Combining Geometric Semantic GP with Gradient-descent optimization

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Outline

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Introduction

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Both kind of operators have randomly selected parameters that are **not optimized** by the search process.

IDEA : combine GSGP with a well-known gradient-based optimizer, *Adam*, in order to leverage:

- the ability of **GSGP** to operate structural changes of the individuals
- the ability of **gradient-based methods** to optimize the parameters of a given structure.

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improve the performance of GP $\Rrightarrow$ integration of **semantic awareness** in the evolutionary process

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Geometric Semantic Genetic Programming (GSGP)

GSGP is an evolutionary technique originating from GP that directly searches the semantic space of the programs

Geometric Semantic Operators

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Geometric Semantic Crossover (GSC)

Given two parents functions $T_1, T_2 : \mathbb{R}^n \rightarrow \mathbb{R}$, **GSC** generates the real function

$$T_{XO} = (T_1 \cdot T_R) + ((1 - T_R) \cdot T_2)$$

where T_R is a random real function whose output range in the interval $[0, 1]$.

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Geometric Semantic Mutation (GSM)

Given a parent function $T : \mathbb{R}^n \to \mathbb{R}$, **GSM** generates the real functions

$$T_M = T + ms \cdot (T_{R1} - T_{R2})$$

where T_{R1} and T_{R2} are random real functions whose output range in the interval $[0, 1]$ and ms is a parameter called mutation step.

The Adam Algorithm

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- efficient
- easy to implement
- little memory usage for its execution
- well suited for problems dealing with a vast amount of data and/or parameter

The Adam Algorithm

Algorithm 1 Pseudocode of the *Adam* algorithm.

Require: $f(\theta),\ \theta_0,\ N,\ \alpha,\ \beta_1 \in [0,1),\ \beta_2 \in [0,1),\ \epsilon$

$m_0 \leftarrow 0$

$v_0 \leftarrow 0$

for $i = 0 \cdots N$ **do**

  $d_{i+1} \leftarrow \nabla_\theta f_{i+1}(\theta_i)$ ▷ gradient of stochastic function f

  $m_{i+1} \leftarrow \beta_1 \cdot m_i + (1-\beta_2) \cdot d_{i+1}$ ▷ update first moment estimates

  $v_{i+1} \leftarrow \beta_2 \cdot v_i + (1-\beta_2) \cdot d_{i+1}^2$ ▷ update second moment estimates

  $\bar{m}_{i+1} \leftarrow m_{i+1}/(1-\beta_1^{i+1})$ ▷ contrast intrinsic initialization bias

  $\bar{v}_{i+1} \leftarrow v_{i+1}/(1-\beta_2^{i+1})$ ▷ contrast intrinsic initialization bias

  $\theta_{i+1} \leftarrow \theta_i - \alpha \cdot \bar{m}_{i+1}/(\bar{v}_{i+1})$ ▷ update parameters

end for

WHY?

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Good jump in promising areas of the solution space (**GSGP**) and subsequent refinement of the solution (**ADAM**).

HOW?

How?

let's consider an equivalent definition for GSOs:

1. GSM: $T_M = T + ms \cdot (R_1 - R_2)$, where $0 \leq m \leq 1$
2. GSC: $T_{XO} = (T_1 \cdot \alpha) + ((1 - \alpha) \cdot T_2)$, where $0 \leq \alpha \leq 1$

where T, T_1 and T_2 are the parent function; R_1 and R_2 are random real functions whose output range in $[0, 1]$.

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- perform **evolutionary steps** (GSM + GSC)
- obtain $T = (T_1, T_2, \cdots, T_N)$ new population, composed of derivable functions
- apply **Adam optimizer**:
  1. objective function $f(\theta) \rightarrow$ the generation considered T
  2. parameter vector $\rightarrow \theta = (\alpha, \beta = (1 - \alpha), ms)$

A practical example

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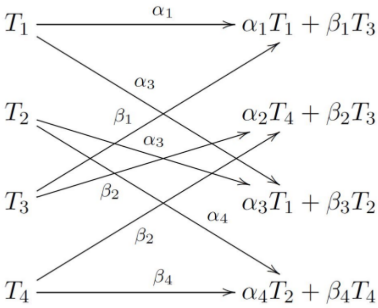
- initial random pop T_1, T_2, T_3, T_4
- only crossover operator

1. 4 new individuals created as linear combination of previous generations through parameters α_i and β_i $(i = 1, \ldots, 4)$
2. perform Adam optimization to update parameters α_i and β_i $(i = 1, \ldots, 4)$



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2. **HeH-GSGP**: (*Half et Half Geometric Semantic Genetic Programming*) initially, all the GSGP genetic steps are performed, followed by an equal number of Adam optimizer steps.

