Gates, GRU

Deep Neural Network

Session 19

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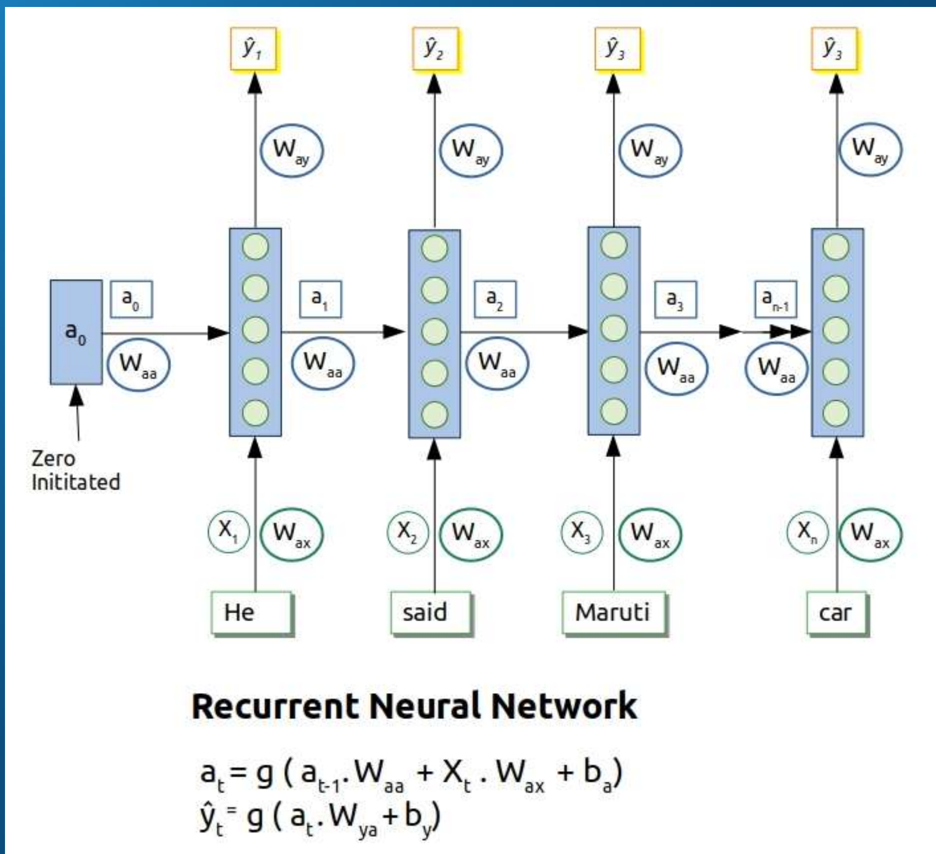
Agenda

- Where we are
- Vanishing and exploding gradients
- Keeping Things Stable
- GRU Units

Recurrent Neural Networks (RNNs)

- Recurrent Neural Networks take the previous output or hidden states as inputs
- The composite input at time "t" has some historical information about the happenings at time T < "t".
- RNNs are useful as their intermediate values (state) can store information about past inputs for a time that is not fixed a priori
- Note that the weights are shared over time
- Essentially, copies of the RNN cell are made over time (unrolling/ unfolding), with different inputs at different time steps

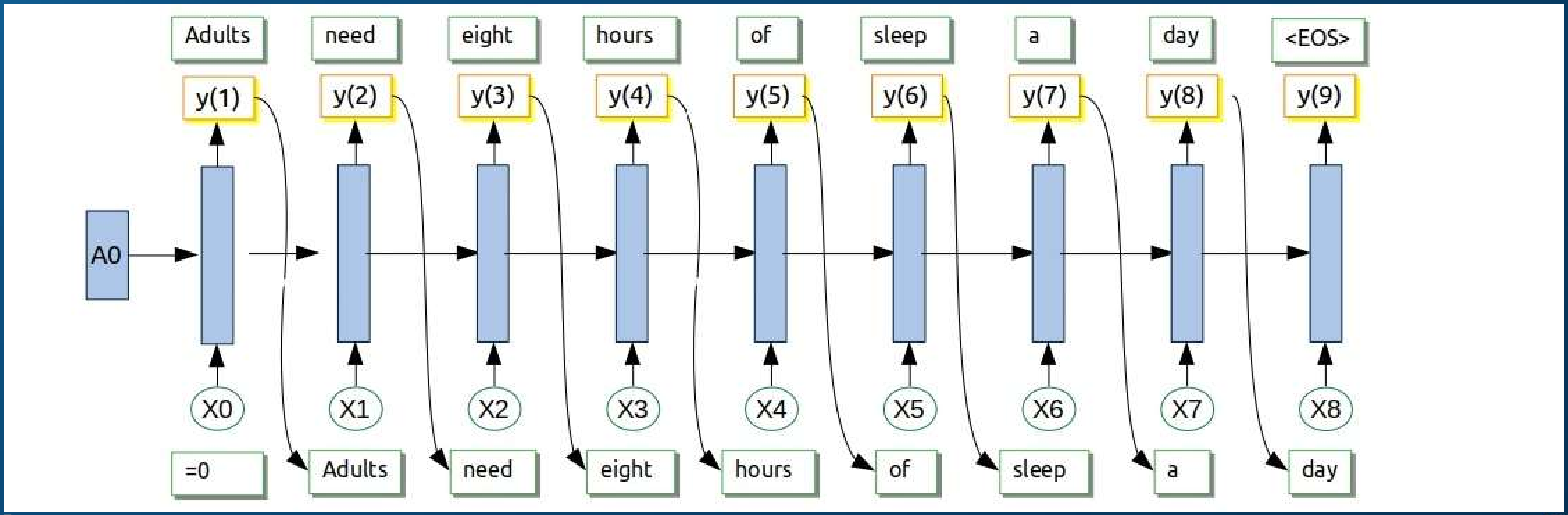
That's is Recurrent Neural Network…



- We worked out math too….

RNN Model – Sampling from Trained Model

- And the Cost functions
- $J(\hat{y}, y) = \Sigma\, \ell\, (\hat{y},\ y)$ BTW: these cost functions are also known as Jacobians
- $J(\hat{y}, y) = -\frac{1}{m}\Sigma\, y * \log(\hat{y})$
  - Which we will be minimizing



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Gradient - a Difficult Terrain

- Have seen how to compute the gradient descent update for using backprop
- In case of RNN the backprop is through time
- At times, gradient descent completely fails because either they explode or vanish
- It’s hard to learn dependencies over long time windows
- How to learn long-term dependencies?

Sentences can be tricky…

"As he crossed toward the pharmacy at the corner he involuntarily turned his head because of a burst of light that had ricocheted from his temple, and saw, with that quick smile with which we greet a rainbow or a rose, a blindingly white parallelogram of sky being unloaded from the van—a dresser with mirrors across which, as across a cinema screen, passed a flawlessly clear reflection of boughs sliding and swaying not arboreally, but with a human vacillation, produced by the nature of those who were carrying this sky, these boughs, this gliding façade."

Vladimir Nabokov, "The Gift." 96 words sentence…

Vanishing and Exploding Gradients

Vanishing Gradient / Exploding Gradient

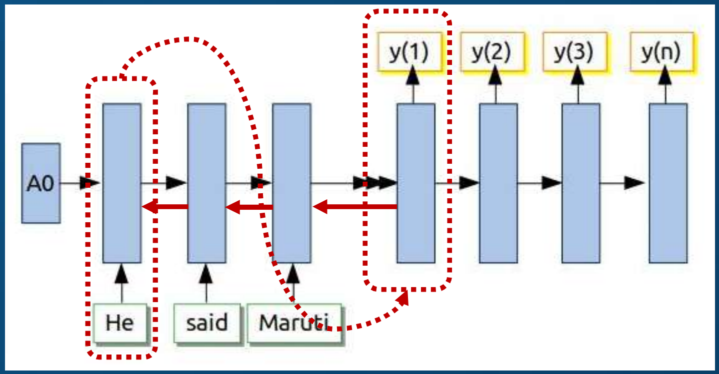
- What happens to the magnitude of the gradients as we back propagate through many layers?
  - If the weights are small, the gradients shrink exponentially
  - If the weights are big the gradients grow exponentially
- Typical feed-forward neural nets can cope with these exponential effects because they only have a few hidden layers
- We can manage gradients by initializing the weights very carefully in feed-forward networks
  - We have already experienced by using appropriate
- Is it applicable to RNNs as well?

