Treinamento LSD

Aulas 2-10 – 2ª Parte

Baseado nos slides de Marco Valente

Agenda

- Aula 2: Linear model and LSD basics
- Aula 3: Random walk model and LSD model structure
- Aula 4: Logistic model and chaotic behavior
- Aula 5: Replicator dynamics model
- Aula 6: Extended replicator dynamics model
- Aula 7: Network externalities model – Part 1
- Aula 8: Network externalities model – Part 2
- Aula 9: Consumers’ model – Part 1
- Aula 10: Consumers’ model – Part 2

Lecture's goal

The use of simulation models allows to assess and compare large series of values. This lecture will focus more on the module dealing with data, statistical and graphical management.

We will focus on a model composed by a simple equation that is known to have a highly complex behaviour, generating linear and chaotic time series.

Using the LSD graphical tools we will explore the model and generate a representation of the results in order to understand the properties of the chaotic function.

Logistic Chaotic Model

Consider the model made of the single equation

$$X_t = m * X_{t-1} * (1 - X_{t-1})$$

To implement this model follow the usual steps:

1. Create a new model using the model browser in **LMM**.
2. Insert the equation's code for the model.
3. Compile and run the model (menu **Model/Run**).
4. Using the **Lsd model program** generate one object and place in it variable X with 1 lag and parameter m.

Logistic Chaotic Model

The model equation can be written as follows

```
EQUATION("X")
/*
Logistic equation
*/
v[0]=VL("X",1);
v[1]=V("m");
v[2]=v[1]*v[0]*(1-v[0]);

RESULT( v[2])
```

Simulating the logistic function

Assuming $X_0 = 0.5$ we obtain different results for different values of m. What happens for $m = 2$?

$$X_t = m * X_{t-1} * (1 - X_{t-1})$$

X_t	$m = 1$	m=2	$m = 2.5$	$m = 2.9$	$m = 3.1$	$m = 3.4$	$m = 3.7$	$m = 3.99$
X_0	0.5	0.5	0.5	0.5	0.5	0.5	0.5	0.5
X_1	0.25	0.5	0.625	0.725	0.775	0.85	0.925	0.9975
X_2	0.1875	0.5	0.585938	0.578188	0.540563	0.4335	0.256687	0.00995006
X_3	0.152344	0.5	0.606537	0.707271	0.7699	0.834964	0.705956	0.0393057
X_4	0.129135	0.5	0.596625	0.600412	0.549178	0.468516	0.768053	0.150666
X_5	0.112459	0.5	0.601659	0.695761	0.767503	0.84663	0.659146	0.510582
X_6	0.099812	0.5	0.599164	0.613865	0.553171	0.441482	0.831289	0.997053
X_7	0.0898497	0.5	0.600416	0.6874	0.766236	0.838357	0.518916	0.0117231
X_8	0.0817767	0.5	0.599791	0.623155	0.555267	0.460749	0.923676	0.0462268
X_9	0.0750893	0.5	0.600104	0.681015	0.765531	0.844762	0.260845	0.175918
X_{10}	0.0694509	0.5	0.599948	0.629977	0.556429	0.445874	0.713378	0.578435

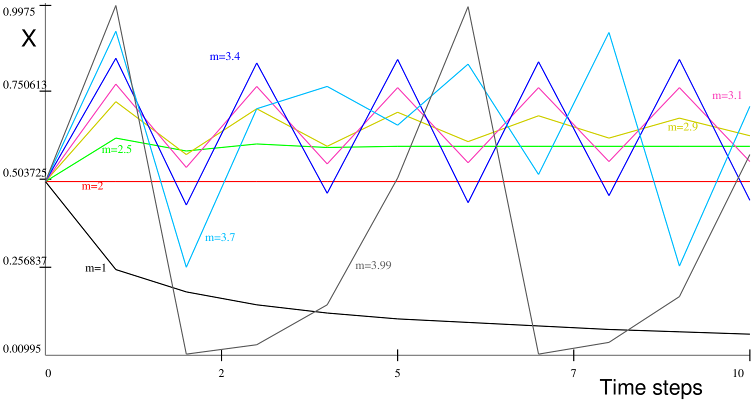
Logistic Chaotic Model

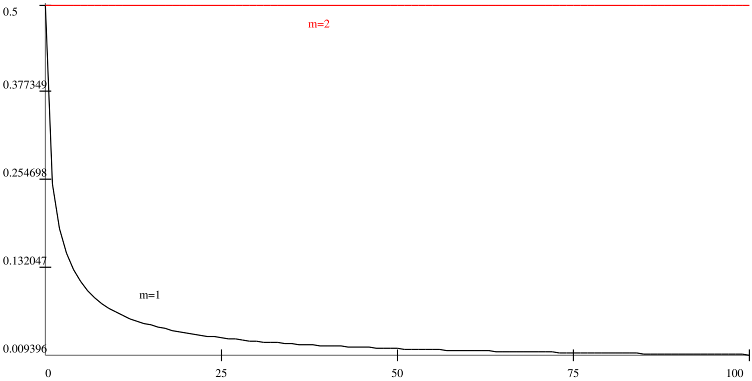
The properties of data series can hardly be appreciated by reading the numbers. As an example consider the following graphs.

