Island Model

Aula 6 – 1ª Parte

Introduction

- The Islands Model (SCED, 2003) is an **endogenous growth model** where bounded-rational, heterogeneous firms interact locally
- The model allows to study under which conditions **self-sustained growth** does emerge
- The model is able to deliver **empirically plausible** output time series
- The models is also employed to study the trade-off between individual **rationality** and macroeconomics **performance**

Bird's eye view of the model

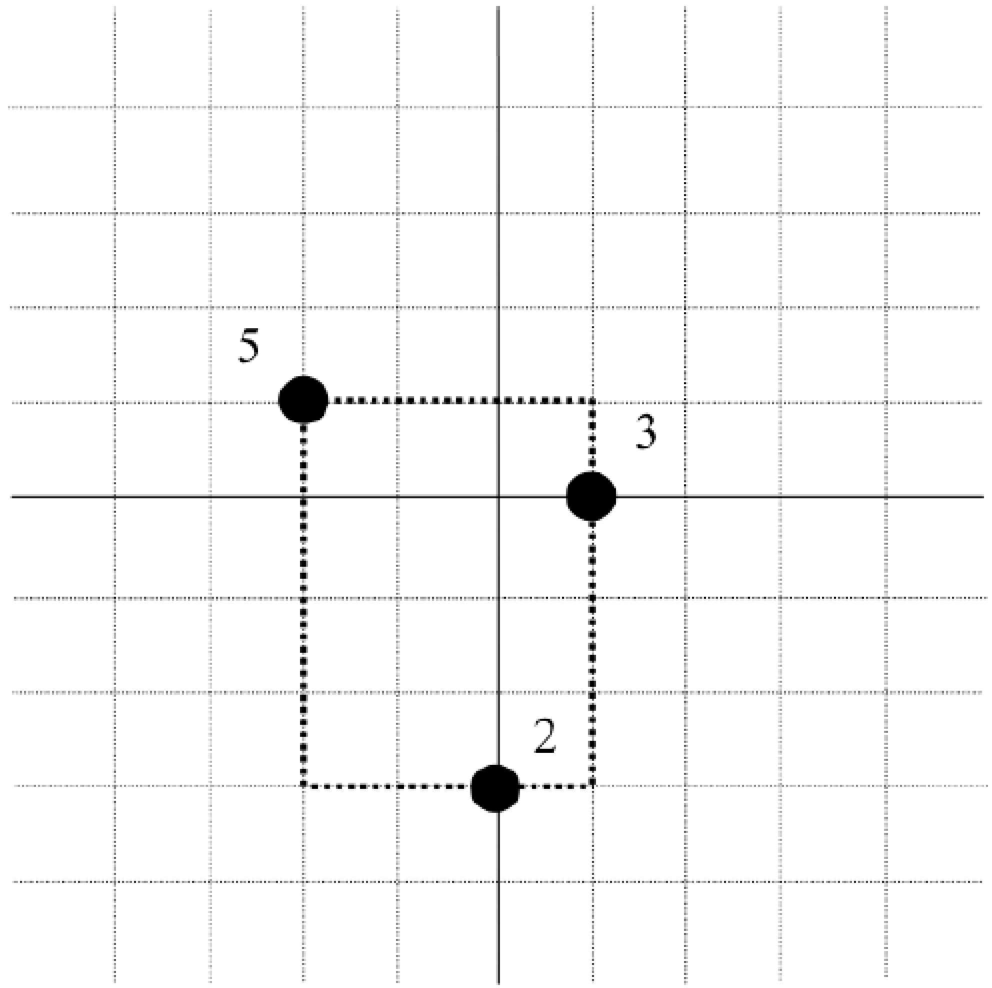
Economic characteristic	Model representation
Technological space	notionally unbounded sea
Technology	island (mine)
Output	homogeneous good
Agents	stylized entrepreneurs
Production	mining/extracting the good
Technological search	exploration of the sea
Technological diffusion	spreading knowledge from islands
Innovation	discovering a new island
Imitation	traveling between discovered islands
Technological difference	distance between islands

World structure

- Discrete time $t = 0,1,2, \ldots, T$
- Finite, constant population of stylized firms $i = 1, 2, \ldots, N$
- Homogeneous good
- Notionally endless, discrete set of technologies (islands)
- Islands:
  - stochastically distributed on a bi-dimensional lattice
  - each node of the lattice can be an "island" with probability π
  - each island (x, y) is characterized by a productivity coefficient

$$s(x, y) = |x| + |y|$$

Example: 3 islands, 10 firms



Production

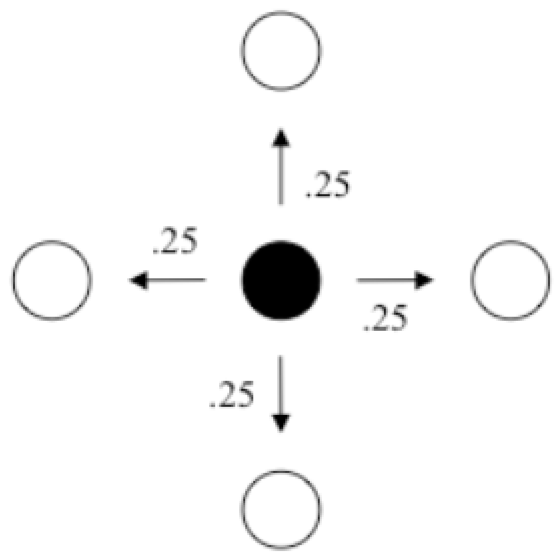
- Set L_0 of initially **known islands** (exploited technologies)
- All N firms **mining** on them (randomly allocated)
- Each firm i working in island (x_j, y_j) produces an **output** $s(x_j, y_j)$ each period
- Each island (x_j, y_j) has a **total output** of

$$Q_t(x_j, y_j) = s(x_j, y_j)\, m_t(x_j, y_j)^{\alpha}$$

  - $m_t(x_j, y_j)$ number of firms currently working on (x_j, y_j)
  - $\alpha > 1$ is an increasing returns-to-scale coefficient

Exploration

- In each t, a "miner" becomes "explorer" with probability ϵ (constant willingness to explore)
- Explorers move around randomly in each period



