

# 10 Must-Know Statistical Concepts for Data Scientists



# 1. Population and sample

**Population** is all elements in a group. For example, college students in the US is a population that includes all of the college students in the US. 25-year-old people in Europe is a population that includes all of the people that fits the description.

It is not always feasible or possible to do an analysis on population because we cannot collect all the data of a population. Therefore, we use samples.

**Sample** is a subset of a population. For example, 1000 college students in the US is a subset of the “college students in the US” population.

## 2. Normal distribution

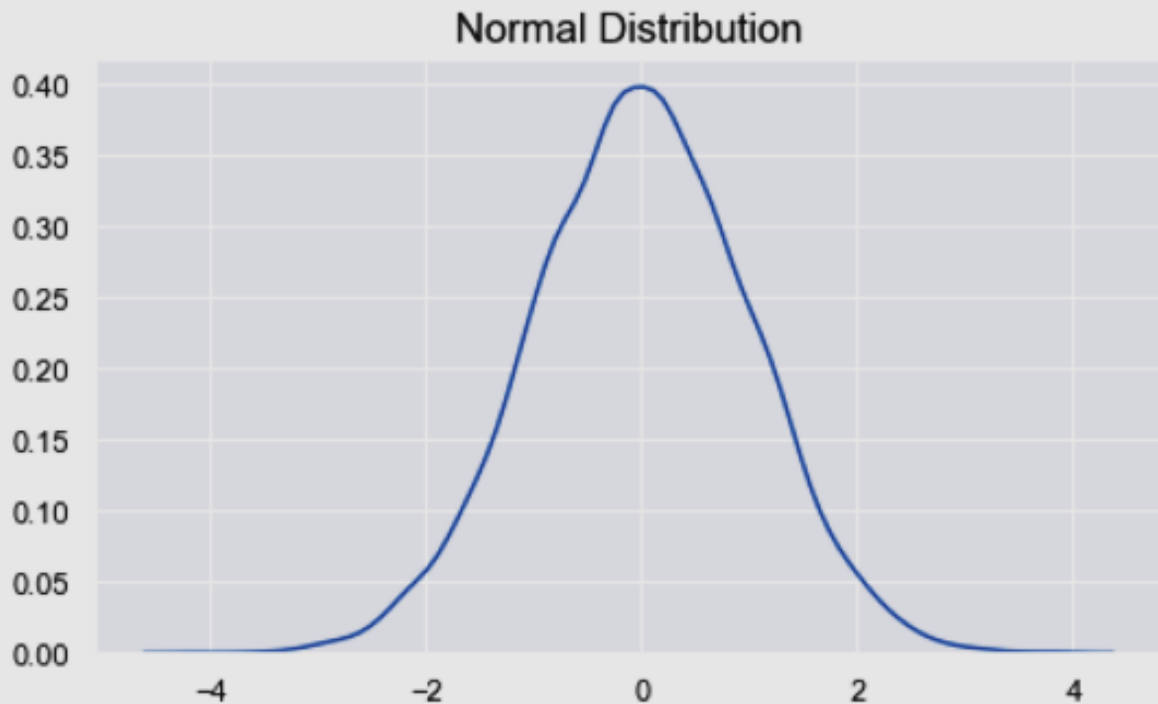
A probability distribution is a function that shows the probabilities of the outcomes of an event or experiment. Consider a feature (i.e., column) in a dataframe. This feature is a variable, and its probability distribution function shows the likelihood of the values it can take.

Probability distribution functions are quite useful in predictive analytics or machine learning. We can make predictions about a population based on the probability distribution function of a sample from that population.

Normal (Gaussian) distribution is a probability distribution function that looks like a bell.

