Finite-Volume Method: Laplacian

Homework 5

Handed out: **October 22**
Due in: **October 31**

Use the 20x20 and 40x40 unstructured grids that you were given for homework 4 to solve the two Laplacian problems in homework 3. Use the standard OpenFOAM `laplacianFoam` solver. Choose your preferred method to calculate the gradient for the non-orthogonal correction based on your results from homework 4 (please specify what you chose).

For each problem below, investigate the number of iterations that are required to fully account for the correction to the discrete Laplace operator. You can find this information in the output of `laplacianFoam` listed after `No Iterations`. For example:

```
DICPCG:  Solving for T, Initial residual = 1, Final residual = 7.16552e-07, No Iterations 43
```

shows that 43 iterations were used to reach the specified residual level for this specific non-orthogonal correction step.

Compare the results on each unstructured grid with those from the solver you wrote in homework 3. Compare for similar number of unknowns (grid size). Show a surface plot of both the converged finite-difference (FD) solution (homework 3) and the converged finite-volume (FV) solution computed with `laplacianFoam`. Plot slices of the FD and FV solutions at a desired constant x or y location to compare the two methods. You can use the `sample` utility like in homework 4 to generate the desired slices with OpenFOAM. Comment on how you compare the different grids (unstructured versus structured).

(1) (5 pts) Solve the Laplacian Dirichlet problem that you solved in homework 3:

$$\begin{aligned} \nabla^2 T(x,y) &= 0 \\ \mathrm{T}(0,y) &= 0 \\ T(x,0) &= 0 \\ T(x,1) &= 0 \\ T(1,y) &= 100 \end{aligned}$$

(2) (5 pts) Solve the Neumann problem that you solved in homework 3:

$$\begin{aligned} \nabla^2 T(x,y) &= 0 \\ \partial_n T(0,y) &= 0 \\ T(x,0) &= 0 \\ T(x,1) &= 100 \\ T(1,y) &= 100 \end{aligned}$$

Note: The unknown u from homework 3 is replaced with T here to be consistent with the `laplacianFoam` nomenclature.