REINFORCE and Policy Gradient Algorithms for Reinforcement Learning

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Reinforcement learning:

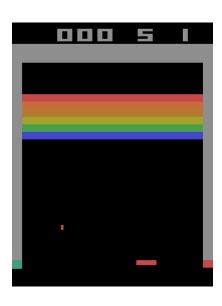
A framework for learning intelligent behaviors

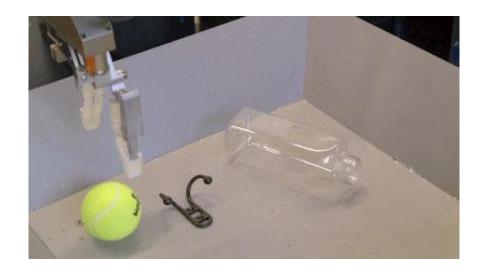
- A completely distinct framework from supervised and unsupervised learning
 - Trained from rewards no supervisor
 - Making decisions to maximize reward not just finding hidden structure
 - Applies to sequential problems that evolve over time

Reinforcement learning:

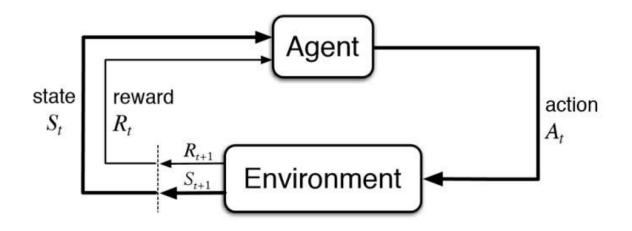
A framework for learning intelligent behaviors

- A completely distinct framework from supervised and unsupervised learning
 - Trained from rewards no supervisor
 - Making decisions to maximize reward not just finding hidden structure
 - Applies to sequential problems that evolve over time
- Example problems
 - Games
 - Robots (real or simulated)
 - Advertising
 - Managing investments
 - Optimizing factory processes
 - Self-driving cars?

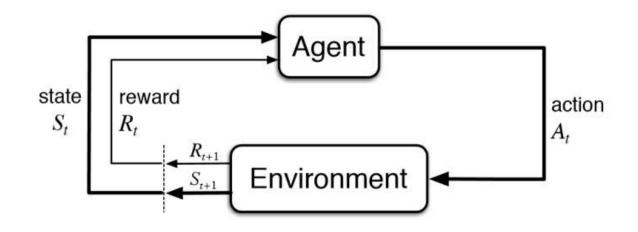




The reinforcement learning problem



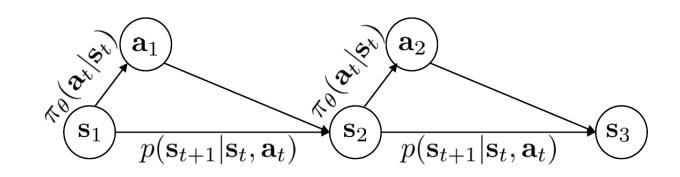
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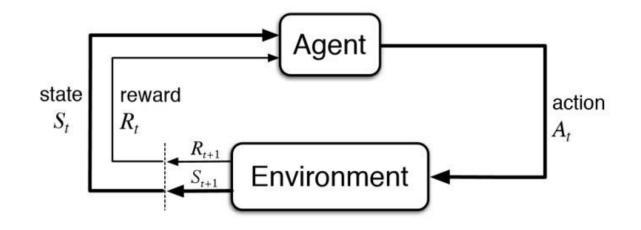
Markov Decision Process – key components

 $p(s_{t+1}|s_t, a_t)$: transition model of world

 $\pi_{\theta}(\boldsymbol{a_t}|\boldsymbol{s_t})$: policy (probability) of choosing actions given states



