assignment_1

2023년 8월 30일 수요일 오후 2:46

1. Rearrange the following expressions so as to avoid loss of significance.

   (a) $y - \sqrt{y^2 - 1}$ for y large

   (b) $1 - \cos^3\theta$ for θ near 0 Hint: Use half-angle formulas.

   (c) $z^2 - 200z + 10001$ for z near 100

(a)
$$y - \sqrt{y^2-1} = \frac{(y-\sqrt{y^2-1})(y+\sqrt{y^2-1})}{y+\sqrt{y^2-1}}$$

$$= \frac{1}{y+\sqrt{y^2-1}}.$$

(b)
$$1 - \cos^3\theta = (1-\cos\theta)(1+\cos\theta+\cos^2\theta)$$

$$= 2\sin^2\frac{\theta}{2}\,(1+\cos\theta+\cos^2\theta)$$

($\because$ half-angle formulas)

(c) $Z^2 - 200Z + 10001$

$= (Z-100)^2 + 1$

2. Write a program to use the bisection method to solve for the root of $\tan\theta = e^{\theta}$ in the interval $[0, \pi/2]$. Terminate the program when the relative difference between two consecutive iteration is less than 10^{-3}.

I submit the code : assignment_1_2.

3. (a) Apply the fixed point iteration to the function $x - f(x)$ to attempt to find approximate zeros of $f(x)$, starting at $p_0 = 11.6$ and performing 4 iterations in each case:

(i) $f(x) = 2552 - 30x^2 + x^3$

(ii) $f(x) = (2552 - 30x^2 + x^3)/(-300)$

(b) In each of the preceding two cases, explain, based on the theory of fixed point iteration, why the method "works" or "fails".

(a). I submit the code: assignment_1_3_a.

(b) Put $g(x) := x - f(x)$

(i) Here,

$$g'(x) = 1 - f'(x)$$
$$= 1 + 60x - 3x^2.$$

Note that

$$|g'(p_0)| = 293.32 > 1,$$

so we cannot apply the fixed point theorem. Hence, the method "fails".

(ii) Here,

$$g'(x) = 1 - f'(x)$$
$$= 1 - \frac{1}{5}x + \frac{1}{100}x^2.$$
$$= \frac{1}{100}(x-10)^2.$$

Hence, for $x \in (5, 15)$

$$|g'(x)| \leqq \frac{25}{100} = \frac{1}{4} < 1.$$

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Note that $p_0 \in [5, 15]$. And $g'(x) \geqq 0$ in $[5, 15]$. Hence, g is monotonically increasing on $[5, 15]$. Here,

$$g(5) \approx 11.4233$$

$$g(15) \approx 12.2566.$$

Hence, if we restrict g on $[5, 15]$, Then

$$g : [5, 15] \to [5, 15].$$

Therefore, we can apply the fixed point theorem, which implies that the method "works". ◇

4. Do the problem 2 using Newton's method.

I submit the code: assignment_1-4. ◇