

**A MULTIDIMENSIONAL ASSESSMENT OF WELL-BEING
BASED ON SEN'S FUNCTIONING APPROACH**

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Abstract

The aim of this paper is to deal with some methodological issues related to the multidimensional analysis of well-being from the theoretical perspective suggested by Amartya Sen. Widely recognised as one of the more satisfying and complete approaches to well-being analysis, Sen's capability approach has found relatively few empirical applications up to now, mainly for its strong informational and methodological requirements. In this paper I try to make some progress towards the possibility of realising a multidimensional assessment of Sen's concept of well-being with the use of the fuzzy sets theory ; the methodology suggested is also tested in the evaluative space of functionings, with an empirical application referred to Italy.

Keywords : capability approach, functionings, multidimensional analysis of well-being, fuzzy set theory

JEL Classification number : I32, I31

Short title: A multidimensional assessment of well-being

1. Introduction

The basic assumption of any multidimensional approach to well-being and poverty analysis is that there are relevant dimensions of well-being that the economic resources are not able to capture. Income and consumption are only rough measures of the quality of life because they are not able to fully describe what people can really achieve with these resources, because they can hide strong differences and inequalities among people and finally because the quality of life is something more than simply a given amount of resources. Health, longevity, knowledge and education, social relations, subjective feelings are constitutive elements of human life that should not be ignored if we are interested in assessing of the people's standard of living.

The opportunity to move from an income-based perspective to account for the constitutive plurality of human life has been widely advocated by the sociological literature on social welfare and poverty that traditionally considers a plurality of indicators to describe the quality of life of individuals and households¹. In recent years, the economic debate on well-being has also been deeply renewed by the essential contribution of Amartya Sen. His capability approach is widely recognised as one of the more complete and comprehensive approaches to welfare analysis, in which a plurality of well-being dimensions are considered, the relationships among them are investigated, and through which poverty, deprivation and inequality assume a new and clear meaning. What mainly characterises capability approach with respect to other multidimensional approaches of well-being is that it is not simply a way to enlarge the evaluative well-being to variables other than income, but it is a radically different way to *conceive* the meaning of well-being. Undoubtedly, the richness of such theoretical argumentation is not easy to translate into practical terms. The capability approach is certainly more demanding at an informational and methodological level if compared with more standard approaches (i.e. income or opulence-centred analysis) to well-being; it is also hard to constrain and to manage in the traditional framework of welfare and poverty analysis, if we want to fully preserve its informative and interpretative contents. These difficulties could partially explain why, up to now, there are relatively few empirical applications that have been able to capture the richness of such a perspective even if many well-being analyses conceptually refer to it².

¹ See, for example, Townsend (1985) or, for the Scandinavian Welfare approach, the contributions of Erikson (1987, 1993), Aberg (1987) and Allardt (1993); in the economic field see also the Leyden School approach with the contributions of Hagenaars (1986) and Van Praag (1991, 1993).

² The most well-know is perhaps the UNDP approach and the related Human Development Index (see the UNDP Reports from 1990 to 1999). Other empirical applications that partly reflect the Sen capability

In this paper a methodological approach for measuring multidimensional well-being in the direction indicated by Sen's theory is proposed and applied. The aim is to try to preserve the richness of this approach and to tackle some methodological questions related to it. Section 2 provides a brief description of the essential features of the capability approach and its informational and methodological requirements. A methodological approach founded on the fuzzy sets theory is suggested in section 3 and empirically tested to depict a well-being assessment in the evaluative space of functionings with reference to Italy. In section 4 the methodological steps and the main results of the empirical application are discussed.

2. Sen's capability approach : constitutive plurality of well-being and methodological issues

The capability approach characterizes individual well-being in terms of what a person is actually able to do or to be (see Sen 1987, 1992, 1993, 1994, 1997). In this perspective, living may be considered as a set of interrelated functionings and an overall evaluation of well-being has to take the form of an assessment of these constitutive elements. Closely related to the notion of functioning is that of the capability to function, i.e. the various combinations of beings and doings that a person can achieve. The capability set is, thus, a set of vectors of functionings that reflects the person's freedom to choose what kind of life to live. So, if achieved functionings constitute a person's well-being, capabilities represent the real opportunities for a person *to have* well-being and include also the freedom to have alternatives other than the chosen combination³.

Capabilities and functionings achieved are strictly related to the intrinsic characteristics of the people (age, gender, health and disability conditions) as well as to environmental circumstances (at the social-economic and institutional level but also referred to the household environment); and the conversion process of the available resources into well-being is strictly related to and dependent on these individual and environmental features.

The substantial difference between the functionings achieved, on the one hand, and the set of feasible vectors of functionings (i.e. capability set) on the other, has a direct effect

approach can be found in Schokkaert, Van Ootegem (1990) and Balestrino (1996). For a recent and more complete assessment of well-being in the functioning space see Brandolini, D'Alessio (1998) and Klasen (2000). Finally, see Desai (1994) for some methodological suggestions on the empirical implementation of this approach.

also in the choice of the evaluative exercise. Sen (1985, 1992, 1993, 1994) suggests three different procedures for the evaluation of capability sets: 1) by the entire set of options open to the person ; 2) by the option actually chosen; 3) by a maximally valued option from the capability set.

If freedom had only instrumental importance and no intrinsic relevance for the individual's well-being, the evaluation of the capability set under procedures 2) or 3) is simply the value of a particular element of it : the chosen one or the best one, respectively ⁴ . If we also assume a maximizing behavior then these procedures will produce the same result. However, if the freedom of choice is seen as a part of living and we think that "doing x" is different from "choosing to do x and doing it", the entire set of options open to the person must be considered ⁵ .

The theoretical, methodological and empirical issues related to the capability approach are deeply discussed in Sen's works as well as in the critical literature on this argument and I will not try to reconsider them again. I want just to briefly summarize the main set of issues that have to be dealt with for an empirical application of the capability approach. Generally speaking, it is necessary to choose:

- a) the adequate evaluative space : capability vs. (achieved) functioning
- b) a list of (essential, relevant) capabilities or functionings
- c) a set of indicators related to the selected dimensions of well-being and adequate criteria to measure and represent them
- d) how (and if) to aggregate the elementary indicators to obtain an overall evaluation for each single dimension (functioning/capability) of well-being
- e) how (and if) to add up all the dimensions and to reach an overall evaluation of well-being.

With reference to the choice of the evaluative space Sen himself has also outlined some real advantages in being able to relate the analysis of well-being on the wider information base of a capability set rather than on a selected element of it (i.e. on the space of the functioning achieved). Nevertheless, in this case, the information requirement increases and observational problems are added because the entire set of available options is not

³ Freedom is conceived here in a positive rather than in a negative sense, and it can be valued for its intrinsic importance as well as for its instrumental role in determining individual well-being. See Sen (1992, 1993, 1994).

⁴ The evaluation of the set by the value of one distinguished element has been defined as "elementary evaluation" by Sen (1985, 1992, 1994).

⁵ "For example, 'fasting' as a functioning is *not* just starving : it *is choosing to starve when one does have other options*. In examining a starving person's achieved well-being, it is of direct interest to know whether he is fasting or simply does not have the means to get enough food". (Sen 1992, p.52)

easily or directly observable, and it can only be estimated on a presumptive basis. For this reason the choice of the achieved functionings set seems to be the more practicable one (see also Sen, 1987, Basu, 1987, Brandolini, D'Alessio, 1998)⁶.

The second issue is related to the possibility of achieving a consensus about a list of functionings to involve into an empirical analysis of well-being. Generally speaking, the more we would like to widen the evaluative space to include all possible constitutive elements of human life, the larger will be the informational basis that we shall need. Thus, a compromise between the aim of giving a satisfying and comprehensive description to the quality of life and the real possibility of assessing it is necessary in the end. When we refer to the more extreme situations of poverty in developing countries, it seems relatively easier to find a consensus about a minimum set of basic functionings, such as to escape avoidable morbidity and premature mortality, to be nourished and sheltered, to receive a primary education and so on. In more developed social and economical contexts this list could be longer and include the above basic and material aspects of human life, but also other relevant dimensions such as to participate in the social life or to engage in social interactions.

