MM2090: Introduction to Scientific Computing

Assignment 4

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1.1 What is Newton's Law of Gravitation?

Newton's law of universal gravitation is usually stated as, " Every particle attracts every other particle in the universe with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between their centers". The mathematical equation that gives the magnitude of the gravitational force between two point objects of mass m_1 and m_2 at a distance 'r' from each other is:

$$F = \frac{Gm_1m_2}{r^2} \tag{1}$$

Here , 'G' is the Universal Gravitational Constant 'F' is the Gravitational force experienced by the two point objects

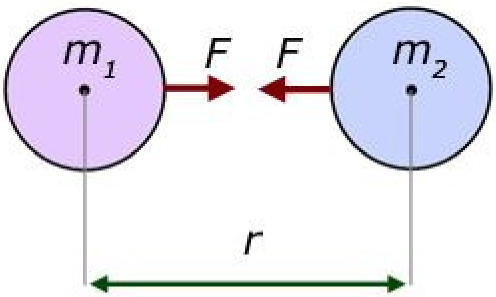

Figure 1: Newton's Law of Universal Gravitation

The Newton's Universal Gravitational Law can be beter understood through figure 3...
Let's study each of the term in mathematical equation 1 of Newton's Law of Gravitation.

1.2 Universal Gravitation Constant

The Universal Gravitational Constant is denoted by the letter 'G'. It is the proportionality constant in the Newtons Law 1 that relates gravitational force between the two bodies to the product of their masses and the inverse of the square of the distance between them. The measured value of the constant in SI Units is $6.674x10^-11m^3kg^{-1}s^{-2}$

Interesting Fact!

The first direct measurement of gravitational attraction between two bodies in the laboratory was performed in 1798, seventy-one years after Newton's death, by Henry Cavendish. He determined a value for G implicitly, using a torsion balance invented by the geologist Rev. John Michell (1753).

1.3 Mass of the Body

Mass is a property of physical body . It is also defined as a measure of its restistance to acceleration when a net force is applied. An object's mass also determines the strength of its gravitational attraction to other bodies.The SI Unit of mass is kilogram (kg). Sometimes people confuse mass with weight , however these are two different terms . Weight is a force which might be different for different magnitudes of gravtitaional force on the body whereas mass is constant for a physical body.

1.4 Distance between two bodies

In equation 1 , 'r' denotes the distance between the two bodies . It measures how much far away the two bodies are from each other. We always consider perpendicular distance between the two bodies as shown in figure 3.The SI Unit of distnace is 'm' metre. However, for planetary bodies , the distance is very huge and cannot be expressed shortly , so we use other units such as 'km' kilometre.

1.5 Force

Force can be defined as any influence that can affect the motion of an object if un-opposed. Newton's Second Law defines force quantitatively as timed rate of change of momentum.The SI Unit of force is Newton. Sir Isaac Newton described the motion of all objects using the concepts of inertia and force, and in doing so he found they obey certain conservation laws which we today know as the "Newton's Laws of Motion".
Force is broadly divided into following categories :

1. Fundamental Forces
    - Gravitaional Force
    - Electromagnetic Force
    - Weak Nuclear Forces
    - Strong Nuclear Forces
2. Non Fundamental Forces
    - Normal Force
    - Friction Force
    - Tension Force
    - Elastic Force
    - Fictitious Force

The force in equation 1 is Gravtitaional Force. So , the force which attracts every particle to every other particle is called the gravitational force.This is a very long range force which exists even between planetry bodies.

Interesting Fact!

The Gravitaional Force is responsible for the motion of planets around sun and the motion of moons around their respective planets!

1.6 Importance of Newton's Law of Universal Gravitation

Newton's Law of Universal gravitation is one of the most important and fundamental laws in classical physics . This is because it can be applied to almost every particle in the universe. It guides the efforts of scientists in their study of planetary orbits. Knowing that all objects exert gravitational influences on each other, the small deviations in a planet's elliptical motion can be easily explained. As the planet Jupiter approaches the planet Saturn in its orbit, it tends to deviate from its smooth path. This deviation is easily explained when considering the effect of the gravitational pul between Saturn and Jupiter. Newton's comparison of the acceleration of the apple to that of the moon led to a surprisingly simple conclusion abou the nature of gravity that is woven into the entire universe.
The equation 1 has been taken from the citation [4]

