capitalists, banks and firms. Now, workers are aggregated all together while a distinction is made between the young capitalists and their elders in accordance with our overlapping generations framework. The ex post accounting are reported in Table 8 (Balance-sheet matrix) and Table 9 (Transactions flow matrix).

[Table 8 around here]

[Table 9 around here]



According to the overlapping generations framework, it is assumed that young capitalists inherit the wealth of the older generation of the previous period. The income of capitalists consists of firms' net profit ( $F_f$ ), banks' profit ( $F_b$ ) and the interests on their deposits.

Both young and older capitalists use their income for consumption ( $C_{yc}$ ,  $C_{oc}$ ) and monetary deposits ( $\dot{M}_{yc}$ ,  $\dot{M}_{oc}$ ). In addition, young capitalists borrow ( $NL$ ) to finance a part of their consumption expenditure. Older capitalists are supposed to commit to service their debt, interests ( $i_l L$ ) and principal ( $\lambda_p PP$ ).<sup>34</sup> The variation of loans is thus  $\dot{L} = NL - \lambda_p PP$ . Besides, the older capitalists' saving is partially employed to buy equities ( $\dot{E}$ ).

Banks collect the monetary deposits which finance the *net* borrowing, that is  $\dot{L} = \dot{M}_c$ , or:

$$NL = \dot{M}_{yc} + \dot{M}_{oc} + \lambda_p PP \quad (74)$$

Gross borrowing is thus partly financed by the deposits and partly by the principal payment. The banks' profit is the difference between the received interests on loans and the paid interests on deposits ( $F_b = i_l L - i_m M$ ). Once again, it clearly appears that the banks and capitalists accountings can be merged together since the former are only connected. In addition, as banks' profit is fully distributed to capitalists, interests on loans paid by capitalists come back to them. One can therefore expect that interests play no role in the macroeconomic model.

The aggregation of the younger and older capitalists accounts leads to:

$$C_c + \dot{M} + i_l L + \dot{E} = F_f + F_b + i_m M + \dot{L} \quad (75)$$

Substituting and rearranging, it comes that:

$$C_c + \dot{E} = F_f \quad (76)$$

Hence firms' profits entirely finance capitalist consumption and their purchase of equities.

#### 4.2. Capitalists' consumption

An overlapping generations framework is assumed once again with  $V$  vintages for the capitalists. The only wealth in the economy is the capital stock ( $K$ ) whose collateral is the amount of equities owned by capitalists. This amount as well as the income ( $F_b + i_m M = i_l L$ ) is supposed to be homogeneously distributed. The young capitalists are assumed to consume a

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<sup>34</sup> This constraint can be weakened if older capitalists bequest their debt as well as their wealth to their posterity.

conventional fraction  $c_c$  of their income and to make monetary deposits with the other fraction. Simultaneously, they borrow to finance a consumption connected to their wealth with the propensity to borrow  $c_{yc}$ . For the younger vintage  $v$  at time  $t$ , we thus have:

$$C_{v,t} = c_c \frac{F_{ft} - \delta K_t + i_l L_t}{V} + c_{yc} \frac{K_t}{V} \quad (77)$$

and

$$L_{v,t} = NL_t = c_{yc} \frac{K_t}{V} \quad (78)$$

The intertemporal budget constraint is satisfied if, in the following periods, these capitalists service their debt before to divide the remaining income between consumption and saving. Consumption for the same vintage  $v$  as it grows older is then:

$$C_{v,t+n} = c_c \left( \frac{F_{ft} - \delta K_t + i_l L_t}{V} - PP_{v,t+n} - i_l L_{v,t+n-1} \right) \quad (v = 1, \dots, V-1) \quad (79)$$

where

$$PP_{v,t+n} = \frac{L_{v,t}}{V-1} \quad (n = 1, \dots, V-1) \quad (80)$$

and

$$L_{v,t+n-1} = L_{v,t+n-2} - PP_{v,t+n-1} \quad (n = 1, \dots, V-1) \quad (81)$$