The third problem, i.e. how to choose a set of appropriate indicators for the representation of the manifold dimensions of well-being, is obviously related to the availability of statistical data or to the decision to conduct *ad hoc* interviews to collect an adequate information set as well as to the kind of assessment we want to pursue. The richer informational requirement involved in the capability approach is one of the reasons often advocated to justify the choice of more traditional approaches to poverty and well-being analysis based on less demanding income measures. However, it doesn't seem to be a real obstacle for a more operative use of this approach. Aggregate information about health, education and other aspects related to the people's quality of life, as well as sample surveys on people's standard of living are quite easily accessible and often of reasonable quality, albeit in more developed countries. Also in the less developed contexts statistical evidence on social and economic indicators is often available; in addition many reports, surveys and poverty assessments are conducted, mainly from international organizations, on specific subgroups or referred to entire populations. Aggregate analysis is clearly less satisfactory

⁶ It is not difficult to understand the reasons that can justify this kind of choice and also to agree with them. However, in so far as the elements that determine the human capabilities (i.e. personal characteristics and environmental conditions), if not the capabilities *per se*, can be observed, an indirect evaluation of the capability set could, to a certain extent, be obtained. On the opportunity to include both capability and functioning spaces in the well-being assessment see also Sudgen (1993) and Chiappero Martinetti (1996, 1999).

in describing the quality of human life because it can hide deep inequalities and internal disparities; data gathered through sample surveys are more informative in this regard but they are also more complex and “time-expensive” on the computational level. Moreover, to a certain extent, the reliability of these variables (aggregate or not) could be sometimes higher with respect to the statistical evidence on the economic resources⁷. So the statistical issue associated with the capability approach seems to be much more a problem of choice of the more adequate data rather than a real lack of statistical evidence. The last two issues refer to aggregation. In the income-related approach, but generally speaking when the attention is just on the economic dimension, the final step of any poverty and well-being analysis is the aggregation of the available data in a synthetic index (in the case of poverty analysis, the number of poor people and/or their income distance from the poverty line). In a multidimensional framework the aggregative question also becomes much more complex because it can be conducted on different and/or subsequent levels. First, we could be interested in moving from the space of elementary indicators to the overall evaluation of a given functioning for each unit of analysis; secondly, the aggregation could involve the whole set of functionings for obtaining an overall picture of the individual or household standard of living ; finally, we could be interested in merging the individual well-being assessments into a synthetic index of social multidimensional well-being⁸.

One of the main advantage of aggregation is the possibility to have an immediate and summarizing picture of the phenomenon analyzed and to obtain under some circumstances a complete ordering. However, this would be a misleading procedure if we first do not clarify what kind of relationship there is among the elementary components of well-being. Two others arguments against the aggregation process refer to the criteria on which aggregation is based (i.e. which kind of procedures follow and how the weights are chosen) and the fact that in any aggregation steps we inevitably lose some important “pieces” of information. Of course, the option to simply present each single component of well-being without merging them into a common index is always available and sometimes recommended (see Sen 1987, Erikson, 1993).

⁷ Characteristics such as health, education, age, professional conditions, housing features, social relations with friends or relatives, or features referred to social environment (i.e. availability of social services) are sometimes easier to observe and to measure and less affected by statistical bias compared to variables such as income, wealth or consumption. On the reliability of the income sample surveys in Italy see Brandolini, D'Alessio (1998).

⁸ On the multidimensional measure of inequality, poverty and well-being see : Maasoumi (1986), Bradburd, Ross (1988), Atkinson, Bourguignon (1982, 1987), Rietveld (1990), Cheli, Lemmi (1995), Tsui (1995).

This, probably incomplete, list of matters involved in the empirical application of the capability approach could give strength to the skepticism of those who affirm that the capability framework is an elegant theoretical scheme but does not offer, on its own, an operational tool for the empirical analysis of well-being⁹. The main critical argument is that capabilities and functionings are rather elusive things to capture and a yardstick to measure them does not exist ; based on these premises, the persistence of the income based perspectives are, once again, justified.

I think that the reasons lying beyond this “methodological conservatism” (see, Sen 1996, p.119) are not only of a practical nature but rather have to be found at a more conceptual level. As a matter of fact, some of these issues, such as the functionings selection and the choice of the most suitable indicators to represent them, do not really seem much more problematic than those that can be met in any other poverty or inequality analysis. The final choice is nothing more than a compromise between the concerns and interests of the researcher and the statistical information available. The solution for the remaining topics, i.e. how to measure and represent each of the constitutive elements of well-being, how to manage the whole set of information and how (and if) to determine a synthetic index of well-being, could effectively appear more complicated in the multidimensional perspective. But they relate to the methodological sphere and are no longer just practical or empirical questions. There is no doubt that in this theoretical framework, poverty, inequality and well-being assume a more complex, broader and ambiguous meaning, but the richness of Sen’s approach lies exactly in his ability to capture, underline and explain the intrinsic complexity of these concepts. In this context, complexity and ambiguity are by no means a weakness of the theory but simply reflect the intrinsic and inescapably complex nature of those concepts. What we need are conceptual and mathematical tools better suited to dealing with these issues, that allow us to capture that ambiguity rather than lose or ignore it¹⁰; and of course, traditional methods such as those based on income measure, the poverty line, equivalence scales or “count of poor” do not seem to be appropriate for the measurement and assessment of well-being.

3. The use of fuzzy sets theory for a fuzzy assessment of well-being

⁹ See, for example, Sudgen (1993), Ysander (1993), Srinivasan (1994).

In the capability approach, well-being can be seen as a broad and fuzzy concept that is intrinsically complex and vague in the sense that it is not possible to contain within clear and unquestionable boundaries. Deprivation too is not an “all-or-nothing condition” that identifies a certain class of people, but rather a vague predicate that manifests itself in different degrees. When we refer to a given functioning, such as healthy living or education and knowledge, we can think that it could be fully achieved by a person or not achieved at all, but more often it will be only partially fulfilled. In all these cases, we are not interested in drawing a clear cut-off between opposite modalities (poor and not poor, sick or healthy, achieved or not achieved) or in representing these concepts in a dichotomous and antithetical way. On the contrary, we need to capture and preserve the interpretative richness of this approach in our description of well-being ¹¹.

At the same time, when we move from an income-based approach to an overall analysis of the individual’s well-being, we presumably work with a wide set of indicators that can assume quantitative or qualitative (dichotomous and ordinal) values or linguistic attributes such as good, bad, low, high and so on.

A useful tool for the treatment of “inexact knowledge” and approximate reasoning is represented by the fuzzy set theory. First introduced by Zadeh (1965) and extensively applied in many areas of research, this theory has also recently gained considerable attention in inequality and well-being analysis and in poverty measurement¹².

Briefly, fuzzy set theory substitutes the characteristic function of a crisp set that traditionally assigns a value of either 1 or 0 to each element in the universal set (discriminating between members and non-members of the crisp set), with a generalized characteristic function (called membership function) which varies between 0 and 1. Larger values denote higher degrees of membership.

In formal terms, if X denotes a universal set, then the membership function μ_A , by which a fuzzy set A is usually defined, has the form

¹⁰ On this point see also Sen (1992, 1993), Nussbaum and Glover (1995).

¹¹ A brief remark on the different meaning of vagueness and ambiguity may be useful. Although both concepts are generally related to uncertainty, the semantic meaning is not the same. Vagueness is associated with the difficulty of making sharp distinctions in some domain of interest. Ambiguity, on the other hand, is related to situations in which the choice between two or more alternatives (that are well defined) is left unspecified. Traditionally, the main mathematical framework for dealing with uncertainty is probability theory which assumes, however, a notion of stochastic uncertainty. As we will see in this section, fuzzy set theory and fuzzy logic “provide a natural way of dealing with problems in which the source of imprecision (in the sense of vagueness) is the absence of sharply defined criteria of class membership rather than the presence of random variables” (Zadeh, 1965).

¹² See Basu (1987), Shorrocks, Subramanian (1994) and Ok (1995) for an attempt to derive fuzzy measures of income inequality. With the aim of representing multidimensional analysis of well-being by fuzzy sets see Cerioli, Zani (1990), Chiappero Martinetti (1993, 1994, 1996), Cheli, Lemmi (1995). On the applicability of the fuzzy set theory to the field of poverty and inequality analysis see also Sen (1996, 1997).

[1] $\mu_A : X \rightarrow [0, 1]$

where $[0, 1]$ is the interval of real numbers from 0 to 1. Hence, $\mu_A(x) = 0$ if the element $x \in X$ does not belong to A , $\mu_A(x) = 1$ if x completely belongs to A and $0 < \mu_A(x) < 1$ if x partially belongs to A ¹³.

Let us assume that the subset A defines the position of each individual according to the degree of achievement of a given attainment or refers to one of the indicators considered for the functioning assessment. In this case, membership values equal to one identify a condition of full achievement with respect to a given functioning, whereas a value equal to zero denotes the opposite situation of total failure. When we consider quantitative variables or qualitative variables measured on an ordinal scale or expressed with linguistic attributes (as in the case of health and physical condition or subjective opinions or perception on one's own conditions), intermediate values between 0 and 1 describe gradual positions within the arrangement¹⁴. In this case, it will be necessary : i) to define an appropriate arrangement of modalities (or values) on the basis of the different degrees of hardship/well-being ; ii) to identify the two extreme conditions such that $\mu_A(x) = 1$ (full membership) and $\mu_A(x) = 0$ (non-membership) ; iii) to specify the membership functions for all the other intermediate positions¹⁵. In Appendix A some possible and very simple membership functions are depicted.