Vanishing Gradient / Exploding Gradient

- In an RNN trained on long sequences (e.g. 100 time steps) the gradients can easily explode or vanish.
- Even with good initial weights, its very hard to detect that the current target output depends on an input from many time-steps ago
  - So RNNs have difficulty dealing with long-range dependencies

Why Gradients Explode or Vanish

- Recall the RNN for machine translation
  - For example, we read an entire English sentence, and then has to output its French translation



- A typical sentence length is 20 words. This means there’s a gap of 20 time steps between when we see some information and when we needs it.

- The derivatives need to travel over this entire pathway

Why Gradients Explode or Vanish…

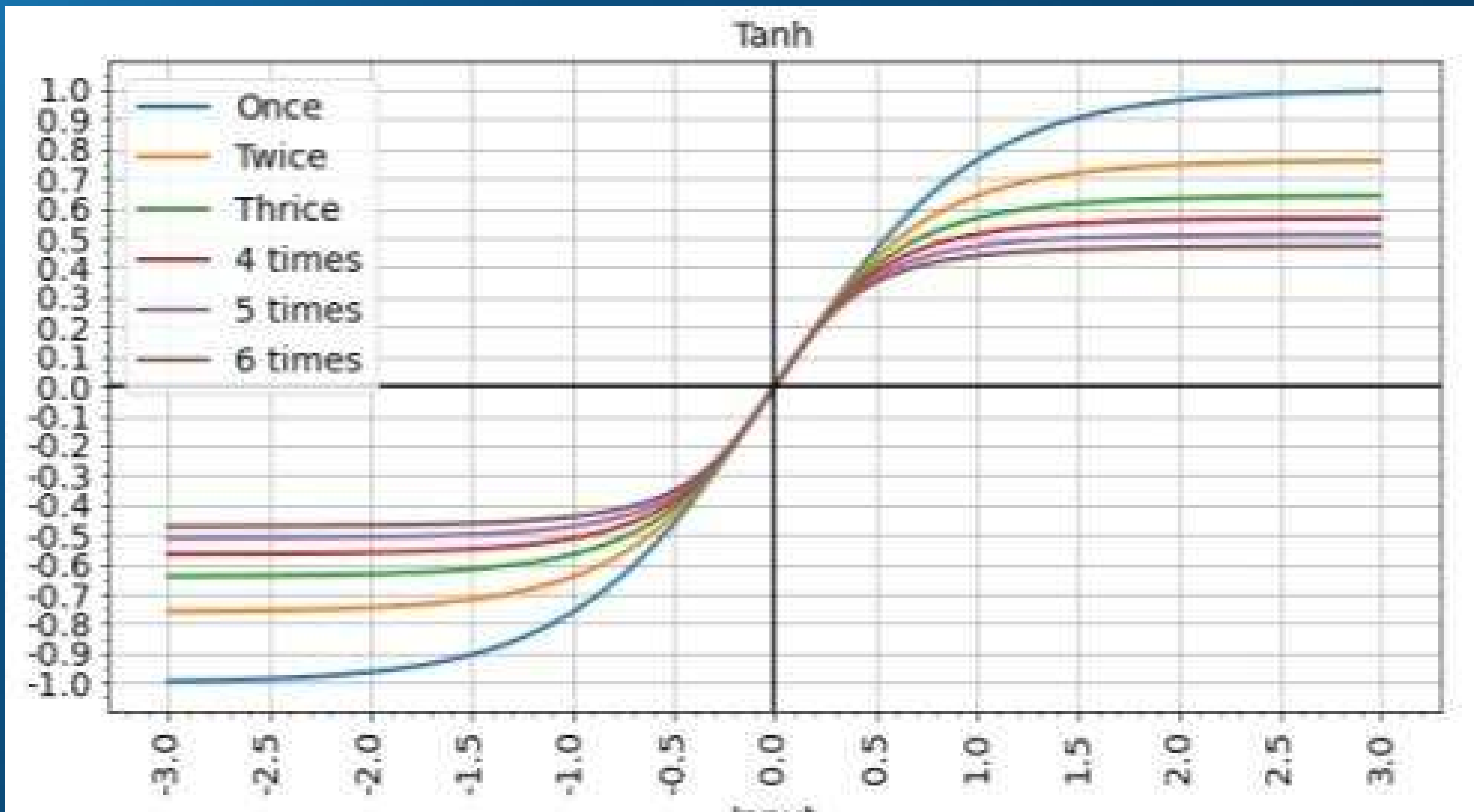
- Please recall:
  - $z = X \,.\, W + b$
  - $\hat{y} = a = \sigma\,(z)$
  - $a_1 = \sigma\,(a_0 \,.\, W_1)$

- That through the time steps will be
  - $\hat{y} = \sigma\,(\,\sigma\,(\,\sigma\,(\,\sigma\,(\sigma\,(a_0 \,.\, W_1)\,.\, W_2).\, W_3\,).\, W_4)$

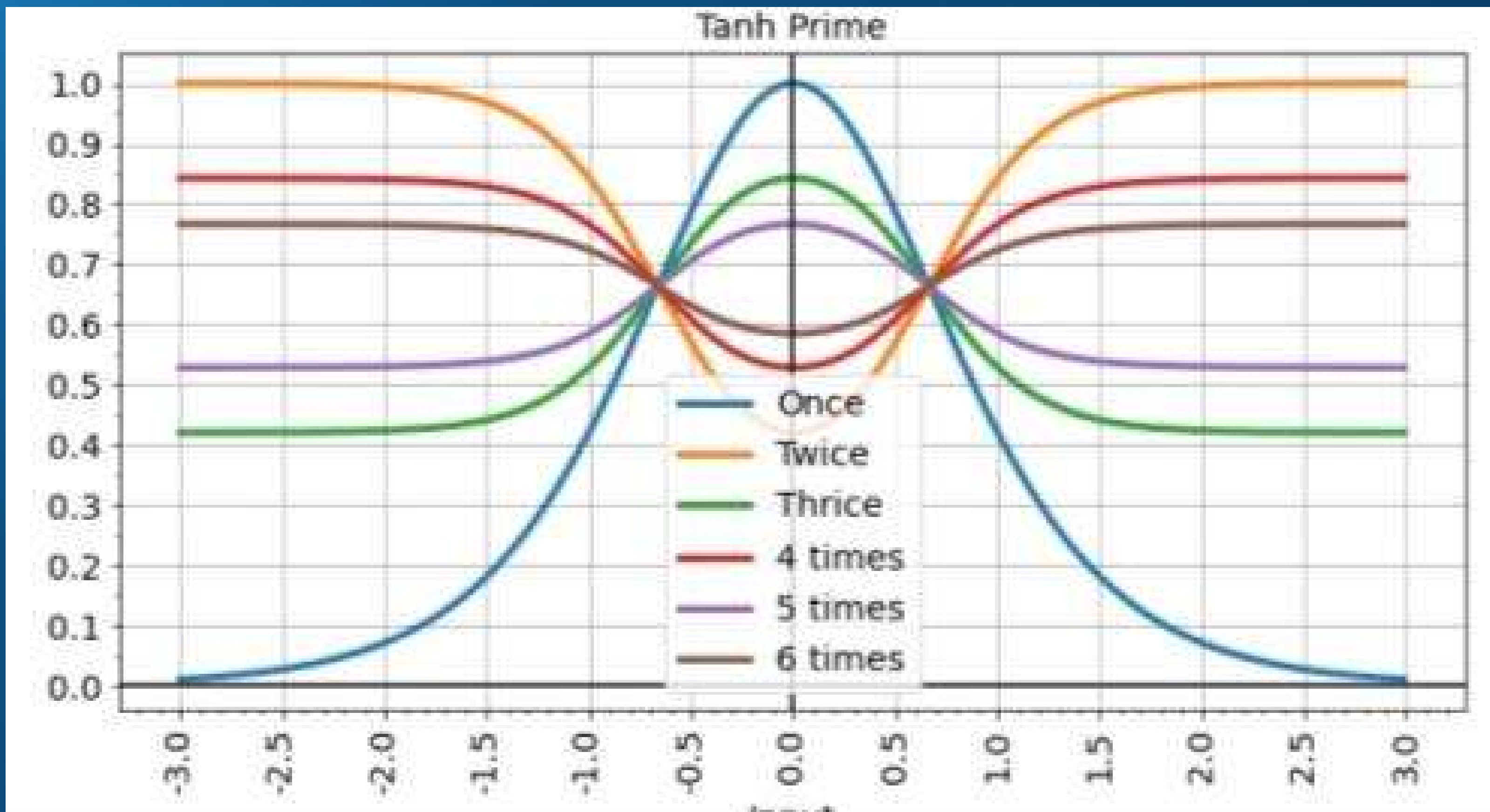
- In backprop, we will be carrying L(ŷ , y) through the activation function iteratively…

- Longer the chain... more iterations on the Ws...