The aggregate amount of principal payment at time  $t$  is then:

$$PP_t = \sum_{v=1}^{V-1} PP_{v,t} = \frac{c_{yc} \Omega K_t}{V} \quad (82)$$

where, as in the two other essays:

$$\Omega = \frac{1}{g(V-1)} [1 - (1+g)^{1-V}] \quad (83)$$

Finally, aggregating for the  $V$  capitalists vintage at time  $t$ , substituting, reintroducing  $\lambda_p$  and adopting continuous time, it comes that:

$$C_c = c_c (F_f - \delta K) + (1 - c_c \lambda_p \Omega) c_{yc} \frac{K}{V} \quad (84)$$

$$\dot{L} = (1 - \lambda_p \Omega) c_{yc} \frac{K}{V} \quad (85)$$

### 4.3. Equilibrium and dynamics analyses

Taking account that workers consume their whole wages, the market goods equilibrium becomes:

$$u^* = \frac{(1-c_c\lambda_p\Omega)\frac{c_{yc}}{V} + \gamma + (1-c_c)\delta}{(1-c_c)\pi} \quad (86)$$

where the major innovation is the absence of the debt ratio  $\lambda$ . The change in the debt ratio is now:

$$\dot{\lambda} = (1 - \lambda_p\Omega)\frac{c_{yc}}{V} - \lambda\gamma \quad (87)$$

which doesn't include the rate of capacity utilization anymore, since the young capitalists borrowing depends on their wealth rather than their income. Consequently, as soon as  $\gamma > 0$ , the system converges without any ambiguity ( $d\dot{\lambda}/d\lambda < -\gamma$ ) towards its equilibrium which just depends on a few parameters:

$$\lambda^* = \frac{(1-\lambda_p\Omega)c_{yc}}{\gamma V} \quad (88)$$

The debt ratio is then an increasing function of the propensity to borrow ( $c_{yc}$ ) and a decreasing function of both  $\gamma$  and  $\lambda_p$ .

The main results of this third essay are threefold. First, the wealth effect (via the  $c_{yc}$  parameter) has a positive impact on the rate of capacity utilization. Second, as the wealth effect is financed by borrowing, it involves an increase in the debt ratio. Finally, the rise in the debt ratio  $\lambda$  implies no feedback on the rate of capacity utilization. That stems from the fact that interests are paid as well as received by capitalists. In other words, contrary to the results in Bhaduri *et al.* (2006), consumer debt has no direct impact on economic activity if the debt is contracted between capitalists for the sake that it involves an income distribution between people whose consumption behaviors are identical. Nevertheless, there is an indirect impact under the assumption that borrowing is necessary for the wealth effect to be effective (capitalists don't have to realize their virtual wealth to finance their consumption).

The other comparative statistics results remain unchanged (see Table 10). Once again, note that both the rate of capacity utilization and the debt ratio are lower if principal is paid ( $\lambda_p = 1$ ) than if it is omitted ( $\lambda_p = 0$ ).

## 5. Conclusion

The aim of this article is essentially methodological as it involves analyzing the properties of the equilibrium of Post Keynesian models including consumer debt and taking the principal payment into account through an overlapping generations framework.

As it has been shown, this innovation doesn't deeply disrupt most of the conclusions of the models available in the existing literature. Our analysis thus confirms that an increase in the young workers propensity to borrow doesn't necessarily imply a rise in the economic activity if workers borrow from capitalists (essay 1). To obtain this positive effect, the rate of accumulation must be higher than the rate of interest,<sup>35</sup> a result which has been highlighted by Dutt (2006), Hein (2012), and Charpe and Flaschel (2013) among other. In addition, as for several authors, our analysis confirms that consumer debt isn't destabilizing *in itself* (that is for a given value of every parameters) provided that the payment of interests corresponds to a discipline device on consumption: a higher debt generates higher interests then a fall of workers consumption that stabilizes the accumulation of debt. In other words, the system instability occurs in two cases: first, if the interests on debt servicing don't play as a discipline device on workers consumption; second, if debt accumulation induces changes in some parameters such as the propensity to borrow (Hein, 2012; Charpe and Flaschel, 2013).

