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Abstract The lack of support for contradictions in fsQCA limits the method's usefulness for conducting inductive research. In this paper, I describe how to extend fsQCA to accommodate contradictory conditions. I review Kirq (Reichert and Rubinson 2011), a new software package for QCA that includes support for fuzzy-set contradictions. For researchers using software that does not support fuzzy-set contradictions, I describe how to identify them by hand.

**Keywords** Qualitative comparative analysis  $\cdot$  Fuzzy sets  $\cdot$  Contradictions  $\cdot$  Comparative research  $\cdot$  Kirq

### 1 Introduction

In almost all respects, fuzzy-set qualitative comparative analysis (fsQCA) is far superior to its older crisp-set siblings. Because many of the phenomena that interest social scientists vary by level or degree, the dichotomization required by crisp-set QCA (csQCA) frequently entails a loss of information. Multi-valued QCA (mvQCA) attempts to ameliorate this situation through the use of multichotomies but researchers are still forced to shoehorn interval and ratio-level measures into a limited number of discrete categories. In contrast, fsQCA permits membership scores across the 0.0–1.0 interval, avoiding lost information and providing for a more nuanced analysis.

There is, however, one area in which fsQCA falters: its support for inductive analysis. Among QCA's distinctive strengths is its usefulness for conducting case-oriented, inductive research. But in comparison to csQCA, <sup>1</sup> fsQCA falls short because it does not support contradictory conditions. In csQCA, the presence of contradictions indicates the possibility of

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<sup>&</sup>lt;sup>1</sup> For the remainder of this paper, my use of the term "csQCA" encompasses both csQCA and mvQCA as both variants make use nominal-level measures.

an insufficiently specified model and the process of resolving them is an important aspect of inductive investigation. fsQCA's lack of support for contradictions is therefore a significant limitation.

In this paper, I describe how to incorporate contradictions into fsQCA. I begin by explaining what contradictions are and their role in conventional csQCA. I then review the historical development of fsQCA that led to the elimination of contradictions. In short: In order to accommodate fuzzy-sets, Ragin needed to develop a new technique for constructing truth tables. But this technique, based upon a measure of *consistency*, does not produce contradictions as in crisp-set QCA. In Sect. 4, I describe how to extend fsQCA to incorporate the identification of contradictions. This technique, which also relies upon the consistency measure, derives directly from Ragin's process of truth table construction using fuzzy sets and introduces two new measures: *observation consistency*, which measures the degree to which an individual observation exhibits consistency with sufficiency, and the *consistency proportion threshold*, which operationalizes contradictions as a a ratio of consistent to inconsistent observations.

Because this technique derives directly from Ragin's process of truth table construction using fuzzy sets, it is relatively straightforward to incorporate it into existing fsQCA software. In Sect. 5, I review Kirq (Reichert and Rubinson 2011), a new software package for QCA that already includes this functionality. For researchers using software that lack this feature, I describe how to manually identify contradictions. I conclude with a discussion of the benefits of incorporating fuzzy-set contradictions into QCA.

## 2 Contradictions in csQCA

Social researchers use QCA to answer questions of causal necessity and sufficiency. At the heart of this research is the construction and analysis of truth tables, which show the connections between different combinations of conditions and an outcome. Each row of a truth table represents one logically possible combination of causal conditions and a complete truth table possesses  $2^k$  rows, where k equals the number of causal conditions.

As an example, consider Brown and Boswell's (1995) analysis of strikebreaking during the 1919 U.S. steel strike. Brown and Boswell (1995) wanted to identify the conditions that determined whether a city's black working class sided with the union or crossed the picket line as strikebreakers. Drawing upon split labor market theory and their own case studies of cities that participated in the strike, Brown and Boswell (1995) hypothesized that black strikebreaking would occur in cities that (1) had recently experience high rates of black in-migration and (2) had a history of union defeats.

As depicted in Table 1, a truth table with two causal conditions possesses four (2<sup>2</sup>) rows. The presence of a causal condition is represented by a "1" in the table; its absence, by a "0." Brown and Boswell (1995) found that those cities with a high rate of black migration and a weak union (row 1) also experienced black strikebreaking. In contrast, black strikebreaking did not occur in cities without high rates of black migration (rows 3 and 4). Row 2 is a contradictory row. Although the cities it represents exhibit an identical causal configuration (a high rate of black migration combined with a weak union), they differ in terms of the outcome condition. Buffalo, Chicago, Gary, and Johnstown all experienced black strikebreaking. Cleveland did not.<sup>2</sup>

<sup>&</sup>lt;sup>2</sup> I indicate inconsistent observations (observations that do not exhibit the outcome) by placing them within (parentheses) in Table 1, a convention that I will follow throughout this paper.



**Table 1** Brown and Boswell's initial truth table for the presence of black strikebreaking, derived from Brown and Boswell (1995, p. 1502, Table 4)

	M	U	Y	Cases
1	1	1	1	East Chicago, Pittsburgh, Youngstown
2	1	0	C	Buffalo, Chicago, Gary, Johnstown, (Cleveland)
3	0	1	0	(Bethlehem, Joliet, McKeesport, Milwaukee, New Castle,
				Reading)
4	0	0	0	(Decatur, Wheeling)

Note: M recent black migration, U weak union, Y black strikebreaking

**Table 2** Brown and Boswell's final truth table for the presence of black strikebreaking, derived from Boswell (1995, p. 1505, Table 5)

	M	U	R	Y	Cases
1	1	1	1	1	East Chicago, Pittsburgh, Youngstown
2	1	1	0	_	
3	1	0	1	1	Buffalo, Chicago, Gary, Johnstown
4	1	0	0	0	(Cleveland)
5	0	1	1	0	(Bethlehem, Joliet, McKeesport, New Castle, Reading)
6	0	1	0	0	(Milwaukee)
7	0	0	1	0	(Decatur)
8	0	0	0	0	(Wheeling)

Note: M recent black migration, U weak union, R political repression, Y black strikebreaking

One of the underlying assumptions of the comparative method is that similar observations should behave in a similar manner. That workers in Cleveland achieved interracial solidarity suggests that Cleveland differed from the other four cities in some way. In order to resolve contradictions, researchers reexamine both their theory and their cases. It may be that measurement error is to blame or that scope conditions were improperly specified. More commonly, the researcher identifies one or more additional causal conditions that explain the contradictory case(s). Brown and Boswell (1995) examined three contextual variables that they suspected might affect whether blacks sided with the union: city size, plant ownership, and local government repression. Only the inclusion of the third condition produced a truth table free of contradictions (Table 2). Returning to their case studies, Brown and Boswell (1995) found that the disposition of local governments was an important predictor of whether unions would be able to organize a cross-racial labor coalition. Pro-industry governments prohibited union meetings, restricted free speech, and physically intimidated union organizers and members. Pro-union governments, on the other hand, intervened on behalf of the union and Cleveland's mayor acted to prevent imported strikebreakers from crossing the picket line.

