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The purpose of the paper is to explore the potential for using fuzzy logic to analyse economic decision-making under Keynesian uncertainty, and in particular in circumstances where variety of opinion is important. Fuzzy logic is shown to apply where expectations may differ because the nature of the subject matter impedes any 'crisp' way of describing the underlying variables. The particular case of the speculative demand for money is considered, since it explicitly reflects variety of opinion as to whether interest rates are 'high' or 'low'.

Keywords: fuzzy logic; liquidity preference; diversity of opinion
JEL codes: B41; C0; E41; E50

Introduction

The focus of this paper is on the analysis of uncertainty, in the Keynesian or Knightian sense of unquantifiable risk. This understanding of uncertainty has been central to a range of non-mainstream literatures, including the Keynesian and Austrian literatures (Shackle 1952, 1955; O'Driscoll and Rizzo 1985; Runde and Mizuhara 2003) and Knightian analysis (Knight 1933; Bewley 1998). Increasingly, mainstream economics has also attempted to incorporate uncertainty in a variety of ways. Yet, since mainstream methodology requires formal mathematical expression and the potential for economists and rational economic agents to quantify variables, it would seem that the scope for incorporating uncertainty is, by its own definition, very limited. The purpose of this paper is to explore the scope for drawing on the branch of mathematics based on fuzzy logic, in order to address economic behaviour uncertainty from a different angle and to suggest a new tool for conceptualising and analysing it.

One of the consequences of uncertainty which we will address in terms of fuzzy logic is that, in the absence of any one true, agreed account of the causal mechanisms underpinning economic events, expectations will vary.[1] Variety of opinion is an everyday fact of life. To attempt to incorporate this consequence of uncertainty in mainstream economics would require an explanation in terms of rational choice applied to given information sets. Rational choice theory suggests that, with identical information sets, rational agents will have identical expectations. Rational expectations theory allows for different models which can yield different expectations, but these too can only be accounted for ultimately by different information sets. In practice, expectations formation requires the intermediate step of interpreting the data, where interpretation opens up scope for difference. While it

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is common in much of the economics literature to conflate information with knowledge, we use the term 'knowledge' here to capture, among other things, the additional element of interpretation.[2]

The concept of fuzziness is one that has been developed in the logic and mathematics literature as well as in a number of other subject areas. Though fuzzy logic has been applied to some branches of economics (e.g. Basu 1984, 1987), and to Keynes's philosophical theory of probability (e.g. Cooke 2004), it has rarely been applied to economic decision-making under Keynesian uncertainty. Where it has been applied is in the context of unobservable variables in industrial economics (McCain 1987). We will be concerned here rather with the meaning attached to an observable variable, the interest rate, as the basis for expectations and for behaviour.[3]

The scant attention paid to the scope for applying fuzzy logic to uncertainty is surprising, given that it is now commonplace to draw implications for economics from a philosophical treatment of uncertainty. For example, Hamouda and Rowley (1996, p. 117) argue with respect to Keynes's *Treatise on Probability* that he

> considered that some probabilities are not comparable while other probabilities are not numerically measurable and non-additive. These limitations remove numerically probabilities from being the effective guides for the conduct of economic agents … This rejection of basic mechanical calculations for characterizing actual choices and actions pervades *The General Theory*, in which most businessman are assumed to be influenced by rational beliefs, intuition, general confidence, speculative anticipations about the 'psychology' of markets, personal circumstances, simple conventions and various imprecisions.

That fuzzy logic has not been applied in this way is surprising further since the related concept of vagueness has been given more extensive treatment, drawing particularly on the work of Marshall and Keynes. Qizilbash (2003) has argued that fuzzy logic is inadequate for analysing the richer concept of vagueness in the context of measuring poverty. But here we argue that logic/mathematics can usefully be combined with the insights from the analysis of vagueness to provide a framework within which to analyse the particular issue of variety of opinion.[4]

We demonstrate the scope for applying the concept of fuzziness in terms specifically of Keynes's theory of the speculative demand for money, since it is explicitly premised on variety of opinion.[5] We are therefore not concerned here with other motives for holding money, and thus with the demand for money overall. While we focus here on variety of opinion about the bond market, the analysis could in principle be extended to opinion about financial conditions more generally, and indeed to other contexts of decision-making where variety of opinion also pertains. The narrower context here is employed in order to explain the conceptual issues as clearly as possible.