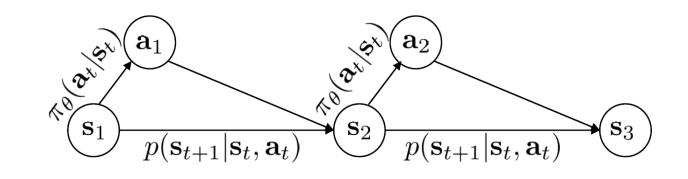
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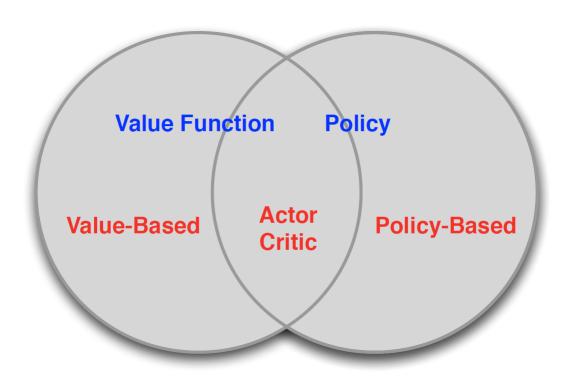
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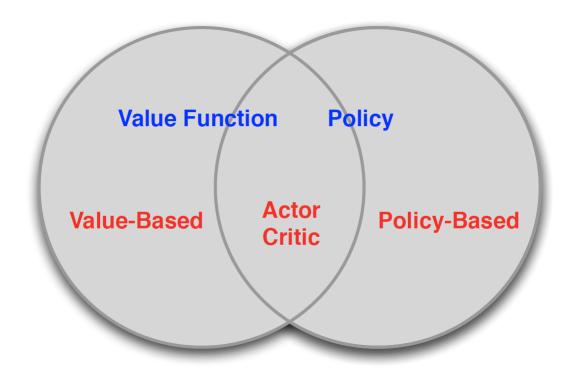


Goal: Learn a policy π_{θ} that will pick actions to maximize reward



Value-based methods

- Learn a value function that estimates expected reward of states V(s) and/or actions Q(s,a)
- No explicit policy = instead choose actions that have the highest value as predicted by the value function
- Examples: TD learning, Q-learning

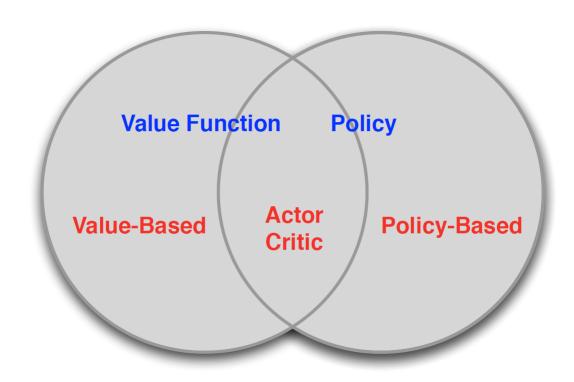


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- Represent policy explicitly using a parametrized function of states $\pi_{\theta}(s)$
- Optimize the policy parameters using gradient descent
- Examples: REINFORCE, Natural Policy Gradient, PPO



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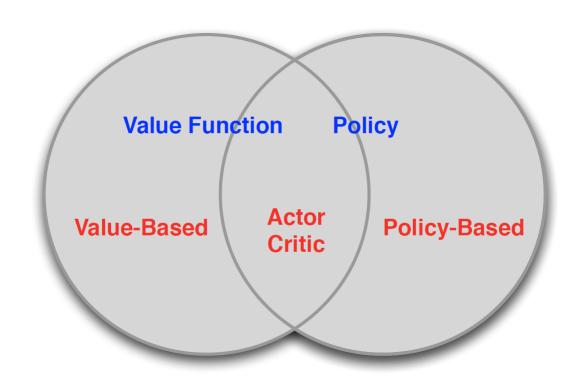
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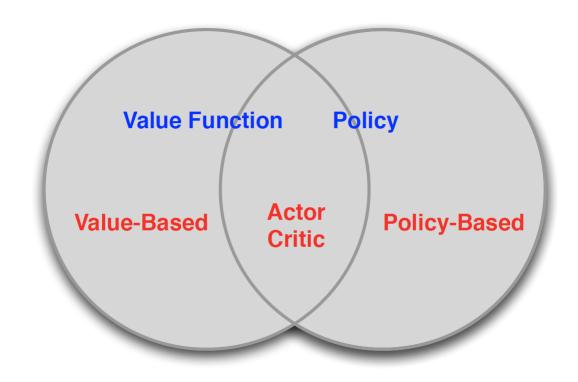
Actor-Critic