This is how contradictions promote inductive analysis. In order to make sense of Cleveland's anomalous character, Brown and Boswell (1995) reinvestigated their data and discovered an important causal condition that they had previously overlooked. It is important to observe how the truth table analysis helped the researchers to adjudicate among competing explanations. In their discussion, Brown and Boswell (1995) present compelling arguments as to why both city size and mill ownership might affect the formation of cross-racial labor



coalitions. But including these conditions in the truth table did not eliminate the contradiction which indicated that they were not, in fact, causally significant.

Contradictions promote inductive analysis in two ways. As already discussed, they alert the researcher of an underspecified model. The existence of contradictions indicates that the researcher still has something to explain. Moreover, their presence in a truth table acts as a "hard stop" that prevents the analysis from proceeding. Most forms of social research do not have such built-in checks. But an incompletely specified truth table cannot be reduced which interrupts the analysis and forces the researcher to confront the contradictions and decide how to resolve them. In this way, QCA structures a close interaction between researchers, theory, and cases.

## 3 The lack of contradictions in fsQCA

But contradictory configurations do not exist in fsQCA. In order to understand why, it is helpful to review the method's historical development. Ostensibly, the 1987 publication of *The Comparative Method* merely formalized techniques that comparative-historical researchers had used for over a century. But in recasting comparative analysis as an application of Boolean algebra, Ragin had laid the groundwork for a progressive methodological program (Lakatos 1980). The technique presented in *The Comparative Method* required dichotomized conditions, a limitation highlighted by a number of reviewers (e.g. Timberlake 1989; Modell 1992; Kiser 2001; Cronqvist 2003). Zadeh (1965), however, had already extended Boolean algebra to encompass fuzzy sets, sets in which elements have varying degrees of membership.

In csQCA, it is conventional to describe conditions as being "present" or "absent." Such terminology is convenient but somewhat misleading. Boolean algebra is the algebra of sets. With a crisp set, an element either belongs to the set or it does not. For example,  $\sqrt{2}$  belongs to the set of real numbers but not the set of rational numbers. Similarly, Brown and Boswell (1995) discovered that during the 1919 steel strike, cities without high rates of recent black migration did not belong to the set of cities that experienced black strikebreaking. What Zadeh (1965, p. 338) realized was that, more often than not, the real world is not so easily classified: "such objects as starfish, bacteria, etc. have an ambiguous status with respect to the class of animals. The same kind of ambiguity arises in the case of a number such as 10 in relation to the 'class' of all real numbers which are much greater than 1."

This same observation underlies Fuzzy-Set Social Science (Ragin 2000). Social phenomena frequently exhibit partial membership. Fuzzy sets allow us to categorize individuals as "slightly religious" or "very rich" and countries as "somewhat democratic" or "not fully developed." Fuzzy sets provide nuance that crisp sets lack. However, it was not immediately obvious how to incorporate fuzzy sets into QCA. In fact, the techniques developed in Fuzzy-Set Social Science do not make use of QCA but, rather, are based upon the direct analysis of subset relationships, using what Ragin (2000) terms "the containment rule." And while the techniques of Fuzzy-Set Social Science do exploit the increased sensitivity of fuzzy sets, they lack the advantages of truth table analysis including, but not limited to, the analysis of contradictions.

Redesigning Social Inquiry (Ragin 2008) presents a technique for QCA using fuzzy sets and largely renders obsolete the methods of Fuzzy-Set Social Science. As in csQCA, fsQCA maps observations onto the rows of a truth table. But in crisp-set analysis, each observation is assigned to one—and only one—row of the truth table. In fuzzy-set analysis, however, observations have varying degrees of membership in each truth table row. Think of the calibrated data set as a multidimensional vector space with one dimension per causal condition. Each



Table 3 Data set for causes of state breakdown during interwar period

Country	Developed	Urban	Literate	Breakdown
Austria	0.81	0.12	0.99	0.95
Belgium	0.99	0.89	0.98	0.05
Czechoslovakia	0.58	0.98	0.09	0.11
Estonia	0.16	0.07	0.98	0.88
Finland	0.58	0.03	0.99	0.23
France	0.98	0.03	0.99	0.05
Germany	0.89	0.79	0.99	0.95
Greece	0.04	0.09	0.13	0.94
Hungary	0.07	0.16	0.88	0.58
Ireland	0.72	0.05	0.98	0.08
Italy	0.34	0.10	0.41	0.95
Netherlands	0.98	1.00	0.99	0.05
Poland	0.02	0.17	0.59	0.88
Portugal	0.01	0.02	0.01	0.95
Romania	0.01	0.03	0.17	0.79
Spain	0.03	0.30	0.09	0.94
Sweden	0.95	0.13	0.99	0.05
United Kingdom	0.98	0.99	0.99	0.05

