Assignment 5

MAS365 Introduction to Numerical Analysis **Fall 2023**
Prof. Chang-Ock Lee **Due date: Nov. 28 (Tue), 2023**

Note: Put your homework in KLMS before the beginning of the class. If you did computer programming work, hand in your code and results in KLMS before the beginning of the class, too. For the plotting work, you may use any plotting tool, but I recommend to use MATLAB.

1. Solve the following equation by hand:

$$\begin{cases} y' = 1 + \dfrac{y}{t} \\ y(1) = 2 \end{cases}$$

2. Solve the above differential equation on the interval [1, 2] using the modified Euler method with $h = 1/10$, $1/20$ and $1/40$. Discuss the reduction of errors.

3. Do the problem 2 using the Runge-Kutta method of order four and Adams Fourth-Order Predictor-Corrector method. Compare accuracy and cost. Draw a graph of errors using any graphic software when $h = 1/40$.

4. This exercise involves solving the system of ordinary differential equations

$$\begin{aligned} \frac{dx_1(t)}{dt} &= x_1(t)[4 - 0.0003x_1(t) - 0.0004x_2(t)] \\ \frac{dx_2(t)}{dt} &= x_2(t)[2 - 0.0002x_1(t) - 0.0001x_2(t)] \end{aligned}$$

   with initial conditions $x_1(0) = x_2(0) = 10000$. The system is to be solved for t from 0 to 4. Write a computer program to solve this system by the Runge-Kutta method of order four using the step size of $h = 0.01$. Does the solution approach constants? If so, what are those?