# Normal distribution

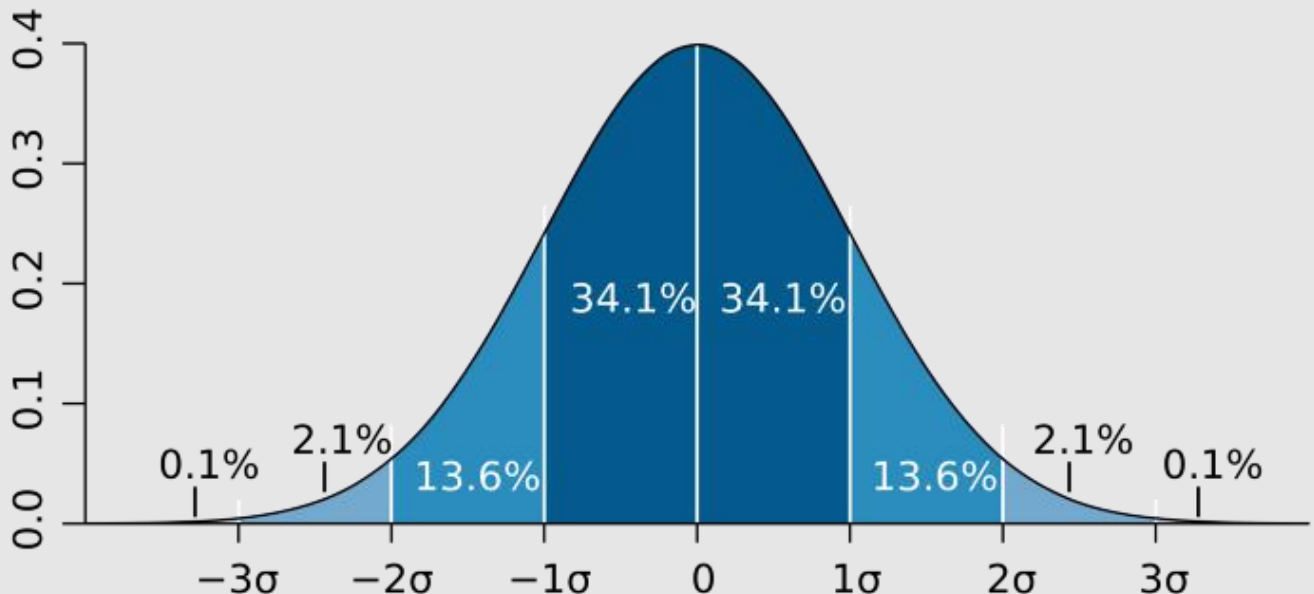
The following figure illustrates the shape of a typical normal distribution curve which was created by using a random sample returned by the `numpy.random.randn` function of NumPy.



# Normal distribution

The peak of the curve indicates the most likely value the variable can take. As we move away from the peak, the probability of the values decrease.

The following is a more formal representation of normal distribution. The percentages indicate the percentage of data that falls in that region. As we move away from the mean, we start to see more extreme values with less probability observed.



### 3. Measures of central tendency

Central tendency is the central (or typical) value of a probability distribution. The most common measures of central tendency are mean, median, and mode.

- **Mean** is the average of the values in series.
- **Median** is the value in the middle when values are sorted in ascending or descending order.
- **Mode** is the value that appears most often.

## 4. Variance and standard deviation

Variance is a measure of the variation among values. It is calculated by adding up squared differences of each value and the mean and then dividing the sum by the number of samples.

$$\text{Variance} = \frac{\sum (x_i - \text{mean})^2}{N}$$

$x_i$  = value i

N = number of values

Mean = average of all values

# Variance and standard deviation

Standard deviation is a measure of how spread out the values are. To be more specific, it is the square root of variance.

**Note:** Mean, median, mode, variance, and standard deviation are basic descriptive statistics that help to explain a variable.