The choice of the proper membership function depends on the application context and on the kind of indicator that we want to describe. For instance, in cases of variables with equidistributed modalities along an ordinal scale, the linear functions indicated in Appendix A can be appropriate. Otherwise, if it is possible to identify a given interval above and below which it is possible to define the opposite conditions of full membership and no-membership, a trapezoidal function can be chosen¹⁶. Finally, a sigmoid function

¹³ In the similar way, the basic assumption of classical logic (or two-valued logic) that every proposition is either true or false, can be extended to many-valued logic and, in particular, to fuzzy logic when the truth values are linguistic variables. Indeed, when certainty about "truth" and "falsity" does not exist, truth value can be expressed through linguistic attributes such as *very true*, *fairly true*, *more or less true*, *mostly false* and so forth, which can be interpreted as *degrees* of truth.

¹⁴ In case of dichotomous variables the only two membership values will be zero or one.

¹⁵ For a wider discussion on fuzzy sets theory and its application to poverty analysis see Chiappero Martinetti (1993, 1994, 1996).

¹⁶ The rational is that, under some circumstances, it can be useful or relatively easy to identify two extreme modalities which act, respectively, as minimum and maximum threshold levels : for example, when it is possible to define, without ambiguity, conditions of total deprivation on one side and a full achieving on the opposite side with reference to a given functioning. Of course, this neither represents a constraint (the option

seems appropriate to describe quantitative and qualitative variables with modalities that are not equidistributed. An analogous specification, directly derived from the distribution functions, has been suggested by Cheli and Lemmi (1995) for representing these kinds of variables:

$$[2] \quad \mu(x^k) = \begin{cases} 0 & \text{if } k = 1 \\ \mu(x_{k-1}) + \frac{F(x_k) - F(x_{k-1})}{1 - F(x_1)} & \text{if } k > 1 \end{cases}$$

where $F(x)$ is the sampling distribution function of the variable x arranged in an increasing order according to k . For each indicator related to a given functioning, membership grades equal to zero and one are respectively assigned to the lowest and the highest position in the rank, and intermediate values are reserved for all the other elements, relating to their position inside the distribution function. As Cheli and Lemmi outline, with this kind of specification any *a priori* and arbitrary choice is avoided and membership functions are the “mirror” of the sample distributions: in this sense, it can be considered as a “totally relative” approach that fully reflects the reality (see Cheli, Lemmi, 1995, p. 124).

As already outlined, in the evaluative process measurement and representation are usually followed by an aggregation operations among i) a subgroup of elementary indicators referred to a given functioning and/or ii) the whole set of functionings. The former allows us to obtain a synthetic evaluation for each achieved functionings f_i , whereas the latter is related to an overall evaluation of well-being w_i . From a general point of view, both operations can be viewed as a suitable aggregation of each elementary fuzzy set with membership degree μ_{kfi} pertaining respectively to the k indicators and the f achievements examined, for each of the i reference units.

The basic operations in crisp sets theory, i.e. union and intersection, have been generalized with reference to the fuzzy sets, so that for both those operations there exists a broad class of function operations, each of which is characterized by a given set of property axioms¹⁷. Let's assume, for simplicity, only two fuzzy sets A and B (with reference to the first aggregation step, they could respectively concern elementary indicators of a given functioning), the most common class of operations of which are the following:

to “fuzzify” the whole space of observations is always open) nor a dichotomous choice as in the case of a poverty line.

¹⁷ On the operations on fuzzy sets see, among others, Dubois, Prade (1980,1985), Klir, Folger (1992), Klir, Yuan (1995).

1. Fuzzy intersection : this requires the simultaneous satisfaction of each elementary condition and corresponds to the logical conjunction “and” :

- | | |
|--|--|
| [1.1] standard (or strong) intersection | $\mu_{A \cap B} = \min [\mu_A, \mu_B]$ |
| [1.2] weak intersection (or algebraic product) | $\mu_{A \cdot B} = [\mu_A \cdot \mu_B]$ |
| [1.3] bounded difference | $\mu_{\underline{A \cap B}} = \max [0, \mu_A + \mu_B - 1]$ |

and the relationship among them is :

$$[3] \quad \mu_{\underline{A \cap B}} \leq \mu_{A \cdot B} \leq \mu_{A \cap B}$$

2. Fuzzy union : the satisfaction of at least one of the conditions is, in this case, required. This notion corresponds to the logical disjunction “or” for which some interchangeability among the arguments is assumed or admitted.

- | | |
|-------------------------------------|---|
| [2.1] Standard (or strong) union | $\mu_{A \cup B} = \max[\mu_A, \mu_B]$ |
| [2.2] weak union (or algebraic sum) | $\mu_{A+B} = [\mu_A + \mu_B - \mu_A \cdot \mu_B]$ |
| [2.3] bounded sum | $\mu_{\underline{A \cup B}} = \min[1, \mu_A + \mu_B]$ |

and the relationship among them is :

$$[4] \quad \mu_{A \cup B} \leq \mu_{A+B} \leq \mu_{\underline{A \cup B}}$$

Since each of these aggregation procedures satisfies a different axiom structure¹⁸ and assumes a distinctive meaning, the choice among them is strictly related to the specific context of analysis¹⁹. However, some brief general remarks about these operations could be useful for understanding their meaning.

Standard intersection and union operations focus, respectively, on the least and the most favourable position, so the membership grades to the composite set will be the lower value of μ to the elementary sets in the former case, and the highest values in the latter. They implicitly excluded that there may be any sort of compensation between indicators, and it can be a proper aggregation in case of a positive correlation between them (i.e. $A \uparrow$

¹⁸ They share some common properties with the aggregation operations on crisp sets theory, such as commutativity and associativity, but they also satisfy some additional interesting properties such as monotonicity and continuity.

¹⁹ See Chiappero Martinetti (1993, 1994) about some general methodological issues involving the specification of membership functions and alternative ways of aggregating deprivation symptoms to obtain an overall fuzzy evaluation of poverty.

$B \uparrow$). On the contrary, the second class of operators (1.2 and 2.2) admits the possibility of compensation, leads to evaluation criteria reflecting both classifications, and could be an adequate operation for aggregating independent indicators (i.e. $A \perp B$). Finally, bounded difference and bounded sum have a more frequent use in the case of a negative correlation between indicators ($A \uparrow B \downarrow$), but they reduce the possibility to “fuzzify” the extreme values.

A different way to consider any kind of aggregation operation on n fuzzy sets (with $n \geq 2$) is to define a function :

$$[5] \quad h : [0,1]^n \rightarrow [0,1]$$

If applied to n fuzzy sets A_1, A_2, \dots, A_n defined on X , function h produces an aggregate (fuzzy) set by operating on the membership grades of these sets for each element x . In other words :

$$[6] \quad \mu_A(x) = h(\mu_{A_1}(x), \mu_{A_2}(x), \dots, \mu_{A_n}(x))$$

The union and intersection operators discussed above represent a special case of this generalisation. If we simply denote with a_1, a_2, \dots, a_n the membership grades of each element belonging to sets A_1, A_2, \dots, A_n the relationship within the class of operators is the following :

$$[7] \quad \min(a_1, a_2, \dots, a_n) \leq h(a_1, a_2, \dots, a_n) \leq \max(a_1, a_2, \dots, a_n)$$

A minimal axiomatic structure is usually associated with the function h and the included boundary condition, monotonicity, continuity and symmetry. A parametric class of operators satisfying this axiomatic structure is the generalised means :

$$[8] \quad h_\alpha(a_1, a_2, \dots, a_n) = \left[\left(a_1^\alpha + a_2^\alpha + \dots + a_n^\alpha \right) / n \right]^{\frac{1}{\alpha}}$$

with α equal to 1 for the arithmetic mean, $\alpha = -1$ for the harmonic mean and $\alpha = 0$ for the geometric mean.

Finally, if we remove the symmetry axiom, a class of weighted averaging operations can be derived :

$$[9] \quad h_\alpha(a_1, a_2, \dots, a_n; w_1, w_2, \dots, w_n) = \left[\sum w_i a_i^\alpha \right]^{\frac{1}{\alpha}}$$

where the weighting structure, expressed by $w_i \geq 0$ and $\sum w_i = 1$ specifies the relative importance assigned to each aggregate set.