2 Anushka Vadhavkar (ME20B028)

Navier-Stokes equation for incompressible flow[1]:

$$\partial_t u_i + u_j \partial_j u_i = -\frac{1}{\rho_0}\partial_i p + \nu \nabla^2 u_i + E_i \tag{2}$$

Equation 2 describes the motion of an incompressible, viscous fluid. u_i are the velociy components, p is the pressure, ρ_0 is the density, E_i are the components of the external forces per unit mass, ν is the coefficient of kinematic viscosity (ratio of viscosity μ to the density of the fluid ρ_0), t is time, and the indices i, j refer to space coordinates. ∂_i denotes differential with respect to i and ∂_t denotes differential with respect to the time t. The Navier-Stokes equation can be applied to many situations, including parallel flow, radial flow, and convection.

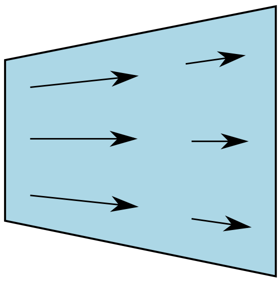

Figure 2: An example of convection

In Figure 2, though the flow may be steady, the fluid decelerates as it moves down the diverging duct.

3 Jay Harish Shah (ME20B088)

$$\omega = \int_{t_1}^{t_2} \alpha dt \tag{3}$$

$$\tau = \int_{\alpha_1}^{\alpha_2} I d\alpha \tag{4}$$

3.1 Analysis

In figure 3 we can see the mechanism of 2 gears and how each work by having direct dependency on the other gears dynamics.
In equation 3 we get the relation between angular velocity(ω) and angular acceleration(α) and time(t) from [5].
In equation 4 we get the relation between the torque(τ) and the angular acceleration(α) and moment of inertia(I) from [2].

Figure 3: ME20B088

4 Sukeerth Ramkumar (ME20B178)

4.1 Landau-Lifshitz-Gilbert equation

In physics, the Landau–Lifshitz–Gilbert (LLG) equation, named after Lev Landau, Evgeny Lifshitz, and T. L. Gilbert, is a differential equation describing the precessional motion of magnetization in a solid.

Various forms of the equation are commonly used in micro-magnetics to model the effects of a magnetic field on ferromagnetic materials. In particular, it can be used to model the time domain behaviour of magnetic elements in presence of a magnetic field.

The LLG equation is:

$$\frac{d\overrightarrow{\mathbf{M}}}{dt} = -\mu_0|\gamma|\left(\overrightarrow{\mathbf{M}} \times \overrightarrow{\mathbf{H}}\right) + \frac{\alpha}{M}\left(\overrightarrow{\mathbf{M}} \times \frac{d\overrightarrow{\mathbf{M}}}{dt}\right) \tag{5}$$

4.2 Physical quantities used in the equation

- $\overrightarrow{\mathbf{M}}$: Magnetisation (Units: A/m)
- μ_0 : Permeability of Free Space (Units: H/m)
- γ : Electron gyromagnetic ratio (Units: rad/s/T)
- $\overrightarrow{\mathbf{H}}$: Intensity of Magnetising Field (Units: A/m)
- α : Phenomenological damping constant (Dimensionless)

Some people may prefer to use $\overrightarrow{\mathbf{H}}$ to represent the magnetic field (In units of Tesla) instead of $\overrightarrow{\mathbf{B}}$. [3] Hence, μ_0 would be absent in that scenario.

4.3 LLG-derived equations

When we limit our focus to the direction of $\overrightarrow{\mathbf{M}}$ (in terms of polar coordinates: θ and φ), eqn. 5 reduces to 2 equations:

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{\gamma\alpha}{\alpha^2+1}H_z \sin\theta \quad \text{and} \quad -\dot{\varphi} = -\frac{d\varphi}{dt} = \frac{\dot{\theta}}{\alpha\sin\theta} = \frac{d\theta}{dt}\frac{1}{\alpha\sin\theta}$$

(Refer fig. 4) Symbols have their usual meanings. [3]

4.4 Gaining more insights into the equation

Magnetization is, by definition, the volume average of the vector sum of the electron spin magnetic moments. Spinning electrons have angular momentum, and it follows, from the law of conservation of angular momentum, that whenever the magnetization changes direction without dissipation, it does so by precession, in a manner analogous to that of a mechanical gyroscope. Dissipation is, in this case, the conversion of magnetic energy into heat. Mathematically, the gyromagnetic equation is:

$$-\mu_0|\gamma|\left(\overrightarrow{\mathbf{M}} \times \overrightarrow{\mathbf{H}}\right)$$

which is nothing but the first term in eqn. 5.