Speculative demand refers to decision-making despite the conditions for certainty not being met, where a positive opinion is formed as to whether to hold money or bonds, and action taken accordingly. The focus of the speculator is on expectations about the future path of the interest rate. This requires a positive judgement as to whether the current rate may be 'high' or 'low', and thus whether a reversal can be expected, a matter over which there will in general be variety of opinion. As Davidson (1978, p. 201) makes clear, such variety of opinion accounts for the level of asset market participation, but is also necessary for the market stability which ensures that money is a stable asset.

The understanding of which rate is to be regarded as 'high' and which 'low' is clearly critical, and it is here that we argue that the concept of fuzziness has direct application. Fuzzy logic thus serves to enhance our understanding of a central concept in Keynesian monetary theory. The next section therefore revisits the speculative demand for money, while the third shows how the concept of fuzzy sets applies.

The speculative demand for money

The shape of the liquidity preference schedule – the inverse relationship between the total quantity of the demand for liquidity and the rate of interest – is one of the fundamental relationships in Keynesian economics (Davidson 2002, chap. 5). The standard orthodox textbook treatment of speculative demand (as in Dornbusch and Fischer 1999, pp. 365–366) bases it explicitly on Tobin's (1958) classic expression of portfolio theory, suppressing the role of diversity of opinion.

The concept of a *normal* rate of interest is the key to the determination of the smooth, continuous shape of the liquidity preference schedule (Chick 1983, chap. 10). This is the rate by reference to which speculators form their expectations, for comparison with the current rate.[6] One way of looking at it is to consider it to be the perceived long-run equilibrium rate, where that equilibrium refers to persistence over time rather than the formal solution to a general equilibrium model. However, '"[t]he normal rate" was not even a single rate, but the collection of entirely *subjectively-determined*, personal normal rates' (Chick 1983, p. 228, emphasis added). Keynes put the focus less on the level of the normal, or 'safe', rate than on divergence from that rate: 'what matters is not the *absolute* level of *r* but the degree of its divergence from what is considered a fairly *safe* level of *r*, having regard to those calculations of probability which are being relied on' (Keynes 1936, pp. 201–202, emphasis in original).[7]

The normal rate determines expected capital gain or loss, so speculators buy bonds when the actual rate is perceived to be high relative to the normal rate, and sell bonds when it is perceived to be low relative to the normal rate. Since there is no basis for this expectation in demonstrable certainty, the speculative demand for money is potentially volatile.[8] It will depend both on variety of opinion and on the degree to which speculators are willing to act (Robinson 1978, p. 43).

In his account of the role of diversity of opinion in Keynes's theory of liquidity preference, Tobin (1958) put the focus on the precise value of what he termed the *critical* rate, a (subjective) equilibrium rate in the standard general equilibrium sense. In his own theory, Tobin moved away from diversity of opinion to assuming a common (subjective) probabilistic expectation of risk (Tobin 1958, p. 71). The difference between the approaches of Keynes and Tobin lies in the basis for the nature and source of the estimations of probability on which the decision to buy or sell bonds is based (see further Chick 1983, pp. 214–218).[9]

Statistical probability theory attempts to reduce uncertainty about economic relations to quantifiable risk, the characteristic of these relations being understood as stochastic. However, in both the type of probability which is based on set theoretic considerations (Kolmogoroff 1950), and the type which refers to the truth value of statements (Koopman 1940), it is assumed that the events (elements of sets, or statements, respectively) are not only all identified in advance but also are well defined (Zimmerman 1985, chap. 1). In the case of liquidity preference, we have to have a clear idea not only about the normal rate, but also about the frequencies at which rates higher or lower than this normal rate are observed in the economy, including the extent of deviation.