The selection of a suitable weighting structure is an old and questionable issue. In a multidimensional approach, if each dimension of human well-being is considered as equally relevant, a neutral choice could be to assign an equal weight to all constitutive elements, as in [8]: in this way we are not called upon to express uneasy judgements or to define a ranking among them. Alternatively, if we want to maintain an “objective” approach to the measurement, a *frequency-based weighting* (see Brandolini, D’Alessio, 1999) can be adopted, and in this case the weighting structure is directly drawn from reality. Desai, Shah (1988), for instance, define w_i as the complement to one to the proportion of deprived people. Cerioli, Zani (1990), suggest defining w_i as an inverse function of the frequency of the corresponding symptom of deprivation :

$$[10] \quad w_i = \ln 1/f_i$$

where the choice of the logarithm is justified with the opportunity of not giving too much importance to the modalities showing a very low frequency.

In a similar way, but with a direct relation to the above membership function [2], Cheli, Lemmi (1995) specify the following weighting structure :

$$[11] \quad w_i = \ln \left[\frac{1}{n} \sum_j \mu_{ij} \right]$$

that represents a generalisation of the previous weighting structure [10] .

4. Some empirical evidence on fuzzy well-being assessment in Italy

In this section an application of the fuzzy methodology briefly described above is presented. The database used for this exploratory analysis is represented by the micro-data of a sample survey conducted in 1994 by the Italian Central Statistical Office (ISTAT, Indagine Multiscopo), a survey that collected a large amount of information on aspects of daily life, habits and social and economic behaviours of the household. For its sample size (61953 individuals that correspond to 21462 households) and for its large amount of information (more than three hundred variables are collected) up to now it perhaps is one of the best statistical sources in Italy for a multidimensional well-being assessment²⁰. In

²⁰ A recent empirical application of the capability approach has been done by Brandolini, D’Alessio (1999) using the micro-data of the Bank of Italy’s survey. As stressed by the same Authors (p. 28) this statistical source is mainly devoted to collecting data on the household economic resources and doesn’t include information *ad hoc* about “non-material” dimensions of well-being. Some information about health, education

the following sub-sections I will briefly describe the choices that have been made with reference to the above a)-e) issues discussed in section 2.²¹ The results are presented in the final part of this section.

4.1 How many well-being dimensions to include and how to represent them : a list of relevant functionings and related indicators

Well-being has been evaluated on the basis of a list of achieved functionings instead of capabilities. As already mentioned, the “elementary evaluation” is generally an escapable choice when the assessment is based on statistical evidence already available and not on data especially collected with the aim of an empirical implementation of the capability approach²². However, with a direct and explicit reference to the meaning of human capabilities, some factors related to personal features, household structures and social and economic environmental indicators will also be considered for analysing what are the most frequent difficulties in achieving a given functioning.

The reference unit is the single individual. This is a direct consequence of the nature of the functionings selected which mainly pertain to the personal situation, with the only exception being the housing conditions for which, as obvious, information is collected at the household level.

Attention has been confined to a set of five functionings - housing, health, education and knowledge, social interactions and psychological conditions - to which corresponds a large spectrum of elementary indicators, mainly qualitative and on an ordinal scale, sometimes dichotomous²³. If the Istat sample survey is very rich regarding the qualitative and subjective aspects of human life, on the other hand no information about household income or wealth is gathered, while some questions about food and drink consumption

and employment status is, however, available and it is mainly on this statistical evidence that the Authors carry out the well-being assessment. This is also one of the few empirical works that tries to assess the well-being in the functioning space, and for this reason I will often refer to it in the following pages.

²¹ It is necessary to stress that the main goal of this exercise is to test the adequacy of a fuzzy approach for an empirical implementation of the capability approach. For this reason I will give more emphasis to the methodological aspects rather than the results of the well-being assessment.

²² For an attempt to obtain a more complete evaluative exercise in which both functioning and capability spaces are jointly considered see Chiappero Martinetti (1999): however, in that work the well-being assessment is only restricted to education and health dimensions.

²³ The list of functionings selected here is frequently suggested or included in the well-being evaluation. For instance, Brandolini, D'Alessio (1999), also consider in their analysis health, education, social relations and housing, but add two other dimensions referred to the labour market status and the household's economic resources. Schokkaert, Van Ootegem (1990), through factor analysis, identify six dimensions of well-being concerning social, psychological, physical and financial aspects, the level of activity and the microsocial

habits are included. In a functioning-oriented assessment the absence of information about income or economic resources doesn't seem to represent a problematic issue. In fact, as a means for achieving well-being the available income assumes only an indirect and derivative role in the evaluation exercise (on this, see Sen, 1992, 1993). This means that inadequate levels of income can explain, with others factors, why a given functioning (for example, housing or education) is not fully achieved by a person, but it doesn't take part of the evaluative space which ought to include only the constitutive elements of well-being.

More interesting for our purpose would be the information available about nutrition ; unfortunately, the collected variables refer to the food and drink habits of Italian households without the possibility of distinguishing the role of constraints and preferences in determining such consumption behaviours. In the capability perspective this distinction is fundamental and it is exactly this that permits us to differentiate between available options and options chosen. The impossibility of capturing these important distinctions persuades us to ignore the statistical evidence on consumption.

The functionings selected and the correlated indicators are, therefore, the following²⁴ :

- 1) Housing is the result of two main indicators : i) a crowding index, defined as the number of rooms available for each family “corrected” by equivalence coefficients to take into account the economies of scale²⁵; ii) a basic housing utilities measure that includes telephone, regular water availability and heating²⁶.
- 2) Health conditions are described by the presence/absence of chronic illnesses. Information available is related to a list of fifteen chronic illnesses with different degrees of seriousness. Three homogenous clusters have been determined : a first group includes chronic illnesses with not very serious disability consequences , the second group refers to severe chronic illnesses that generates a partial disability, while the last group includes the more serious or incurable illnesses.
- 3) Education and knowledge are measured by means of three ordinal indicators : the higher educational attainment and two variables that refer to personal knowledge in a

contacts. However, even if the “labels” assigned to the functionings are quite similar to our analysis, the variables used and the methodology applied are totally different.

²⁴ See Appendix B for a description about features and modalities of each indicators included in this analysis.

²⁵ The rationale of this choice is that the house is perhaps the most relevant “household public good”, so the standard procedure to simply consider the number of rooms per capita doesn't seem too correct. The coefficients applied are those proposed by Carbonaro (1985) : it is an equivalence scale estimated on the basis of household food expenditure and frequently adopted in Italy for inequality and poverty analysis.

²⁶ More specifically, the total absence of a heating system or an inappropriate heating system for households living in the northern and central regions of Italy has been considered a symptom of inadequacy.

wider sense, that is the number of books read during the last twelve months and the frequency of reading newspapers during a week .

- 4) Social interactions are depicted by two subsets of indicators that respectively refer to the social relationships during the leisure time (frequency of contact and meeting with friends) and to the participation in the social life. This latter is measured by fifteen variables divided into three groups of indicators referring to: i) passive participation (eight dichotomous variables related to political, cultural or associative meetings participation, public demonstrations, etc.) ; ii) active participation (six dichotomous variables concerning the membership or a direct involvement in associations, political parties, and other kinds of organisation)²⁷ ; political interest (a categorical variable that roughly describes the degree of interest in political issues).
- 5) Psychological conditions are described by a plurality of indicators that express a subjective perception on one's own situation or a personal judgement about the level of satisfaction regarding some relevant aspects of one's own life. Nine variables measured on ordinal scales have been included in our analysis and re-arranged in five homogenous groups that refer to : i) economic conditions; ii) personal and social relations; iii) health conditions; iv) working conditions ;v) leisure time.

The necessity, but sometimes also the opportunity, to take into account the subjective perception of people in well-being assessment is quite controversial. The most recurrent argument against this choice is that people are influenced by their actual condition as well as by their aspirations, and these elements unavoidably introduce a bias in the evaluative exercise. A second, but strictly related, question concerns the meaning that people can assign to the verbal labels assigned to describe different levels of satisfaction ranked along an ordinal scale²⁸. Finally, many elements included in the psychological functioning could overlap with other dimensions included in our analysis and measured based on objective criteria.

However, the decision to consider also the subjective dimension of well-being is justified by the fact that it doesn't substitute but just complements our assessment and permits us to compare the results derived from the use of objective and subjective evaluation criteria. At the same time, the concept of "being well" in the sense of being happy, having self-respect, and satisfying one's own desires assumes an important role in determining the

²⁷ For the last two groups of variables a sum of the score has been calculated and a reliability analysis has been conducted to verify the item's "goodness". The α coefficients of the reliability analysis are respectively equal to 0,65 and 0,60.