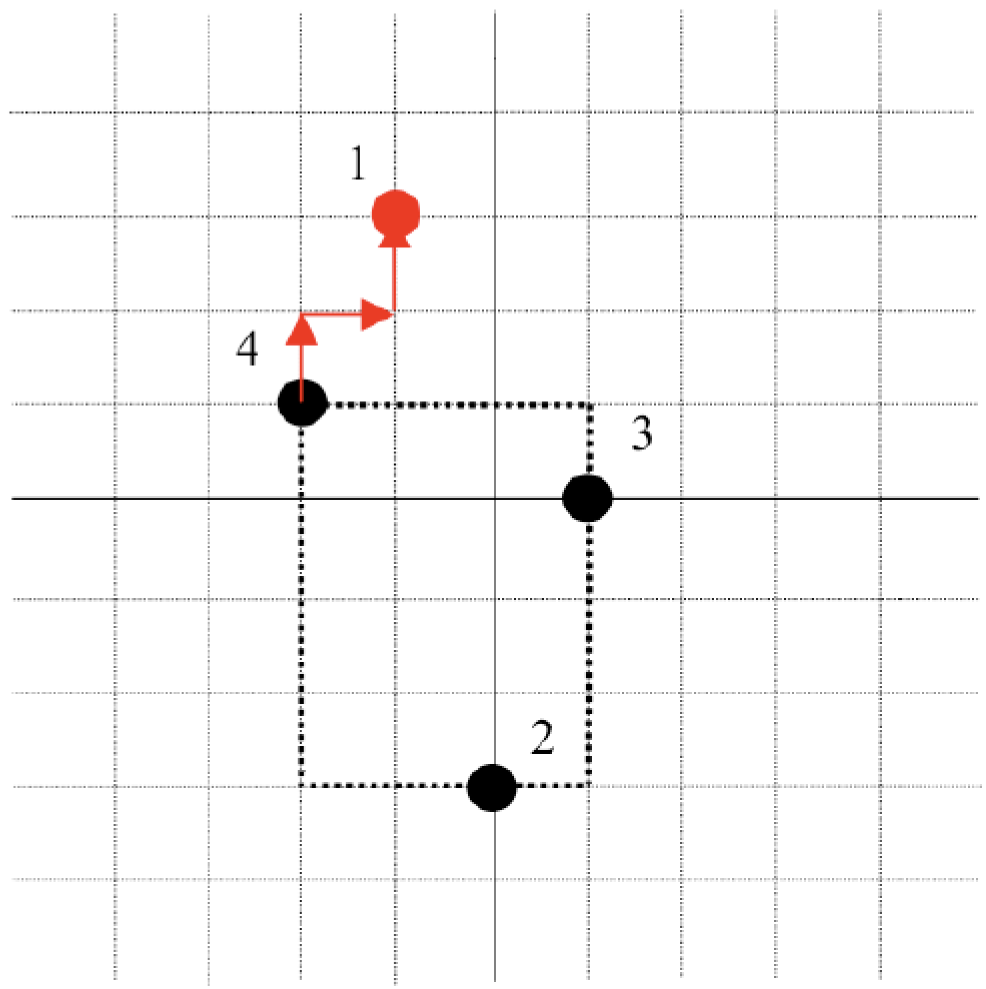
Innovation

- In each exploration period, explorers find a **new island** with a probability π
- The **productivity** of the newly discovered island is

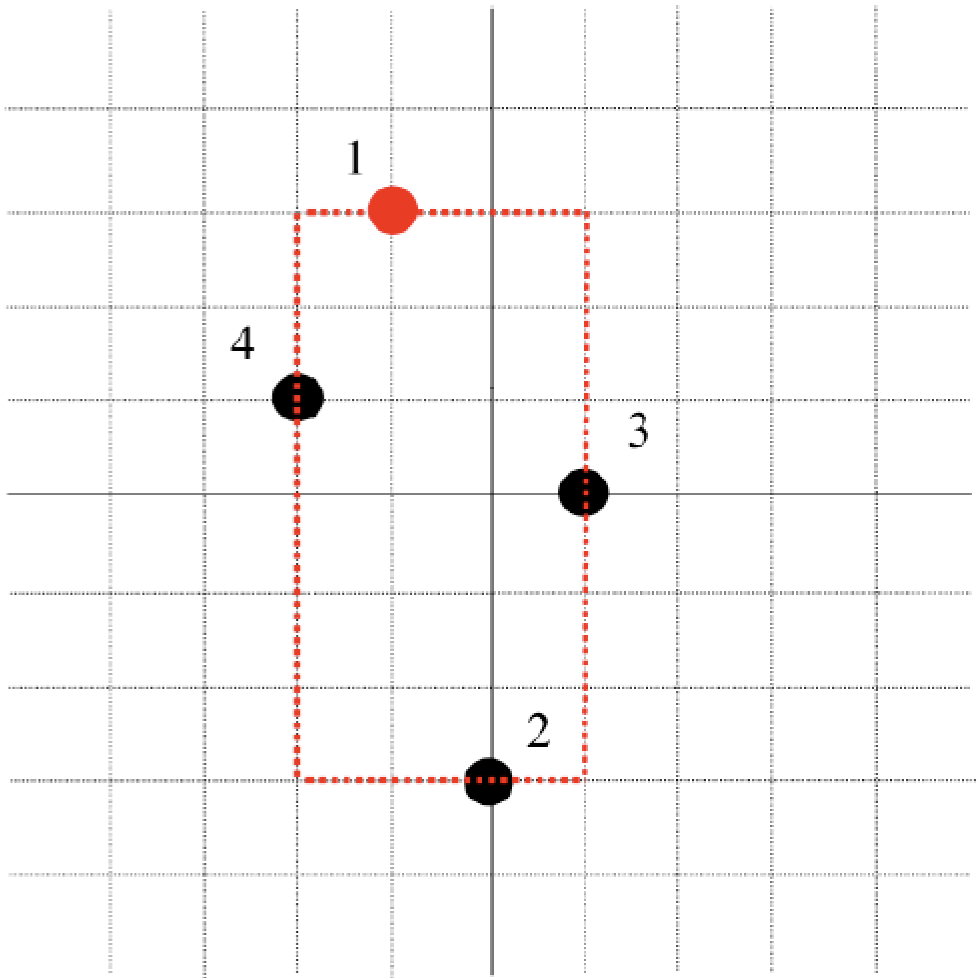
$$\begin{aligned} s^* &= s(x^*, y^*) \\ &= (1\ + W)(|x^*| + |y^*| + \varphi q_{i,t} + \xi) \end{aligned}$$

  - W: Poisson random variable (low probability, high jumps)
  - $|x^*| + |y^*|$: distance from the origin
  - $\varphi q_{i,t}$: cumulative learning effect
  - ξ: zero-mean random variable (high probability low jumps)

Example: exploration + innovation



Expanded technological space



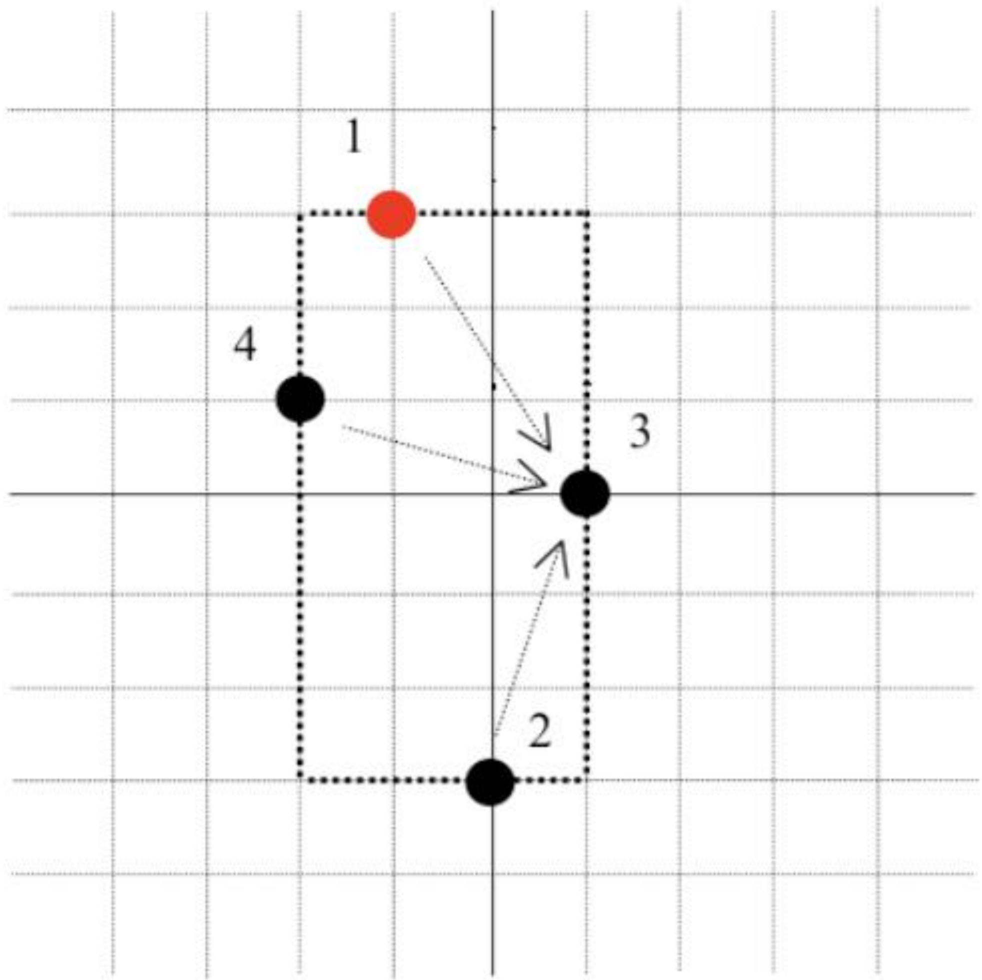
Imitation

- In each t, every currently exploited island ($m(x_j, y_j) > 0$) send a **signal** about its productivity $s(x_j, y_j)$
- Any miner on island (x_k, y_k) **follows** the signal from (x_j, y_j) with a probability proportional to

