P Systems inference via Grammatical Evolution

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CMC 2022, Trieste (Italy)





- P systems can be exploited to simulate theoretical and application-focused investigations
- But their design is non-trivial, time-consuming, and requires precision and expertise
- Would it be easier to specify their behavior and automatically infer the rules from that? Certainly!

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Outline



- P Systems
- ② Grammatical Evolution
- Inferring P Systems with GE
- 4 Conclusion

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P Systems: Definition



Definition

A P system with active membranes and cooperative rules, of initial degree $d \ge 1$, is a tuple

$$\Pi = (\Gamma, \Lambda, \mu, w_{h_1}, \dots, w_{h_d}, R)$$

where

- I is an alphabet
- Λ is a finite set of labels
- \bullet μ is a membrane structure (represented as a rooted unordered tree) consisting of d membranes labeled by elements of Λ
- w_{h_1}, \ldots, w_{h_d} , with $h_1, \ldots, h_d \in \Lambda$, are multisets describing the initial contents of each of the d regions of μ
- R is a finite set of rules

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Cooperative rewriting rules:

$$[u \to v]_h$$
 for $h \in \Lambda$ and $u, v \in \Gamma^*$

Cooperative communication send-in rules:

$$u []_h \rightarrow [v]_h$$
 for $h \in \Lambda$ and $u, v \in \Gamma^*$

Cooperative communication send-out rules:

$$[u]_h \to v$$
 $[]_h$ for $h \in \Lambda$ and $u, v \in \Gamma^*$

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- Distinction between inner representation (genotype) and actual solution (phenotype)
- Mapping function to move from genotype to phenotype
- Fitness measure (computed on the phenotype) to quantify the performance of a solution



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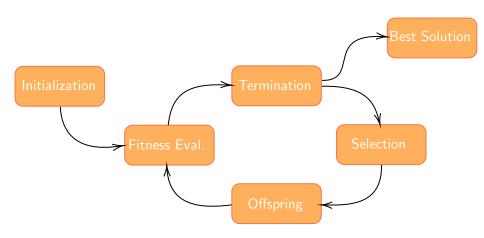
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Evolutionary Cycle







Definition

- Genotype → bit string (or integer string, considering 8 bits for conversion)
- ullet Phenotype o string of the language
- ullet Mapping o use of the modulus operator (%)
- ullet Fitness o depends on the problem



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Context-Free Grammar (CFG)



Definition

A CFG is defined as a tuple $\mathcal{G}=(N,T,s,P)$, where N is the set of non-terminal symbols, T is the set of terminal symbols (with $N\cap T=\emptyset$), $s\in N$ is the starting symbol, and P is the set of production rules.

Example: a CFG for mathematical expressions

```
\langle \exp r \rangle ::= (\langle \exp r \rangle \langle op \rangle \langle \exp r \rangle) | \langle num \rangle | \langle var \rangle \langle op \rangle ::= + | - | * | / \langle var \rangle ::= x | y \langle num \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
```

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Example: a CFG for mathematical expressions.