Table 4 Truth table for causes of democratic breakdown during interwar period

	Developed	Urban	Literate	N	Consistency w/Breakdown	Observations
1	1	1	1	4	0.37	DE, (BE, GB, NL)
2	1	1	0	1	0.53	(CZ)
3	1	0	1	5	0.44	AT, (FI, FR, IE, SE)
4	1	0	0	0	n/a	_
5	0	1	1	0	n/a	_
6	0	1	0	0	n/a	_
7	0	0	1	3	0.84	EE, PL, (HU)
8	0	0	0	5	0.98	ES, GR, IT, PT, RO

row of the truth table represents a corner in this vector space. As an example, consider the data set presented in Table 3, which is adapted from Rihoux and Ragin (2009, Chap. 5) and examines the causes of democratic breakdown in inter-war Europe. Figure 1 provides a visual representation of the vector space. With three causal conditions, there are eight (2<sup>3</sup>) corners in the vector space and eight rows in the corresponding truth table (Table 4). The truth table reports the distribution of observations across the vector space—specifically, the corner that each observation is closest to. For example, with scores of 0.89, 0.79, and 0.99, Germany is located toward the upper-right corner of Fig. 1, which corresponds to row 1 of Table 4. Belgium, the Netherlands, and the UK are also located in this corner. Each corner of the vector space (row of the truth table) represents one logically possible combination of



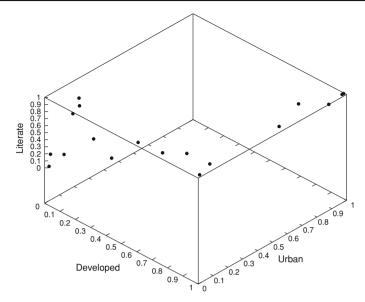


Fig. 1 Visual representation of truth table vector space

causal conditions. Together, they represent all possible combinations of causal conditions. QCA helps us to identify which of these combinations are associated with the presence (and absence) of the outcome.

As is the case in Fig. 1, observations are typically distributed unequally across the vector space. Such "limited diversity" (Ragin 1987, pp. 104–113) is characteristic of social phenomena—many logically possible combinations of causal conditions do not exist in reality and some corners will be empty or nearly empty. In QCA, these corners are referred to as "remainders" and indicated with a dash (—) in the truth table. It is up to the researcher to determine a frequency threshold below which a corner is classified as a remainder. Ragin (2008, p. 133) lists a number of criteria that researchers should consider when establishing frequency thresholds. One important consideration is total sample size. As sample size decreases, the relative value of each observation increases. Small-N studies, therefore, typically warrant lower frequency thresholds than large-N studies. This is merely a rule of thumb and it is ultimately up to the researcher, relying upon his or her substantive and theoretical knowledge of the phenomenon under investigation, to determine an appropriate frequency threshold.

The QCA researcher must also establish a consistency threshold. *Consistency* measures the strength of subset relationships and enables researchers to assess the degree to which cases sharing a particular combination of conditions also exhibit the same outcome (Ragin 2006). For example, a social researcher might observe that religious fundamentalists tend to be politically conservative. Such an observation is set-theoretic in nature: religious fundamentalists constitute a subset of political conservatives. Of course, subset relationships are not necessarily perfect and some fundamentalists are liberal. The consistency measure assesses the degree of overlap between the two sets:

Consistency
$$(X_i < Y_i) = \sum [\min(X_i, Y_i)] / \sum (X_i)$$
 (1)

Clearly defined subset relationships produce high consistency scores. If all religious fundamentalists were politically conservative, consistency would equal 1.0. As the subset



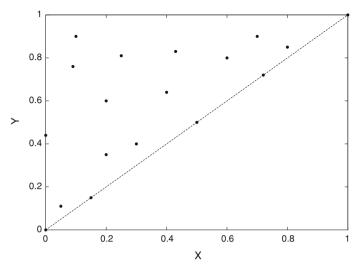


Fig. 2 Hypothetical scatterplot displaying perfect consistency

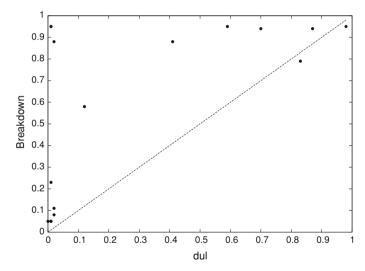


Fig. 3 Degree of membership in dul by degree of membership in breakdown

relationship attenuates, consistency drops. A consistency score of 0.9 would indicate that most—but not all—fundamentalists were politically conservative. Generally speaking, consistency scores below 0.75 indicate substantial inconsistency (Ragin 2008, pp. 143–144). A score of 0.0 indicates that the phenomena under investigation are unrelated to each other.

Figure 2 presents a hypothetical scatterplot that is perfectly consistent with sufficiency, with all of the observations on or above the diagonal. Consistency is calculated by measuring the degree to which observations depart from this relationship, with highly inconsistent cases—cases that fall further from the diagonal—accruing greater penalties than "near misses." Row 8 of Table 4 exhibits near-perfect consistency and is plotted in Fig. 3. Portugal and Romania both fall below the diagonal but only slightly. In contrast, row 1 exhibits



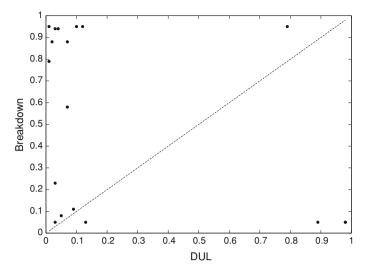


Fig. 4 Degree of membership in DUL by degree of membership in breakdown

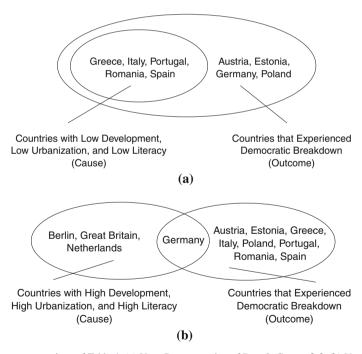


Fig. 5 Venn representations of Table 4. (a) Venn Representation of Row 8, Corner dul. (b) Venn Representation of Row 1, Corner DUL

substantial inconsistency, with Belgium, Great Britain, and the Netherlands all falling significantly below the diagonal (Fig. 4). We can also represent the sufficiency relationship as a Venn diagram. As Fig. 5a illustrates, a sufficient condition is one in which the cause—membership in the set of countries characterized by low levels of development, low urbanization, and low levels of literacy—is a subset of the outcome—membership in the set of countries



	M	U	Consist w/Y	Observations
1	1	1	1.00	East Chicago, Pittsburgh, Youngstown
2	1	0	0.80	Buffalo, Chicago, Gary, Johnstown, (Cleveland)
3	0	1	0.00	(Bethlehem, Joliet, McKeesport, Milwaukee, New Castle, Reading)
4	0	0	0.00	(Decatur, Wheeling)

**Table 5** Brown and Boswell's initial truth table for the presence of black strikebreaking with consistency scores, derived from (1995, p. 1502, Table 4)

Note M recent black migration, U weak union, Y black strikebreaking

that experienced state breakdown. The subset relationship need not be perfect, of course, and row 1's low consistency score reflects that, within the set of developed, urban, and highly literate countries, only the German state broke down, as illustrated in Fig. 5b.