²⁸ In defence of a subjective approach to poverty and well-being analysis see, in particular, Hagenaars (1986) and Van Praag (1993).

personal well-being achievement (see Sen, 1992,1993) . Of course, these aspects are only partially reflected in our available data.

4.2 From the elementary indicators to the functioning assessment

Subsequent to the selection of a list of relevant functionings and their related indicators, a first step in the fuzzy assessment of well-being requires us to define the membership functions most suitable to represent them. The general rule adopted here is to reduce as much as possible the arbitrariness of the choices and to rely on the methodological questions regarding the observed data : that also allow us to assume a “relativist” perspective in describing the functionings achievement which is in line with capability approach²⁹. Following this prescription, most of the membership functions have been defined through the above specification [2] or by linear functions in the case of equally distributed modalities.

With the aim of realising a more defined picture of the achieved functionings, some elementary indicators have subsequently been merged: for instance, information concerning the availability of the telephone, water and heating, put together into a synthetic index of basic housing utilities, or for groups of chronic illnesses included in the health-functioning, or in the case of variables related to the social participation as well as for indicators embodied in some components of the psychological condition.

If aggregation could be considered a problematic exercise among dimensions that are substantially heterogeneous or for which there is uncertainty about their exact relationship, a synthesis within elementary components of a given functioning seems to be less controversial, as this concerns elements that in most cases are homogeneous and comparable to each other. In this regard, the union and averaging operators described in section 3 have been applied and, in the case of weighted averaging operators, the weighting structure chosen refers to both specifications [10] and [11].

Following the prescription mentioned above, also the choice of the more adequate aggregation operators has been done on the basis of empirical observation whenever possible, or by referring to the meaning of each operation. For instance, the indicators of basic utilities included in the housing-functioning have been aggregated by a weighted

²⁹ I refer to the distinction between the functionings selected, each of which can be considered relevant in absolute terms, and the ways and the resources required to achieve them, which are strictly related to the social and economic context. The rationale is that “being relatively poor in a rich country can be a great capability handicap” (Sen, 1997, p.212) so it is important to evaluate the individual position along a scale which describes the functionings achievement for the reference society .

averaging operator with weights equal to the (logarithm) of the inverse of the frequency of each elementary indicator. In the case of chronic illnesses, a standard union operator has been applied and the choice is justified by the need to assign a degree of membership to the composite fuzzy set that reflected the less favourable position (i.e. the worst health condition). Finally, an equal relevance and hence an equal weight has been assigned to the elementary indicators included in the psychological condition evaluation. Through this operation, fifteen elementary indexes are obtained with reference to the five functionings included in the well-being assessment : the results are reported in Table 1 and will be presented in the next section.

Analogous criteria have been adopted for determining the further step of aggregation from the fuzzy elementary subsets to the five composite fuzzy sets, each of which refers to the dimensions of well-being considered here. A purely objective approach has also been followed in this case, and the elementary indexes are put together for determining each single functioning by a weighting averaging operator with the weighting structure indicated in [11]³⁰. Finally, an overall index of well-being has been computed for each reference unit through union, intersection and average operators. As already outlined, there is a trade-off between synthesis and detailed knowledge due to the fact that for each subsequent step in the aggregation procedure some “piece of information” is inevitably lost. This is particularly true for multidimensional analyses where dimensions of well-being that are qualitatively and intrinsically distinct are assessed. In this case, especially for the design of public actions, more helpful and effective information can be derived from an articulated picture rather than a misleading index.

4.3 *The results*

Tables 1 - 4 depict the main results of our analysis. The first table reports the central tendency measures of the membership degrees for the fifteen elementary indicators included in our assessment exercise. Two main factors stand out : a relatively high degree of achievement in the material dimensions as well as in the health conditions, and a relatively low fulfilment in education and knowledge as well as in the participation in the social life outside of family and friends. These results are not surprising if we consider that Italy is one of the industrialised countries with the highest life expectancies but lowest

³⁰ A detailed picture about the membership functions and the aggregation procedures applied for each functioning indicator is given in Appendix 3 while the results are reported in table 2.

educational attainment ³¹. Nevertheless, the degree of satisfaction regarding the main spheres of their own life, and especially for health and personal and familiar relationships, is significantly high.

The average membership degrees for subgroups of the population (see table 2) emphasises inequality and bias in the functioning achievements. A slight gender disparity, mainly in health, education, knowledge and participation in the social life that, however, does not affect the subjective perception of their own condition, as the differences between male and female are minimal. Health, education and social life have the lowest performance for elderly people. With the exception only of health conditions and the relational life there are large differences in functioning achievements between the North and the South of Italy. The own condition with respect to the labour market affects in a considerable way the achievement of material, social and personal dimensions of human life : emblematic is the low level realised by housewives in most functionings which reveals that the gender inequality issue is something more than a labour market discrimination.

Table 3 offers a more aggregate picture related to the distribution of membership degrees for each of the five dimensions of well-being, while table 4 shows the average of these membership degrees by subgroups of population defined on the basis of personal and social characteristics. The aggregation procedure chosen (i.e. weighted averaging operation) seems to preserve to a large extent the information content of the elementary indicators. Generally speaking, there is a polarisation of the results : nearly forty per cent of the population has a high level of achievement with respect to housing conditions and more than 65% fully achieve the health functioning. On the contrary, over six and seven people out of every ten have, respectively, a membership degree equal or lower than 0,400 in the education and social interactions spheres. Finally, concerning the psychological dimension of well-being, more than 70 per cent of the population has a membership degree belonging to the middle classes of distribution, that is between 0,400 and 0,700.

The picture is more exhaustive if we observe the results in terms of functionings achieved for different groups of the population which, however, to a large extent reflects the above results related to elementary indicators. With reference to the subjective and relational dimensions of well-being, the membership degrees to the psychological functioning are, on average, relatively high for all groups without meaningful differences, while low and

³¹ Less than 5% of the population has a university degree and more than 10% has no qualification. The former value goes up to 7 per cent and the latter goes down to less than 5 per cent for the population aged between 25 and 65 years.

more variable values are associated with the social interactions. The elderly, mainly if they live alone, and housewives show a great isolation in their social life. At first sight it seems to be a relation directly between social interactions on one side and education and knowledge on the other, as the population more deprived sub-groups are approximately the same.

To verify the degree of association among elementary indicators as well as among functionings we can refer to the Pearson correlation coefficients shown in tables 5 and 6. From a general point of view, their absolute values are rather low : only a few elementary indicators are correlated, as in the case of active/passive participation and political interest or education and readings or, as obvious, between objective and subjective conditions of health. A low degree of correlation exists also in the functionings space except for education and knowledge, as already outlined above. However, if we observe the correlation coefficients among each of the elementary indicators referring to these functionings, it is quite evident that education attainment is correlated only with the degree of participation and the political interest, but not with the fields of personal and friend relationships. Therefore, it seems possible to affirm that the list of relevant functionings chosen has allowed us to depict a richer picture and to capture complementary dimensions of human well-being.

The last step of our analysis refers to an overall index of individual well-being. In table 7 we report the average membership degrees to the composite fuzzy set obtained by union, intersection and average operators on the five elementary fuzzy subsets³². The different meaning of these operators has to be considered when we look at their absolute value: standard intersection operator refers to the worst performance in the functionings space, so it could be interpreted as a sort of deprivation index in (at least) one of the elementary dimensions of well-being; on the other hand, the union operator highlights the better performance, and it could be assumed to be the distance from a full achievement in (at least one) well-being dimension; finally, the un-weighted average lies in-between and postulates a condition of symmetry among elementary sub-sets³³. However, in one aspect the results are fully concordant. If we look at the rankings among subgroups of population obtained from the three aggregation operators we can see that women, the elderly (mainly

³² At this stage, with the aim to simplify our analysis, only standard union and intersection operators have been calculated.

³³ As already outlined, important pieces of information are lost in any kind of aggregation exercise and/or strong and arbitrary hypotheses are introduced about the role and the weight of the elements considered. For these reasons, in a multidimensional assessment, a description of each single dimension can give us much more information than an overall index.

if they live alone), people living in the South of Italy, housewives and blue-collar workers are in less favourable conditions no matter how the overall well-being has been determined.