Tobin's (1958, note 12) analysis drew explicitly on the work of Savage (1954), who argued that, even when it is impossible to construct an objective probability distribution, people do manage to make decisions by maximising (subjective) expected utility in light of the axioms of probability. But, in order for (objective or subjective) numerical probabilities to be derived, the subject matter must satisfy certain conditions; these are the conditions for a closed system (Chick and Dow 2005). The economic structure must be (or be understood to be) such as to yield (potentially even if not actually) objective probability distributions which can serve as a guide to the future (see further Runde 1995). Where these conditions do not hold, then the general case will be variety of opinion. Indeed, as we have seen already, the stability of the system and its sensitivity to change in monetary conditions are crucially dependent on the existence of variety of opinion about what is uncertain. Although mass psychology may at times limit variety, there is normally some range of opinion about which market rate of interest is the 'highest' relative to the 'normal' or 'safe' rate, in the sense that the next direction of the rate is confidently expected to be in the upward or downward direction.[10]

The conception of the normal rate, and its distribution, is unlikely to be precise. But, since action is taken at a discrete point in time in response to the interest rate reaching a specific level, individuals must have point values in mind at that time for what they regard as 'high'. What is important for us to understand here is that what is high (low) for one speculator is not high (low) for the others. This contrasts with Tobin's (1958) approach where there is only one rational expectation in given circumstances, and only one rational action given a degree of risk-aversion.

The critical issue is how to conceptualise 'variety of opinion about what is uncertain'. The categories involved are fuzzy rather than crisp. Lofti A. Zadeh originated the fuzzy logic framework in 1965, which has proved to have greater potential than Shackle's framework as an alternative means of dealing with these situations.[11] As Hamouda and Rowley (1996, pp. 72–73) point out, Zadeh 'holds that conventional quantitative techniques are intrinsically unsuited for dealing with complex humanistic situations'. The potential for applying fuzzy logic is also evident in Davidson's (1988, p. 333) argument that decision makers hesitate in making their decisions and 'the desire to postpone commitments *varies with a person's assessment of the current situation and perception of the uncertain future*' (emphasis added).

Our purpose here is to consider fuzzy logic as a framework for conceptualising aspects of the Keynesian analysis of liquidity preference where

> the individual, who believes that future rates of interest will be above the rates assumed by the market, has a reason for keeping actual liquid cash, whilst the individual who differs from the market in the other direction will have a motive for borrowing money for short periods in order to purchase debts of longer term. (Keynes 1936, p. 170)

The smooth negatively sloped liquidity preference schedule for the whole economy is an envelope of the agents' individual liquidity preference functions which are L-shaped. In other words each individual's speculative demand for money represents a choice between holding either money or bonds, but not both at the same time. Each individual chooses to hold money when the interest rate is judged to be 'low' and bonds when the interest rate is judged to be 'high'. Extending the framework to other assets, and taking account of developments in the financial sector, would suggest a more complex account (see for example Chick 1983, pp. 209–210). But we focus here on a more straightforward framework in order to provide a more clear, preliminary, account of how fuzzy logic might be applied to analysing Keynes's concept of uncertainty.

Fuzziness and variety of opinion

We start with the question, how high is 'high'? Even though the conditions are not met for constructing a probability distribution of interest rates, we are concerned with that (numerical) level of the interest rate which triggers action. The 'uncertainty as to the future rate of interest' Keynes talked about is due substantially to different agents estimating the prospect of the interest rate being 'high' differently. It is not (just) that there are different subjective estimates of the probability of a high rate, and different degrees of confidence in those estimates, but different understandings of what that 'high' rate actually is. In this context, we can understand that, quite apart from other sources of uncertainty, uncertainty here also results from the *vagueness* of the term 'high' in terms of identifying its empirical counterpart.