• Combine both: learn a value function (critic) that is used to guide improvements of the policy (actor)



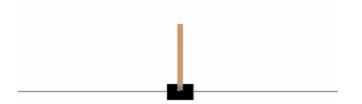
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Gaining intuition: two toy problems

- Cartpole task
 - Goal: keep the pole straight (within 12 degrees) and the cart within the screen
 - States are 4-dimensional vector: $[x, \dot{x}, \theta, \dot{\theta}]$
 - Actions are discrete: [move left, move right]



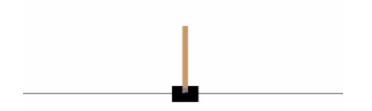
Gaining intuition: two toy problems

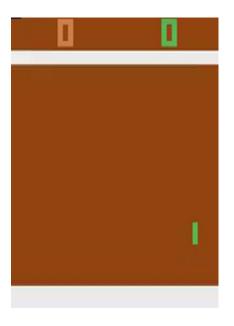
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Pong from Atari

- Goal: maximize score relative to opponent
- States are images [210 x 160 x 3]
- Actions are discrete: [move up, move down]

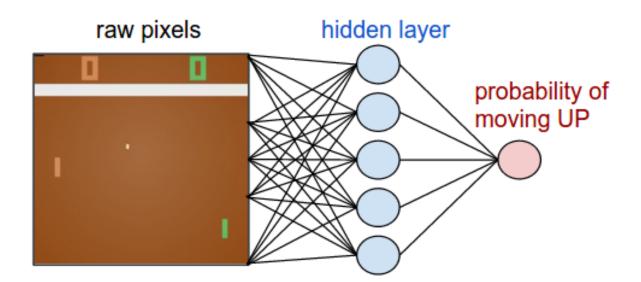




Policy parametrization

For discrete actions, often a softmax policy is used: $\pi_{\theta}(a|s) \propto e^{f_{\theta}(s)}$

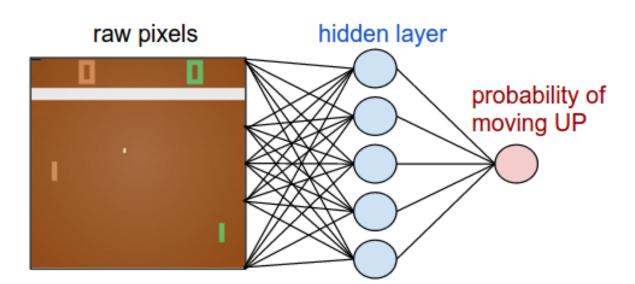
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Policy parametrization

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For continuous actions, usually a Gaussian policy is used:

 $\pi_{\theta}(a|s) \sim Normal(f_{\theta}(s), \sigma)$



Supervised learning for classification

Maximum likelihood objective:

$$J(\theta) \propto \sum_{i=1}^{N} \log p(y_i|x_i;\theta)$$

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Algorithm: move in direction of gradient, weighted by some learning rate α

Supervised learning for classification

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Supervised learning of a policy

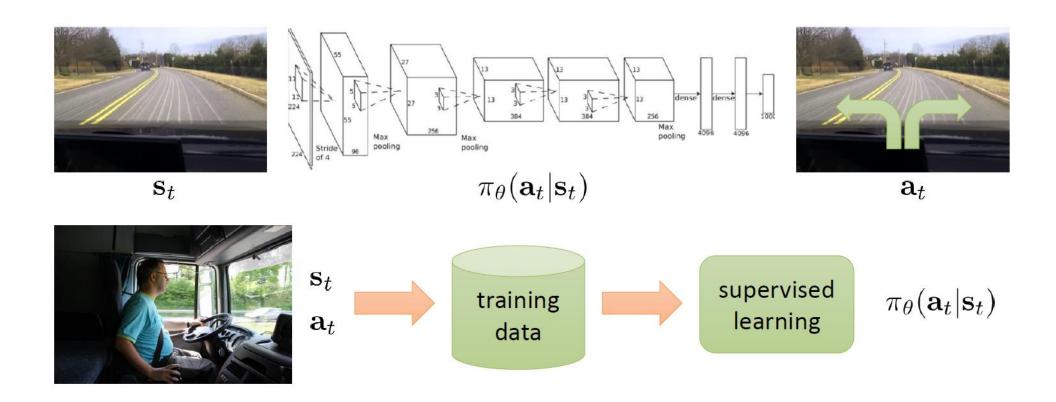
If given the correct actions a^* , our objective becomes:

$$J(\theta) \propto \sum_{i=1}^{N} \log \pi_{\theta}(a_i^*|s_i)$$

Gradient of ML objective:

$$\nabla_{\theta} J(\theta) \propto \sum_{i=1}^{N} \nabla_{\theta} \log \pi_{\theta}(a_i^*|s_i)$$

 π_{θ} = policy that outputs a probability distribution over actions, given states



Policy Gradient vs Supervised Learning

But we don't know the correct actions \rightarrow we only know that the sequence of actions we tried:

$$s_1, a_1, r_1, s_2, a_2, r_2, s_3, a_3, r_3, \dots$$

Policy Gradient vs Supervised Learning

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The basic idea of policy gradient is to use the same supervised learning gradient – but instead use the *actual* actions we actually took, **weighted by the reward** (summed over the future)

$$\nabla_{\theta} J(\theta) \propto \sum_{i=1}^{N} \nabla_{\theta} \log \pi_{\theta}(a_i^*|s_i) * R$$

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Intuition: Increase the probability of "good" actions, decrease the probability of "bad" actions. A formalized algorithm for trial-and-error!

function REINFORCE

```
Initialise \theta arbitrarily for each episode \{s_1, a_1, r_2, ..., s_{T-1}, a_{T-1}, r_T\} \sim \pi_{\theta} do for t=1 to T-1 do \theta \leftarrow \theta + \alpha \nabla_{\theta} \log \pi_{\theta}(s_t, a_t) \boldsymbol{g_t} end for end for return \theta end function
```

1) Run the policy for 1+ episodes

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```

return θ

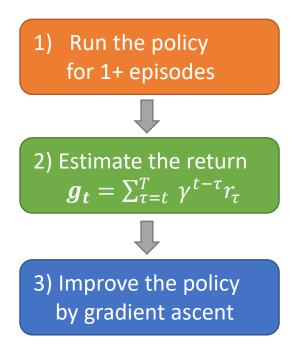
end function

1) Run the policy for 1+ episodes

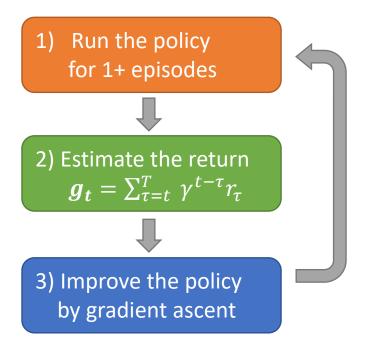


2) Estimate the return $\boldsymbol{g_t} = \sum_{\tau=t}^{T} \gamma^{t-\tau} r_{\tau}$

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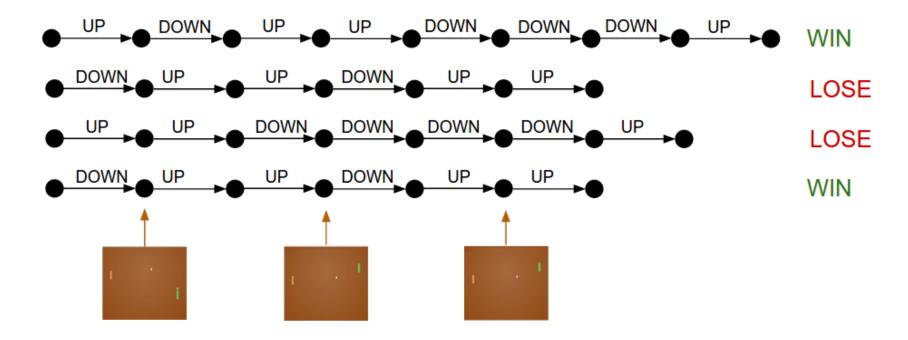
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```