Although initially developed for fsQCA, consistency applies equally to csQCA. In csQCA, consistency has a straightforward interpretation: it reports the proportion of cases that exhibit the outcome. Applied to Brown and Boswell's (1995) analysis of black strikebreaking, for example, the measure produces the consistency scores presented in Table 5. Row 1 exhibits perfect consistency because East Chicago, Pittsburgh, and Youngstown all experienced black strikebreaking. Rows 3 and 4 produce consistency scores of 0.0 because none of the cities described by these rows experienced black strikebreaking. Row 2 produces a consistency score of 0.80 because black strikebreaking occurred in four of the five cities: Buffalo, Chicago, Gary, and Johnstown but not Cleveland.

The consistency measure is important because it allows QCA researchers to recognize, accommodate, and analyze invariant relationships (relationships of necessity and sufficiency) that are imperfect. A common criticism of QCA (and comparative research, in general) is that the method is deterministic (Mahoney 2003). Lieberson (1992, 2001) has famously argued that QCA and related methods are vulnerable to a single disconfirming case. Common sense, of course, suggests otherwise. It is not unreasonable to argue that something might "almost always" be necessary or sufficient for a particular outcome to occur. People almost always marry people who are sociodemographically similar to themselves, wealthy individuals almost always come from wealthy families, and criminals are almost always poor (especially the ones who get caught).

But until Ragin introduced the consistency measure, QCA researchers did not have a means of accommodating imperfect invariance. Instead, we had "contradictions." A contradiction—more precisely, a contradictory row—is a row of a truth table in which some cases exhibit the outcome and others do not.

Because contradictions serve as a "hard stop" that prevent truth table reduction, Ragin (1987, pp. 113–118) devotes considerable discussion in *The Comparative Method* to the process of their resolution. He encourages researchers to use the presence of contradictions as an opportunity to engage in what he would later term "retroductive research" (Ragin 1994)—the revisiting, reexamination, and reconsideration of one's theory and data:

To follow the case-oriented approach, then, is to treat any specification of relevant causal conditions as tentative and to use theoretical and substantive knowledge to achieve a proper specification of causal conditions before reducing the truth table (Ragin 1987, p. 113).

Ragin (1987) specifically cautions against resolving contradictions mechanically. One such option is to simply assign contradictory rows an outcome score of 0, a conservative approach



that retains only those truth table rows that are unambiguously associated with the outcome. Another option is to assign contradictory rows a score of 1, an expansive solution that allows for greater causal complexity. Neither of these solutions is ideal, argues Ragin (1987, p. 118), because they obscure the cases under investigation and ultimately hinder the analytic process; such solutions "should be used only when it is impossible to return to the original cases and construct a better truth table." Unfortunately, using the consistency measure as described in *Redesigning Social Inquiry* encourages precisely this type of rote resolution.

Consider, again, Table 5, which presents Brown and Boswell's (1995) initial investigation of the determinants of black strikebreaking. Following the analytic procedure described in *Redesigning Social Inquiry*, Brown and Boswell would have first specified a frequency threshold, the number of observations below which a truth table row would be classified as a remainder. As there are only 16 cities in the study, we can safely assume that they would have used a frequency threshold of 1, a threshold that would not affect the subsequent analysis (because each row of the truth table has at least one observation associated with it). The same cannot be said for the second step, the specification of a consistency threshold.

As discussed above, consistency scores of less than 0.75 indicate substantial inconsistency. The consistency score of 0.80 for Row 2 of Table 5 is close to this boundary and Brown and Boswell would have had to decide whether to classify this row as consistent or inconsistent with sufficiency. That is, Brown and Boswell would have had to determine whether causal recipes with consistency scores of 0.80 were "consistent enough" to treat as "usually leading to the outcome." Specifying a consistency threshold of greater than 0.80 would classify Row 2 as inconsistent with sufficiency and lead to the conclusion that the combination of recent black migration and a weak union was not sufficient to produce black strikebreaking during the 1919 steel strike. Conversely, specifying a threshold of 0.80 or less would classify the row as consistent with sufficiency and lead to the conclusion that this combination of conditions was sufficient to produce black strikebreaking.

The point here is not that Brown and Boswell would have needed to establish a consistency threshold but that having done so, their analysis would be effectively finished. Classifying Row 2 as inconsistent is equivalent to assigning contradictory rows a score of 0, the conservative solution that Ragin (1987) cautions against. And classifying Row 2 as consistent is equivalent to assigning contradictory rows a score of 1, the expansive solution cautioned against. In either case, the truth table would be complete, free of contradictions, and reducible. Using the conservative consistency threshold, the final solution would be MU, that the combination of recent black migration and the presence of a weak union was sufficient to produce black strikebreaking. The more liberal consistency threshold would produce a final solution of M, that recent black migration was sufficient to produce black strikebreaking. In both instances, applying the consistency threshold as prescribed in *Redesigning Social Inquiry* would have led Brown and Boswell to a premature and erroneous conclusion.

But because Brown and Boswell were conducting conventional crisp-set QCA, they were forced to resolve the contradiction of Row 2. To resolve the contradiction, Brown and Boswell returned to their cases and searched for additional causal conditions that could have affected the presence of black strikebreaking. What they found was that the presence of political repression was important. Including political repression produced a truth table free of contradictions (Table 2) and a final solution of MR: it was the combination of recent black migration and political repression that was sufficient to produce black strikebreaking. If the presence of the contradiction had not forced Brown and Boswell to reexamine their cases, it is likely that they would not have recognized the importance of political repression to black strikebreaking.