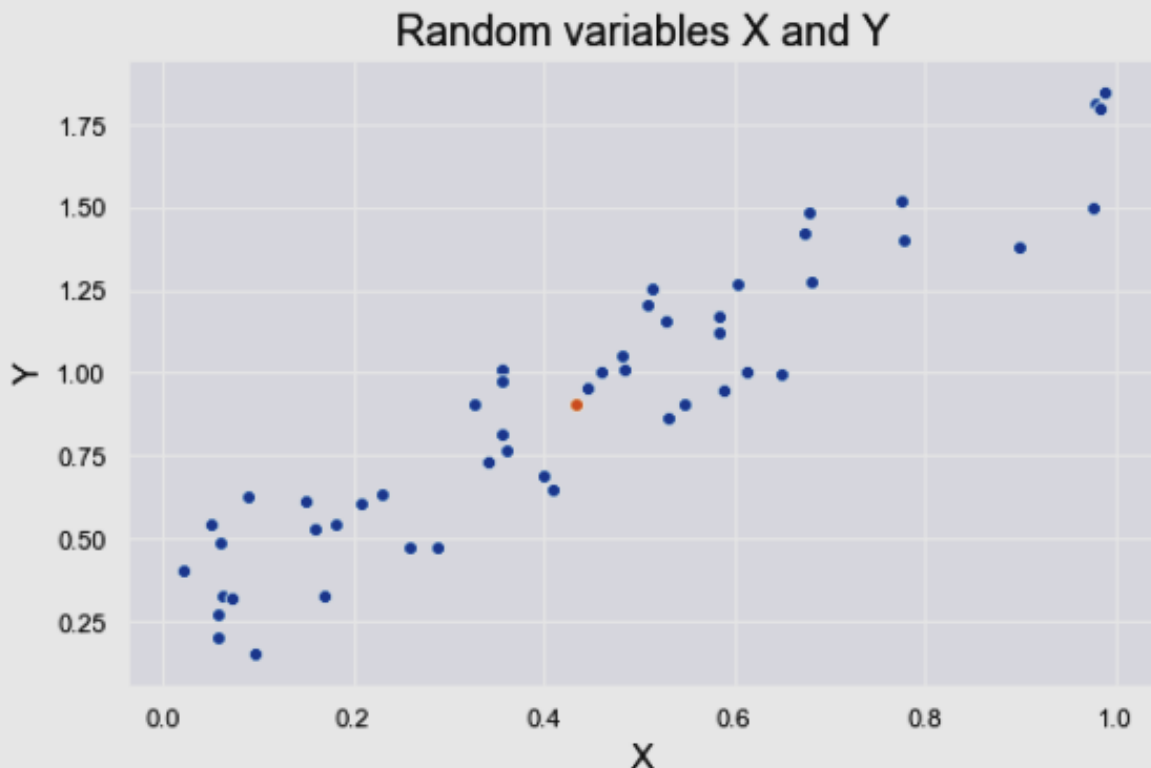


## 5. Covariance and correlation

Covariance is a quantitative measure that represents how much the variations of two variables match each other. To be more specific, covariance compares two variables in terms of the deviations from their mean (or expected) value.

# Covariance and correlation

The figure below shows some values of the random variables  $X$  and  $Y$ . The orange dot represents the mean of these variables. The values change similarly with respect to the mean value of the variables. Thus, there is positive covariance between  $X$  and  $Y$ .



# Covariance and correlation

The formula for the covariance of two random variables:

$$\text{Cov}(X, Y) = E[(X - \mu_x)(Y - \mu_y)]$$

where E is the expected value, and  $\mu$  is the mean.

**Note:** The covariance of a variable with itself is the variance of that variable.

Correlation is a normalization of covariance by the standard deviation of each variable.

# Covariance and correlation

$$\text{Corr}(X, Y) = E\left[\frac{(X - \mu_X)}{\sigma_X} \frac{(Y - \mu_Y)}{\sigma_Y}\right]$$

where  $\sigma$  is the standard deviation.

This normalization cancels out the units, and the correlation value is always between 0 and 1. Please note that this is the absolute value. In the case of a negative correlation between two variables, the correlation is between 0 and -1. If we are comparing the relationship among three or more variables, it is better to use correlation because the value ranges or units may cause false assumptions.

## 6. Central limit theorem

In many fields, including natural and social sciences, when the distribution of a random variable is unknown, a normal distribution is used.

The central limit theorem (CLT) justifies why normal distribution can be used in such cases. According to the CLT, as we take more samples from a distribution, the sample averages will tend towards a normal distribution regardless of the population distribution.

# Central limit theorem

Consider a case that we need to learn the distribution of the heights of all 20-year-old people in a country. It is almost impossible and, of course not practical, to collect this data. So, we take samples of 20-year-old people across the country and calculate the average height of the people in the samples. CLT states that as we take more samples from the population, the sampling distribution will get close to a normal distribution.

