Discretization for Linear Advection Equation

Homework 6

Handed out: **October 29**
Due in: **November 12**

Consider the linear advection equation:

$$\partial_t T + c\partial_x T = 0$$

Solve this equation for $T(x,t)$ on a domain $x \in [0:2]$. The velocity is constant $c = 1$ m/s. The initial conditions are:

$$T^0(x) \equiv T(x,0) = \begin{cases} 0 & \text{if } x \leq 0.5, \\ 1 & \text{otherwise} \end{cases}$$

The boundary condition is $T(0,t) = 0$. Discretize the flow domain with a spacing of $\Delta x = 0.01$ (see sample case `line`). OpenFOAM has a solver named `scalarTransportFoam`, that solves the following equation:

$$\frac{\partial T}{\partial t} + \nabla \cdot \mathbf{U}T - \nabla \cdot (\nu \nabla T) = 0$$

Use this solver by setting the diffusion coefficient to zero, and set $\mathbf{U} = c\mathbf{i}$. Compare the following eight discretization schemes:

- A: `linear` implicit
- B: `upwind` implicit
- C: `linearUpwind` implicit
- D: `linearUpwind, cellLimited` implicit
- E: `upwind` explicit
- F: `linear` explicit
- G: `linearUpwind` explicit
- H: `linearUpwind, cellLimited` explicit

(1) (5 pts) Test each scheme using a time-step size corresponding to Courant numbers of $\sigma = 0.5, 1, 2$. The implicit solver is part of the OpenFOAM library, and is called `scalarTransportFoam`. The custom explicit version is provided to you as `explicitScalarTransportFoam`. Compare the eight schemes. Hand in three figures, one for each time-step size. Plot the solution at $t = 1$, as a function of x. You do not need to plot the curve if the solution is unstable, but in that case you must state that the solution is not stable. Comment about which scheme you prefer.

(2) (5 pts) Solve the scalar transport equation, with zero diffusion coefficient, on a square unstructured domain of size 1 x 1 (see case `squareUnstructured`). The advection velocity is $\mathbf{U} = 1\mathbf{i} + 1\mathbf{j}$. The left boundary has a value of $T(0,y) = 0$, the bottom boundary has the value of $T(x,0) = 1$. First use the explicit solver, and use your favorite space discretization. Plot the solution at $t = 1$ along the line $x = 0.5$, and compare with the upwind space discretization. Report the time step size that you use, and the corresponding Courant number (since it is a spatially varying grid size, use the utility `Co`). Repeat for the implicit solver. Again report the time-step size that you used, in terms of seconds and in terms of maximum Courant number. You will hand in two plots for this question, each with two curves: one for your favorite scheme, and one for the upwind.