The concept of vagueness is one that has been given extensive treatment in Keynesian analysis (see Coates 1996; see also Davis 1999). Following Wittgenstein, vagueness of language refers to the clusters of meaning attached to ordinary language, contrasted with the precise singular meaning presumed to attach to formal language. There is a direct counterpart with fuzzy logic, where sets (e.g. of meaning) lack clear boundaries. But here our concern is more with vagueness in terms of empirical counterparts; even if there were a singular meaning of the term 'high', there would still be scope for variety of opinion as to which interest rates fell into that category. The two sources of vagueness are interconnected by their ontological foundation in what gives rise to expectations. As Davis (1999, p. 511) argues, vagueness of language for Keynes arose from the 'complexities and interdependencies of the real world' (its openness), which also account for variety of opinion as to empirical counterparts. Fuzzy logic thus addresses vagueness as to empirical meaning as well as to conceptual meaning.

Qizilbash (2003) sees fuzzy logic as contributing to our understanding of poverty by representing degrees to which a statement as to a measure of poverty is true. But he argues that fuzzy logic, as an exercise in measurement, falls short, in that there is a higher-order vagueness as to the whole notion of measuring poverty, still less measuring the degree to which something is a true measure of poverty. But the vagueness of language and of empirical counterparts that we are concerned with here refers to variety in the formation of expectations about a specific quantity, the rate of interest. The quantification involved in fuzzy logic is therefore less problematic, as we will explain below.

The theory introduced by Zadeh (1965) to deal with uncertainty in the sense of vagueness is therefore distinct from the concept of probability. In this seminal paper, Zadeh introduced the concept of a *fuzzy set*, in contrast to the classical concept of a set, which is sometimes distinguished by the term '*crisp set*'. While the boundaries of a crisp set are required to be precise, the boundaries of a fuzzy set are not sharp or precise. As Zadeh (1965, p. 339) puts it:

> The notion of a fuzzy set provides a convenient point of departure for the construction of a conceptual framework which parallels in many respects the framework used in the case of ordinary sets, but is more general than the latter and, potentially, may prove to have a much wider scope of applicability … Essentially, such a framework provides a natural way of dealing with problems in which the source of imprecision is the absence of sharply defined criteria of class membership rather than the presence of random variables.

Zimmermann (1985, p. 5) reinforces the distinction between fuzziness about the object, potentially, of probability analysis and probability analysis itself:

> 'Imprecision' here is meant in the sense of *vagueness* rather than the lack of knowledge about the value of a parameter as in tolerance analysis. Fuzzy set theory provides a strict mathematical framework … in which vague conceptual phenomena can be precisely and rigorously studied.

Let us try to elaborate. We take the example of an ordinary die. The number of dots on each side of the die belongs to a crisp set. The boundaries of this set are strictly defined – these are whole numbers from one to six. If we come across number of dots of, say, seven or nine we can say categorically that these dots are not members of the crisp set whose boundaries lie between one and six. In other words, after counting the number of dots, we can decide definitely whether these are members of a set represented by the number of dots on a regular die. Being a member of a fuzzy set, however, is not a simple matter of being definitely in or definitely out: *A member may be inside the set to a greater or lesser degree*. For example, the concept of a 'crowd' in a gathering includes a wide range of numbers of people *depending on the context*. In addition to context as understood by the social scientists, there is further the context (or background) of the subject matter.

