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Logistic Regression

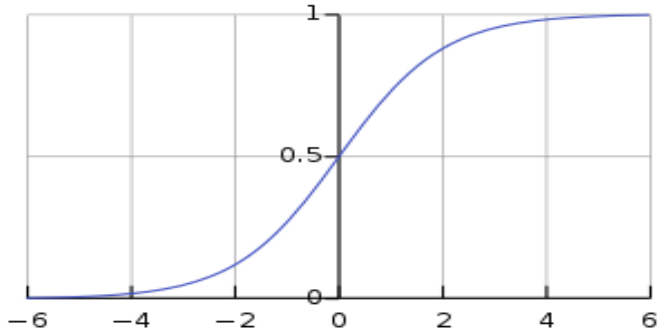
Logistic Regression

- This algorithm is used for classification type problems
- Types of Logistic Regression:
  - Binary
  - Multinomial
  - Ordinal
- We are going to cover Binary & Multinomial Logistic Regression

Logistic Response Function

- Standard logistic function on 2-dimensional plane is given by the following expression given on the right.
- From the graph, it is evident that the value of the f(x) ranges between 0 and 1.
- This function is also called sigmoid function and has a wide usage in various other algorithms such as neural network.

$$y = f(x) = \frac{1}{1 + e^{-x}}$$



Logistic Response Function

- The same function in the m-dimensional space can be written in the following way:

$$y = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m)}}$$

Where

$\beta_0, \beta_1, \beta_2, \ldots \beta_m$: Coefficients of the variables in m-dimensional space

- For any values of $\beta_0, \beta_1, \beta_2, \ldots \beta_m$ and $x_1, x_2, \ldots x_m$, the value of y always between 0 and 1.
- We can denote y by probability p.

Odds

$$p = \frac{1}{1+e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2+\cdots+\beta_m x_m)}}$$

$$1-p = \frac{e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2+\cdots+\beta_m x_m)}}{1+e^{-(\beta_0+\beta_1 x_1+\beta_2 x_2+\cdots+\beta_m x_m)}}$$

$$\frac{p}{1-p} = e^{(\beta_0+\beta_1 x_1+\beta_2 x_2+\cdots+\beta_m x_m)}$$

- The ratio $\frac{p}{1-p}$ is called odds. For any values of $\beta_0, \beta_1, \beta_2, \ldots \beta_m$ and $x_1, x_2, \ldots x_m$, odds always ranges from 0 to ∞.

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Interpreting Logistic Function

- In our binary classification, let us consider 0 and 1 as two possible outcomes, with 0 as non-occurrence of a particular event and 1 as occurrence of the particular event.
- p in our expression, is considered as probability of occurrence of the event and $1 - p$ as non-occurrence of the event
- Hence, the ratio $\frac{p}{1-p}$ is ratio of probability of occurrence to the probability of non-occurrence of the event.

Logit Function

$$\frac{p}{1-p} = e^{(\beta_0+\beta_1 x_1+\beta_2 x_2+\cdots+\beta_m x_m)}$$

$$\log(odds) = \log\left(\frac{p}{1-p}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_m x_m$$

- The ratio $\log\left(\frac{p}{1-p}\right)$ is called logit function. For any values of $\beta_0, \beta_1, \beta_2, \ldots \beta_m$ and $x_1, x_2, \ldots x_m$, log(odds) always range from -∞ to ∞.

Parameter Calculation

- Parameters $\beta_0, \beta_1, \beta_2, \ldots \beta_m$ are calculated with the help of maximum likelihood method.
- In Python, we make use of the function LogisticRegression() from sklearn.linear_model

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Questions?