Assignment 4

MAS365 Introduction to Numerical Analysis **Fall 2023**
Prof. Chang-Ock Lee **Due date: Nov. 14 (Tue), 2023**

Note: Put your homework in KLMS before the beginning of the class. If you did computer programming work, hand in your code and results in KLMS before the beginning of the class, too. For the plotting work, you may use any plotting tool, but I recommend to use MATLAB.

1. Find the integral
$$\int_0^1 s\sqrt{1-s}\,ds$$
with four decimal place accuracy, using the following methods.
   (a) The composite trapezoidal rule.
   (b) The composite Simpson's rule.

   Compare the costs and efficiencies of two methods.

2. Using the adaptive Simpson's rule, find the integral in Problem 1 with $\mathrm{TOL} = 10^{-7}$.

3. To find the integral
$$\int_0^{\pi/4} \cos^2 x\,dx\,,$$
we will use Gaussian quadrature. To get an accurate answer, there are two ways; one is use the higher order quadrature and another is use the composite Gaussian quadrature with, for example, three nodes. Discuss efficiency of the two approaches in terms of accuracy, cost, and ease of programming. See the table in the next page for the nodes and weights of Gaussian quadrature.

4. Use the composite Gaussian quadrature to approximate a triple integral
$$\iiint_S xy\sin(yz)\,dV,$$
where S is the solid bounded by the coordinate planes and the planes $x = \pi, y = \pi/2$ and $z = \pi/3$. To use the composite Gaussian quadrature, divide the interval on each axis into two subintervals and apply Gaussian quadrature with three nodes for each subinterval. Compare this approximation to the exact result.

Nodes and weights of Gaussian quadrature

Number of nodes	Nodes	Weights
$n \leq 5$	See Table 4.12 in the textbook	
6	$\pm$.9324695142	.1713244924
	$\pm$.6612093865	.3607615730
	$\pm$.2386191861	.4679139346
7	$\pm$.9491079123	.1294849662
	$\pm$.7415311856	.2797053915
	$\pm$.4058451514	.3818300505
	0	.4179591837
8	$\pm$.9602898565	.1012285363
	$\pm$.7966664774	.2223810345
	$\pm$.5255324099	.3137066459
	$\pm$.1834346425	.3626837834