Iterated functions…



Iterated functions…



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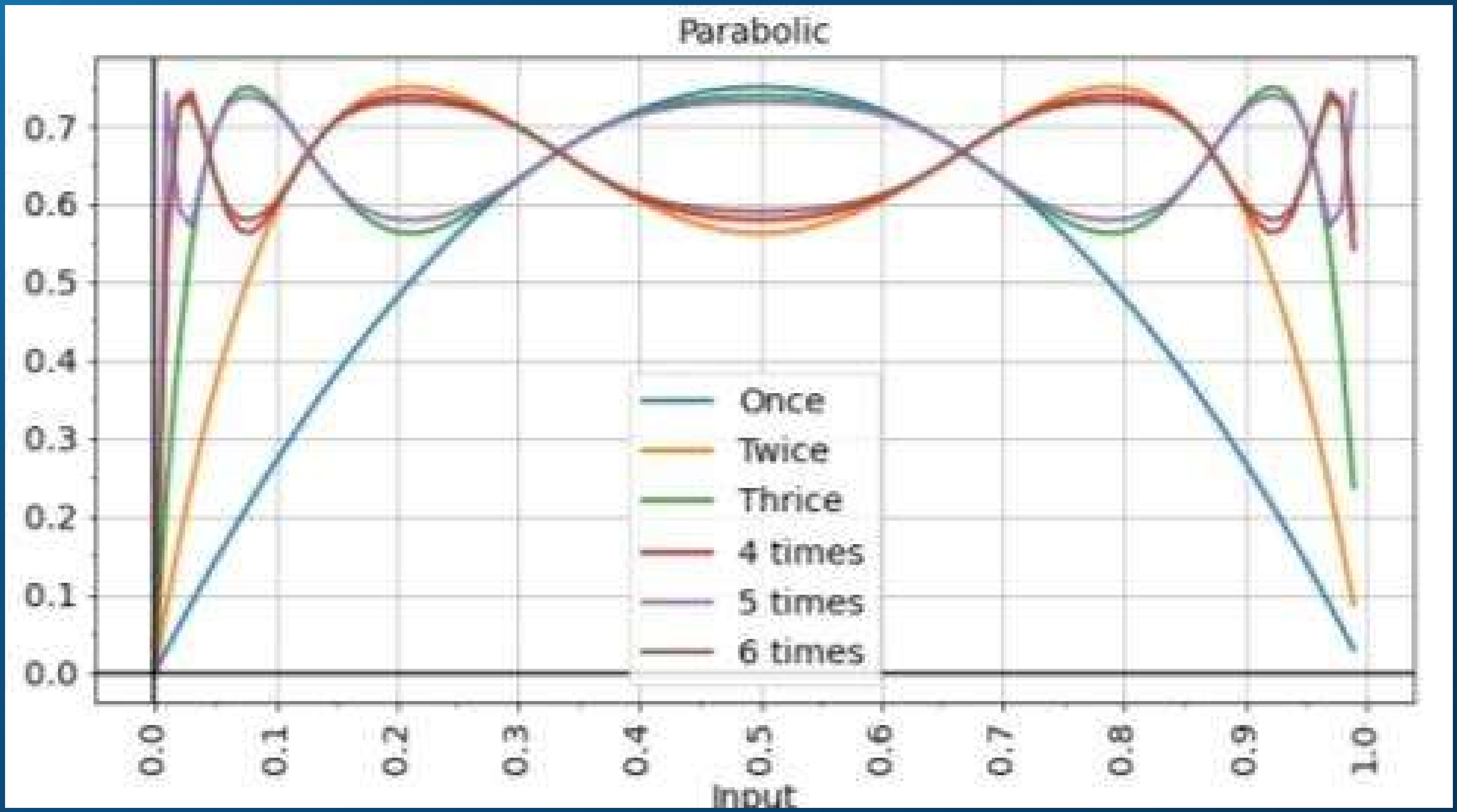
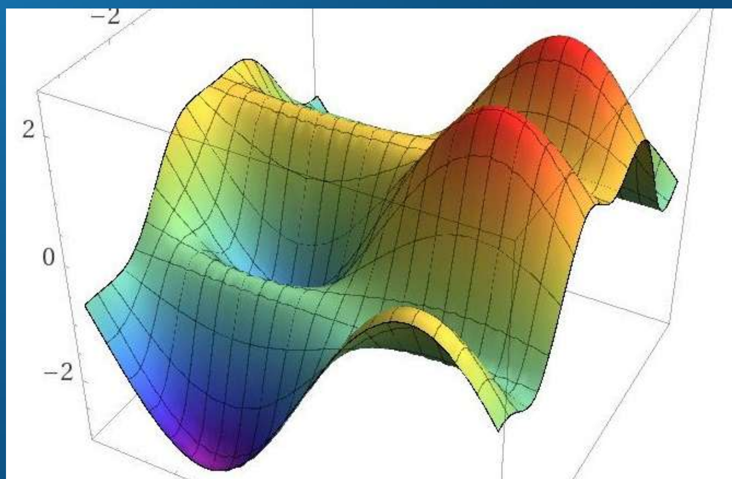
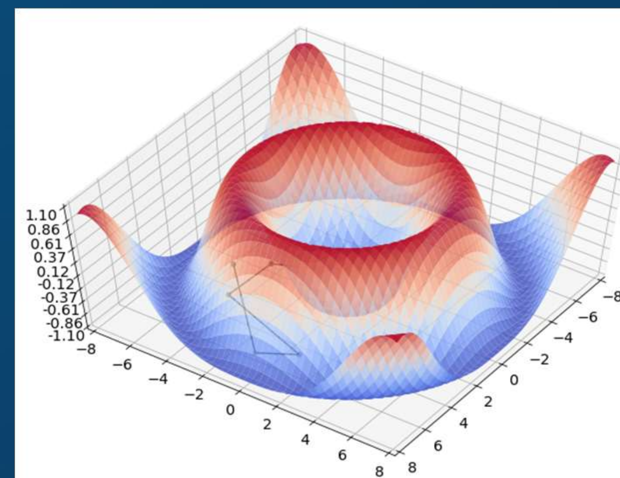
Iterated functions…



Why Gradients Explode or Vanish

- Let's imagine an RNN's behavior as a dynamical system, which has various slopes and valleys





- Within one of the colored regions, the gradients vanish because even if you move a little, you still wind up at the same valley

- If you're on the boundary, the gradient blows up because moving slightly moves you from one side to another and subsequently to other valley

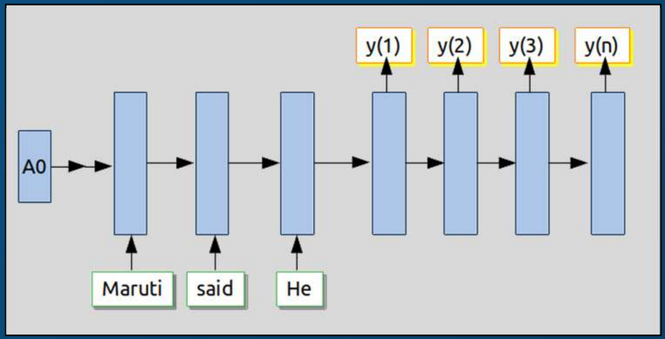
Keeping Things Stable

Keeping Things Stable - Gradient Clipping

- Clip the gradient
- Clip the gradient ‘g’ so that it has a norm of at most ‘η’:
  - if $||g|| > \eta$: then $g = \eta * \frac{g}{||g||}$
  - Where ‘η’ is another parameter you may want to tune
- The gradients are biased, but at least they don’t blow up.

Keeping Things Stable - Reverse the Input Sequence

- Applicable in similar languages
  - Hindi → Marathi,
  - English →French,
  - Spanish → Portuguese



- No point in using situation like
  - Hindi → Mandarin or 'Hiragana' or 'Kanji' ; may be 'Katakana'

- This way, there's only one time step between the first word of the input and the first word of the output.

- The network can first learn short-term dependencies between early words in the sentence, and then long-term dependencies between later words.