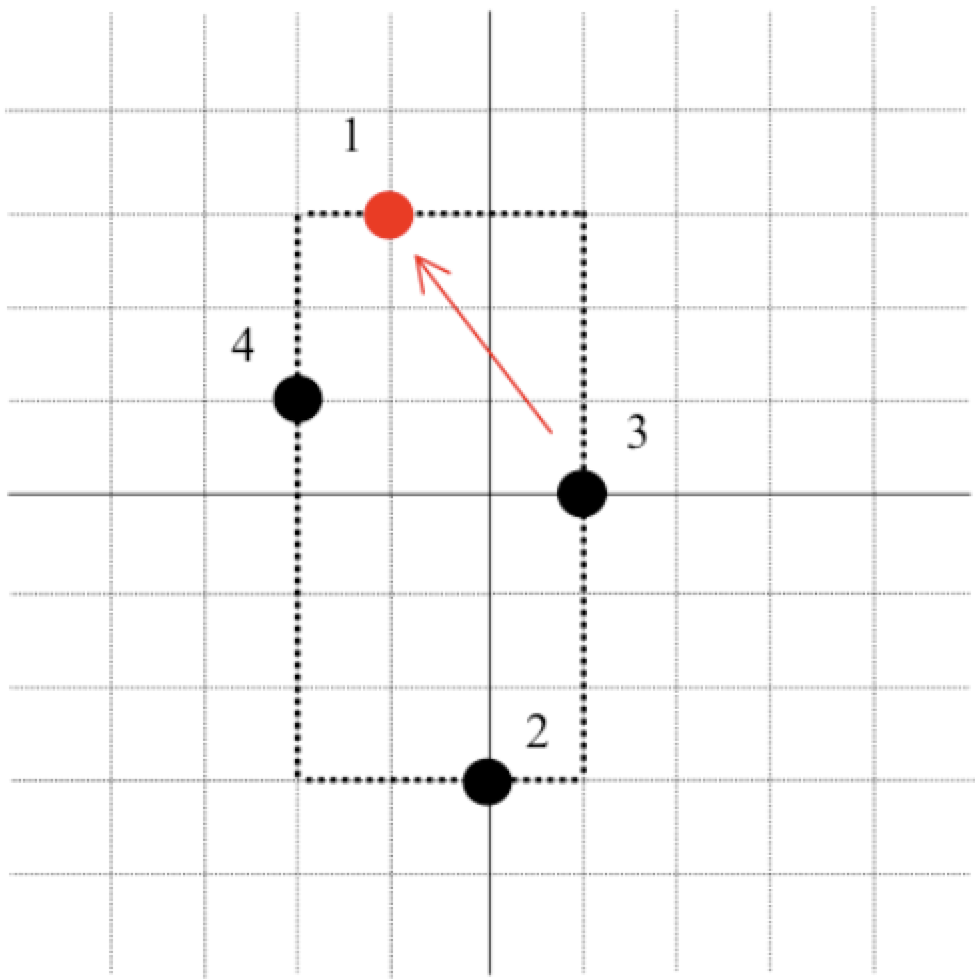
$$Q_t(x_j, y_j)\, e^{-\rho \mathrm{d}\left((x_j, y_j), miner\right)}$$

  - the current output $\mathrm{Q}_t(x_j, y_j)$ of the island the signal comes from
  - the distance between island (x_j, y_j) and miner
- The higher (smaller) ρ the more **global** (local) is information and knowledge diffusion
- Imitators move toward the imitated island following the **shortest path** leading to it (one step per period)