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Have been considered and tested dataset:

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- widely used as benchmarks for GP

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Dataset	Variables	Instances	Area	Task
%F	242	359	Pharmacokinetic	Regression
LD50	627	234	Pharmacokinetic	Regression
%PPB	627	131	Pharmacokinetic	Regression
yac	7	308	Physics	Regression
slump	10	102	Physics	Regression
conc	9	1030	Physics	Regression
air	6	1503	Physics	Regression

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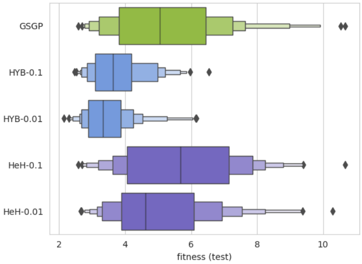
- 100 runs for each problem.
- $70:30$ training-test partition.

Outline

Slump



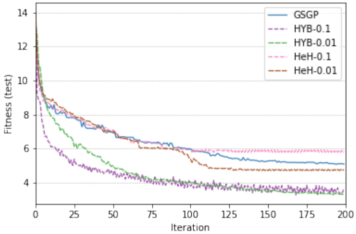
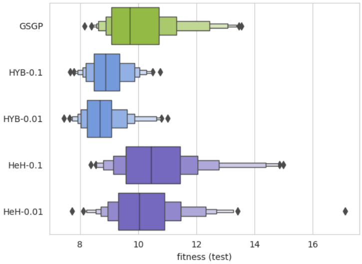


Figure: Boxplots of testing RMSE and median of the fitness over epochs obtained over 100 independent runs for **slump** problems.

Concrete



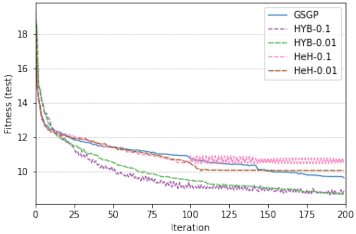
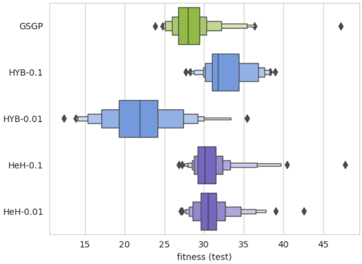


Figure: Boxplots of testing RMSE and median of the fitness over epochs obtained over 100 independent runs for **concrete** problems.

Airfoil



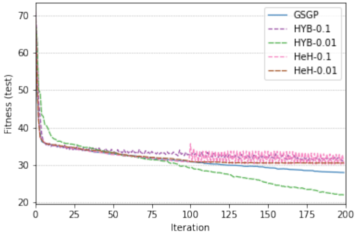


Figure: Boxplots of testing RMSE and median of the fitness over epochs obtained over 100 independent runs for **airfoil** problems.

Results

		GSGP	HYB-0.1	HYB-0.01	HeH-0.1	HeH-0.01
%F	Train	38.08	37.74	**36.80**	39.61	40.60
	Test	40.15	40.48	**39.61**	40.85	41.23
LD50	Train	2118.00	**2086.56**	2128.22	2144.27	2161.00
	Test	2214.78	**2203.25**	2229.87	2221.72	2215.09
%PPB	Train	30.15	27.00	**24.32**	34.79	33.26
	Test	328.1	401.43	263.81	**213.86**	235.53
yac	Train	**11.83**	11.92	12.48	12.28	12.31
	Test	11.92	**11.83**	12.52	12.38	12.48
slump	Train	4.56	3.47	**2.92**	5.19	4.41
	Test	5.08	3.63	**3.32**	5.77	4.76
conc	Train	9.62	8.86	**8.50**	10.59	10.05
	Test	9.65	8.88	**8.69**	10.47	10.07
air	Train	27.76	31.54	**21.98**	30.37	30.46
	Test	27.94	31.71	**21.97**	30.15	30.53

Table: Training and testing fitness (RMSE) for the considered benchmark problems. **Bold** font indicates the best results.

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- to consider a different kind of crossover where more than 2 parents are involved to get a structure more similar to a neural network one.
- to test this technique considering other optimizer.

thanks for your attention!!