Keeping Things Stable – Identity initialization

- Redesign the architecture, since the exploding/vanishing problem highlights a conceptual problem with vanilla RNNs

- The hidden units are a kind of memory. Therefore, their default behavior should be to keep their previous value.
  - The function at each time step should be close to the identity function.
  - It's hard to implement the identity function if the activation function is nonlinear!

- If the function is close to the identity, the gradient computations are stable

Keeping Things Stable – Identity initialization

- The identity RNN architecture : [Le et al., 2015. A simple way to initialize recurrent networks of rectified linear units.]
  - The activation functions are all ReLU,
  - Recurrent weights are initialized to the identity matrix
- Proof: This simple initialization trick achieved some neat results;
- For instance, it was able to classify MNIST digits which were fed to the network one pixel at a time, as a length-784 sequence

Applicability

- Discussed three mechanism for training RNNs
  - All pretty widely used.
  - But the identity initialization trick actually eludes to something much more fundamental
  - Keep their previous value, unless it is necessary to change

- Ask: the ability to preserve information over time until it’s needed

GRU Units

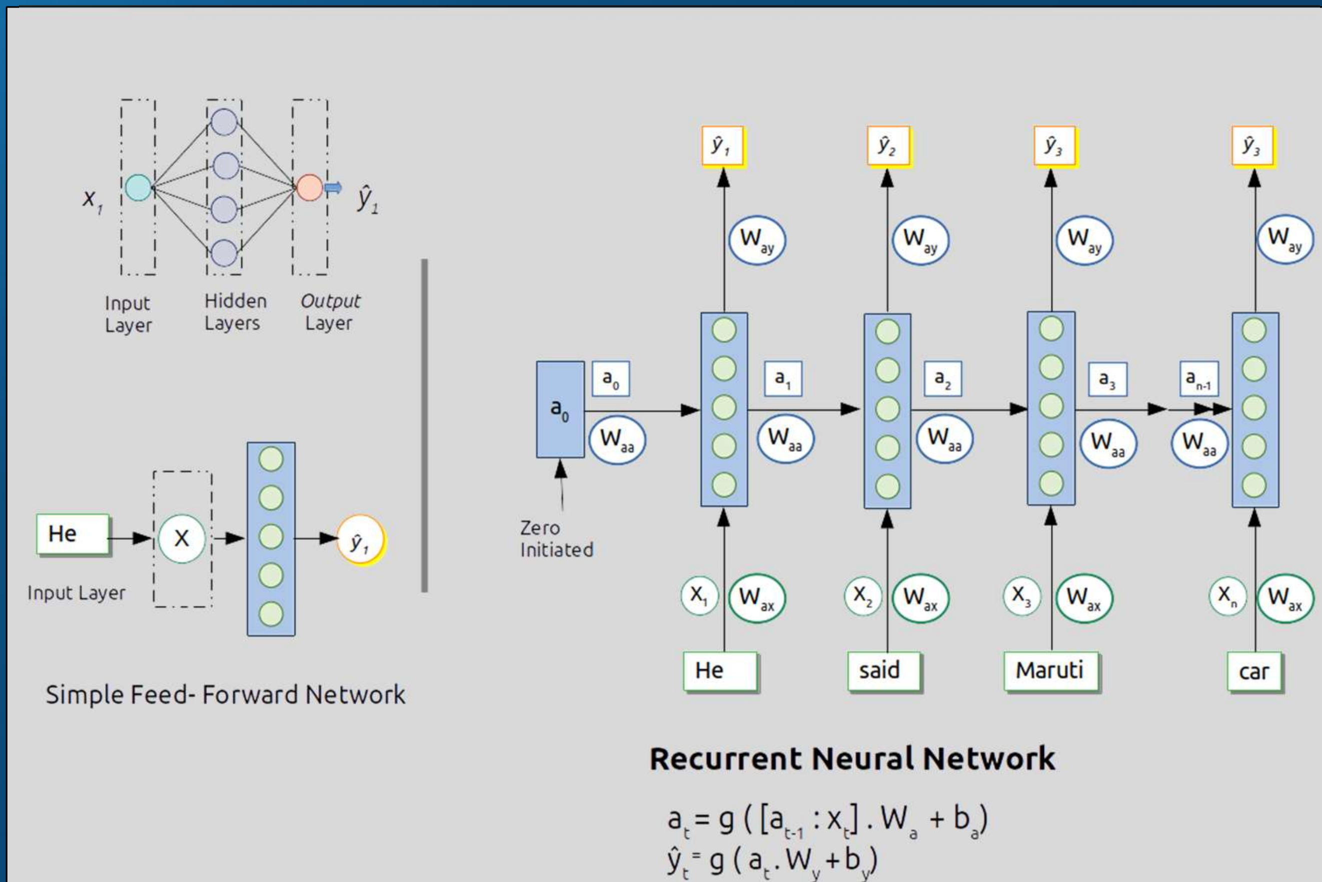
Long Short Term Memory (LSTM)

- Architecture designed to make it easy to remember
- Keep it until it's needed.
- The name refers to the idea:
  - The activations of a network correspond to short-term memory,
    - (Changing very fast with every new incoming record)
  - The weights correspond to long-term memory.
- If the activations can preserve information over multiple time steps
  - That makes them long-term short-term memory
- It's composed of memory cells which have controllers governing when to store or forget information

Gated Recurrent Unit

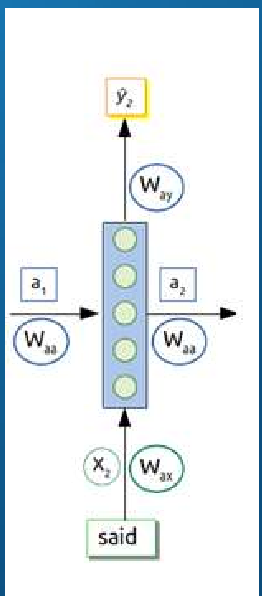
- Before we get to LSTM, lets look at its simplified version…
- Introduced by Cho, et al. in 2014, and Chung et al. in 2014 in their respective papers
- GRU (Gated Recurrent Unit)

Gated Recurrent Unit



Converted our simple feed forward network to Recurrent Network

Gated Recurrent Unit



- RNN Equation
  - $a_t = g_1\,(\,[a_{t-1} : x_t\,] \cdot W_a + b_a)$

    and
  - $\hat{y}_t = g_2\,(\,a_t \;\cdot W_y + b_y)$
- We look at an alternate way to paint the network

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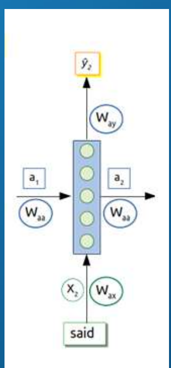
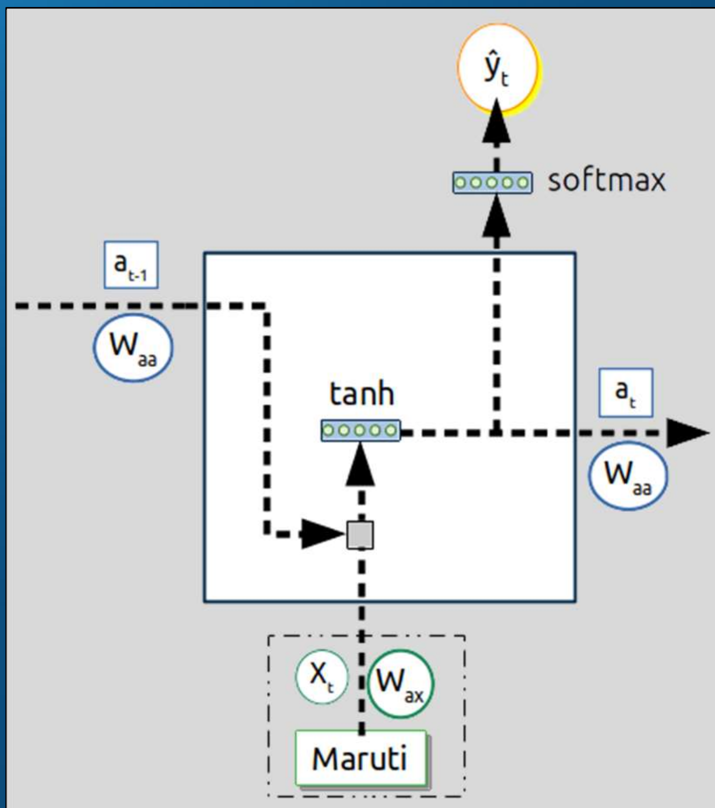
RNN