Similarly, consider the set, COLD, of many different temperature readings. In a given context, say a weather report, we might consider different possible temperature readings, in centigrade, such as $-35°$, … $-15°$, $-10°$, $-5°$, $0°$, $5°$, $10°$, $15°$, …, $35°$ and ask to what extent each reading is compatible with the concept 'cold'. Depending on our background and the specifics of the context – place, season, day, night, whether windy or not, etc. – we might say that $-15°$ definitely belongs inside the set COLD. It is likely that we would make the same sort of judgement for $-5°$, $0°$, maybe even for $5°$, though this is not certain for every situation. However, a temperature of $15°$ would not appear to all of us to be clearly in this set. For those accustomed to living in the Arctic region it may, perhaps, even be warm. But for someone living in a tropical country $15°$ is definitely rather on the cold side. Accordingly, saying that the weather 'is cold' is at least somewhat true for the temperature $10°$; certainly it is not clearly true and it is not clearly false. However, even for an inhabitant of the Arctic, $-35°$ should be on the cold side while $35°$ is definitely not cold for someone from a tropical country. For each individual concerned, the answer may be clear-cut as to how to classify temperatures. The problem of aggregating assignments arises from variety of opinion. Because $15°$ and other temperatures are in the set COLD to various degrees, this set is a fuzzy set.

Probability theory and fuzzy sets address different kinds of uncertainty. One way to describe the difference is to say that (quantitative) probability theory deals with the *expectation* of some event, on the basis of something known now. Uncertainty then arises where the conditions are not met for quantifying probability. The sense of uncertainty represented by fuzziness, however, is not the uncertainty of the actual formation of the expectation at the individual level. It is the uncertainty as to the nature of the subject matter (in the past as in the future) which is reflected in the imprecision of meaning of a concept expressed by a linguistic term in natural language, such as 'very high', 'high', 'low', 'very low', etc., where a precise assignment to sets is required. Another important way to draw the distinction between probability and fuzzy sets is to point out that quantitative probability theory is the theory of random events, so that it is concerned with the likelihood of the relevant event (or assignation to a particular set). Fuzzy set theory, on the other hand, is not concerned with events at all. It is concerned with concepts.[12]

Novak (2005) expresses the distinction as being between uncertainty, which is addressed by probability theory, and vagueness addressed by fuzzy logic. In fact traditional probability theory is not capable of dealing with uncertainty when it arises from vagueness, in the sense of either meaning or measurement. But, taking Novak's distinction as being between vagueness on the one hand, and uncertainty (without vagueness) on the other: 'Unlike uncertainty where we always have to consider whether some phenomenon *occurs or not*, vagueness concerns the way how the *phenomenon itself* is delineated, no matter whether it should occur or not' (Novak 2005, pp. 343–344, emphasis in original). But uncertainty in its wider sense, which includes vagueness, cannot provide the necessary ingredient of probability theory, which is a well-defined set of possibilities.

We can define the fuzzy set COLD as described above by assigning to each temperature a number between 0 and 1, which indicates the *degree* or *grade of their membership* in the set. The assignment of grade 0 to a particular temperature (say, 20°) means that this temperature definitely does not belong to the set COLD. While the assignment of 1 means that the temperature definitely belongs to the set (say, a temperature of −20°). Assignment of a grade membership between 0 and 1 implies the degree of the particular temperature belonging to the set. A temperature assigned a grade closest to 1 implies that it is highly likely for this particular temperature to belong to the set. Obviously as the degree of membership assigned approaches 0, the likelihood of the temperature belonging to the set diminishes. We are using the term 'likelihood' here in a broader sense than quantifiable probability, allowing for cases where the sum of likelihoods need not be constrained to one.

The conditions where probability cannot be quantified include those where the full range of possibilities is not known.[13] This was for Shackle (1952, 1968) a general case, which he analysed as being better handled in terms of whether or not the agent was surprised, rather than how high was the numerical probability. Potential surprise measures the possibility of an event; when an event is perfectly possible the degree of potential surprise assigned to it is zero. However, 'there is, in general no limit to the number of mutually exclusive hypotheses to all of which simultaneously a person can, without contradiction, attach zero potential surprise' (Shackle 1952, p. 31). See further Hamouda and Rowley (1996, chap. 4).