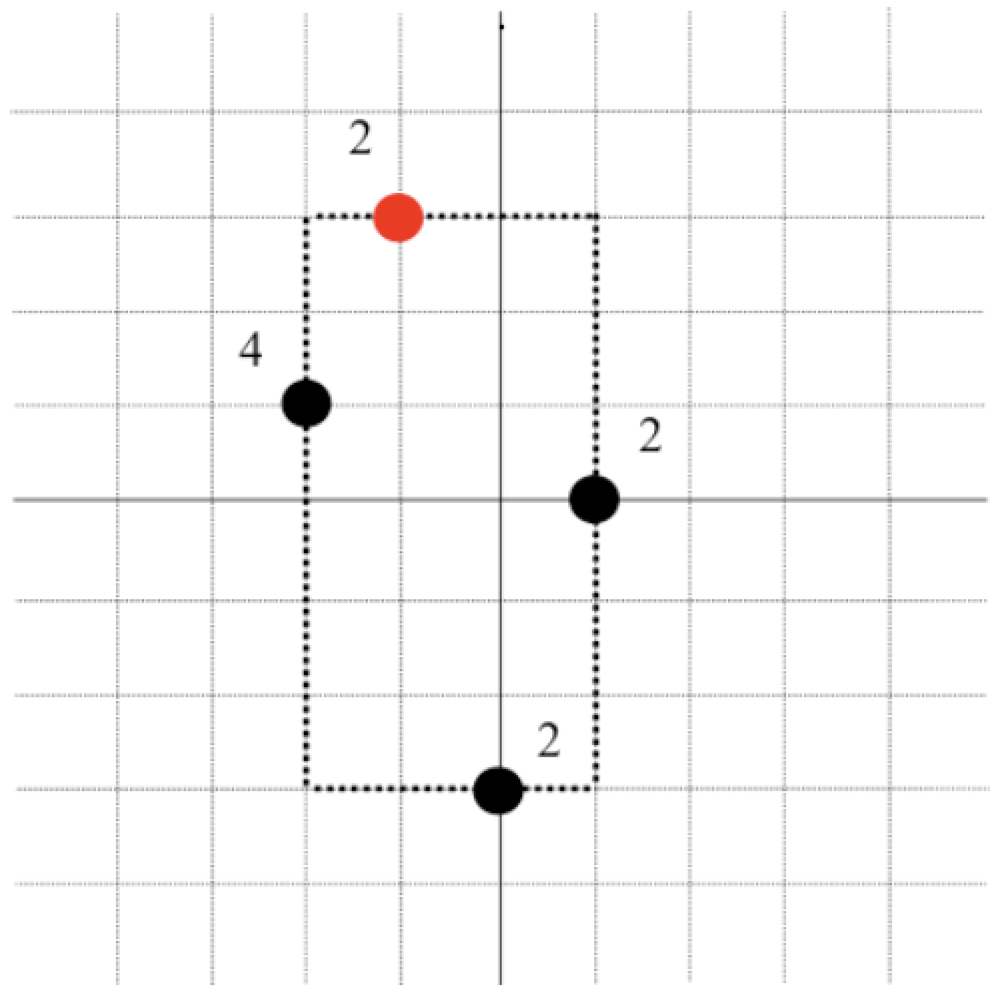
Example: imitation



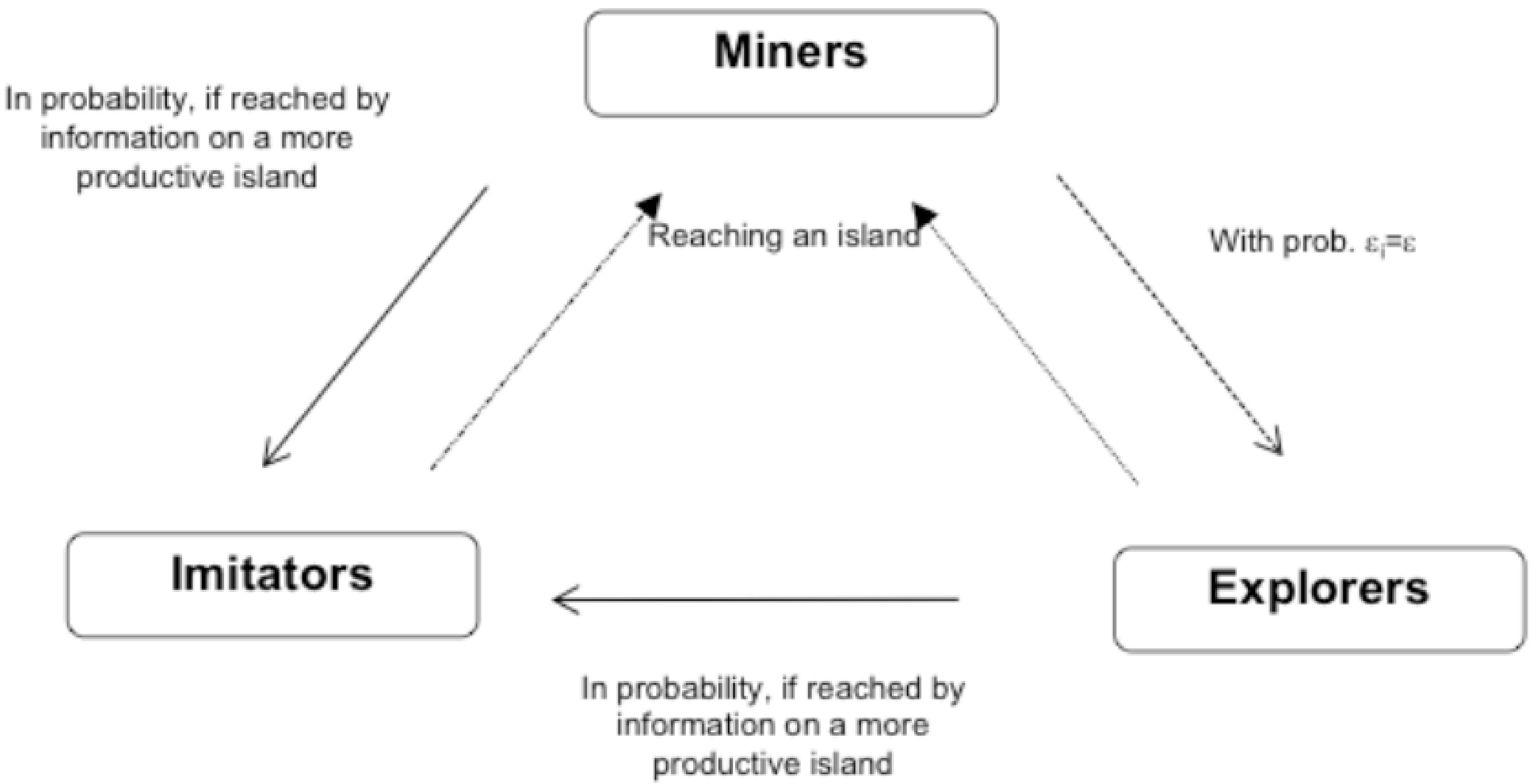
Example: imitation



Example: imitation



Agent's states



Timeline and aggregation

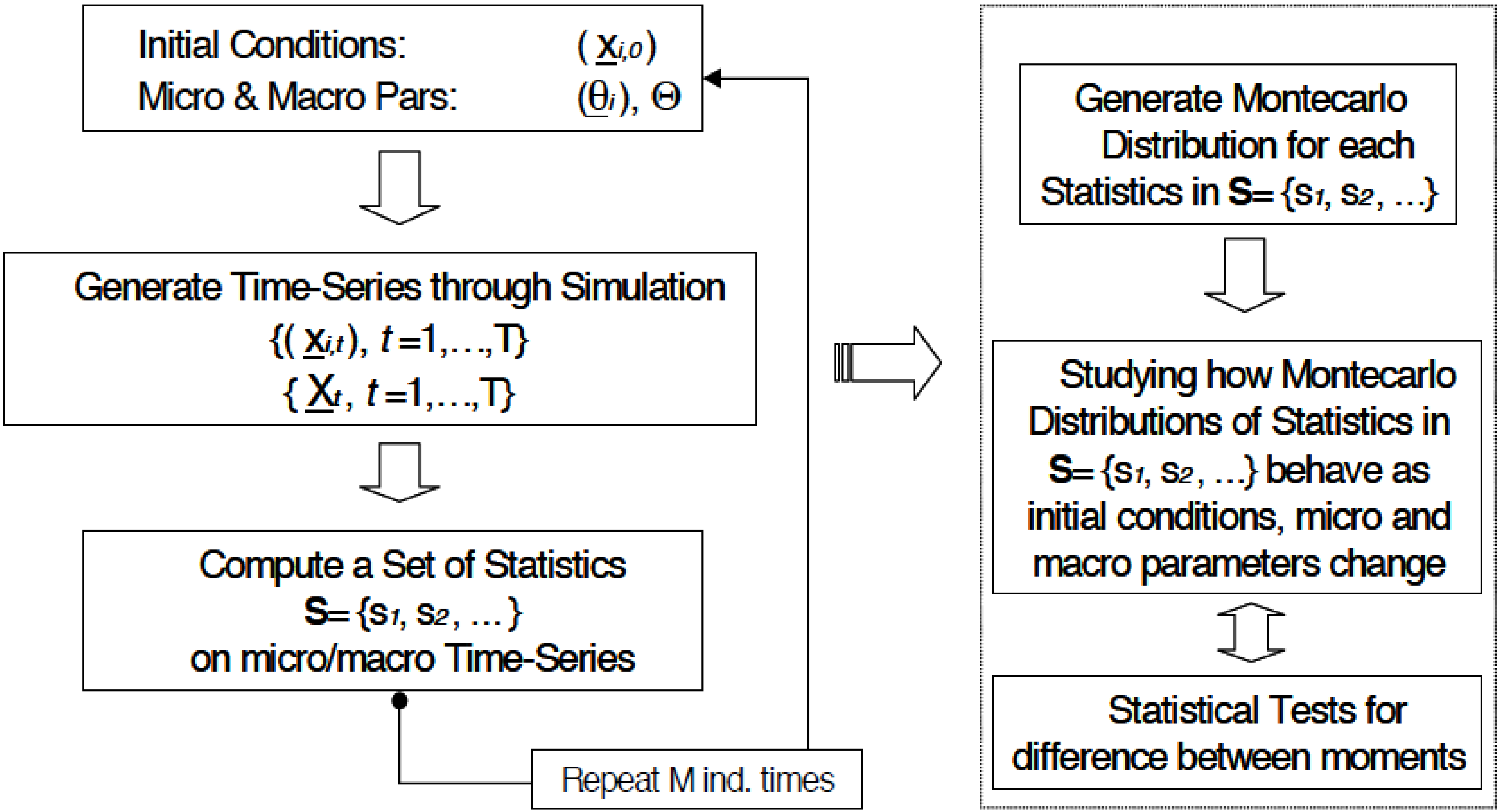
Focus on:

- **aggregate output** (sum of firms' output) and **growth rates**
- number of explorers, imitators, miners

Parameters

Parameter	Meaning
ρ	globality of information diffusion
φ	path-dependency in learning
λ	likelihood of radical innovations
π	baseline opportunity conditions
α	increasing returns to scale in exploitation
ϵ	willingness to explore
N	population size
T	time horizon