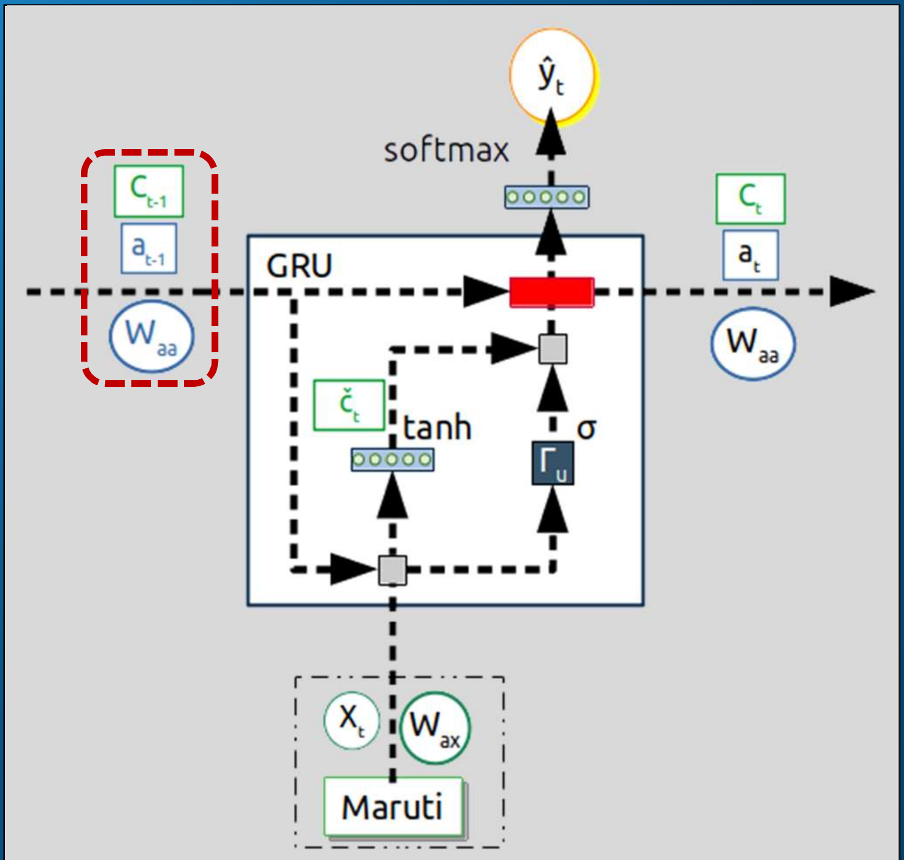


Basic RNN

- RNN Equation
  - $a_t = g_1\,(\,[a_{t-1} : x_t\,] \cdot W_a + b_a)$
    and
  - $\hat{y}_t = g_2\,(\,a_t \;\cdot W_y + b_y)$

- Alternate layout for one of the recurrence

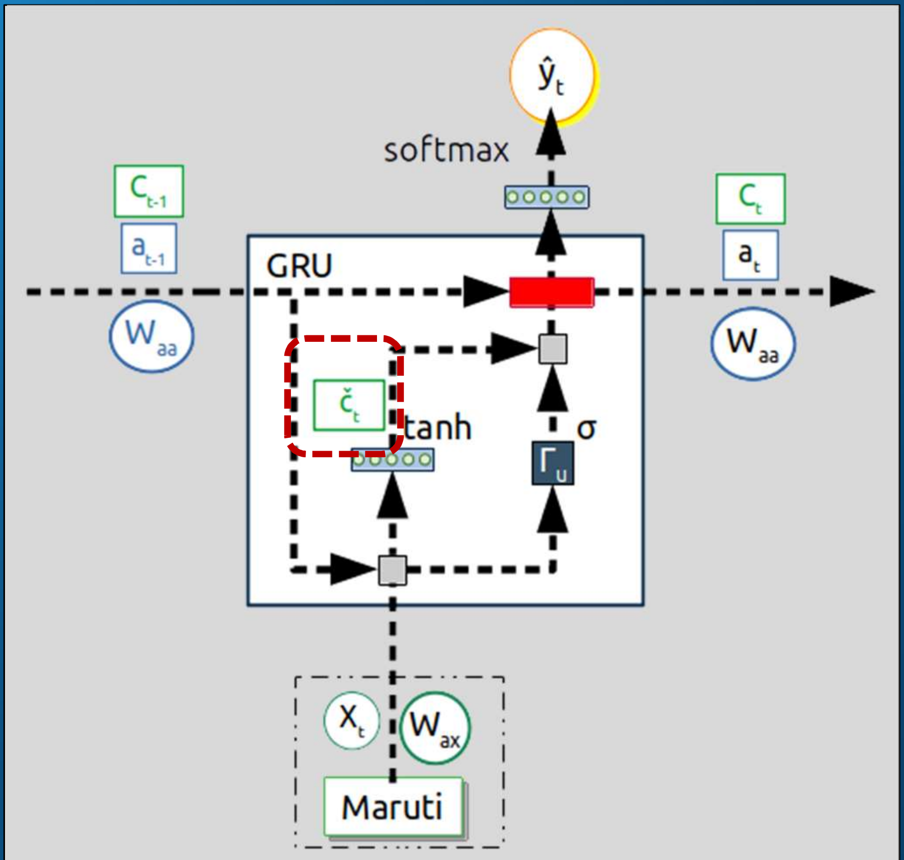
GRU Cell



Gated recurrent units (GRUs)

- GRU will have a cell which we will use to **retain** values from earlier iterations.
- Cell is called memory cell and is represented by ‘c’.
- In this case , memory cell value ‘c’ and activation ‘a’ value will be same.

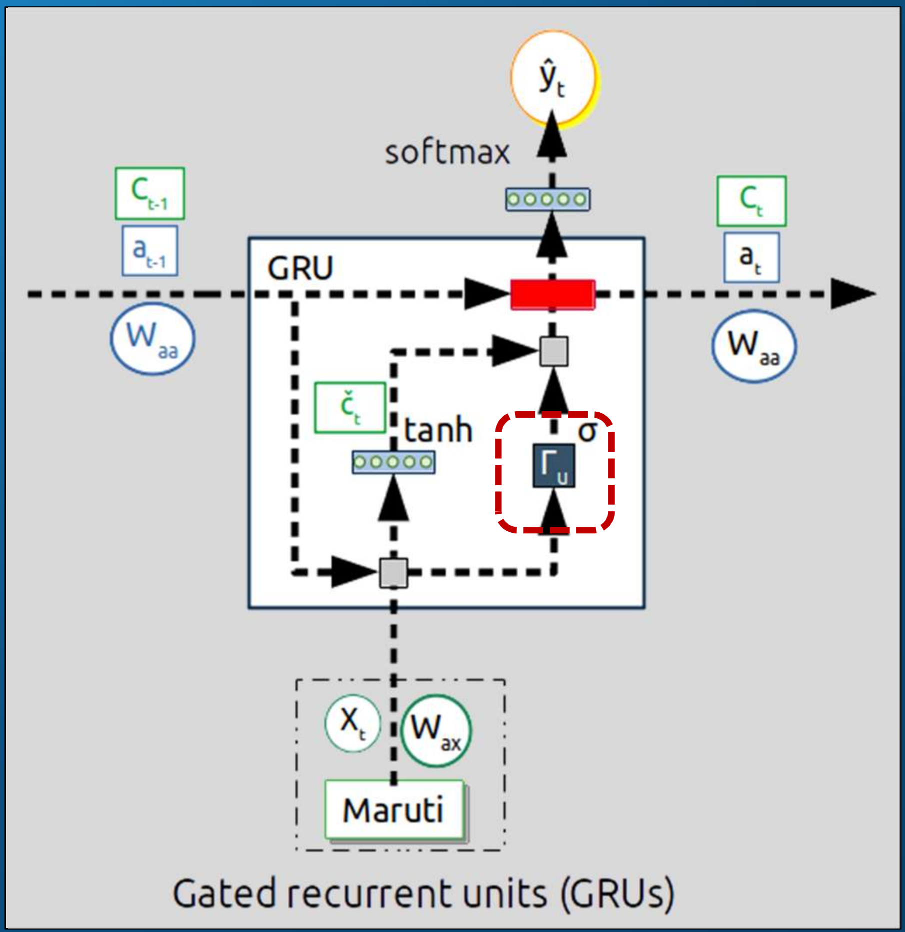
GRU Cell



Gated recurrent units (GRUs)