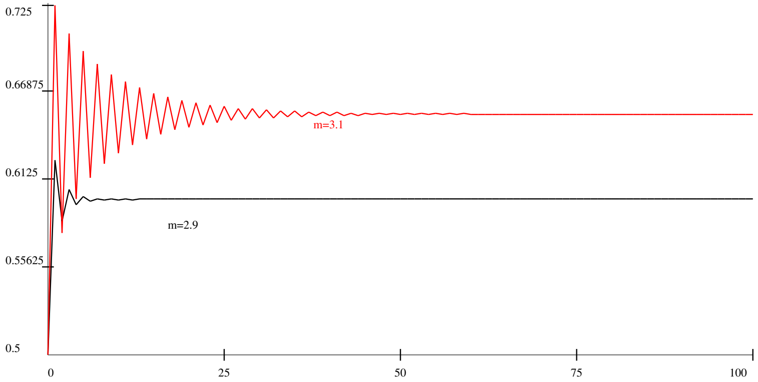
We show firstly all the series above, computed over 10 time steps.

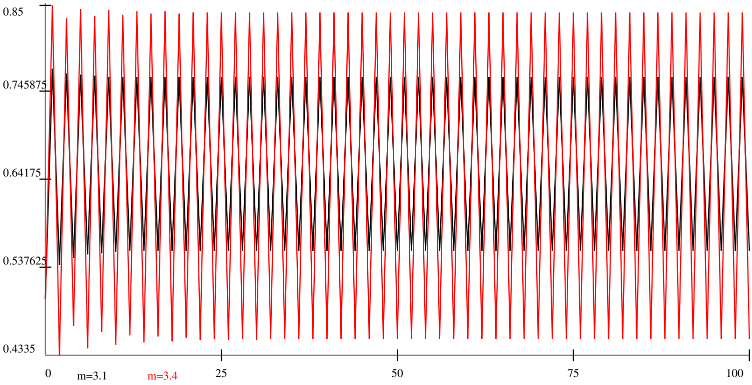
We then see the 100 series for each couple of series, for easier reading

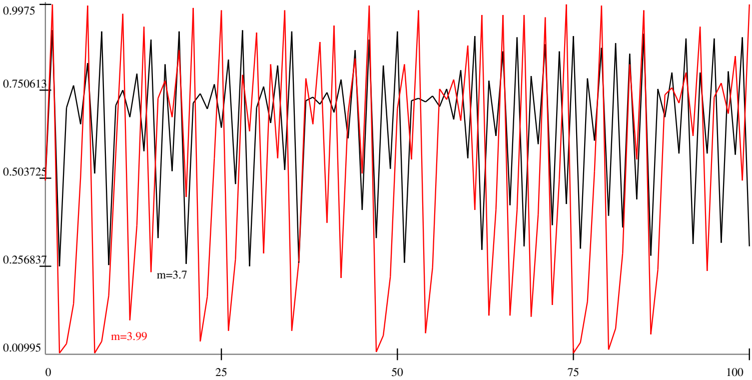
Remind: menu **Run/Sim.Settings** sets the number of steps in the simulation. Menu **Data/Init.Values** sets the initial values for the data in an object. Menu **File/Reload** reloads a fresh configuration at the end of a simulation run.


0.9975
X
0.750613
0.503725
0.256837
0.00995
m=3.4
m=3.1
m=2.9
m=2.5
m=2
m=3.7
m=3.99
m=1
0
2
5
7
10
Time steps


0.5
0.377349
0.254698
0.132047
0.009396
0
25
50
75
100
m=2
m=1


0.725
0.66875
0.6125
0.55625
0.5
0
25
50
75
100
m=3.1
m=2.9


0.85
0.745875
0.64175
0.537625
0.4335
0
25
50
75
100
m=3.1
m=3.4


0.9975
0.750613
0.503725
0.256837
0.00995
m=3.7
m=3.99
0
25
50
75
100

Logistic Chaotic Model

The function produces extremely different results depending on the value of m. To understand how the results depend on m let's make a systematic analysis of the simulation results.

We will create a large number of series computed independently. We will set m to a slightly different value for each series, so that we can have an understanding of the function's behaviour as if we were testing the whole continuum for m.

Finally, we will set the initial points for the X to random values, so that results do not depend on specific values of the initial X.

Logistic Chaotic Model

Let's set the configuration to compute 10,000 series over 1,000 time steps.

For each series we assign a random value to X chosen in the range (0,1).

For each series we assign a different and increasing value of m starting from 2.8 and reaching, for the 10000^{th} series, 3.99.

Finally, we save all values of m and of X for post-simulation analysis. The operations required for the above setting are listed below.