# 4 Incorporating contradictions into fsQCA

Contradictions are valuable because they signal the possibility of an insufficiently specified model and their absence in fsQCA is a significant limitation. Fortunately, it is relatively straightforward to incorporate contradictions into fsQCA. In csQCA, a contradiction is a row of a truth table in which some of the observations exhibit the outcome and others do not. The fsQCA corollary employs the consistency measure: In fsQCA, a contradiction refers to a corner of the vector space (row of the truth table) in which some of the observations are consistent with sufficiency and others are not.

As discussed, consistency measures the degree to which the observations that correspond to a particular combination of causal conditions exhibit a subset relationship that is consistent with sufficiency. That is, the measure is used to describe *sets* of observations. But it can also be applied to individual observations:

Observation consistency 
$$(X \le Y) = \min(X, Y)/X$$
 (2)

Observation consistency measures how far off of the diagonal a specific observation is. Observations on or above the diagonal have a score of 1.0. As with Eq. 1, observations falling further from the diagonal are more severely penalized than near misses. Figure 6 presents a hypothetical scatterplot, with Table 6 presenting the corresponding consistency scores. Overall consistency of the subset relationship is 0.93; the distribution of this hypothetical data is strongly consistent with the conclusion that X is a sufficient cause of Y. But overall

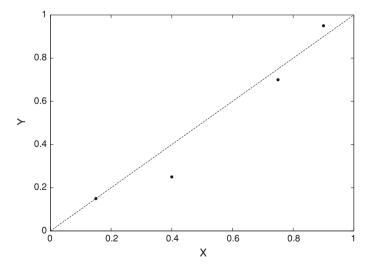


Fig. 6 Hypothetical scatterplot

**Table 6** Consistency scores for hypothetical data from Fig. 6

Observation	X	Y	Observation consistency		
A	0.90	1.00	1.00		
В	0.15	0.15	1.00		
C	0.75	0.70	0.93		
D	0.40	0.25	0.62		



Table 7	Observa	ation	
consisten	cies for	duL co	untries

Country	Membership in <b>duL</b> (X)	Membership in Breakdown (Y)	Consistency w/Breakdown
Estonia	0.84	0.88	1.00
Hungary	0.84	0.58	0.69
Poland	0.59	0.88	1.00

consistency—which we might also term *set consistency* to distinguish it from observation consistency—is a composite measure that masks differences between the individual observations: observations A and B fall on or above the diagonal, observation C falls slightly below the diagonal, and observation D falls significantly below the diagonal.

Consider Row 7 of Table 4. Three observations occupy this corner of the vector space (**duL**), which represents countries with low levels of development, low urbanization, and high literacy rates. Set consistency for this corner is 0.84 but as Table 7 makes clear, there are important differences between these countries. Estonia and Poland both exhibit high (in fact, perfect) consistency with the outcome while Hungary exhibits substantial inconsistency.

The situations presented in Tables 6 and 7 are analogous to the one faced by Brown and Boswell (1995): most of the observations are consistent with sufficiency but one is not. A researcher following the procedure described in *Redesigning Social Inquiry* would decide whether to classify all of row 7 as consistent or inconsistent with sufficiency based upon the set consistency score of 0.84. But this ignores the differences among the corner's observations. Hungary is apparently unique in some way and it may be that the analysis could be improved by returning to the cases and searching for additional causal conditions that would eliminate the **duL** contradiction.

How strictly should contradictions be defined? A conservative approach would define as contradictory any corner of the vector space that includes at least one consistent observation and at least one inconsistent observation. Such a definition corresponds to the model of *The Comparative Method*, in which a single negative observation is sufficient to classify a truth table row as contradictory. For small-N studies, a conservative approach is frequently appropriate. If there are only five observations occupying a particular corner of the vector space, it is probably important to understand why only one of them inconsistent. For large-N studies, however, such a restrictive definition is more problematic, as the likelihood that a given vector space corner will include both consistent and inconsistent observations tends to increase with sample size, particularly for individual-level data.

For this reason, I recommend employing proportions rather than absolute values. A *consistency proportion threshold* specifies the minimum ratio of consistent to inconsistent observations required to classify a vector space corner as consistent or inconsistent with sufficiency. The consistency proportion threshold complements the existing frequency and consistency thresholds. A consistency proportion threshold of 0.90, for example, would categorize a vector space corner as consistent if (a) the number of observations in the corner meets or exceeds the specified frequency threshold, (b) overall consistency of the corner meets or exceeds the specified consistency threshold, and (c) at least 90% of the observations belonging to that corner are consistent with sufficiency. Conversely, if at least 90% of the observations belonging to that corner were inconsistent, the corner would be categorized as inconsistent. If neither the number of consistent or inconsistent observations exceeds the consistency proportion threshold—that is, if the number of consistent and inconsistent observations are both substantial—the vector space corner would be categorized as contradictory.



	Developed	Urban	Literate	N	Consistency w/Breakdown	Outcome	Observations
1	1	1	1	4	0.37	Contradiction	DE, (BE, GB, NL)
2	1	1	0	1	0.53	False	(CZ)
3	1	0	1	5	0.44	False	AT, (FI, FR, IE, SE)
4	1	0	0	0	n/a	Remainder	_
5	0	1	1	0	n/a	Remainder	_
6	0	1	0	0	n/a	Remainder	-
7	0	0	1	3	0.84	Contradiction	EE, PL, (HU)
8	0	0	0	5	0.98	True	ES, GR, IT, PT, RO

**Table 8** Truth table for causes of state breakdown during interwar period, consistency proportion threshold = 0.80

Note that the same threshold is used for establishing both set consistency and observation consistency. While it is technically possible to use different thresholds for set consistency and observation consistency, I caution against doing so. My experience indicates that using different consistency thresholds (e.g., 0.85 for set consistency and 0.90 for observation consistency) can be confusing and that it is not particularly helpful. Using different thresholds will not usually affect the results of the analysis significantly. Furthermore, it obscures what is meant for an observation or set of observations to be "consistent with sufficiency." For these reasons, I recommend using a single consistency threshold to establish both the consistency of individual observations and of vector space corners.