Analyzing results



Five questions

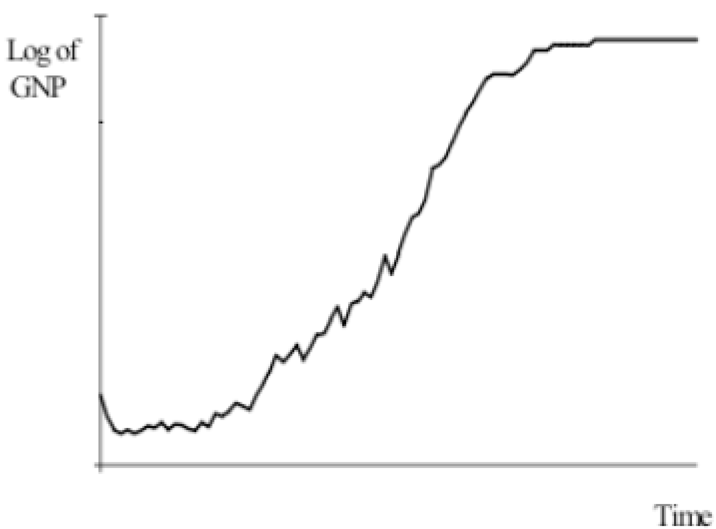
1. Under which general conditions is the economy able to generate **self-sustaining growth** as the outcome of the joint processes of exploitation and exploration?
2. In self-sustaining growth regime, do log(GNP) time-series display empirically observed statistical properties?
3. In self-sustaining growth regime, what are the roles played by system parameters (i.e. by the sources of growth)?
4. Does the self-sustaining growth process lead to explosive growth patterns? Does the variability of growth rates increase over time and tends to infinity?
5. What happens in we inject in the economy more rational firms?

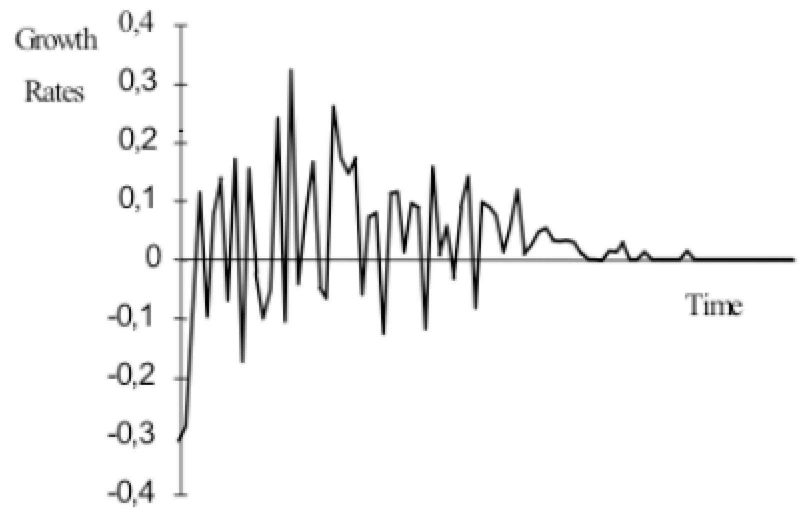
Closed economy **without** exploration

- **Shutting down** exploration and innovation:
  - a given initial set of islands (e.g., only 2)
  - firms initially mining on them (50%, 50%)
  - they can only exchange information among the 2 existing technologies (initial set of islands cannot be expanded)
- **Diffusion** of information drives growth:
  - in this case the model is analytically solvable!
  - whenever an island manages to capture all agents the growth process stops (growth rates are zero)
  - the process is path-dependent and possibly inefficient (convergence toward an inefficient level of output is a non- zero probability event)

Closed economy **without** exploration

- Growth is always a **transitory** phenomenon
- **Lock-in** may occur on the ex-ante less efficient island



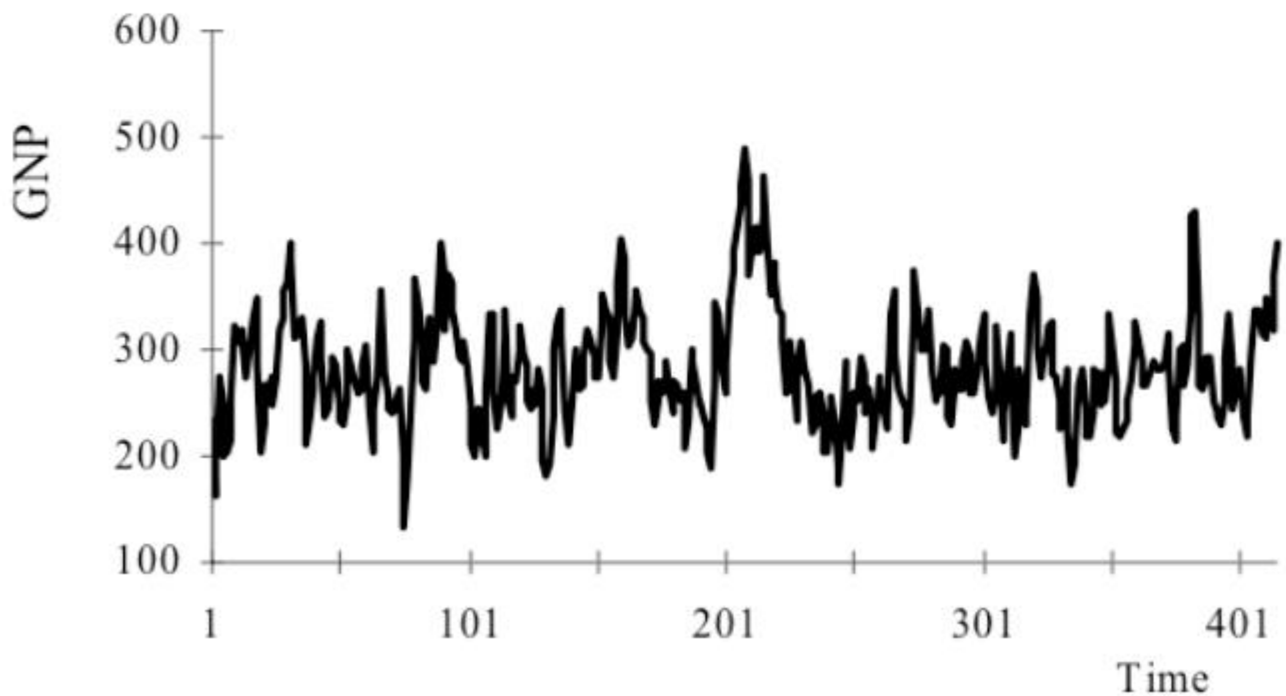


Closed economy **with** exploration

- Allowing for exploration in a **closed box**:
  - initial set of islands cannot be expanded (no innovation)
  - explorers are allowed to search only inside initial box
  - imitation still occurs as before
- **Diffusion** of information still drives growth:
  - process driven by information diffusion
  - steady states can be destabilized by “irrational” agents who
  - decide to leave their island even if everyone is there