If workers borrow from their peers (essay 2), as in Kim *et al.* (2014a), borrowing has a greater impact on economic activity if workers are assumed to consume a conventional fraction of their gross wage income whereas they regard saving as a luxury (*scenario b*). In such configuration however, the direction of the stream of funding is a matter of parameter: capitalists may contribute to the financing of the workers debt or, conversely, the workers saving may contribute to the financing of investment.

Finally, if capitalists borrow from themselves (essay 3), borrowing can make it possible to consume out of wealth without the necessity to realize their virtual wealth. However, contrary to Bhaduri *et al.* (2006), we don't find any other impact of debt on the level of economic activity because neither this debt nor its servicing involve some income distribution between agents with different consumption behaviors.

However, introducing principal payment through an overlapping generations framework improves the analysis in many ways. First, it makes the models more consistent because, contrary to states, households can't transform their loans in perpetual debt. Especially, the proof has been made that the intertemporal budget *constraint* (*i.e.* workers don't consume more than their income during their lifetime is fully consistent with the *result* that workers (taken as an aggregate) consume more than their income at every period. The reason lies in economic growth that makes it possible for young workers to consume today a fraction of

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<sup>35</sup> More exactly, assuming that capitalists have a positive propensity to consume, the rate of accumulation must be higher than the fraction of the interest rate that is devoted to capitalists saving.

their further, growing wages while older workers pay back the principal which is proportional with their previous, lower wages.

Second, it makes easier the study of the consequences of defaults as they result from households who can't pay back the principal to their creditor. It has been shown that defaults imply an income redistribution that has a positive impact on economic activity as it increases the disposable income of debtors and then their level of consumption. In addition, defaults don't involve system instability in itself. These counterintuitive properties mean that the negative expected effect of defaults doesn't results from their direct impact on economic activity but from an indirect impact: for instance, defaults deteriorate the state of confidence that induce either credit rationing or a continuous rise in the rate of interest.

Third, as the increase in consumption resulting from young workers' borrowing occurs to the detriment of their elders who have to service a higher debt, the proposal that consumer debt refers to the relative income hypothesis is questionable. As it had be claimed, the relative income hypothesis is more consistent with households increasing their lower than one propensity to consume than households increasing their propensity to borrow.

Most importantly, introducing principal payment implies a fall in the multiplier effect (in the first two essays), hence a fall in the impact of an increase in the propensity to borrow on both the rate of capacity utilization and the debt ratio. The positive impact of consumer debt on economic activity is therefore overestimated in the models where the principal payment is omitted, which casts some doubts about the genuine impact of consumer debt. In addition, it had been shown that this reduced positive effect suffers from another fall when the economic rate of growth is low. In that case, the positive effect on consumption of the young workers' borrowing is exactly offset by the negative effect resulting from their elders' principal payment.

In other words, the consumer debt impact on the *long-run* equilibrium is probably rather small. This is because of the negative effect of principal payment on the rate of capacity utilization. This is also because, in the long run, the debt ratio is stabilized at its equilibrium level. However, empirical data show a dramatic increase in consumer debt in many countries for a few decades. Such increase may result from the gradual but permanent shift in the value of an exogenous parameter: for instance, a year after year increase in the propensity to borrow stemming either from proactive banks behavior (and other supply-side factors), or from conspicuous consumption (and other demand-side factors). Such increase may also result

from debt unsustainability stemming from equilibrium instability: for instance, over-indebtedness due to the destabilizing behavior of households having to borrow in order to service their debt. Whatever the reason, the rise in consumer debt corresponds to a situation in which the economic system isn't at its long run equilibrium yet. This *rise* in consumer debt may have a greater impact on economic activity than what has been calculated at the long-run equilibrium. Indeed, if more households enter into debt every year, the positive impact of new loans keep one step ahead of the negative impact stemming from the ensuing principal payment, therefore a positive *transitory* impact on economic activity.<sup>36</sup> However, the *rise* in consumer debt ought to reach a ceiling where the *transitory* impact vanishes, the long-run properties assert themselves, and economy is more vulnerable to a vicious circle of lending contraction.<sup>37</sup>

