Estimating Dynamic Models Without Solving Value Functions

Gabriel Petrini

October 1st, 2020

Contents

(hotzMiller1993)

Dynamic discrete choice models are complicated to estimate because of the future value terms. **hotzMiller1993** show:

- Differences in conditional value functions $v_j - v_{j'}$ can be mapped into conditional choice probabilities (p_j's)
- We can pull the p_j's from the data in a first stage
- **Empirical example:** optimal stopping with respect to couples' fertility

Difference in v's and logit errors

Consider an individual who faces two choices where the errors are T1EV. The probability of choice 1 is:

$$p_1 = \frac{\exp(v_1)}{\exp(v_0) + \exp(v_1)}$$

The ratio of p_1/p_0 is then:

$$\frac{p_1}{p_0} = \frac{\exp(v_1)}{\exp(v_0)} = \exp(v_1 - v_0)$$

implying that:

$$\ln(p_1/p_0) = v_1 - v_0$$

General structure

The inversion theorem of Hotz and Miller says that there exists a mapping, ψ, from the conditional choice probabilities, the p's, into the differences in the conditional valuation functions, $v_j - v_k$:

$$\begin{aligned} V_{t+1} &= v_{0t+1} + \mathbb{E}\max\{\epsilon_{0t+1}, v_{1t+1} + \epsilon_{1t+1} - v_{0t+1}, ..., \\ &\quad v_{Jt+1} + \epsilon_{Jt+1} - v_{0t+1}\} \\ V_{t+1} &= v_{0t+1} + \mathbb{E}\max\{\epsilon_{0t+1}, \psi_0^1(p_{t+1}) + \epsilon_{1t+1}, ..., \psi_0^J(p_{t+1}) + \epsilon_{Jt+1}\} \end{aligned}$$

The p's can be taken from the data. However:

1. We need the mapping, ψ,
2. We need to be able to calculate the expectations of the ϵ's
3. We need to do something with the v_0's