The second term of eqn. 5 takes in damping into consideration. The damping constant is termed "phenomenological" because it is not derived from a heat or energy transfer mechanism or model.

Fig 5 [3] represents the orientation of magnetisation vector at an arbitrary instance, whereas fig 4 shows one of the trajectories that can be taken by the magnetisation vector.

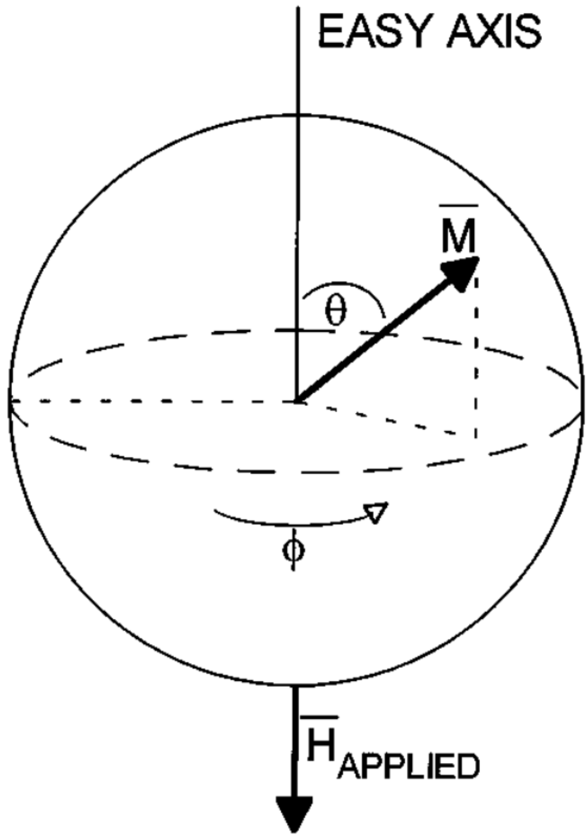

Figure 4: The polar coordinate system showing the magnetization vector M and both the uniaxial anisotropy easy axis and the applied field direction.

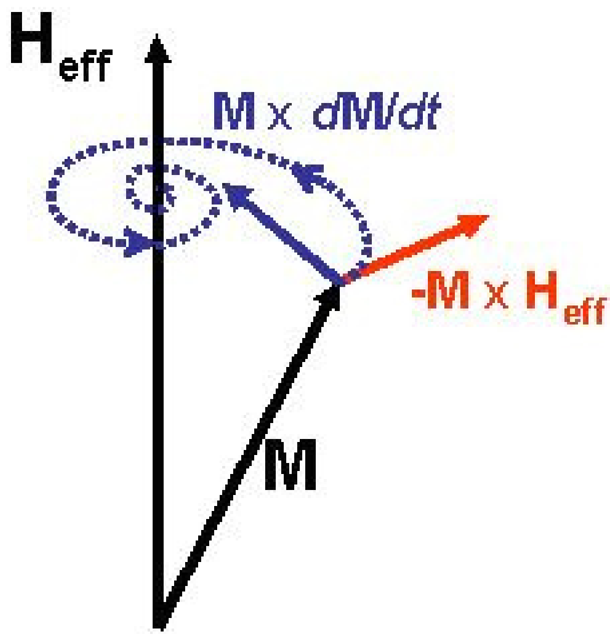

Figure 5: The trajectory of the magnetization & the terms of the Landau–Lifshitz–Gilbert equation: Precession (red) and damping (blue)

4.5 Development of the LLG equation

An earlier, but equivalent, equation (the Landau-Lifshitz equation) was introduced by Landau and Lifshitz in 1935, considering only very small damping:

$$\frac{d\overrightarrow{\mathbf{M}}}{dt} = |\gamma| \frac{1}{1+\alpha^2} \left(\overrightarrow{\mathbf{M}} \times \overrightarrow{\mathbf{B}}_{\text{eff}} + \frac{\alpha}{M} \left(\overrightarrow{\mathbf{M}} \times \left(\overrightarrow{\mathbf{M}} \times \overrightarrow{\mathbf{B}}_{\text{eff}} \right) \right) \right) \tag{6}$$

In 1955, Gilbert modified the equation to get eqn. 5.

An additional term was later added to eqn. 5 to account for the spin-transfer torque, i.e. the torque induced upon the magnetization by spin-polarized current flowing through the ferromagnet.

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