

## Assignment 3

MAS365 Introduction to Numerical Analysis

Fall 2023

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Due date: Oct. 31 (Tue), 2023

Note: Put your homework in KLMS before the beginning of the class. If you did computer programming work, hand in your code and results in KLMS before the beginning of the class, too. For the plotting work, you may use any plotting tool, but I recommend to use MATLAB.

1. By hand calculation, find the Hermite polynomial for the data:

|      |    |   |   |
|------|----|---|---|
| $x$  | -1 | 1 | 2 |
| $y$  | 2  | 0 | 2 |
| $y'$ | 1  | 1 | 3 |

2. Let  $S$  be the natural cubic spline that interpolates function  $f(x) = (x^2 + 1)^{-1}$  at 21 equally spaced knots in the interval  $[-5, 5]$ . Write a computer program to evaluate  $S(x) - f(x)$  at 51 equally spaced points on the interval  $[-5, 5]$ . This is the same problem as Problem 4 of Assignment 2. Compare the results to discuss the difference.
3. For  $f(x) = \cos x$ ,
  - (a) Find  $f'(0.25)$  as accurately as possible using

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

- (b) Find  $f'(0.25)$  as accurately as possible using Richardson extrapolation formula  $N_3(h)$  described in the class.
4. Derive Simpson's Three-Eighths rule with error term.