As an example, consider the democratic breakdown data from Table 3. Specifying a frequency threshold of 1, a consistency threshold of 0.80, and a consistency proportion threshold of 0.80 would produce the truth table presented in Table 8. Table 8 is identical to Table 4, except for the addition of an "Outcome" column, the values for which are computed from the frequency, consistency, and consistency proportion thresholds:

- Row 1: Categorized as a contradiction because its proportion of inconsistent observations (3/4 = 0.75) falls below the specified consistency proportion threshold of 0.80.
- Rows 2 and 3: Categorized as "False," indicating "inconsistent with sufficiency," because (a) the number of observations belonging to these corners of the vector space exceeds the specified frequency threshold of 1, (b) the overall consistency of these corners fall beneath the specified consistency threshold of 0.80, and (c) at least 80% of the observations belonging to these corners are inconsistent.
- Rows 4, 5, and 6: Remainders
  - Row 7: Categorized as a contradiction because, although its frequency and consistency scores exceed the specified thresholds, only two-thirds of the observations belonging to this corner of the vector space exceed the specified observation consistency threshold.
  - Row 8: Categorized as "True", indicating "consistent with sufficiency," because (a) the number of observations belonging to this corner of the vector space meets the specified frequency threshold of 1, (b) the set consistency of this corner exceeds the specified consistency threshold of 0.80, and (c) the





Fig. 7 Kirq, data set view, democratic breakdown data

number of consistent observations belonging to this corner of the vector space exceeds the specified consistency proportion threshold of 0.80.

Observe that if we dropped the consistency proportion threshold to 0.75, Row 1 would then be classified as "False" and the only contradiction in the table would be Row 7. Also observe that the valid range of the consistency proportion threshold is >0.50-1.00 and that specifying a consistency proportion threshold of 0.50 is equivalent to applying none at all. Because it is always true that at least 50% of observations will be consistent or inconsistent, specifying a consistency proportion threshold of 0.50 effectively means that only the frequency and consistency thresholds will be used to calculate the value of the outcome column. This characteristic is useful when one wishes to mimic the traditional fsQCA algorithms in software that supports fuzzy-set contradictions.

## 5 Software

Support for fuzzy-set contradictions is available in software packages based on libfsqca (Rubinson 2011), a software library<sup>3</sup> for QCA. Two such packages currently exist: acq (Rubinson and Reichert 2011) and Kirq (Reichert and Rubinson 2011), both of which are available for free download from http://grundrisse.org/qca/. acq is designed to be used from the Unix commandline (e.g., a Linux/BSD shell, Terminal.app on Mac, or Cygwin on Windows). Most users will probably prefer to use Kirq, which is cross-platform (running on Windows, Mac, and Unix platforms) and provides a full graphical interface.

To use Kirq to conduct a sufficiency analysis, one loads a data set, specifies the outcome and causal conditions, and sets the frequency, consistency, and proportion thresholds (Fig. 7). Clicking "Truth Table" will then generate the corresponding truth table, automatically

<sup>&</sup>lt;sup>3</sup> In computer science jargon, a "software library" is a collection of related routines that can be used to create specific software programs. libfsqca, for example, could be used to create an fsQCA module for the R statistical system.



low A	Dev	Urb	Lit	N	Consist	Outcome	ConsistObs	InconsistObs	History:
1	True	True	True	4	0.37	Con	DE	BE;NL;GB	v-breakdown/Brk
2	True	True	False	1	0.53	False		cz	- Little Manager
3	True	False	True	5	0.44	False	AT	FI;FR;IE;SE	
4	True	False	False	0	n/a	Rem		*	
5	False	True	True	0	n/a	Rem			Necessity Sufficiency
6	False	True	False	0	n/a	Rem			Frequency Threshold Simplificati
7	False	False	True	3	0.84	Con	EE;PL	HU	Consistency Threshold
8	False	False	False	5	0.98	True	GR:IT:PT:R0:ES		0.80
									Proportion Threshold
									0.80

Fig. 8 Kirq, truth table view, democratic breakdown data

low A	Dev	Urb	Lit	N	Consist	Outcome	ConsistObs	InconsistObs	History:
1	True	True	True	4	0.37	Con	DE	BE;NL;GB	v-breakdown/Brk truth table 1
2	True	True	False	1	0.53	False		cz	trutti_tuble_1
3	True	False	True	5	0.44	False	AT	FI;FR;IE;SE	
4	True	False	False	0	n/a	Rem			
5	False	True	True	0	n/a	Rem			Necessity Sufficiency
6	False True	True F	False	0	n/a	Rem			Frequency Threshold Simplifica
7	False	False	True	3	0.84	Con v	EE;PL	HU	2 Consistency Threshold
8	False	False	False	5	0.98	True False Imp Rem	GR:IT:PT:R0:ES	-	Proportion Threshold  0.80

Fig. 9 Kirq, editing truth table outcome, democratic breakdown data

calculating the values of the Outcome column (Fig. 8). This behavior is different from Ragin's fs/QCA software, in which the researcher must always manually code the outcome column's values. Kirq does allow the researcher to manually override the calculated Outcome value, by double-clicking on the cell in question (Fig. 9). A second feature that distinguishes Kirq from fs/QCA is that Kirq lists the observations that belong to each vector space corner as part of the truth table, identifying whether each is consistent or inconsistent.