The models presented in this article may be developed in at least three directions. First, as in Charpe and Flaschel (2013), the propensity to borrow should be made endogenous, relating to supply-side or demand-side factors of debt increase. Second, some simplifying hypothesis may be relaxed, as that of the exogenous rate of accumulation. Eventually, the analysis may be extended to the issue of housing debt, knowing that it would require distinguishing consumption goods from housing goods, the latter being durable and subject to transactions on the secondary market.

## **References**

- Barba, A. and Pivetti, M. 2009. Rising household debt: its causes and macroeconomic implications—a long-period analysis, *Cambridge Journal of Economics*, vol. 33, no. 1, 113–37
- Bhaduri, A. 2011. A contribution to the theory of financial fragility and crisis, *Cambridge Journal of Economics*, vol. 35, no. 6, 995–1014
- Bhaduri, A., Laski, K. and Riese, M. 2006. A model of the interaction between the real and the virtual economy, *Metroeconomica*, vol. 57, 412–27

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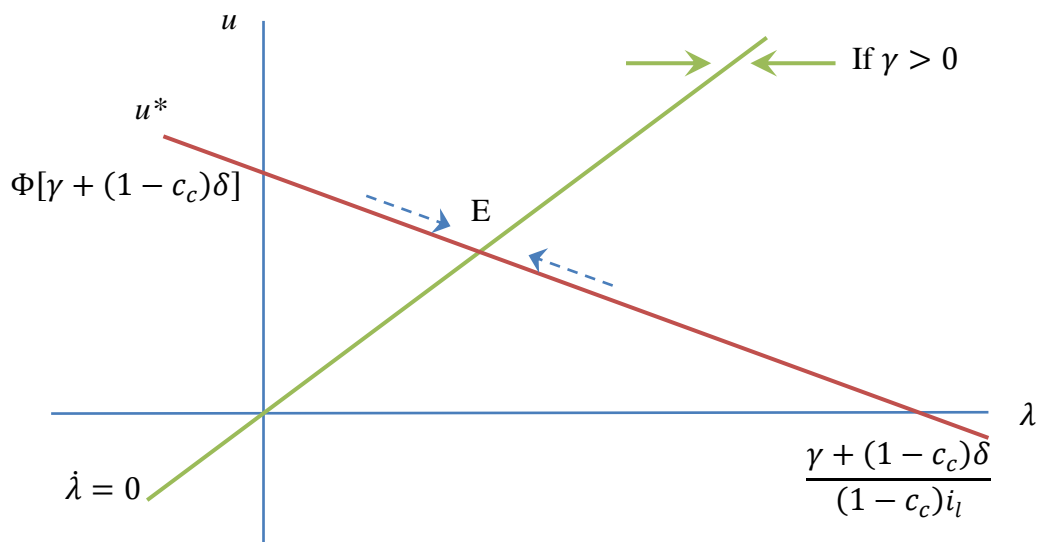
<sup>36</sup> Let us remind however that, in the first essay, a rise in the debt ratio has a positive impact on economic activity only if the economic rate of growth is higher than the rate of interest (times the capitalists propensity to save).

<sup>37</sup> See Palley (2002, p. 22) who points out that it could cause a debt-driven business cycle (Palley, 1994).