- At every time step, we will calculate 'ĉ', a candidate which may replace 'c_{t-1}'
  - $\hat{c}_t$ = tanh ([c_{t-1} : x_t] . W_c+ b_c)
- May be not…
- Next we need some parameter which will tell us if we need to replace the value of 'c_t' with candidate '$\hat{c}_t$' or not
- Note: At present both $c_t$and a_t are same… keeping them separate for consistency

GRU Cell



- Important change is called ' Gate'
- Gate, represented by 'Γ' Gamma is:
  - $\Gamma_u = \sigma\,(\,[\,c_{t-1} : x_t\,]\,.\,W_u + b_u)$
- In above equation, we are using separate Weights and Biases for update gate.
- The activation used here is sigmoid so for most part our Gamma will be 1 or 0

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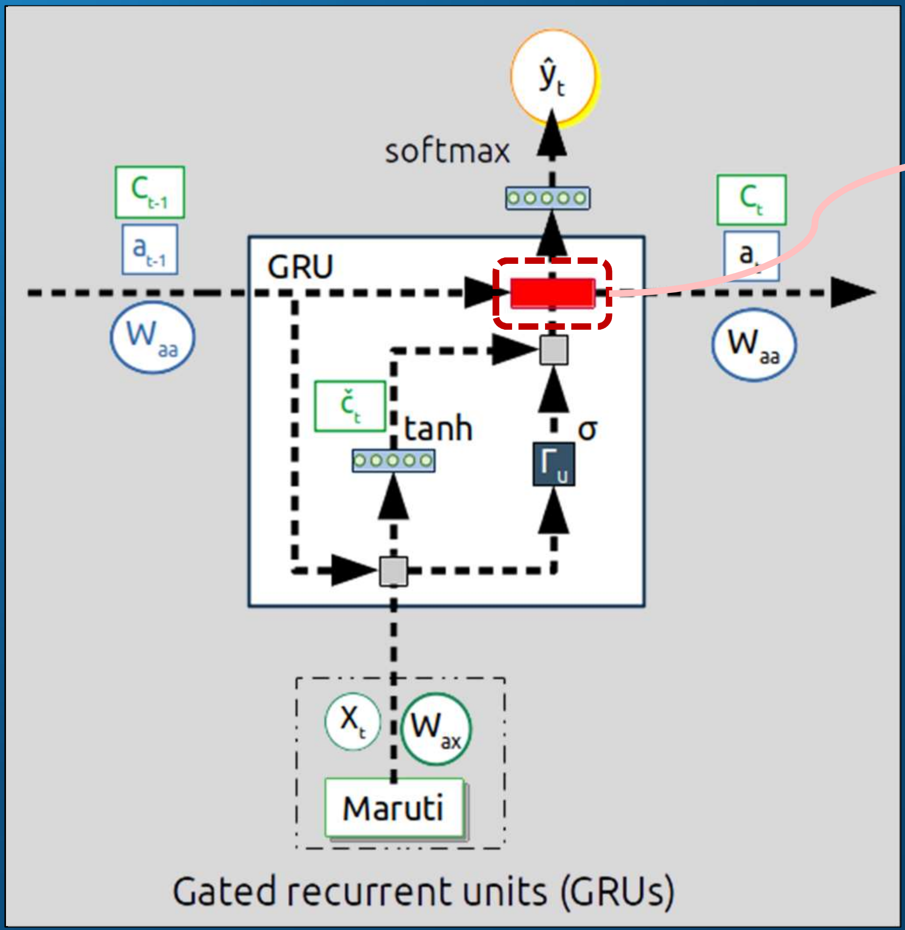
GRU Cell



- With 'Γ_u' Gamma update, we can calculate c_t as follows:
  - $c_t = \Gamma_u * \hat{c}_t + (1-\Gamma_u) * c_{t-1}$
  - The Red box represents the above equation
- Intuition: how Γ_u will be used????

P P

"I **felt happy** because I saw the others **were happy**

P

and because I knew I should **feel happy,** but I **wasn't**

N

really happy."

- Create or keep c_t P
- Time to Replace c_t N

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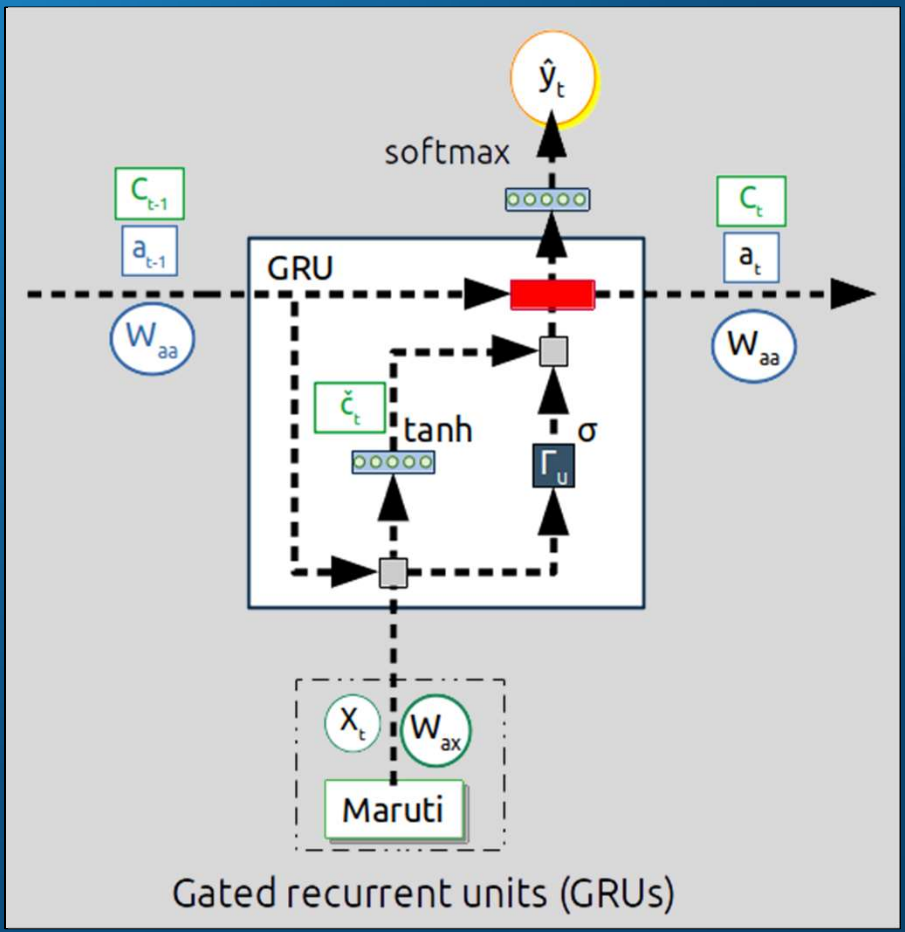
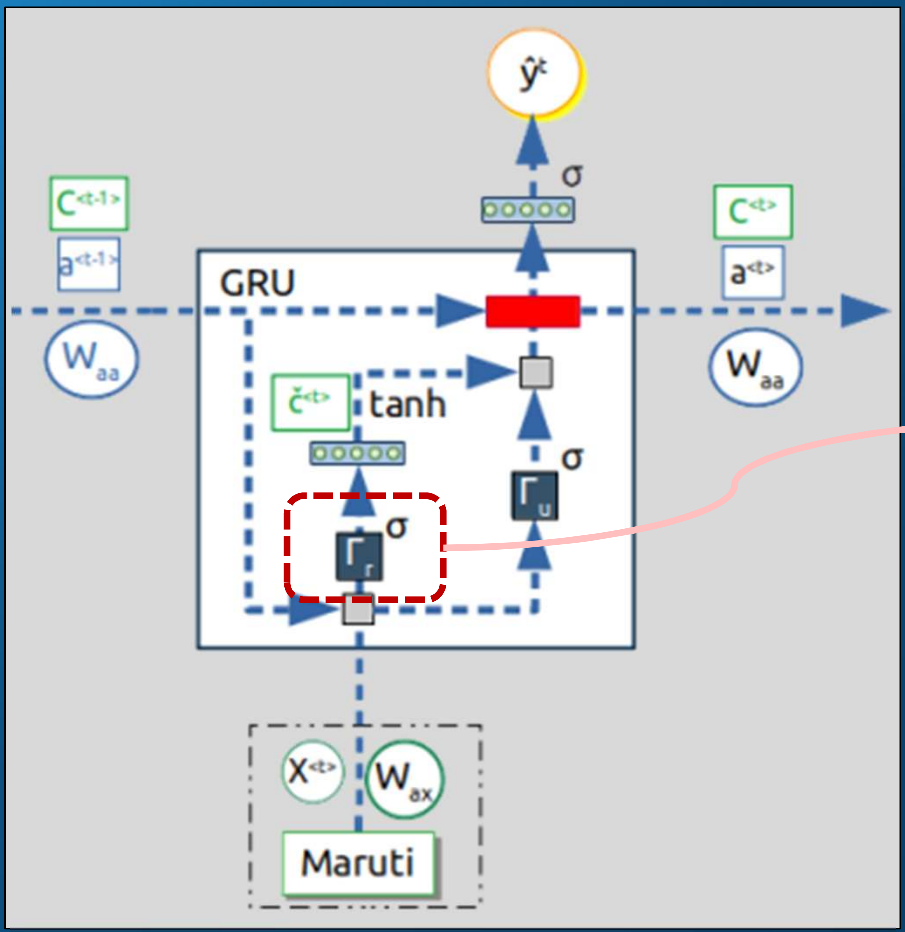
GRU Cell