It is irrelevant to the temperature example whether or not the assignation of membership is objectively established or not; what impedes the quantification of probability is the absence of sets that are well defined in the sense that the only possibilities are membership, or not, of the set, such that the probability refers to the probability of membership, or not. In a fuzzy set, the identification of membership is not crisp, but is itself the subject of a distribution. The assignment of a grade membership in the fuzzy set COLD to each considered temperature is called a *membership grade function* of this fuzzy set. We can thus see that each set is uniquely determined by a particular membership grade function, which assigns to each object of interest its grade of membership in the set. Although it is not necessary, the convention in the literature on fuzzy sets is to express the grade membership by numbers between 0 and 1 (Klir and Folger 1988, chap. 1, sec. 3)[14].

Let us consider the standard example of a membership structure of a committee, a case where membership is a matter of rules rather than opinion (Ragin 2000, p. 156). A person can be either a member of the committee or not a member of the committee. We assign a score of 1 when the person is 'in' and a score of 0 when the person is 'not in'. This is an example of a crisp set. However, now consider that the committee has different grades of membership. These cover the range of full members, associate members, part-time members, observers and non-members. The membership grades one can assign to persons belonging to each of these categories can be as shown in Table 1.

The above is an example of what is called a five-value fuzzy set. By extending the principle we can describe a 'continuous' fuzzy set.

We can now think in terms of a continuous fuzzy set from the point of view of the community of speculators in an economy. In Table 2 below we show an example of the membership grades which refer to the community of speculators or are reflected in the behaviour of speculators in a particular set of circumstances, or set of contexts. The interest rate is the nominal rate. In this example, when the interest rate is 3%, the interest rate is 'high' in the opinion of 10% of the speculators, giving it a membership grade (an M-grade) of 0.1, and so on.

From Table 2 we can see that only at an interest rate of 9.5% and above does the opinion of the community converge that this rate is 'high'. Each individual, or group of individuals, arrives at an interpretation based on their background such that a particular rate would be regarded by them as 'high'. Since uncertainty prevents the identification of a critical rate, in Tobin's terms, against which any deviation of the actual rate can be understood as causing the rate to be 'high' or 'low' within a probability distribution around the critical rate, there is scope for variety of opinion. The set of opinions as to whether a particular rate is high is thus a fuzzy set.

While this is a hypothetical example, a fuzzy set can be identified empirically. According to Watanabe (1979, p. 589) the techniques for construction of membership functions can be divided into two broad categories.[15] These are (1) use of frequencies; and (2) by direct estimation. The frequency method, which we have used in assigning the membership grades in our hypothetical example above, assigns the membership grades by measuring the percentage of people in a group (which could be a sample of savers/investors or experts in the field) who answer yes to a question about whether a specific rate of inflation belongs to the set 'high' ('low').

Table 1. Committee membership as a fuzzy set.

Category	M-grade	Description
Full member	1	fully in
Associate member	0.75	more in than out
Part-time member	0.50	cross-over, neither in nor out
Observer	0.35	more out than in
Non-member	0	completely out

Table 2. Opinion that the interest rate is 'high' as a fuzzy set.

Rate of interest	M-grade
1.0%	0
1.5%	0
2.0%	0
2.5%	0
3.0%	0.1
3.5%	0.15
4.0%	0.25
4.5%	0.35
5.0%	0.45
6.0%	0.5
7.0%	0.6
7.5%	0.7
8.0%	0.8
8.5%	0.85
9.0%	0.9
9.5%	1
10.0%	1
12.5%	1
15.0%	1

We have plotted the membership function corresponding to Table 2 in Figure 1. As we can see, it looks like the mirror image of a liquidity preference schedule. For interest rates less than 3%, the whole community wants to hold only money, while at an interest rate of 9.51% or above the community wants to hold only bonds. When the interest rate is 7.5%, 70% of speculators want to hold cash and the remaining 30% want to hold bonds, and so on. Of course, by selecting smaller intervals of increase in the rate of interest, the function depicted could be made smoother.