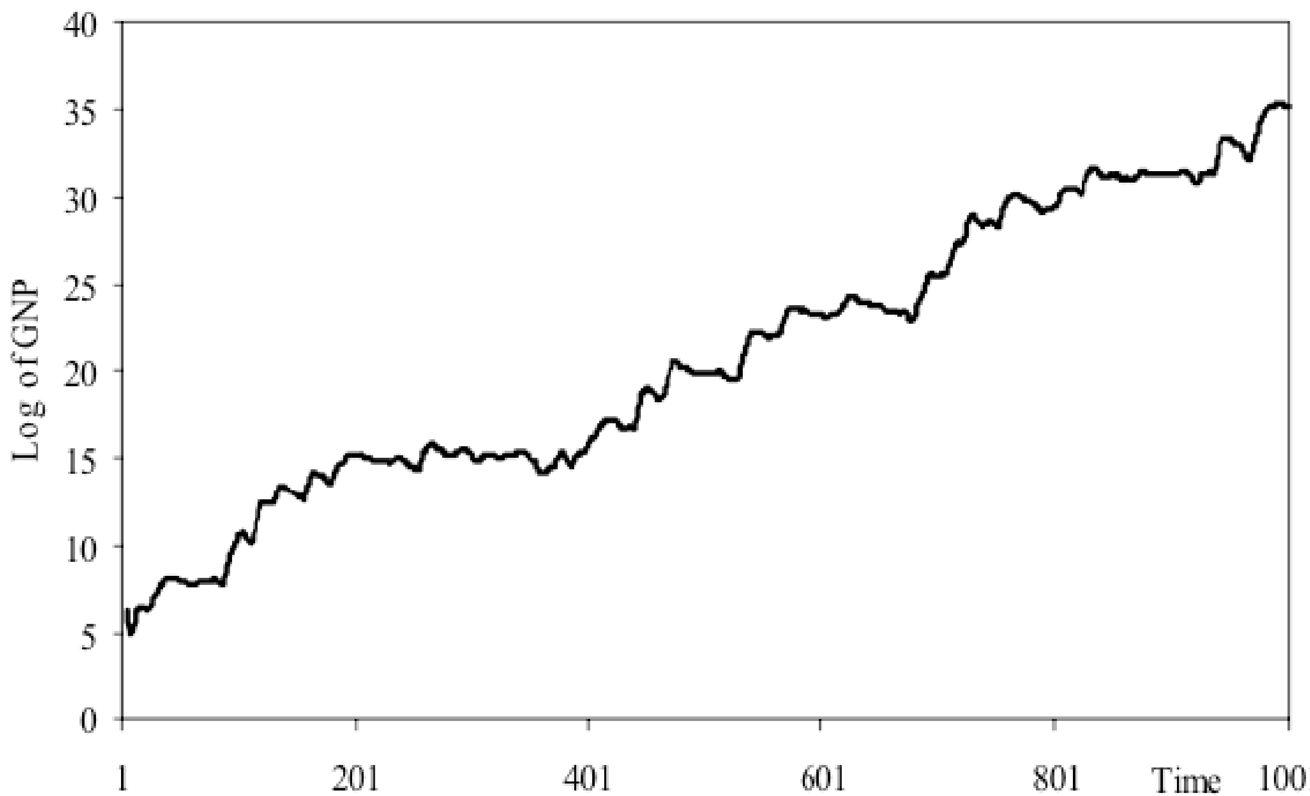
Closed economy **with** exploration

- Absorbing states become basins of attraction: growth is a **transitory** phenomenon, but fluctuations can arise



Open-ended economy

- In the **full-fledged model** self-sustaining growth can arise



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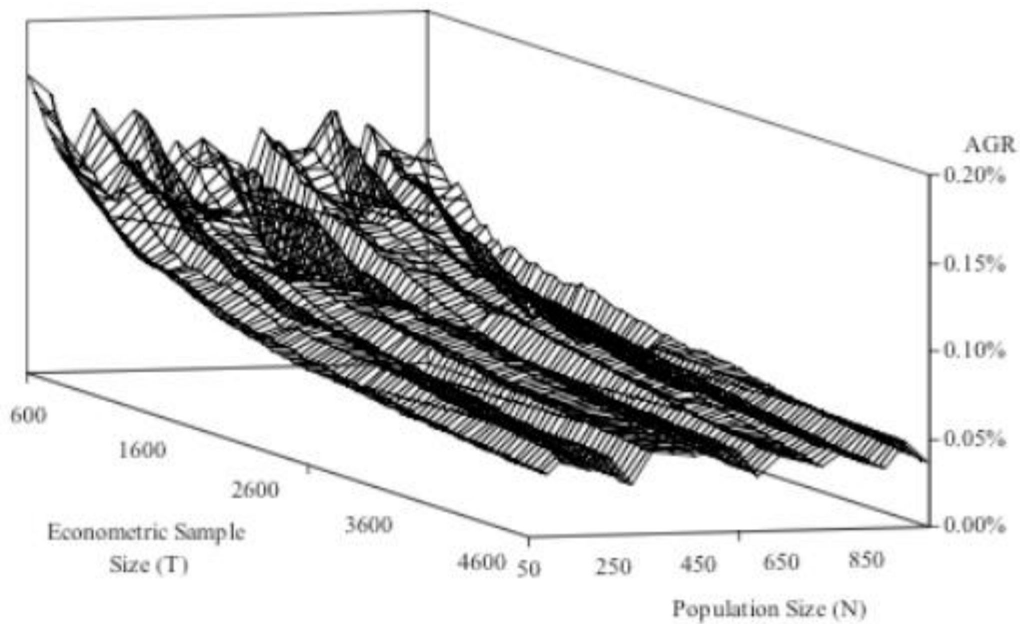
1. Under which general conditions is the economy able to generate self-sustaining growth as the outcome of the joint processes of exploitation and exploration?
2. In self-sustaining growth regime, do log(GNP) time-series display **empirically-observed** statistical properties?
3. In self-sustaining growth regime, what are the roles played by system parameters (i.e. by the sources of growth)?
4. Does the self-sustaining growth process lead to explosive growth patterns? Does the variability of growth rates increase over time and tends to infinity?
5. What happens in we inject in the economy more rational firms?

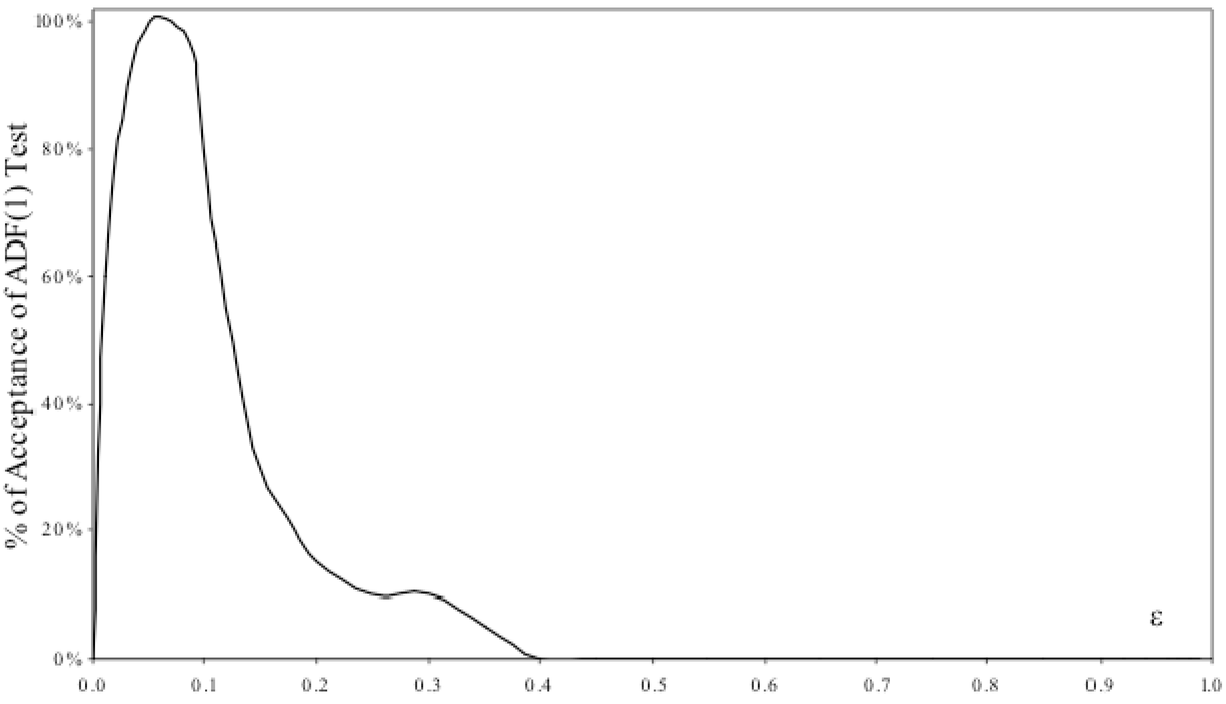
Empirical validation

- Yes, if **self-sustaining growth** does emerge:
  - log(GNP) time series are I(1), i.e. difference-stationary
  - growth rates are positively correlated over short horizons
  - persistence of shocks are in line with empirical evidence
- **Scale-effects** are not present
  - as in reality, unlike in many endogenous growth models!