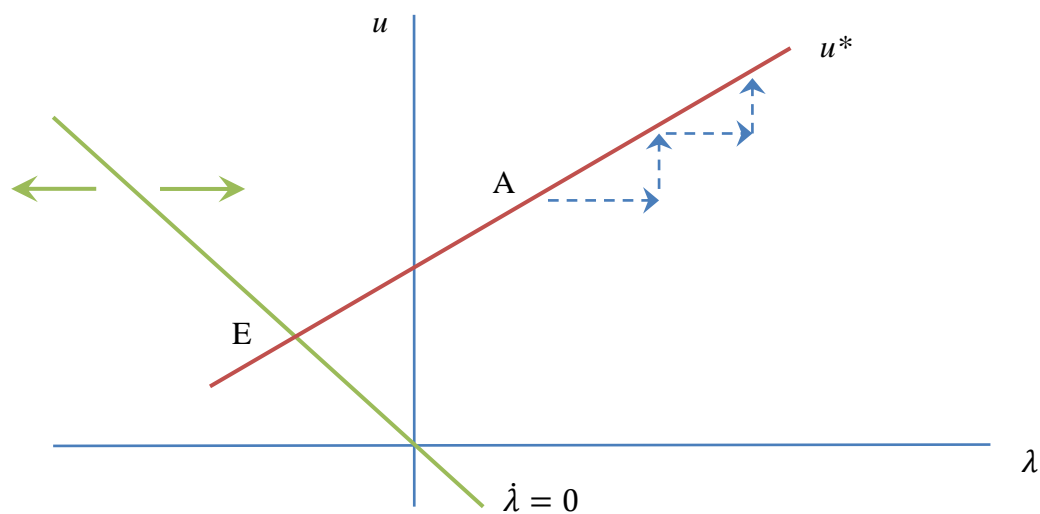
- Charpe, M. and Flaschel, P. 2013. Workers' debt, default and the diversity of financial fragilities, *Structural Change and Economic Dynamics*, vol. 27, 48–65
- Cynamon, B. Z. and Fazzari, S. M. 2008. Household debt in the consumer age: source of growth—risk of collapse, *Capitalism and Society*, vol. 3, no. 2, article 3
- Cynamon, B. Z. and Fazzari, S. M. 2013. The end of the consumer age, pp. 129–57 in Cynamon, B. Z., Fazzari, S. M. and Setterfield, M. (eds), *After the Great Recession: The Struggle for Economic Recovery and Growth*, New York, Cambridge University Press
- Dutt, A. K. 2006. Maturity, stagnation and consumer debt: A steindlian approach, *Metroeconomica*, vol. 57, no. 3, 339–364
- Hein, E. 2012, Finance-dominated capitalism, re-distribution, household debt and financial fragility in a Kaleckian distribution and growth model, *PSL Quarterly Review*, vol. 65, no. 260, 11–51
- Kapeller, J. and Schütz, B. 2014. Conspicuous Consumption, Inequality and Debt: The Nature of Consumption-driven Profit-led Regimes. *Metroeconomica*, forthcoming
- Kim, Y., Setterfield, M. and Mei, Y. 2014a. A theory of aggregate consumption, *European Journal of Economics and Economic Policies: Intervention*, vol. 11, no. 1, 31–49
- Kim, Y., Setterfield, M. and Mei, Y. 2014b. Aggregate consumption and debt accumulation: an empirical examination of US household behavior, *Cambridge Journal of Economics*, forthcoming
- Lusardi, A., Schneider, D. J. and Tufano, P. 2011. 'Financially Fragile Households: Evidence and Implications', NBER Working Paper no. 17072
- Palley, T. I. 1994. Debt, Aggregate Demand, and the Business Cycle: An Analysis in the Spirit of Minsky and Kaldor, *Journal of Post Keynesian Economics*, vol. 16, no. 3, 371–390
- Palley, T. I. 2002. Economic contradictions coming home to roost? Does the US economy face a long-term aggregate demand generation problem? *Journal of Post Keynesian Economics*, vol. 25, 9–32
- Palley, T. I. 2010. The relative permanent income theory of consumption: a synthetic Keynes–Duesenberry–Friedman model, *Review of Political Economy*, vol. 22, 41–56
- Setterfield, M. and Kim, Y. 2013. 'Debt Servicing, Aggregate Consumption, and Growth', Working Papers no. 1316, Trinity College, Department of Economics