5. Conclusions

The main purpose of this paper was to suggest an empirical implementation of Sen's capability approach that was able to preserve its interpretative richness and attempt some methodological problems that couldn't find an operative solution in more standard approaches to poverty and well-being analysis. A strategy based on fuzzy sets theory has been suggested and step by step we have tried to discuss and show why this kind of approach could be useful for dealing with the complexity of a multidimensional assessment of well-being. With the aim of testing this methodology, an empirical exercise based on micro-data referred to Italy has been done and a richer and more clarifying picture of well-being dimensions has been obtained. To a certain extent our results are not too dissimilar from those obtained from more traditional income-based approaches to well-being analysis: once again, the poorer groups in the population are mainly the elderly, people living in the South or with a lower education level. However, there are some meaningful differences. First, even if deprivation and inequality in the functioning space always exist, they seem to be relatively lower if compared with deprivation and inequality in the income space (cfr., Rossi, 1998): the differences between North and South, male and female, young and elderly, and within occupational groups are not so remarkable. Second, good results are obtained in more material aspects of human life, such as housing or health, together with a worse performance in non-material but equally important spheres of well-being (education and knowledge as well as social relationships). Third, the individual assessment of well-being permitted us to highlight deprivation conditions (i.e. housewife) otherwise hidden in the income inequality or poverty analysis, when the household is assumed as the unit of analysis.

To sum up, there is no doubt that this paper has to be intended as a very preliminary step towards a more complete and satisfying application of Sen's theory, and many information and methodological issues need further research to be fully clarified and solved. However, the possibility of extending our knowledge to some fundamental aspects

of human life and measuring deprivation and inequality not only in the income space can help us to obtain a better and deeper knowledge of personal well-being.

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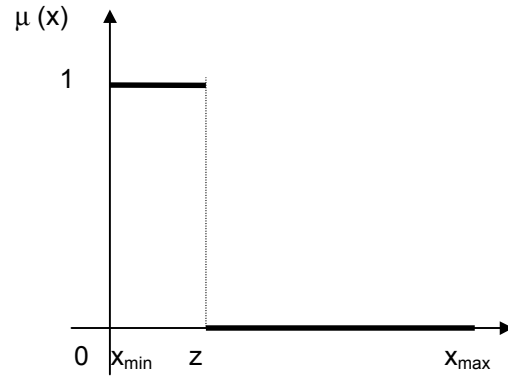
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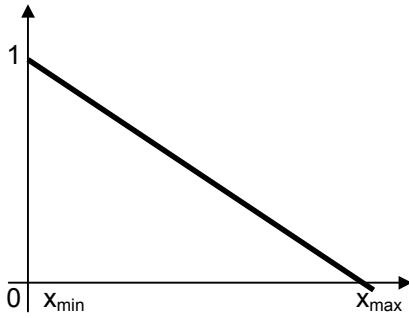
Appendix A - Some membership functions

1. traditional approach (i.e. poverty line)

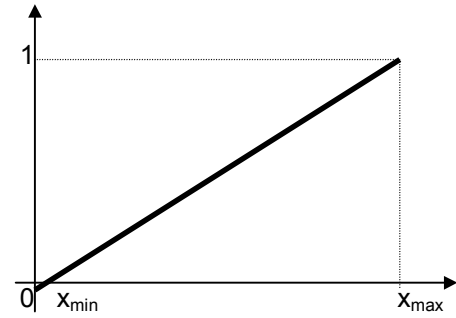
$$\begin{aligned}\mu(x) &= 1 && \text{if } 0 \leq x \leq z \\ \mu(x) &= 0 && \text{if } z \leq x \leq x_{\max}\end{aligned}$$



2. Linear function



$$\mu(x) = \frac{x_{\max} - x}{x_{\max} - x_{\min}}$$

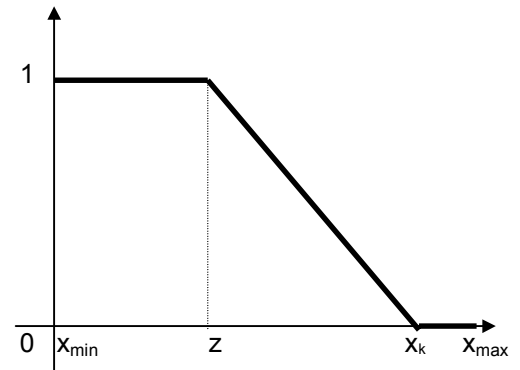


$$\mu(x) = \frac{x - x_{\min}}{x_{\max} - x_{\min}}$$

3. Trapezoidal function

$$\begin{aligned}\mu(x) &= 1 && \text{if } x_{\min} \leq x \leq z \\ \mu(x) &= 0 && \text{if } x_k \leq x \leq x_{\max}\end{aligned}$$

$$\mu(x) = \frac{x_k - x}{x_k - z} \quad \text{if } z \leq x \leq x_k$$



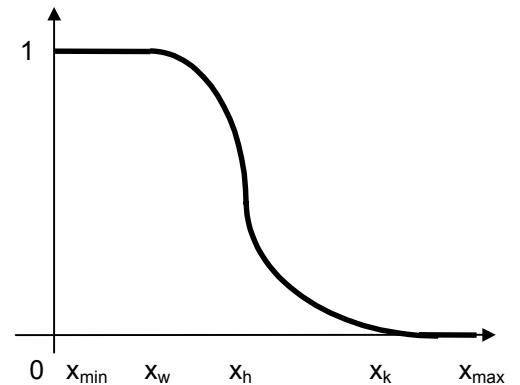
4. Sigmoid (or logistic) function

$$x_k = \mu(x) = 0 \quad x_h = \mu(x) = 0.5 \quad x_w = \mu(x) = 1$$

$$\begin{aligned}\mu(x) &= 1 && \text{if } x_{\min} \leq x \leq x_w \\ \mu(x) &= 0 && \text{if } x_k \leq x \leq x_{\max}\end{aligned}$$

$$\mu(x) = 1 - \frac{1}{2} \left[\frac{x_w - x}{x_w - x_h} \right]^2 \quad \text{if } x_w \leq x \leq x_h$$

$$\mu(x) = \frac{1}{2} \left[\frac{x_k - x}{x_k - x_h} \right]^2 \quad \text{if } x_h \leq x \leq x_k$$



Appendix B - The elementary indicators included in the functionings assessment

♦ HOUSING :	Crowding index (μ 11)	- numbers of room/equivalence coefficient ^(a)
	Basic housing utilities (μ 12)	- telephone (dichotomous variable) - regular water availability (dichotomous variable) - adequate heating availability (categorical variable)
♦ HEALTH :	Chronic illnesses (μ 21)	- group 1 (hypertension, allergy, asthma, bronchitis, osteoarthritis) - group 2 (diabetes, liver or kidneys stones, ulcer) - group 3 (nervous disease, cirrhosis, coronary , cancer)
	level of education (μ 31)	- from none qualification to university degree (8 modalities)
	knowledge1 (μ 32)	- numbers of book read during the last year (5 modalities)
♦ EDUCATION & KNOWLEDGE	Knowledge2 (μ 33)	- regularity in reading newspapers (5 modalities)
	friends (μ 41)	- frequency of contact and meeting with friends (7 modalities)
♦ SOCIAL INTERACTIONS	passive participation (μ 42)	- participation to political, cultural or associative meetings, etc. (8 dichotomous variables rearranged for considering the sum of scores)
	active participation (μ 43)	- direct, active participation to associations, political parties etc. (6 dichotomous variables rearranged for considering the sum of scores)
	political interests (μ 44)	- degree of interest and speech about political issues (5 modalities)
♦ PSYCHOLOGICAL CONDITIONS	economic conditions (μ 51)	- degree of satisfaction about personal and household economic resources and comparison of the last year economic condition (3 variables, 4/5 modalities)
	personal/social rel. (μ 51)	- degree of satisfaction about friend and familiar relationships (2 variables, 4 modalities)
	health conditions (μ 53)	- degree of satisfaction about own health and perception of own health status (2 variables, 4/5 modalities)
	working conditions (μ 54)	- degree of satisfaction about own job (4 modalities)
	leisure time condit. (μ 55)	- degree of satisfaction about own leisure time (4 modalities)

(a) Equivalence scale (Carbonaro, 1996) : no. of components
coefficient

1	2	3	4	5	6	7+
1	1,67	2,22	2,72	3,17	3,60	4

Appendix C - membership functions and aggregation operations

1. Housing conditions

- $$\begin{aligned}
1.1. \quad & \text{crowding index } (\mu_{11}) = 0 \text{ if rooms for equivalent adult } \leq 0.59^{(a)} \\
& = 1 \quad \quad \quad " \quad \quad \quad " \quad \quad \quad \geq 4^{(b)} \\
& = [2] \text{ otherwise} \\
1.2. \quad & \text{basic housing utilities } (\mu_{12}) = 0 \text{ if no basic utilities are available} \\
& = 1 \text{ if all basic utilities are available} \\
& = \frac{\sum \mu(x) w_i}{\sum w_i} \text{ otherwise with } w_i \text{ as [10]}
\end{aligned}$$