Empirical validation

- log(GNP) time-series is **I(1)** if:
  - increasing returns to scale, opportunities, path-dependency and globality of information are strong enough
  - and if the exploitation-exploration trade-off is solved



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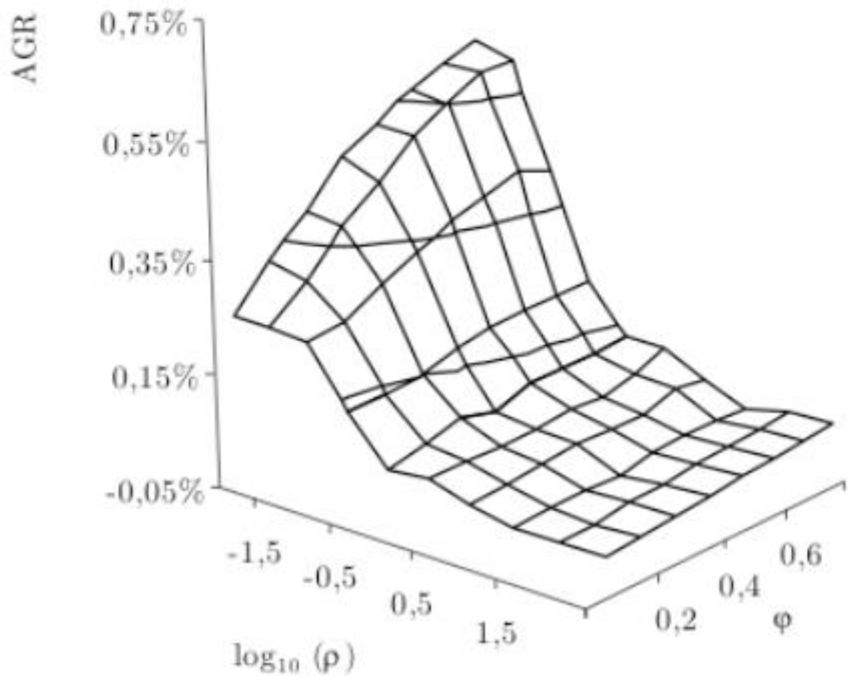
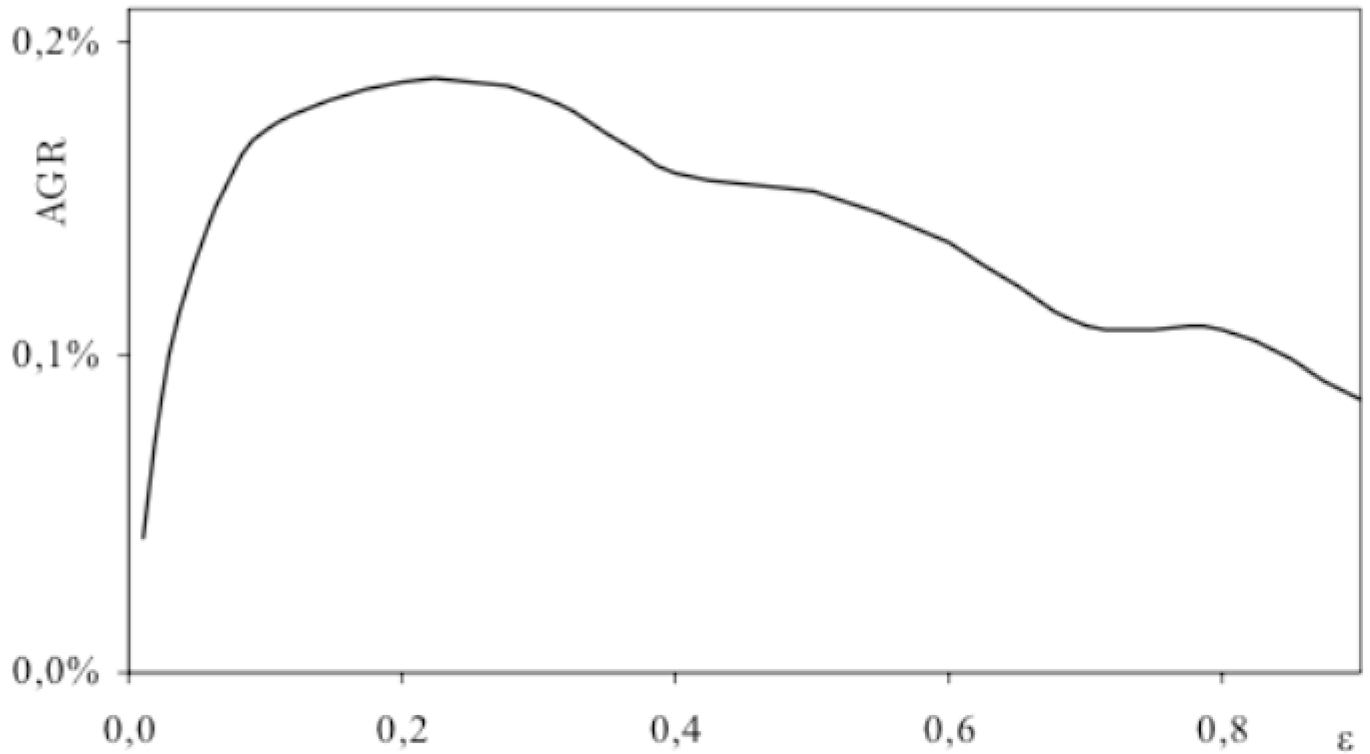
Sensitivity analysis

- Average growth rates (AGRs) **increasing** in:
  - path-dependency in knowledge accumulation
  - globality of information diffusion
  - returns-to-scale strength and opportunities



Sensitivity analysis

- AGRs are **maximized** only if there is a balance between
  - resources devoted to exploration and resources devoted to
  - exploitation



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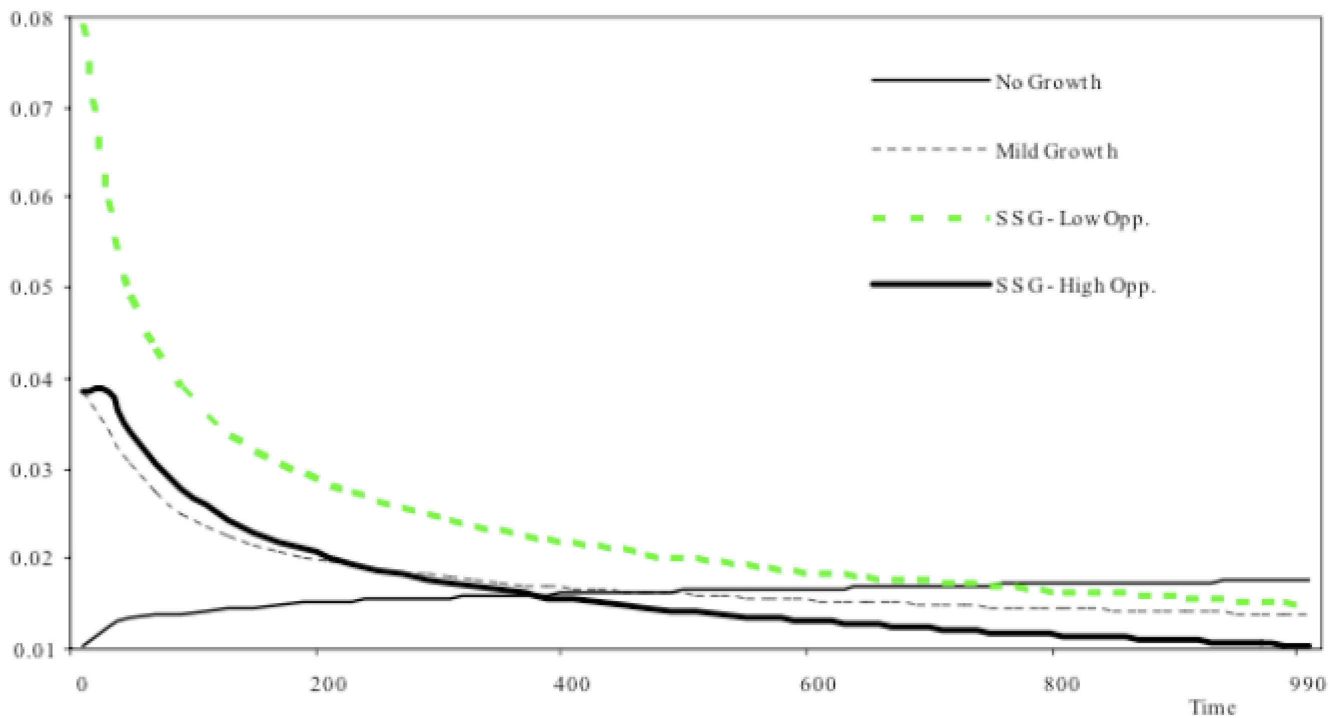
Time dynamics