Guaranteed by the Policy Gradient Theorem to eventually converge on the optimal policy! (Marbach and Tsitskilis 1998, Sutton 200)

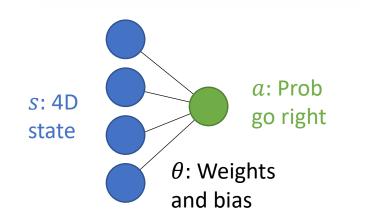
Intuition for Pong



For each episode we **WIN**: increase probability of the actions a little bit in those states For each episode we **LOSE**: decrease probability of the actions a little bit in those states

CartPole task

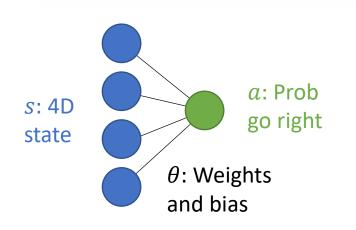
Can solve this with a logistic regression classifier (aka a single neuron network)



CartPole task

Can solve this with a logistic regression classifier (aka a single neuron network)

```
class ReinforceBinaryLR(RLAgent):
   def build model(self):
       # initialize weights
        self.weights = np.random.randn(self.state_size[0])
        self.bias = np.random.randn(1)
   def act(self, state):
       # sample action using output probabilities of classifier
        logit = np.dot(self.weights, state) + self.bias
        p = 1/(1 + np.exp(-logit))
        action = 1 if p_action > np.random.rand() else 0
        return action
```



1) Run the policy for 1+ episodes

```
env = gym.make('CartPole-v0') # Load environment
agent = ReinforceBinaryLR(env.observation space.shape, env.action space.n) # create agent
for episode in range(num episodes):
   state = env.reset() # restart environment
   done = False
   while not done:
        action = agent.act(state) # choose action
        next state, reward, done, info = env.step(action) # apply action to env
        agent.remember(state, action, reward, done) # store experience in memory
        state = next state
    agent.train() # train after each episode (or batch of episodes)
```

```
def train(self):
    # compute action probabilities
    predictions = 1/(1 + np.exp(-(np.dot(states, self.weights) + self.bias)))
    # compute gradient wrt weights using X * (Y-Yhat)
    states = np.hstack((np.ones((state_size, 1)), states)) # append ones for bias
    gradient = states * (actions-predictions)
                                                                 2) Estimate the return
    gradient *= returns # but weighted by the returns
                                                                    \boldsymbol{g_t} = \sum_{\tau=t}^T \gamma^{t-\tau} r_{\tau}
    gradient = gradient.sum(axis=0)
    # move up the aradient
                                                                 3) Improve the policy
    self.bias += self.learning rate*gradient[0]
                                                                   by gradient ascent
    self.weights += self.learning rate*gradient[1:]
```

For supervised learning, the update rule for logistic regression is: $\theta \coloneqq \theta + \alpha x_i (y_i - y_{pred})$

For policy gradient, the update rule is now weighted by the return: $\theta \coloneqq \theta + \alpha \, s_t (a_t - a_{prob}) g_t$

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Demo

Problems with REINFORCE: it is inefficient!

- Main problem: high variance
 - Extremely, extremely noisy compared to supervised learning
 - Input data (samples from episodes) is highly correlated
- Ways to reduce variance
 - Use a **baseline** -- subtract rewards by baseline value
 - Use a **critic** learn a value function → lower variance but high bias
 - Use smarter step sizes e.g. natural gradient, TRPO, PPO
- Inefficient exploration

What if one learning step makes your policy change dramatically (e.g., 500X)?

Too small step size: **very slow training process**Too high step size, **too much variability in training.**

Any Solution?