## Appendix: Figures and Tables

**Figure 1. Long-run equilibrium  
if the payment of interests plays as a discipline device on consumption**

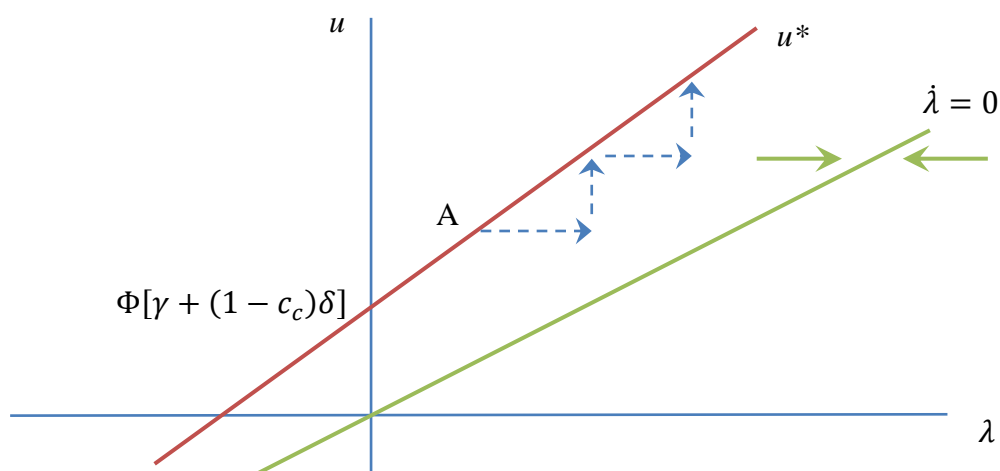


**Figure 2. Long-run equilibrium if the payment of interests  
doesn't play as a discipline device on consumption: instability (case a)**



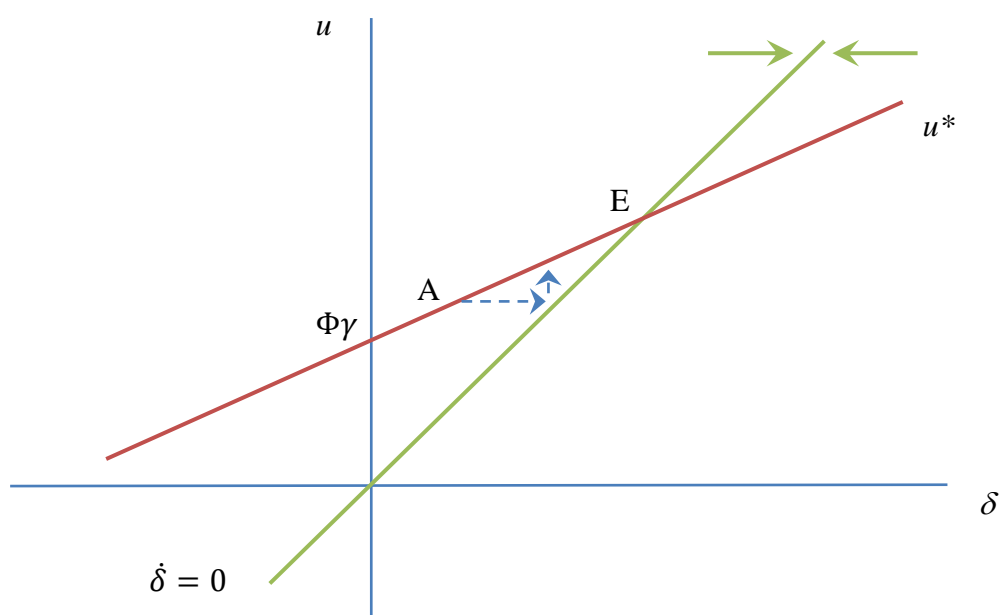


**Figure 3. Long-run equilibrium if the payment of interests doesn't play as a discipline device on consumption: instability (case b)**



Nota : The horizontal arrows correspond to the convergence condition for a *given* rate of capacity utilization, that is:  $d\dot{\lambda}/d\lambda = -\gamma$ . However, the negative sign of this derivative (provided that  $\gamma$  is positive) doesn't ensure that the whole system converges towards its equilibrium. In this case, the calculus of the derivative for an *endogenous* rate of capacity utilization is positive (since  $\Psi_{i_l} > \gamma$ ), therefore the unstable equilibrium.