The researcher will need to resolve any contradictions present in the truth table before it can be reduced. There are three ways of doing so: by modifying the data set, by changing the parameters of the analysis, and by manually coding the Outcome column. Returning to the data often results in a modification of the data set. Researchers frequently uncover one or more previously omitted causal conditions, as was the case for Brown and Boswell (1995). Recalibrating one's fuzzy sets or revising one's operationalizations can also help to resolve contradictions. Changing the values of the frequency, consistency, and proportion thresholds will affect the distribution of observations across rows of the truth table and examining the differences between more conservative and more expansive solutions can help in identifying and isolating the source of a contradiction. Also remember that setting the proportion threshold to 0.50 will produce a truth table without any contradictions. This is particularly useful



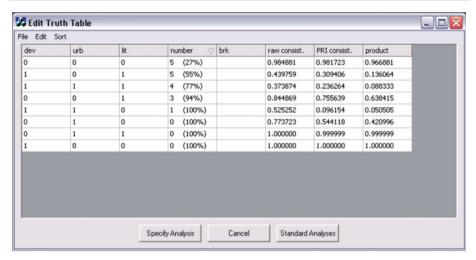


Fig. 10 fs/QCA truth table editor, democratic breakdown data

for large-N, individual-level studies and other instances in which one expects a fair amount of inconsistency within the vector spaces corners. Finally, the researcher can always manually recode a contradictory row. It may be that, for one reason or another, a contradiction cannot be eliminated empirically. The researcher may be unable to collect additional data. Or there may be random variation among the observations. In such circumstances, if the researcher has a strong theoretical and/or empirical basis for classifying a particular combination of causal conditions as consistent or inconsistent, he or she should do so. The point of incorporating contradictions into fsQCA is not to make researchers' tasks more difficult but, rather, to provide them with an additional tool—another arrow in the quiver—for their investigations. It is up to the researcher to determine how to deploy this tool. A researcher might, for example, choose to drop some contradictory rows from the analysis by setting them as remainders, while casting others as "True" or "False." No single method of resolution is always preferred and all methods are appropriate under particular circumstances. It is ultimately up to the researcher, relying upon his or her substantive and theoretical knowledge, to discern which methods are appropriate under which circumstances.

#### Identifying contradictions manually

Although fs/QCA does not include support for fuzzy-set contradictions, users can identify them manually using spreadsheet software such as Microsoft Excelor LibreOffice Calc. Begin by using fs/QCA's truth table algorithm to generate a truth table (Fig. 10). By default, fs/QCA sorts the truth table by the number of observations belonging to each vector space corner, which makes it easy to identify those corners that meet your specified frequency threshold criteria. With a frequency threshold of 1, there are five non-remainder corners in Fig. 10: dul, DuL, DUL, duL, and DUI.

Open the data set in your spreadsheet and create two columns for each non-remainder corner. The first column will record each observation's degree of membership in the corner; the second, its observation consistency. Label the columns as in Fig. 11. To calculate an observation's degree of membership in a corner, take the observation's *minimum* membership score of the causal conditions that make up that corner (Fig. 12). For example, Austria's



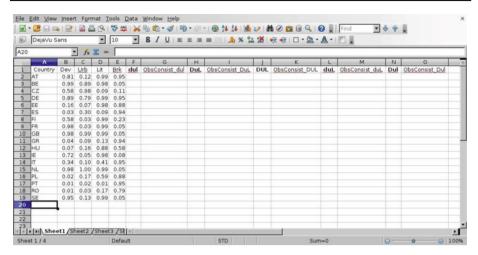


Fig. 11 LibreOffice Calc, democratic breakdown data

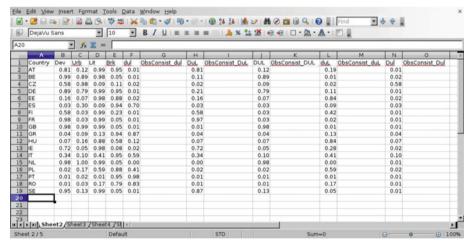


Fig. 12 LibreOffice Calc, calculating membership scores

degree of membership in corner **DUL** is 0.12 because its degree of membership in the set of urbanized countries is 0.12. Conversely, Austria's degree of membership in corner **dul**—the set of not-developed, not-urbanized, and not-literate countries—is 0.01, which is found by negating its 0.99 membership in the set of literate countries. Next, use Eq. 2 to calculate the observation consistency for each observation-corner combination (Fig. 13). Observe that when an observation's corner membership is 0.0, the spreadsheet will generate a division-by-zero error; this is of no concern since the observation, by definition, does not belong to that corner.

From here, it is a simple manner to identify any contradictions by computing the proportion of consistent and inconsistent observations in each corner. To test corner **dul**, first apply a descending sort to that column. As depicted in Figs. 14 and 15, I find it helpful to bold the cells of the observations belonging to that corner (that is, observations with  $\geq 0.50$  corner membership). Calculate the proportion of consistent observations by dividing the number of



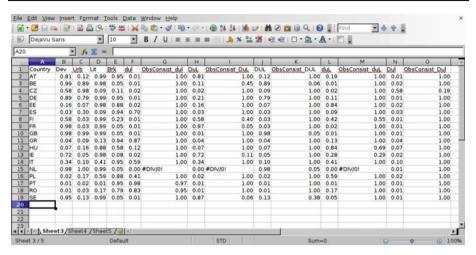


Fig. 13 LibreOffice Calc, calculating observation consistencies

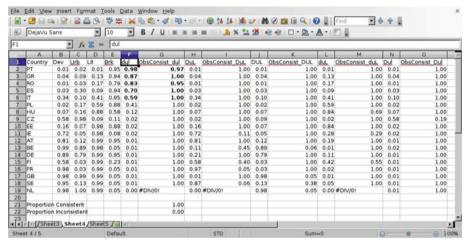


Fig. 14 LibreOffice Calc, identifying consistent and inconsistent observations in corner dul

observations with an observation consistency meeting your specified consistency threshold by the total number of observations belonging to that corner. To calculate the proportion of inconsistent observations, simply substitute the number of inconsistent observations for the numerator. The sum of these two values must of course equal 1.0.

In Fig. 16, corners for which either the proportion of consistent or inconsistent observations meets the specified consistency proportion threshold of 0.80 are highlighted. Corners **DUL** and **duL** are identified as contradictions, the same result provided by Kirq and presented in Table 8.