Trust Region Policy Optimization (TRPO)

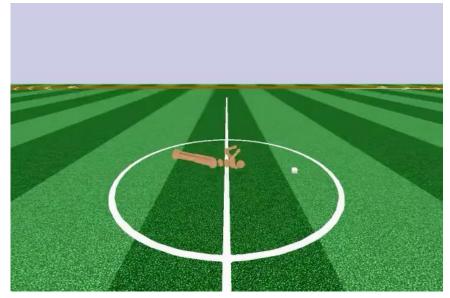
- A family of policy optimization methods
- Avoids large policy change by controlling parameter updates
- Enforcing parameter updates by a constraint on the KL divergence between $\pi_{ heta_{old}}(a_t|s_t)$ and $\pi_{ heta}(a_t|s_t)$
- Computationally intensive

Proximal Policy Optimization (PPO)

- A family of policy optimization methods
- Multiple epochs of stochastic gradient ascent for each policy update
- Same concept as well as stability and reliability of TRPO but simpler implementation
- A clipping term to the objective $[1 \epsilon, 1 + \epsilon]$

Clipped Surrogate Objective Multiple epochs of stochastic gradient ascent for policy update

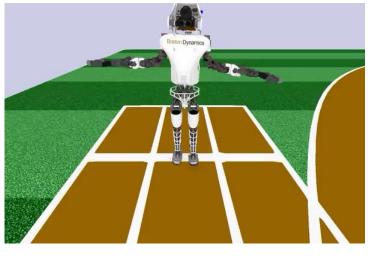
Any Solution?



"learning to walk, run, turn, use its momentum to recover from minor hits, and how to stand up from the ground when it is knocked over."



"flexible movement policies that let them improvise turns and tilts as they head towards a target location"



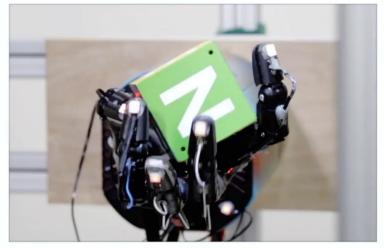
"the 'Atlas' model from Boston Dynamics shown below; the model has 30 distinct joints"

Clipped Surrogate Objective

Multiple epochs of stochastic gradient ascent for policy update

Any Solution?







FINGER PIVOTING

SLIDING

FINGER GAITING

Objective Function

- Improve training stability by limiting the change you make to your policy at each step

$$\underline{L^{PG}(\theta)} = \underbrace{E_t[\log \pi_{\theta}(a_t|s_t) * \underline{A_t}]}_{\text{Expected}} \underbrace{\log \operatorname{probability of}_{\text{taking that action at}}}_{\text{that state}}$$

Advantage if A>0, this action is better than the other action possible at that state

REINFORCE

Gradient ascent step on $L^{PG}(\theta)$ w.r. to network parameters will promote actions led to higher reward

$$r_t(\theta) = \frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta \ old}(a_t|s_t)}$$
 Action is more probable for current policy than old one
$$0 < r_t(\theta) < 1 \text{ Action is less probable for current policy than old one}$$

Trust Region Policy Optimization (TRPO)

- Improve training stability by limiting the change you make to your policy at each step

$$L^{TRPO}(\theta) = \hat{\mathbb{E}}_t \left[\frac{\pi_{\theta}(a_t|s_t)}{\pi_{\theta old}(a_t|s_t)} \hat{A}_t \right] = \hat{\mathbb{E}}_t \left[r_t(\theta) \hat{A}_t \right]_{\mathsf{TRPO}}$$

What if the action is much more probable (e.g., 500X) for your current policy?

Enforcing parameter updates by a constraint on the KL divergence between $\pi_{\theta_{old}}(a_t|s_t)$ and $\pi_{\theta}(a_t|s_t)$

Can we instead build these properties into the objective function? Yes, PPO

Proximal Policy Optimization (PPO)

Clipped Surrogate Objective

$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t \left[\frac{\sin \left(r_t(\theta) \hat{A}_t, \operatorname{clip} \left(r_t(\theta), 1 - \epsilon, 1 + \epsilon \right) \hat{A}_t \right)}{\sin \left(r_t(\theta) \hat{A}_t, \operatorname{clip} \left(r_t(\theta), 1 - \epsilon, 1 + \epsilon \right) \hat{A}_t \right)} \right]$$

min of the same objective from before and the clipped one

$$\epsilon = 0.2$$
, *Clip*: (0.8, 1.2)

```
def clipped_objective(new_log_probs, old_log_probs, advs, epsilon):
    """
    Compute the component-wise clipped PPO objective.
    """
    prob_ratio = tf.exp(new_log_probs - old_log_probs)
    clipped_ratio = tf.clip_by_value(prob_ratio, 1-epsilon, 1+epsilon)
    return tf.minimum(advs*clipped_ratio, advs*prob_ratio)
```