Conclusion

We have presented the argument that Keynes's analysis of the speculative demand for money can usefully be understood in terms of fuzzy sets. Indeed, the concept of

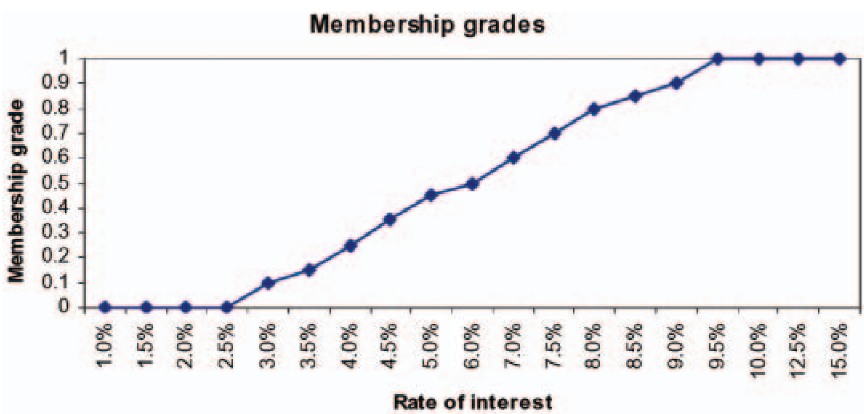

Figure 1. Opinion that the rate of interest is 'high' as a fuzzy set.

fuzziness builds on the understanding of vagueness which has already been developed in the Keynesian literature. It could even be argued that the concept of fuzziness is implicit in Keynes's theory of uncertainty more widely. But we have restricted our attention here to the specific issue of variety of opinion as it appears in the theory of speculative demand for money.

We have therefore not attempted here to discuss the full range of Keynesian uncertainty, or of liquidity preference, to which the more general concept of vagueness applies. Rather we have focused on a situation where decision-making requires that a stand be taken on whether the interest rate is 'high' or 'low'. Since, given uncertainty, a range of stands can be taken simultaneously, we have seen that the resulting diversity of opinion can be understood as a fuzzy set. While the context of application, speculative demand for money, has for expositional purposes been narrow, there is scope for much wider application in terms of behaviour in financial markets more generally, particularly at a time when there is variety of opinion as to whether asset prices, and official interest rates, are or are not 'high'.

Such variety is one of the implications of considering the economy as an open system that cannot, in general, yield certain knowledge or quantifiable risk, but where decision-making requires some provisional closure to justify action. Keynes had shown that rationality alone (based on classical logic) cannot justify action where the social system is such that knowledge is in general held with uncertainty. Human logic, rather, includes a range of devices employed for coping with this uncertainty. We have seen here that fuzzy logic provides an analytical device for conceptualising the variety of opinion which is a natural outcome of human logic.

Notes

1. The basis for variety of opinion in economics is explored in Dow (2007).
2. We will discuss also the sense in which 'knowledge' reflects the background of the 'knower', drawing on Searle (1999); see also Dow (1998).
3. A rare use in monetary economics is Chick's (1983, p. 202) reference to the boundary between speculators and other wealth holders as 'fuzzy'.
4. While the ambiguity literature might be thought to be relevant (as in Camerer and Weber 1992, for example), the emphasis there is on information about probability distributions which is either known or not known, but always knowable, not on knowledge as we are using the term.
5. We show how fuzzy logic can be applied to another situation of variety of opinion, monetary policy-making, in Dow and Ghosh (2007).
6. Chick (1983, pp. 228–229) points out that Keynes did not provide a theory of the normal rate.
7. Keynes (ibid., p. 197) inferred evidence of the smoothness of the liquidity preference function from the capacity of the Bank of England to successfully conduct open market operations. What gives rise to this shape is the fact that each individual potentially has a different opinion of the normal rate. In Keynes's words: '[O]pinion about the future of the rate of interest may be so unanimous that a small change in present rates may cause a mass movement into cash. It is interesting that the stability of the system and its sensitiveness to changes in the quantity of money should be so dependent on the existence of a *variety* of opinion about what is uncertain' (ibid., p. 172, emphasis in original).
8. It is possible that the use of the word 'expectation*s*' by Keynes in a statement like 'so also *expectations* as to the future of the rate of interest as fixed by mass psychology have their

reactions on liquidity-preference' (Keynes 1936, p. 170, emphasis added) has caused some misunderstanding in the literature, as in Ackley (1961 p. 179, n. 6), for example.