#### 6 Conclusion

Among QCA's distinctive strengths is that it is case-oriented. QCA perceives social reality as comprised of set-theoretic relations and models social relationships in terms of multiple



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23			· fx	Σ =	•										
	Α	В	C	D	E	F	G	H		1	K	L	M	N	0
	Country	Dev				dul	ObsConsist_dul					duL		Dul	ObsConsist Dul
	CZ	0.58				0.02	1.00		1.00	0.09	1.00	0.02	1.00		0.19
	IT	0.34	0.10		0.95	0.59	1.00		1.00	0.10	1.00	0.41	1.00		
-	HU	0.07	0.16			0.12			1.00	0.07	1.00	0.84	0.69	0.07	1.00
	GR ES	0.04	0.09			0.87	1.00		1.00	0.04	1.00	0.13	1.00		1.00
-	EE	0.03				0.70			1.00	0.03	1.00	0.84	1.00		
	PL	0.16		0.59		0.02			1.00	0.07		0.59	1.00	0.02	
÷	IF.	0.02				0.02			0.11	0.02	1.00	0.39	0.29	0.02	
	BE	0.99				0.01	1.00		0.45	0.89	0.06	0.01	1.00		1.00
ä	FI	0.58				0.01	1.00		0.40	0.03		0.42	0.55		1.00
	AT	0.81	0.12				1.00		1.00	0.12		0.19			1.00
l de	RO	0.01	0.03			0.83	0.95		1.00	0.01	1.00	0.17	1.00		1.00
	DE	0.89	0.79	0.99	0.95	0.01	1.00	0.21	1.00	0.79	1.00	0.11	1.00	0.01	1.00
1	SE	0.95	0.13	0.99	0.05	0.01	1.00	0.87	0.06	0.13	0.38	0.05	1.00	0.01	1.00
N.	FR	0.98	0.03	0.99	0.05	0.01	1.00	0.97	0.05	0.03	1.00	0.02	1.00	0.01	1.00
	GB	0.98	0.99	0.99	0.05	0.01	1.00	0.01	1.00	0.98	0.05	0.01	1.00	0.01	1.00
	PT	0.01	0.02		0.95		0.97	0.01	1.00	0.01	1.00	0.01	1.00		1.00
	NL	0.98	1.00	0.99	0.05	0.00	#DIV/0!	0.00	#DIV/0!	0.98	0.05	0.00	#DIV/0!	0.01	1.00
	Proportio						1.00		0.20		0.25		0.67		0.00
	Proportio	in Incor	nsisten	t .			0.00		0.80		0.75		0.33		1.00

Fig. 15 LibreOffice Calc, identifying consistent and inconsistent observations in all corners

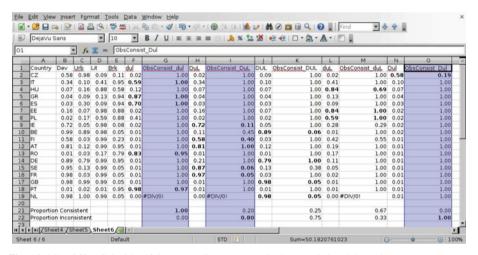


Fig. 16 LibreOffice Calc, identifying contradictory corners in democratic breakdown data

conjunctural causation. It treats cases as holistic, complex configurations and is sensitive to anomalous and nonconforming observations. As currently practiced, however, fsQCA frequently renders cases invisible. It is not uncommon for the observations conforming to a particular configuration of causal conditions to vary in their expression of the outcome, particularly with individual-level data. Ragin's consistency measure accommodates this circumstance, permitting researchers to model relationships that are "almost always" consistent with sufficiency. But it has the unfortunate side effect of veiling anomalies and discouraging researchers from investigating nonconforming observations, a hallmark of case-oriented research. Fuzzy-set contradictions solve this problem. Incorporating contradictions into fsQCA helps researchers to identify anomalous observations. And the process of resolving them encourages researchers to return to their data to reexamine and reconceptualize their cases and theories. Fuzzy-set contradictions improve fsQCA by enhancing its case sensitivity.



QCA is sometimes referred to as a "family" of related techniques (Rihoux and Ragin 2009). In fact, csQCA is but a "special form" of fsQCA—the methods, techniques, and algorithms of fsQCA apply equally to csQCA. libfsqca, for example, has a single routine for the construction of truth tables; it does not matter whether the data is crisp or fuzzy. Contradictions were the one element of csQCA not supported by fsQCA. As the analysis of the democratic breakdown data makes clear, however, it is not that contradictions are absent in fsQCA but, rather, that their existence has been obscured by the consistency measure. In this paper, I have sought to draw attention to the presence of contradictions in fsQCA and have provided techniques for recognizing and identifying them.

Fuzzy-set contradictions encourage researchers to closely engage with their analyses. Many have criticized what Franzosi (1995, p. 19, emphasis in original) terms the *regression blender*: "We toss everything in (the more the better, because thereby we think that we are *controlling* for as many factors as possible), we let the blender mix up the data, and then we write up the results." As QCA grows increasingly popular, it faces a similar threat. fs/QCA, the de facto QCA software, provides a "standard analysis" of three solutions—one of maximum complexity, one of maximum parsimony, and an intermediate one that seeks to balance complexity and parsimony. The intent of providing three solutions is to assist the researcher in examining the range of valid solutions that can be used to explain the occurrence of a particular outcome. Ideally, the researcher uses his or her theoretical and substantive knowledge to adjudicate among the various solutions and to select the one that makes the most explanatory sense. Unfortunately, far too many researchers automatically pick the intermediate solution, assuming that it must be best.<sup>4</sup>

Similarly, *Redesigning Social Inquiry* devotes considerable discussion to the consistency measure and how to pick a proper consistency threshold. What is missing is any discussion that it may be useful to examine why a particular combination of causal conditions is producing high—but imperfect—consistency scores. The danger is that researchers will accept highly consistent scores at face value, drawing erroneous conclusions because they neglected to subject their analysis to further interrogation. To understand social phenomena, it is frequently important to study so-called "negative cases"—instances of nonoccurrence and inconsistency. Identifying contradictions is the first step in doing so.

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<sup>&</sup>lt;sup>4</sup> One reason that some researchers default to the intermediate solution is due to a misreading of Ragin (2008, pp. 171–172), which appears to prescribe the intermediate solution as always superior to both the complex and parsimonious solutions: "Many researchers who use QCA either incorporate as many simplifying assumptions (counterfactuals) as possible or they avoid them altogether. They should instead strike a balance between complexity and parsimony…"



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