Proximal Policy Optimization (PPO)

Clipped Surrogate Objective

same objective same, but $r(\theta)$ is clipped from before between (1 - e, 1 + e)

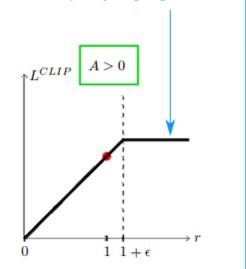
$$L^{CLIP}(\theta) = \hat{\mathbb{E}}_t \left[\min \left(\overline{r_t(\theta)} \hat{A}_t, \operatorname{clip}(r_t(\theta), 1 - \epsilon, 1 + \epsilon) \hat{A}_t \right) \right]$$

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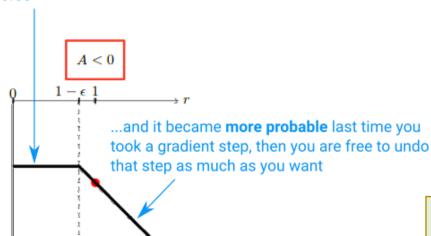
If the action was good....

...and it became **more probable** the last time you took a gradient step, don't keep updating it too far or else the policy might get worse



If the action was bad

...and it became **less probable**, don't keep making it too much less probable or else the policy might get worse



Why the minimum of clipped and unclipped terms?

Surrogate Function vs. Probability Ratio r

 L^{CLIP}

Multiple Epochs for Policy Updating

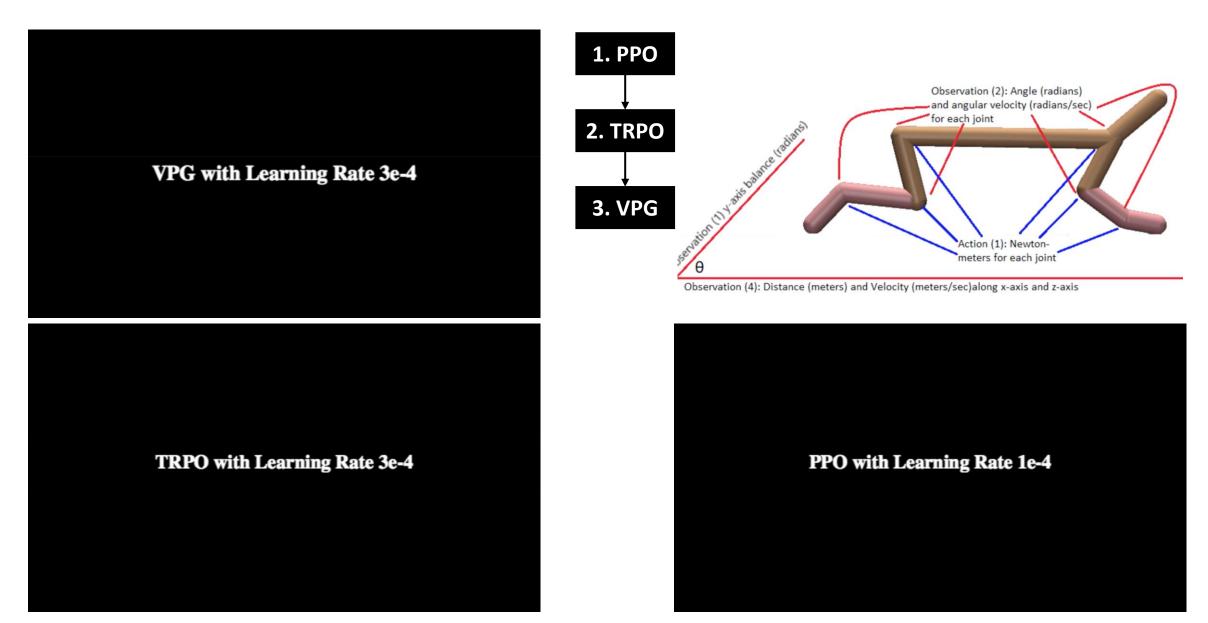
Proximal Policy Optimization (PPO)