2. Health

- 2.1. chronic illnesses (μ_{21}) . Aggregation procedure : standard union [1.2]
 = 0 for illnesses belonging to the group 3
 = 0,33 for illnesses belonging to the group 2
 = 0.66 for illnesses belonging to the group 1
 = 1 in case of absence of illnesses

3. Education and knowledge

- | | | |
|------|---------------------------|--|
| 3.1. | education (μ_{31}) | = 0 for no qualification
= 1 for the higher level of qualification
= [2] otherwise |
| 3.2. | knowledge1 (μ_{32}) | = 0 if no books are read during the last year
= 1 if > 10 books are read
= [2] otherwise |
| 3.3. | knowledge2 (μ_{33}) | = 0 if no newspaper is read during the week
= 1 if newspapers are read daily
= [2] otherwise |

4. Social interactions

- | | | |
|------|-----------------------------------|--|
| 4.1. | friends (μ_{41}) | = 0 if no friends
= 1 for daily meeting or contacts with friends
= [2] otherwise |
| 4.2. | passive part. (μ_{42}) | = 0 if minimum score (=8)
= 1 if maximum score (=16)
= [2] otherwise |
| 4.3. | active part. (μ_{43}) | = 0 if minimum score (=6)
= 1 if maximum score (=12)
= [2] otherwise |
| 4.4. | political interest (μ_{44}) | = 0 for the lower modality (no political interest)
= 1 for the higher modality (strong political interest)
= [2] otherwise |

5. Psychological conditions

- 5.1. economic cond. (μ_{51}). Aggregation procedure as [8] with $\alpha=1$ (arithmetic mean):
- = 0 for the lower level of satisfaction
 - = 1 for the higher level of satisfaction
 - = $0 < (\mu_{511}) < 1$ for intermediate cases
- 5.2. personal/social rel. (μ_{52}). Aggregation procedure as [8] with $\alpha=1$ (arithmetic mean)

(a) Less than 0,3% of total cases has a value $\leq 0,59$.

- = 0 for the lower level of satisfaction
- = 1 for the higher level of satisfaction.
- = $0 < (\mu_{512}) < 1$ for intermediate cases
- 5.3. health conditions (μ_{53}). Aggregation procedure as [8] with $\alpha=1$ (arithmetic mean)
 - = 0 for the lower level of satisfaction
 - = 1 for the higher level of satisfaction.
 - = $0 < (\mu_{513}) < 1$ for intermediate cases
- 5.4. working conditions (μ_{54})
 - = 0 for the lower level of satisfaction
 - = 1 for the higher level of satisfaction.
 - = $0 < (\mu_{514}) < 1$ for intermediate cases
- 5.5. leisure time condit. (μ_{55})
 - = 0 for the lower level of satisfaction
 - = 1 for the higher level of satisfaction.
 - = $0 < (\mu_{515}) < 1$ for intermediate cases

^(b) 4,6% of total cases has a value ≥ 4

TABLE 1 - Membership degrees to the elementary subsets: central tendency measures					
FUNCTIONINGS	ELEMENTARY SUBSETS		mean	mode	median
HOUSING	crowding index	μ_{11}	0,440	0,374	0,374
	basic housing utilities	μ_{12}	0,891	1,000	1,000
HEALTH	chronic illnesses	μ_{21}	0,799	1,000	1,000
EDUCATION & KNOWLEDGE	level of education	μ_{31}	0,418	0,131	0,420
	knowledge1 (books)	μ_{32}	0,253	0,000	0,000
	knowledge2 (newspapers)	μ_{33}	0,432	0,000	0,366
SOCIAL INTERACTIONS	friends	μ_{41}	0,560	1,000	0,608
	passive participation	μ_{42}	0,303	0,000	0,000
	active participation	μ_{43}	0,167	0,000	0,000
	political interest	μ_{44}	0,441	0,000	0,534
PSYCHOLOGICAL COND.	economic resources	μ_{51}	0,487	0,607	0,523
	personal/social relations	μ_{52}	0,746	0,660	0,660
	health	μ_{53}	0,755	1,000	0,830
	working	μ_{54}	0,614	0,660	0,660
	leisure time	μ_{55}	0,574	0,660	0,660
Source: own elaboration on Istat sample survey (Indagine Multiscopo)					

TABLE 2 - Average membership degrees for each elementary subsets by personal and social characteristics

	μ_{11}	μ_{12}	μ_{21}	μ_{31}	μ_{32}	μ_{33}	μ_{41}	μ_{42}	μ_{43}	μ_{44}	μ_{51}	μ_{52}	μ_{53}	μ_{54}	μ_{55}
SEX															
male	0,429	0,891	0,817	0,438	0,211	0,503	0,610	0,374	0,195	0,542	0,489	0,756	0,733	0,638	0,591
female	0,451	0,892	0,782	0,399	0,291	0,367	0,512	0,237	0,140	0,347	0,485	0,735	0,737	0,594	0,557
Total	0,440	0,891	0,799	0,418	0,253	0,432	0,560	0,303	0,167	0,441	0,487	0,746	0,755	0,614	0,574
AGE															
18-35	0,413	0,889	0,925	0,626	0,322	0,461	0,627	0,325	0,168	0,464	0,486	0,770	0,825	0,626	0,585
36-65	0,463	0,902	0,732	0,409	0,223	0,488	0,449	0,331	0,195	0,484	0,488	0,735	0,694	0,612	0,535
66 and over	0,645	0,866	0,452	0,190	0,125	0,330	0,426	0,165	0,101	0,292	0,488	0,698	0,506	0,555	0,627
Total	0,472	0,892	0,759	0,450	0,244	0,455	0,509	0,304	0,171	0,449	0,487	0,742	0,713	0,614	0,566
GEOGR. AREA															
North-West	0,450	0,932	0,787	0,430	0,314	0,523	0,496	0,296	0,187	0,484	0,513	0,759	0,743	0,642	0,606
North-East	0,530	0,928	0,811	0,420	0,315	0,521	0,508	0,331	0,226	0,480	0,523	0,768	0,764	0,667	0,615
Centre	0,488	0,908	0,775	0,427	0,256	0,450	0,561	0,302	0,177	0,462	0,483	0,756	0,737	0,608	0,580
South	0,357	0,877	0,814	0,412	0,175	0,296	0,620	0,294	0,114	0,377	0,464	0,713	0,764	0,574	0,521
Islands	0,386	0,756	0,803	0,391	0,203	0,396	0,624	0,286	0,128	0,402	0,438	0,739	0,762	0,558	0,550
Total	0,440	0,891	0,799	0,418	0,253	0,432	0,560	0,303	0,167	0,441	0,487	0,746	0,755	0,614	0,574
MARITAL STATUS															
single	0,385	0,886	0,926	0,474	0,343	0,404	0,754	0,328	0,156	0,440	0,479	0,767	0,864	0,630	0,637
married	0,422	0,901	0,740	0,416	0,211	0,461	0,450	0,309	0,179	0,463	0,496	0,746	0,698	0,612	0,541
divorced	0,628	0,867	0,754	0,525	0,357	0,555	0,457	0,330	0,192	0,505	0,464	0,669	0,716	0,616	0,551
widow/widower	0,708	0,857	0,473	0,182	0,134	0,288	0,399	0,148	0,107	0,248	0,471	0,686	0,524	0,573	0,590
Total	0,440	0,891	0,799	0,418	0,253	0,432	0,560	0,303	0,167	0,441	0,487	0,746	0,755	0,614	0,574
WORK STATUS															
employee	0,453	0,907	0,852	0,556	0,272	0,544	0,501	0,384	0,221	0,546	0,514	0,756	0,773	0,638	0,524
unemployed	0,339	0,844	0,901	0,595	0,267	0,387	0,711	0,267	0,113	0,403	0,366	0,751	0,809	-	0,642
housewife	0,434	0,873	0,717	0,315	0,192	0,314	0,424	0,163	0,099	0,276	0,463	0,719	0,681	0,555	0,538
student	0,371	0,916	0,958	0,758	0,445	0,406	0,824	0,383	0,154	0,461	0,480	0,799	0,869	-	0,669
pensioner	0,604	0,886	0,516	0,231	0,152	0,397	0,478	0,229	0,140	0,378	0,496	0,726	0,557	-	0,647
other	0,506	0,843	0,524	0,283	0,142	0,287	0,445	0,203	0,103	0,278	0,437	0,670	0,513	-	0,560
Total	0,464	0,892	0,769	0,447	0,249	0,444	0,530	0,303	0,167	0,441	0,487	0,746	0,721	0,614	0,574
OCCUPATIONAL GROUPS															
manager	0,540	0,947	0,855	0,872	0,483	0,764	0,428	0,510	0,347	0,701	0,558	0,750	0,780	0,688	0,517
office worker	0,471	0,926	0,867	0,722	0,399	0,634	0,496	0,450	0,266	0,616	0,532	0,760	0,780	0,658	0,546
blue-collar work.	0,398	0,880	0,861	0,394	0,154	0,414	0,527	0,345	0,171	0,471	0,487	0,759	0,774	0,601	0,542
other employee	0,397	0,889	0,835	0,451	0,211	0,476	0,544	0,307	0,151	0,447	0,496	0,759	0,776	0,633	0,544
self-employed	0,486	0,910	0,828	0,489	0,220	0,551	0,485	0,339	0,214	0,536	0,519	0,749	0,763	0,645	0,474
Total	0,453	0,907	0,852	0,556	0,272	0,544	0,501	0,384	0,221	0,546	0,514	0,756	0,773	0,638	0,524