**Figure 4. Long-run equilibrium if the payment of interests doesn't play as a discipline device on consumption: stability (case c)**



**Table 1. Capitalists finance the workers' debt: balance-sheet matrix**

	Workers	Capitalists	Banks	Firms	Sum
Capital				$+K$	$+K$
Deposits		$+M$	$-M$		0
Loans	$-L$		$+L$		0
Equities		$+E$		$-E$	0
Sum (net worth)	$-L$	$+M + E$	0	0	$+K = +E$

**Table 2. Capitalists finance the workers' debt: transactions flow matrix**

	Workers		Capitalists	Banks		Firms	Sum
	young	older		Current	Capital	Current	Capital
Consumption	$-C_{yw}$	$-C_{ow}$	$-C_c$			$+C$	0
Investment						$+I$	0
Capital depreciation						$-\delta K$	0
Wages	$+W_y$	$+W_o$				$-W$	0
Net profits			$+F$	$-F_b$		$-F_f$	0
Interests on deposits			$+i_m M$	$-i_m M$			0
Interests on loans		$-i_l L$		$+i_l L$			0
$\Delta$ Deposits			$-\dot{M}$		$+\dot{M}$		0
$\Delta$ Loans	$+NL$	$-(1 - \theta)\lambda_p PP$		$-\theta\lambda_p PP$	$-\dot{L}$		0
$\Delta$ Equities			$-\dot{E}$				$+\dot{E}$
Sum	0	0	0	0	0	0	0

**Table 3. Capitalists finance the workers' debt:  
short-run impact effects on  $u^*$  assuming  $\gamma > 0$**

$\gamma, \delta$	+	
$c_c, c_{yw}$	+	
$\pi$	–	
$\lambda_i, \lambda_p$	–	
$i_l, \lambda$	$\pm$	An increase in $\lambda_i$ implies a decrease both in $du^*/di_l$ and in $du^*/d\lambda$ $\lambda_i = 0 \Rightarrow du^*/di_l > 0$ and $du^*/d\lambda > 0$ $\lambda_i = 1 \Rightarrow du^*/di_l < 0$ and $du^*/d\lambda < 0$
$V$	–	
$\theta$	+	

**Table 4. Capitalists finance the workers' debt:  
long-run impact effects assuming  $\gamma > 0, \lambda_i = 1$  and  $\theta = 0$**

Positive shock on	$\dot{\delta} = 0$	$u^*$		Impact on	
	Slope	Intercept	Slope	$u$	$\lambda$
$\gamma$	$\uparrow$	$\uparrow$	0	+	?
$\delta$	0	$\uparrow$	0	+	+
$i_l$	0	0	$\downarrow$	–	–
$c_{yw}$	$\downarrow$	$\uparrow$	$\downarrow$	+ if $\gamma > (1 - c_c)i_l$ – if $\gamma < (1 - c_c)i_l$	+
$c_c$	0	$\uparrow$	$\downarrow$	+	+
$\pi$	$\uparrow$	$\downarrow$	$\uparrow$	– if $\gamma > (1 - c_c)i_l$ ? if $\gamma < (1 - c_c)i_l$	–
$\lambda_p, V$	$\uparrow$	$\downarrow$	$\uparrow$	– if $\gamma > (1 - c_c)i_l$ + if $\gamma < (1 - c_c)i_l$	–

**Table 5. Capitalists finance the workers' debt:  
long-run impact effects assuming  $\gamma > 0, \lambda_i = \lambda_p = 1$  and  $\theta > 0$**

Positive shock on	$\dot{\delta} = 0$	$u^*$		Impact on	
	Slope	Intercept	Slope	$u$	$\lambda$
$\theta$	0	$\uparrow$	$\downarrow$	+	+
$c_{yw}$	$\downarrow$	$\uparrow$	$\downarrow$	+ if $\gamma > (1 - c_c)i_l$ ? if $\gamma < (1 - c_c)i_l$	+
The other results of Table 4 remain unchanged.					