- Multiple epochs of gradient ascent on samples avoiding large policy updates
- Extract more from your data and improving sample efficiency
- Parallel actors parts does not work with VPG properly

```
Algorithm 1 PPO, Actor-Critic Style
                                                                  K = 3 - 15, M = 64 - 4096, T = 128 - 2048
    1 for iteration=1, 2, \dots do
           for actor=1, 2, \dots, N do
               Run policy \pi_{\theta_{\text{old}}} in environment for T timesteps
               Compute advantage estimates \hat{A}_1, \dots, \hat{A}_T
           end for
           Optimize surrogate L wrt \theta, with (K epochs) and minibatch size M \leq NT
           \theta_{\text{old}} \leftarrow \theta
      end for

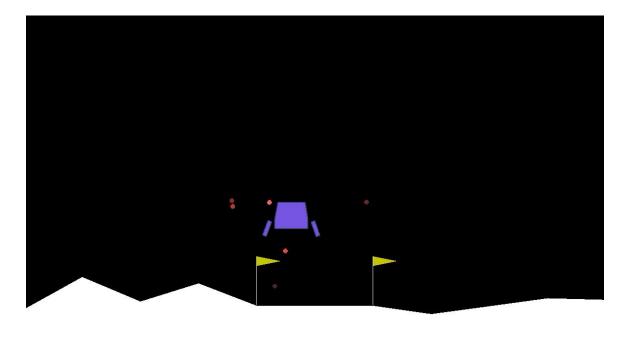
ightharpoonup Sample the environment with \pi_{	heta_{old}}
   Start the optimization and objective will start hitting the clipping limits
```

VPG vs. TRPO vs. PPO



Example with PPO - LunarLander-v2

"Spinning Up is an educational resource produced by OpenAI that makes it easier to learn about deep reinforcement learning (deep RL)."



Spinning Up requires Python3, OpenAl Gym, and

python -m spinup.run ppo --hid "[32,32]" --env LunarLander-v2 --exp_name lunarTest --gamma 0.999 python -m spinup.run test policy data/lundarData

- Landing pad is always at coordinates (0,0).
- Coordinates are the first two numbers in state vector.
- Reward for moving from the top of the screen to landing pad and zero speed is about 100-140 points.
- If lander moves away from landing pad it loses reward back.
- Episode finishes if the lander crashes or comes to rest, receiving additional -100 or +100 points.
- Each leg ground contact is +10.
- Firing main engine is -0.3 points each frame.
- Landing outside landing pad is possible.
- Fuel is infinite, so an agent can learn to fly and then land on its first attempt.
- Four discrete actions available:
 - do nothing
 - > fire left orientation engine
 - fire main engine
 - fire right orientation engine.

References

Learning material

Great blog post intro by Andrej Karpathy: http://karpathy.github.io/2016/05/31/rl/
Great course material:

- Reinforcement Learning at UCL by David Silver: http://www0.cs.ucl.ac.uk/staff/d.silver/web/Teaching.html
- Deep RL at Berkeley by Sergey Levine: http://rail.eecs.berkeley.edu/deeprlcourse/

Best intro book: Reinforcement Learning by Sutton & Barto (free pdf): http://incompleteideas.net/book/the-book.html
Great YouTube explanations by Arxiv Insights: https://www.youtube.com/channel/UCNIkB2IeJ-6AmZv7bQ1oBYg

More about PPO

- John Schulman, Filip Wolski, Prafulla Dhariwal, Alec Radford, Oleg Klimov. Proximal Policy Optimization Algorithms, arXiv:1707.06347
- Youtube video on PPO: https://www.youtube.com/watch?v=5P7I-xPq8u8&t=48s
- RL Proximal Policy Optimization (PPO) Explained [https://goo.gl/dH2Uwn]

Code bases

- OpenAl Spinning Up: https://spinningup.openai.com/en/latest/
 - See also Baselines https://github.com/openai/baselines)
- garage/rllab (Berkeley): https://github.com/rlworkgroup/garage
- Deep RL course with ipynb examples: https://github.com/simoninithomas/Deep_reinforcement_learning_Course
- Minimal and clean code examples: https://github.com/rlcode/reinforcement-learning