9. In Chapter 12 of the *General Theory* Keynes said 'The state of long-term expectation upon which our decisions are based, does not solely depend, therefore, on the most probable forecast we can make. It also depends on the *confidence* with which we make this forecast' (ibid., p. 148, emphasis in original). Later in the same chapter he goes on to say 'Nor can we rationalise our behaviour by arguing that to a man in a state of ignorance errors in either direction are equally probable, so that there remains a mean actuarial expectation based on equi-probabilities. For it can easily be shown that the assumption of arithmetically equal probabilities based on a state of ignorance leads to absurdities' (ibid., p. 152).
10. If confidence in expectations is too low to form such an opinion, then a speculative act does not take place; then the relevant motive is precautionary demand, and Savage's axiom cannot apply (see Runde 1994, 1995).
11. As Hamouda and Rowley (1996, pp. 84–85) argue: 'Formal structures play a much more important role in Zadeh's approach to possibility theory, which also gives greater attention to communication and the fuzziness of language than is exhibited by Shackle. Thus, approximate reasoning as partially characterised by possibilities is markedly different for the two alternative schools of imprecise thought which are associated with potential surprise and fuzzy concepts. They share major objections to the ubiquity of probability for complex (human) situations, but reveal two distinct responses to their dissatisfaction with earlier conventions. Both offer firm realistic justifications for their substantial interest in the significance of imprecision, but the rapid development of fuzzy mathematics was more effective (and more actively sought) than any formalization of potential surprise and related concepts beyond the initial geometry offered by Shackle' (parentheses in original).

They then point out that while Shackle's treatment of possibilities from a kaleidic perspective remained close to its original conception, Zadeh's approach has much wider application in both theoretical and empirical literature.

12. An alternative approach is to draw on the theory of imprecise probabilities, as advocated by Weatherson (2002), on the grounds that it best captures Keynes's intuition about uncertainty. In imprecise probability, an agent's credence (belief) in a proposition *p* is vague over a set of values. We are using the term credence in the sense Weatherson has used it. Gerrard (1995, p. 194) uses the term credence as 'a measure of the degree of absolute belief'. He has derived the concept of credence from Keynes's concept of 'weight' or 'confidence'. However, Keynes wrote that in our decision-making we are 'guided to a considerable degree by the facts about which we feel *somewhat* confident'. (Keynes 1936, p. 148, emphasis added). In Weatherson's example, 'for some rational agents and some proposition *p*, the agent's epistemic state will determine that they [*sic*] believe *p* to a greater degree than 0.2, and a lesser degree than 0.4, but there will be no more facts about the matter' (Weatherson 2002, p. 49). This notion of imprecise probability is consistent with the general discussion of uncertainty in Keynes as unquantifiable risk (as in Carabelli 1995, for example), where the conditions are not met for frequency distributions as a basis for quantifying probability. But the concept of imprecise probability is not so directly applicable to the basis for liquidity preference in expectations with respect to a variable which itself is imprecise. It is the latter to which fuzzy set theory is addressed.
13. In contrast, conceptualising unknown variables as stochastic shocks reflects, not uncertainty, but knowledge that anything unidentified is stochastic.
14. Formally stated, each fuzzy set A is defined in terms of a relevant universal set, X, by a function called a membership function. A membership function assigns to each element x of X a number A(x), in the closed unit interval of [0, 1] that characterizes the degree of

membership of x in A. Membership functions are thus functions of the form: A: X → [0, 1]. In defining a membership function, the universal set X is always a classical set.

15. See Turksen (1991) for a review of these two as well as other methods of constructing membership functions.

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