Logistic Chaotic Model

1. Use menu **Data/Set.Obj.Num/All Obj** to generate 10000 copies of the object.
2. Place the Browser to show the object's content.
3. Open the initial values interface with **Data/Init.Values**.
4. Use the buttons **Set All** on the side of m and X to assign values to all the elements.
5. Set m using the option **Range** between 2.8 and 3.99.
6. Set X using the option **Random (Uniform)** between 0.01 and 0.99.
7. Exit the initial values window and set the options for m and X to be saved
8. Use menu **Run/Sim.Settings** to set 1000 time steps.

Now, run the simulation using menu **Run/Run**.

Analyse the results

At the end of the simulation open the analysis of results module (menu **Data**/**Analysis**). The window shows 20000 series available (all the *m*'s and *X*'s).

If this is not the case, exit the analysis of results module and set on the option to save the missing item. Then re-run the simulation and open the Analysis of Results module.

As suggested from the pilot experiments, choosing a series among those computed with $m < 3$, the series rapidly converges. For subsequent values, the series oscillates between two values, and for still higher values the series becomes chaotic.

To make sense of the function overall limit behaviour the best option is to generate a scatter plot graph with all the values of m on the horizontal axis, and the corresponding value of X, at the final step, on the vertical one.

Analyse the results

LSD is designed intuitively to perform the most common operations, but also to provide sophisticated tools for special cases.

We now face the problem of selecting all the *m*'s, that is, 10,000 series in a list of 20,000. Clicking on each of them is obviously impossible. One option is to click on the button **Sort** to re-arrange the series in alphabetical order, and then click on the first and last series, keeping the key shift pressed, to select all series in between.

However, even this route this is tedious, and does not solve all the possible selection problems one may need. LSD offers a more flexible and fast tool for series selection. Click with the right button of the mouse when the pointer is over one of the series *m*.

Analyse the results

The new window offers many selection criteria. We need the simplest, already selected by default, that is, “select all series of this type”. Click on **Ok** to confirm.

As you will see, all the series with the label *m* have been placed in the central box.

After this, replicate the same operation, placing all the series for *X* in the **Series Selected** box.

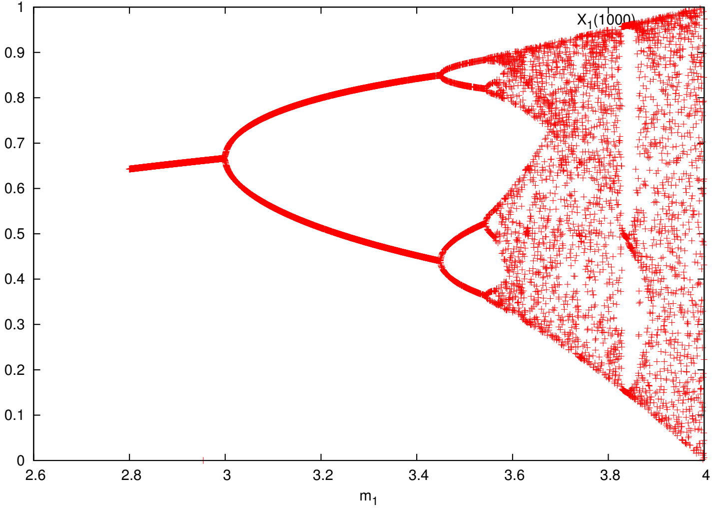
Scatter plots

The analysis of results module is controlled setting the options on the lower right side of the window. We need to perform a **Cross section**, **XY plot**.

The plot we wish to create is more readable using **Points**, rather than lines. Set these options as indicated and press the button **Plot**.

A new window will appear asking for the time step at which the values of the selected series must be used. Leave the default value of 1000, the latest time step of the simulation, and press **Continue.**

The following graph will be created and shown.


X1(1000)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
2.6
2.8
3
3.2
3.4
3.6
3.8
4
m1

Scatter plot

The graph contains one point for each value of X_{1000} with the coordinate of the corresponding m value on the horizontal axis.

The graph shows that, for increasing values of m, we see firstly the convergence of the series to a unique point.

For larger values of m the series oscillate indefinitely between two points, generating a biforcation in the plot because the series will be differently located at the two poles of their oscillations.

For a short span of m, the function generates a cycle across 4 points. Subsequently it becomes chaotic, although in some areas the density of the points seem to be very thin.

Analyse the results: conclusions

Though LSD's main goal is to create efficiently simulations, the possibility to generate basic graphic representations of simulation results is very important to appreciate.

In particular, LSD graphical tools manage easily massive amounts of data generated during simulation runs. More sophisticated graphical or statistical analyses can be obtained exporting the LSD data in external packages such as STATA or R.

The analysis of results module, included in any LSD model program, can be activated both at the end of a simulation, as well as during an interrupted simulation run. It is possible to plot various types of graphs, generate statistics, or simply export data sets for further elaboration. See the Help manual page for details.