- Higher growth is always associated to smaller growth-rate variability
  - self-sustained growth is a self-organized process leading to ordered growth patterns



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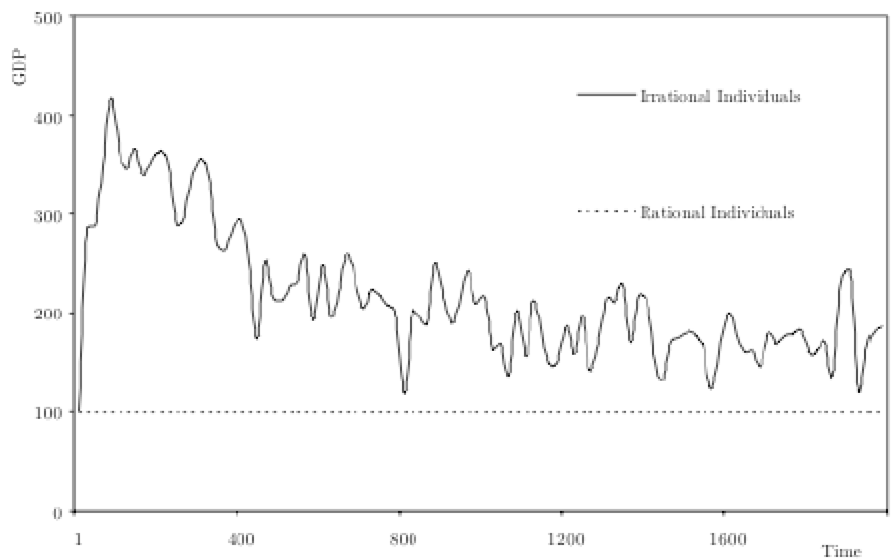
Rational agent coordination failure

- Simple setup:
  - CRTS, no info diffusion, no path-dependency
  - injecting in the economy a representative rational firm (RRF) who decides whether to exploit or explore by maximizing expected returns
  - RRF knows the structure of the economy and the direction where best islands are (but not where they are)



ISLAND MODEL IN LSD

Island Model in LSD

- What is it?
  - A **full** version of Fagiolo & Dosi 2003 available to be run inside the LSD environment and analyzed in R
- How was it done?
  - It's a full **recode**, using all LSD advanced resources like objects, networks and lattices
- Is it really the same model?
  - Model outputs were not carefully compared with the original version (**bugs** may still exist!)

Why LSD?

- Why Island Model in LSD would be better?
  - **Multiple** parallel “seas/countries” for free
  - Easy **interface** (configuration, output data, graphs)
  - Transparently run in Windows/**Mac**/**Linux**
  - More **productivity**: all simulation tasks done in a single graphic interface (no command line!)
  - Enhanced **tools**: configuration sets mgmt., exception mgmt., quick graphic data analysis, sensitivity analysis, statistical packages interface

Exploring the Island Model

- Inside LSD folder structure, look for the folder containing the Island Model in the 'Examples/SantAnna' folder
  - The folder contains the main file '**fun_Island.cpp**'
- The code is organized in sections, associated to the respective object types:
  - **Sea**: single object instance representing one sea
  - **Island**: one instance per island (known/unknown)
  - **KnowIsland**: one instance per known island
  - **Miner**: one dummy instance per agent mining on island
  - **Agent**: one instance per agent

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Object Sea

- **Init**: initializes the model, run **once**
  - Create the lattice, if appropriate
  - Create the random initial islands (known/unknown)
  - Draw the initially known islands
  - Allocate agents in known islands randomly
- **Step**: forces agents to decide **what to do** first
- **l**: counts the known islands
- **m**: counts the number of miners
- **Q**: accumulates the sea's production (GDP)
- **J**: expands and counts the islands set
  - Expand each frontier as required (N, S, W, E)

Object KnownIsland

- **_m**: counts the miners in the island
  - Ignore "inactive" Miner object instances
  - Adjust island color on the lattice (empty/colonized)
- **_Qisland**: accumulates the island production
- **_c**: compute the island productivity

Object Miner

- **_Qminer**: compute miner production
- **_cBest**: the best signal productivity received
  - Evaluates all signals received on the island

Object Agent

- **_a**: define the state of the agent
  - If an Explorer, navigate to a new random position
  - If an Imitator, navigate the shortest path to target
  - If navigator, update position on the lattice
  - If arrive in an island, become a Miner
  - If a Miner decide if become Explorer or Imitator

Data structure (1)

- Object **Root**: technical flags & parameters
  - latticeOpen, showSea, simSpeed, sizeLattice
- Object **Sea**:
  - Parameters: N, alpha, epsilon, phi, lambda, pi, rho, l0radius, minSgnPrb
  - Flags: seaShown,
  - State parameters: xxxxFrontier
  - Technical variables: Init, Step
  - Variables: l, m, J, Q

Data structure (2)

- Object **Island**:
  - Flag: _known
  - State parameters: _idIsland, _xIsland, _yIsland
- Object **KnowIsland**:
  - State parameters: _idKnown, _s
  - Variables: _c, _m, _Qisland
- Object **Miner**:
  - Flag: _active
  - State parameters: _agentId, _xBest, _yBest
  - Variables: _cBest, _Qminer

Data structure (3)

- Object **Agent**:
  - State parameters: _idAgent, _knownId, _xAgent, _yAgent, _xTarget, _yTarget, _Qlast
  - Variable: _a

The lattice

- Can be turned off for speed: **showSea**
- Can show the entire sea or just the central area: **sizeLattice**
  - Sea size = 2 * total time steps + 1
- Reduce simulation speed: **simSpeed**

Topology configuration

- Set the maximum radius for initial known islands: **l0radius**
- Define the minimum signal probability to consider (speed-up): **minSgnPrb**

‘hook’ pointers

- LSD ‘hook’ pointers connect all the objects to maximize performance:
  - KnowIsland -> Island
  - Miner -> Agent
  - Agent -> Miner

Diffusion network

- Each KnownIsland instance is the **hub** of a **star network** to all other relevant known islands
  - Link creation depends on the signal intensiti threshold defined by **MinSgnPrb**
  - All links are bidirectional
  - Link weight is defined by the exponential decay factor between the two islands

Bibliografia

- FAGIOLO, G.; DOSI, G. Exploitation, exploration and innovation in a model of endogenous growth with locally interacting agents. *Structural Change and Economic Dynamics*, v. 14, p. 237-273, 2003.