Source: own elaboration on Istat sample survey (Indagine Multiscopo)

TABLE 3 - Functionings' membership degrees (% values; aggregation by weighted averaging operator)

μ classes	HOUSING	HEALTH(*)	EDUCATION- KNOWLEDGE	SOCIAL INTERACTIONS	PSYCHOLOG. CONDITIONS
0,000-0,100	2,9	8,8	18,6	22,6	0,1
0,101-0,200	12,8	-	15,3	20,3	0,4
0,201-0,300	20,0	-	12,5	16,3	1,7
0,301-0,400	1,1	8,8	16,3	11,6	5,8
0,401-0,500	25,0	-	8,3	9,5	15,2
0,501-0,600	0,2	-	6,3	5,0	27,5
0,601-0,700	12,9	16,5	9,5	5,3	31,2
0,701-0,800	2,1	-	6,4	4,8	12,0
0,801-0,900	11,3	-	4,0	3,4	5,8
0,901-1,000	11,7	65,9	2,8	1,2	0,3
TOTAL	100,0	100,0	100,0	100,0	100,0
μ(x) average value	0,496	0,799	0,358	0,304	0,590

Source: own elaboration on Istat sample survey (Indagine Multiscopo)

() only one elementary indicator (chronic illnesses) with linear membership function*

TABLE 4 - Average membership degrees for each functionings by personal and social characteristics

	HOUSING	HEALTH(*)	EDUCATION- KNOWLEDGE	SOCIAL INTERACTIONS	PSYCHOLOG. CONDITIONS
SEX					
male	0,486	0,817	0,369	0,360	0,601
female	0,506	0,782	0,347	0,251	0,580
Total	0,496	0,799	0,358	0,304	0,590
AGE					
18-35	0,472	0,925	0,448	0,327	0,612
36-65	0,518	0,732	0,348	0,320	0,573
66 and over	0,672	0,452	0,199	0,198	0,566
Total	0,524	0,759	0,359	0,304	0,586
GEOGR. AREA					
North-West	0,509	0,787	0,411	0,310	0,611
North-East	0,580	0,811	0,406	0,336	0,623
Centre	0,540	0,775	0,364	0,312	0,589
South	0,421	0,814	0,284	0,275	0,559
Islands	0,431	0,803	0,321	0,284	0,561
Total	0,496	0,799	0,358	0,304	0,590
MARITAL STATUS					
single	0,447	0,926	0,455	0,331	0,622
married	0,498	0,740	0,337	0,304	0,580
divorced	0,659	0,754	0,459	0,324	0,561
widow/widower	0,726	0,473	0,189	0,183	0,549
Total	0,496	0,799	0,358	0,304	0,590
WORK STATUS					
employee	0,509	0,852	0,428	0,364	0,595
unemployed	0,402	0,901	0,404	0,289	0,577
housewife	0,488	0,717	0,260	0,193	0,560
student	0,439	0,958	0,585	0,363	0,647
pensioner	0,639	0,516	0,241	0,254	0,587
other	0,549	0,524	0,220	0,209	0,523
Total	0,517	0,769	0,358	0,304	0,590
OCCUPATIONAL GROUPS					
manager	0,591	0,855	0,669	0,469	0,610
office worker	0,527	0,867	0,554	0,412	0,610
blue-collar work.	0,458	0,861	0,294	0,322	0,591
other employee	0,459	0,835	0,360	0,302	0,595
self-employed	0,538	0,828	0,387	0,344	0,579
Total	0,509	0,852	0,428	0,364	0,595

Source: own elaboration on Istat sample survey (Indagine Multiscopo)

(*) only one elementary indicator (chronic illnesses) with linear membership function

TABLE 5 - Pearson correlation coefficients among elementary indicators

	μ_{11}	μ_{12}	μ_{21}	μ_{31}	μ_{32}	μ_{33}	μ_{41}	μ_{42}	μ_{43}	μ_{44}	μ_{51}	μ_{52}	μ_{53}	μ_{54}	μ_{55}
μ_{11}	1,000														
μ_{12}	0,082	1,000													
μ_{21}	-0,161	0,045	1,000												
μ_{31}	0,032	0,138	0,265	1,000											
μ_{32}	0,059	0,128	0,148	0,438	1,000										
μ_{33}	0,098	0,154	0,088	0,353	0,293	1,000									
μ_{41}	-0,087	-0,013	0,201	0,022	0,075	0,011	1,000								
μ_{42}	0,028	0,088	0,097	0,269	0,225	0,261	0,108	1,000							
μ_{43}	0,076	0,082	0,039	0,196	0,216	0,206	0,035	0,486	1,000						
μ_{44}	0,034	0,135	0,115	0,343	0,260	0,396	0,075	0,462	0,290	1,000					
μ_{51}	0,170	0,166	0,088	0,131	0,096	0,158	0,004	0,060	0,097	0,089	1,000				
μ_{52}	0,016	0,071	0,142	0,120	0,126	0,136	0,245	0,090	0,087	0,100	0,135	1,000			
μ_{53}	-0,173	0,044	0,622	0,245	0,172	0,116	0,278	0,126	0,062	0,149	0,128	0,291	1,000		
μ_{54}	0,078	0,091	0,107	0,124	0,080	0,145	0,057	0,057	0,085	0,083	0,363	0,244	0,199	1,000	
μ_{55}	0,080	0,050	0,039	0,020	0,093	0,085	0,228	0,031	0,029	0,034	0,134	0,324	0,134	0,200	1,000

Note: all coefficients are statistically significant at a 1% level

Source: own elaboration on Istat sample survey (Indagine Multiscopo)

TABLE 6 - Pearson correlation coefficients among functionings

	HOUSING	HEALTH	EDUCATION AND KNOWLEDGE	SOCIAL INTERACTIONS	PSYCHOLOGICAL CONDITIONS
HOUSING	1,000				
HEALTH	-0,156	1,000			
EDUCATION/KNOWLEDGE	0,077	0,247	1,000		
SOCIAL INTERACTIONS	0,061	0,123	0,412	1,000	
PSYCHOLOGICAL COND.	0,108	0,257	0,252	0,173	1,000

*Note: all coefficients are statistically significant at a
1% level*

Source: own elaboration on Istat sample survey (Indagine Multiscopo)

TABLE 7 - An overall well-being assessment. Average membership degrees by personal and social characteristics

	STANDARD UNION OPERATOR	STANDARD INTERSECTION OPERATOR	UN-WEIGHTED AVERAGING OPERATOR
SEX			
male	0,911	0,232	0,552
female	0,901	0,190	0,520
Total	0,906	0,210	0,536
AGE			
18-35	0,957	0,224	0,560
36-65	0,870	0,175	0,499
66 and over	0,805	0,080	0,417
Total	0,891	0,178	0,508
GEOGR. AREA			
North-West	0,907	0,216	0,545
North-East	0,922	0,246	0,575
Centre	0,898	0,215	0,539
South	0,900	0,185	0,507
Islands	0,899	0,191	0,513
Total	0,906	0,210	0,536
MARITAL STATUS			
single	0,958	0,284	0,606
married	0,871	0,166	0,492
divorced	0,917	0,210	0,553
widow/widower	0,837	0,081	0,424
Total	0,906	0,213	0,536
WORK STATUS			
employee	0,924	0,223	0,551
unemployed	0,940	0,186	0,519
housewife	0,861	0,110	0,445
student	0,973	0,262	0,600
pensioner	0,818	0,110	0,447
other	0,803	0,010	0,409
Total	0,896	0,180	0,512
OCCUPATIONAL GROUPS			
manager	0,950	0,320	0,639
office worker	0,938	0,272	0,595
blue-collar work.	0,919	0,175	0,508
other employee	0,910	0,182	0,514
self-employed	0,912	0,206	0,536
Total	0,924	0,223	0,551
TOTAL	0,906	0,210	0,536

Source: own elaboration on Istat sample survey (Indagine Multiscopo)