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\begin{split} \langle \mathsf{expr} \rangle &::= (\langle \mathsf{expr} \rangle \langle \mathsf{op} \rangle \langle \mathsf{expr} \rangle) \mid \langle \mathsf{num} \rangle \mid \langle \mathsf{var} \rangle \\ \langle \mathsf{op} \rangle &::= + \mid - \mid * \mid / \\ \langle \mathsf{var} \rangle &::= x \mid y \\ \langle \mathsf{num} \rangle &::= 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{split}
```



- The next unused value g_i of the genotype is taken (and marked as used)
- ② The value is divided by the number of possible replacement options $|R_s|$
- The remainder of the division j is used to select the new symbol (e.g.
 0 corresponding to the first substitute, and so on)



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Mapping example

 $\langle num \rangle ::= 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9$



 $g = 231 \, 15 \, 133 \, 142 \, 178 \, 224$

	i	gi	$ R_s $	j	W	Phenotype <i>p</i>
•	0	231	3	0	0	⟨expr⟩
	1	15	3	0	0	($\langle expr \rangle \langle op \rangle \langle expr \rangle$)
	2	133	3	1	0	$((\langle expr \rangle \langle op \rangle \langle expr \rangle) \langle op \rangle \langle expr \rangle)$
	3	142	10	2	0	$((\langle num \rangle \langle op \rangle \langle expr \rangle) \langle op \rangle \langle expr \rangle)$
	4	178	4	2	0	$((2 \langle \mathbf{op} \rangle \langle \mathbf{expr} \rangle) \langle \mathbf{op} \rangle \langle \mathbf{expr} \rangle)$
	5	224	3	2	0	((2 * \(\mathbf{expr} \) \(\lambda \rho \rangle \(\lambda \mathbf{expr} \rangle \))
	0	231	2	1	1	((2 * ⟨ var ⟩) ⟨op⟩ ⟨expr⟩)
	1	15	4	3	1	$((2*y)\langle \mathbf{op}\rangle \langle \mathbf{expr}\rangle)$
	2	133	3	1	1	((2*y)/ (expr))
	3	142	10	2	1	$((2*y)/\langle num\rangle)$
$\langle expr \rangle ::= (\langle expr \rangle \langle op \rangle \langle expr \rangle) \langle num \rangle \langle var \rangle$						((2*y)/2)
⟨op⟩ ::= + - * /						,, ,
$ \langle var \rangle ::= x y$						

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Inferring P Systems with GE



Scenario

We observe C, the set of subsequent configurations pairs (C_i, C_{i+1}) of the P system, for $0 \le i < n$, and we want to apply GE to find the ruleset whose application would give rise to the sequence C_0, \ldots, C_n .

To apply GE we need to define

- A CFG to express the ruleset of a P system
- The fitness of a ruleset

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CFG for a P system ruleset



Mapping example



$g = 231\ 55\ 113\ 17\ 35\ 144$							
i	gi	$ R_s $	j	W	р	\times \text{multiset} \text{sendout} \multiset\\ \ \text{multiset} \text{division} \multiset\\ \multiset\\ \text{multiset} \text{cobject} \cup \multiset\\ \text{multiset} \text{cobject} \text{multiset} \text{(object} \text{(multiset)} \text{(object)} \text{(membrane} \qquad \qqquad \qqqq \qqqqq \qqqq \	
0	231	2	1	0	⟨ruleset⟩		
1	55	1	0	0	$\langle rule \rangle$	⟨object⟩ ::= o1 oΓ	
2	113	10	3	0	(membran	$\mathbf{e} \rangle \langle core \rangle$	
3	17	4	1	0	m2 (core)		
4	35	2	1	0	m2 (multiset) sendin (multiset)		
5	144	10	4	0	m2 (object) sendin (multiset)		
6	231	2	1	1	m2 o3 sendin (multiset)		
7	55	10	5	1	m2 o3 sendin (object)		
					m2 o3 sendin o4		
{o ₃ } [] _{m₂}					$_{02} \rightarrow \left[\left\{ o_4 \right\} \right]_{m_2}$		



It is necessary to introduce a fitness to quantify how well the rules inferred by the evolutionary algorithm approximates the observed behavior.

For i = 0, ..., n, we define the fitness of a ruleset as the distance between

- \circ C_{i+1}
- C_{i+1} , obtained from C_i applying the ruleset under evaluation.

The membrane structure of a P system is a rooted unordered tree \rightarrow define a distance inspired by the edit distance between labeled trees

Edit distance between membrane structures



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Edit distance between membrane structures



The **edit distance** is based on the following operations:

- Addition of a membrane and its content
 - ightarrow cost = number of membranes added
- Removal of a membrane and its content
 - \rightarrow cost = number of membranes removed
- Ochange of the objects contained in a membrane
 - \rightarrow cost = Jaccard distance

$$d_J(A,B) = 1 - \frac{|A \cap B|}{|A \cup B|}$$

- ▶ value is comprised between 0 and 1
- $d_J(A, B) = 0$ if A and B are the same multiset
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Thanks!

Any unanswered question?

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