**Table 6. Workers partially finance their own debt: balance-sheet matrix**

	Workers	Capitalists	Banks	Firms	Sum
Capital				$+K$	$+K$
Deposits	$+M_w$	$+M_c$	$-M$		0
Loans	$-L$		$+L$		0
Equities		$+E$		$-E$	0
Sum (net worth)	$+M_w - L$	$+M_c + E$	0	0	$+K = +E$

**Table 7. Workers partially finance their own debt: transactions flow matrix**

	Workers		Capitalists	Banks		Firms		Sum
	young	older		Current	Capital	Current	Capital	
Consumption	$-C_{yw}$	$-C_{ow}$	$-C_c$			$+C$		0
Investment						$+I$	$-I$	0
Capital depreciation						$-\delta K$	$+\delta K$	0
Wages	$+W_y$	$+W_o$				$-W$		0
Net profits			$+F$	$-F_b$		$-F_f$		0
Interests on deposits	$+i_m M_{yw}$	$+i_m M_{ow}$	$+i_m M_c$	$-i_m M$				0
Interests on loans		$-i_l L$		$+i_l L$				0
$\Delta$ Deposits	$-\dot{M}_{yw}$	$-\dot{M}_{ow}$	$-\dot{M}_c$		$+\dot{M}$			0
$\Delta$ Loans	$+NL$	$-\lambda_p PP$			$-\dot{L}$			0
$\Delta$ Equities			$-\dot{E}$				$+\dot{E}$	0
Sum	0	0	0	0	0	0	0	0

**Table 8. Capitalists finance their own debt: balance-sheet matrix**

	Workers	Capitalists	Banks	Firms	Sum
Capital				$+K$	$+K$
Deposits		$+M$	$-M$		0
Loans		$-L$	$+L$		0
Equities		$+E$		$-E$	0
Sum (net worth)	0	$+E$	0	0	$+K = +E$

**Table 9. Capitalists finance their own debt: transactions flow matrix**

	Workers	Capitalists		Banks		Firms		Sum
		young	older	Current	Capital	Current	Capital	
Consumption	$-C_w$	$-C_{yc}$	$-C_{oc}$			$+C$		0
Investment						$+I$	$-I$	0
Capital depreciation						$-\delta K$	$+\delta K$	0
Wages	$+W$					$-W$		0
Net profits		$+F_{yc}$	$+F_{oc}$	$-F_b$		$-F_f$		0
Interests on deposits		$+i_m M_{yc}$	$+i_m M_{oc}$	$-i_m M$				0
Interests on loans			$-i_l L$	$+i_l L$				0
$\Delta$ Deposits		$-\dot{M}_{yc}$	$-\dot{M}_{oc}$		$+\dot{M}$			0
$\Delta$ Loans		$+NL$	$-\lambda_p PP$		$-\dot{L}$			0
$\Delta$ Equities			$-\dot{E}$				$+\dot{E}$	0
Sum	0	0	0	0	0	0	0	0

**Table 10. Capitalists finance their own debt: comparative statistics**

Positive shock on	Impact on	
	$u$	$\lambda$
$\gamma$	+	–
$\delta, c_c$	+	0
$\lambda^{(a)}$	0	not relevant
$i_l$	0	0
$c_{yc}$	+	+
$c_c$	+	0
$\lambda_p, V$	–	–
<sup>(a)</sup> Only concerns the short-run impact of an exogenous shift in the debt ratio ( $\lambda$ ) on the rate of capacity utilization ( $u$ ).		