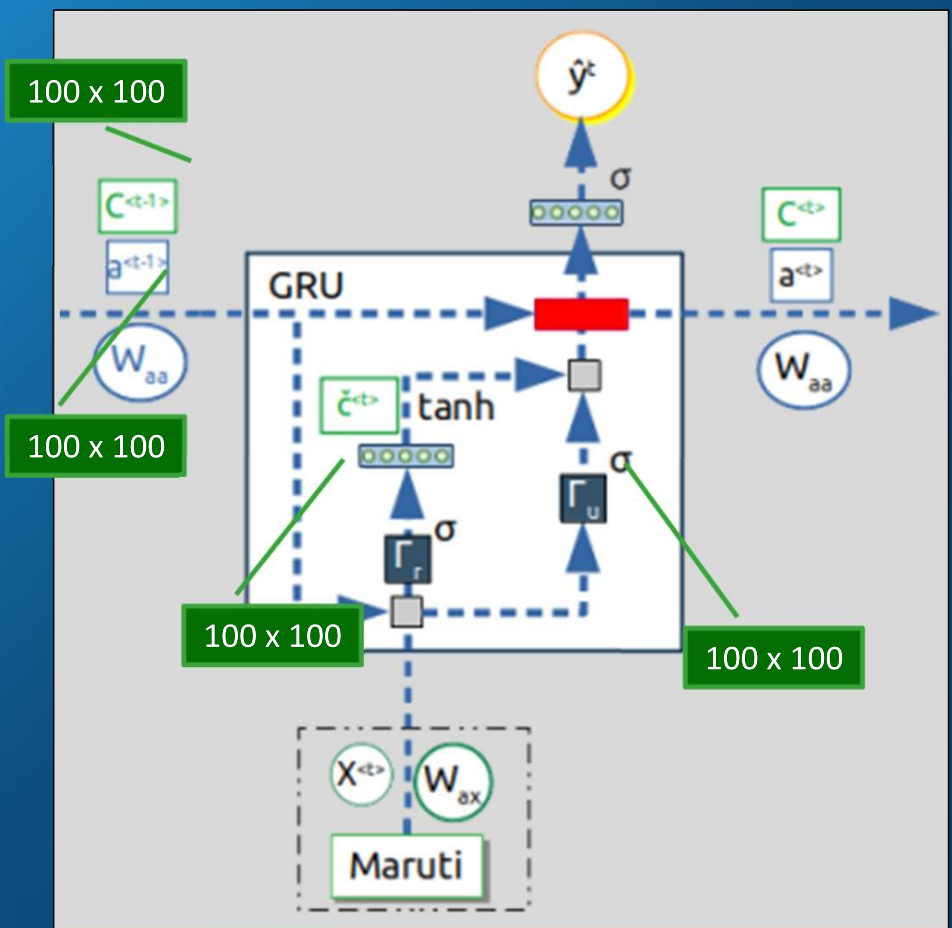
- So we can maintain c_t and their values through many layers and use them repeatedly.
- And more importantly, when to change it
- I **felt happy** because I saw the others **were happy** and because I knew I should **feel happy,** but I **wasn't really happy.**

GRU Cell



- That was GRU in simplified form.
- Actual implementation has one additional parameter Γr Gamma Relevance
  - $\hat{C}_t = \tanh([\Gamma_r * c_{t-1} : x_t].W_c + b_c)$
  - $\Gamma_u = \sigma([c_{t-1} : x_t].W_u + b_u)$
  - $\Gamma_r = \sigma([c_{t-1} : x_t].W_r + b_r)$
  - $C_t = \Gamma_u * \hat{c}_t + (1 - \Gamma_u) * c_{t-1}$

GRU Cell



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  - $\Gamma_r = \sigma\,([\,c_{t-1} : x_t\,] \,.\, W_r + b_r)$
  - $c_t = \Gamma_u * \hat{c}_t + (1 - \Gamma_u) * c_{t-1}$

Element wise multiplications

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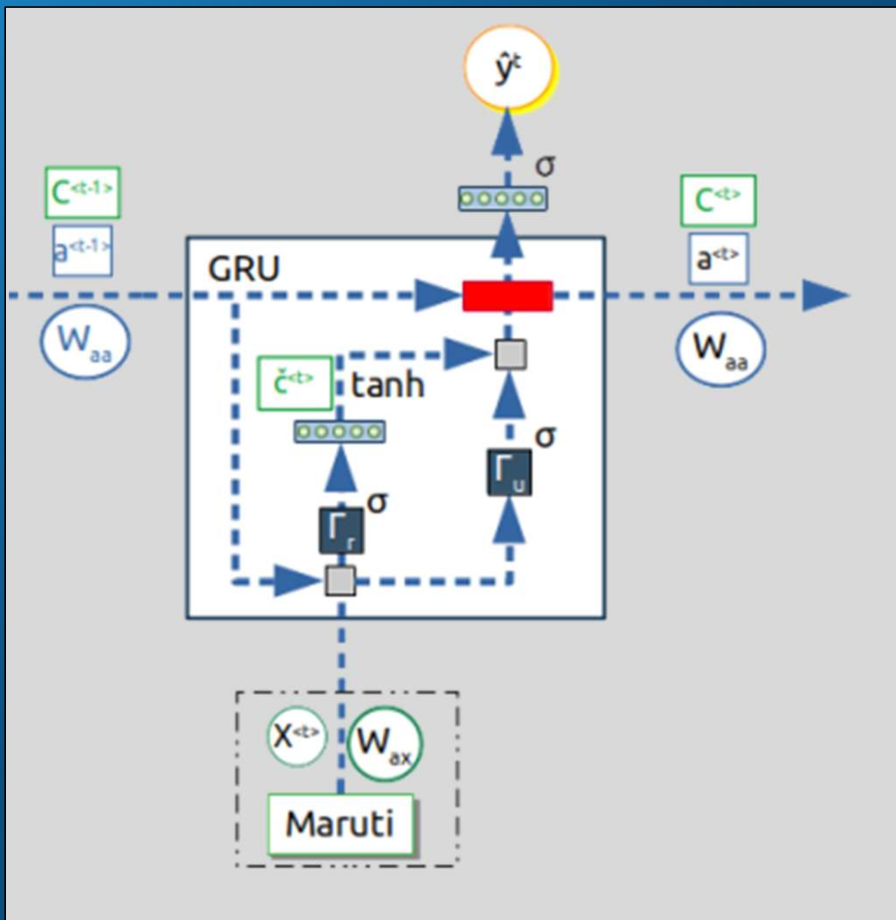
GRU Cell



Extended GRU:

$\check{c}_t = \tanh([\Gamma_r * c_{t-1} : x_t] . W_c + b_c)$

$\Gamma_u = \sigma([c_{t-1} : x_t] . W_u + b_u)$

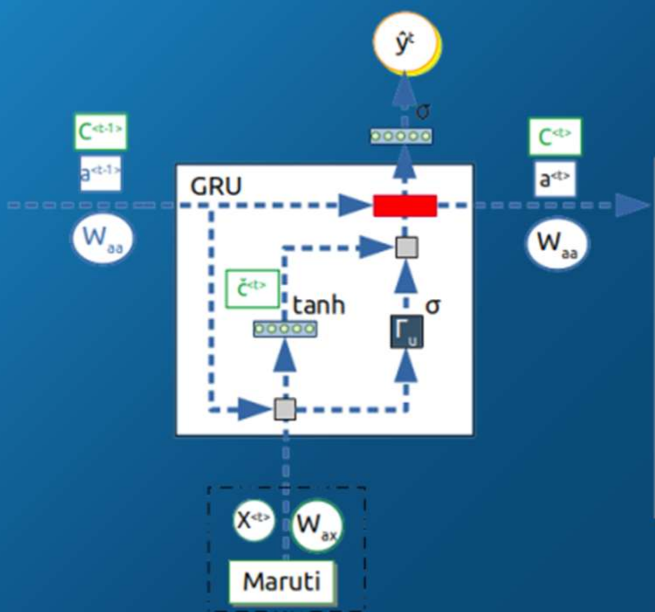
$\Gamma_r = \sigma([c_{t-1} : x_t] . W_r + b_r)$

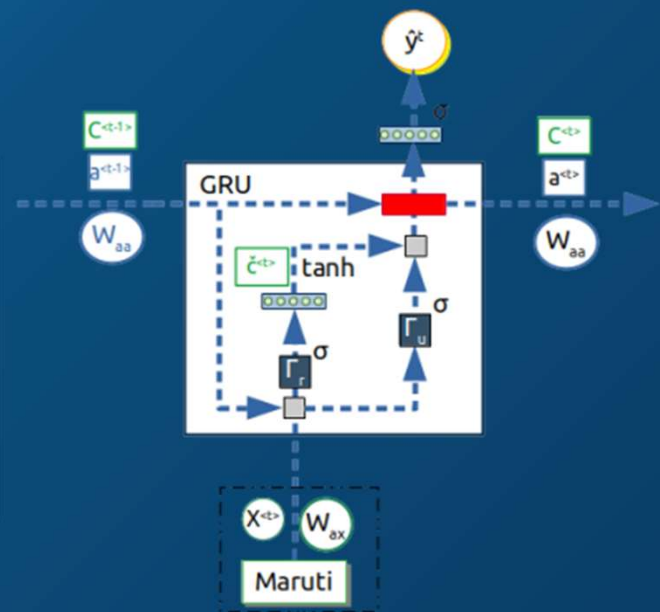
$c_t = \Gamma_u . \check{c}_t + (1 - \Gamma_u) . c_{t-1}$

If $\Gamma_u = 1$ then c_t will be equal to $\check{c}_t$,

If $\Gamma_u = 0$ then c_t will be equal to c_{t-1}

GRU Cell





$$a_t = g^1 (\, \underset{[100,100]}{a_{t-1}} . \quad \underset{[100,\,100]}{W_{aa}} \quad + \quad \underset{[100,10000]}{X_t} . \quad \underset{[10000,\,100]}{W_{ax}} . \quad + b_a)$$

$$\hat{y}_t = g^2 (\, a_t . W_{ya} . + b_y \,)$$

$$a_t = g^1 (\, [\, a_{t-1} : X_t] . W_a . + b_a)$$
$$\hat{y}_t = g^2 (\, a_t . W_y + b_y)$$

$$\underset{[100 \times 10100]}{[\, a_{t-1} , \; X_t]} \quad = [\, \underset{[100]}{a_{t-1}} : \; \underset{[10000]}{X_t}] \; [100]$$

$$W_a \quad = \quad \begin{array}{cc} [100] & \\ \begin{pmatrix} W_{aa} \\ W_{ax} \end{pmatrix} & \begin{array}{l} [100] \\ [10000] \end{array} \end{array}$$

Simple GRU:

$\check{C}^{<t>} = \tanh (\, [\, C^{<t-1>} : X^{<t>}] . W_c + b_c)$
$\Gamma_u = \sigma ([\, C^{<t-1>} : X^{<t>}] . \; W_u + b_u)$
$C^{<t>} = \Gamma_u . \check{C}^{<t-1>} + (\, 1 - \Gamma_u) . C^{<t-1>}$

If $\Gamma_u = 1$ then $C^{<t>}$ will be equal to $\check{C}^{<t>}$,
If $\Gamma_u = 0$ then $C^{<t>}$ will be equal to $C^{<t-1>}$.

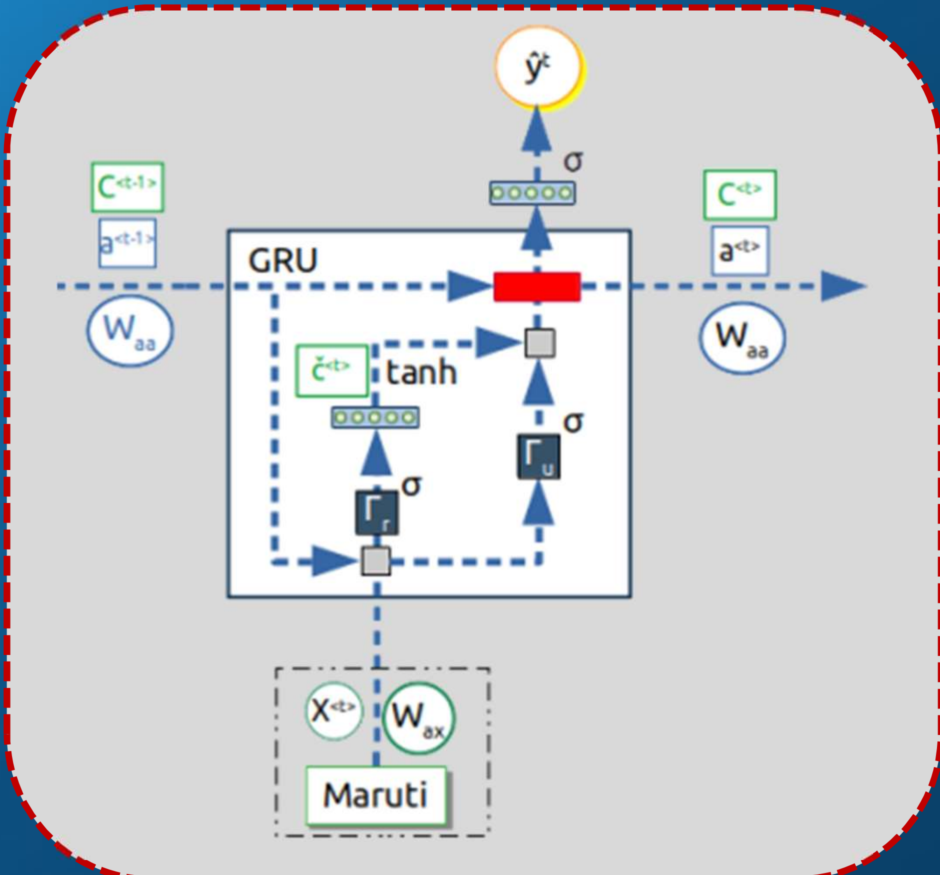
GRU:

$\check{C}^{<t>} = \tanh (\, [\, \Gamma_r * C^{<t-1>} : X^{<t>}] . W_c + b_c)$
$\Gamma_u = \sigma (\, [\, C^{<t-1>} : X^{<t>}] . W_u + b_u)$
$\Gamma_r = \sigma (\, [\, C^{<t-1>} : X^{<t>}] . W_r + b_r)$
$C^{<t>} = \Gamma_u . \check{C}^{<t>} + (\, 1 - \Gamma_u) . C^{<t-1>}$

If $\Gamma_r = 1$ then consider $C^{<t-1>}$ in $\check{C}^{<t>}$ calculations,
If $\Gamma_r = 0$ then do not consider $C^{<t-1>}$ in $\check{C}^{<t>}$ calculations.

If $\Gamma_u = 1$ then $C^{<t>}$ will be equal to $\check{C}^{<t>}$,
If $\Gamma_u = 0$ then $C^{<t>}$ will be equal to $C^{<t-1>}$.

GRU Cell



- And that, my friends, is GRU….

$$\hat{c}_t = \tanh([\Gamma_r * c_{t-1} : x_t] \cdot W_c + b_c)$$
$$\Gamma_u = \sigma([c_{t-1} : x_t] \cdot W_u + b_u)$$
$$\Gamma_r = \sigma([c_{t-1} : x_t] \cdot W_r + b_r)$$
$$c_t = \Gamma_u * \hat{c}_t + (1 - \Gamma_u) * c_{t-1}$$

- Coming up next… LSTM!

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THANK YOU