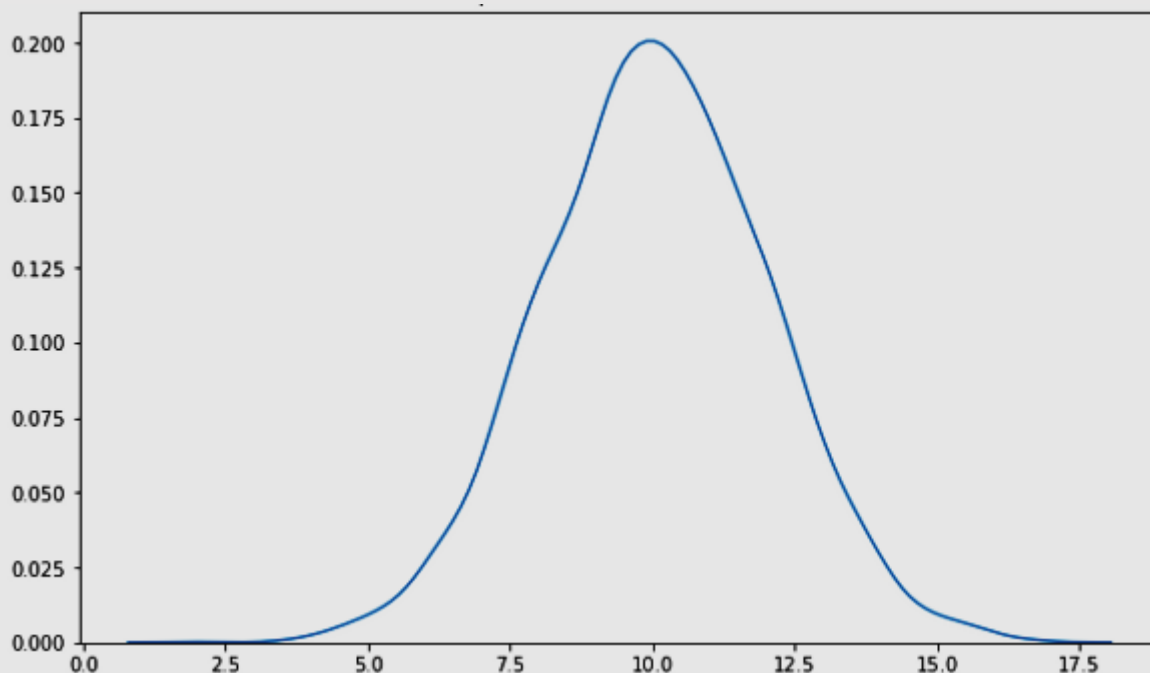
Why is it so important to have a normal distribution? The normal distribution is described in terms of mean and standard deviation, which can easily be calculated. And, if we know the mean and standard deviation of a normal distribution, we can compute pretty much everything about it.

## 7. P-value

**P-value is a measure of the likelihood of a value that a random variable takes. Consider we have a random variable  $A$  and the value  $x$ . The p-value of  $x$  is the probability that  $A$  takes the value  $x$  or any value that has the same or less chance to be observed.**

# P-value

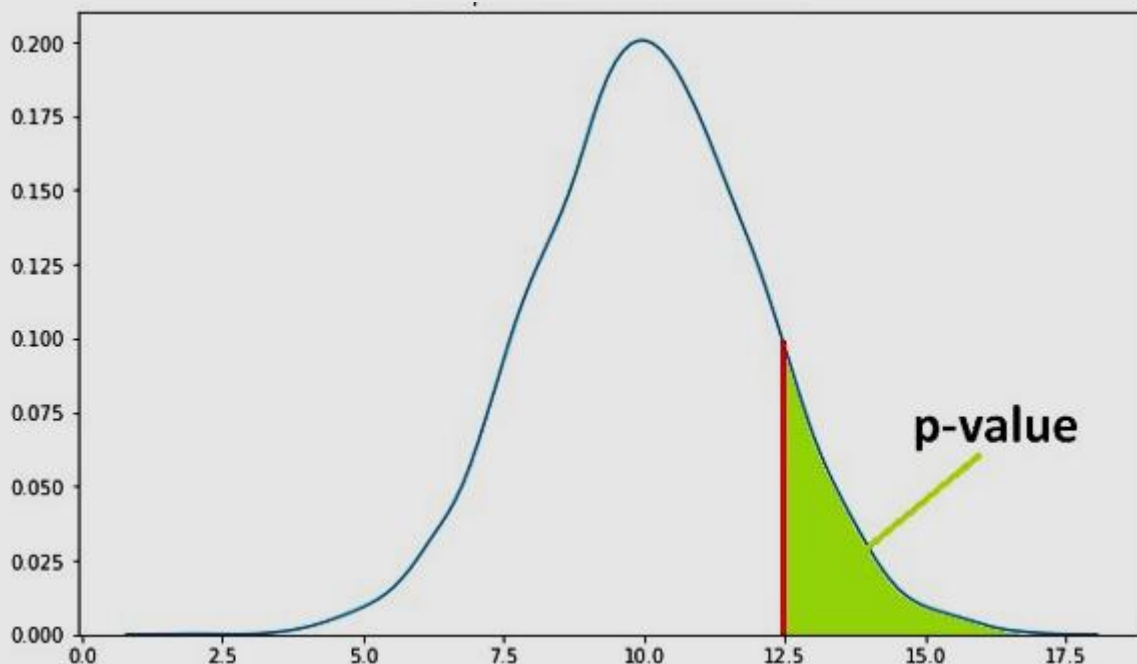
The figure below shows the probability distribution of A. It is highly likely to observe a value around 10. As the values get higher or lower, the probabilities decrease.





# P-value

We have another random variable, B, and want to see if B is greater than A. The average sample means obtained from B is 12.5. The p-value for 12.5 is the green area in the graph below. The green area indicates the probability of getting 12.5 or a more extreme value (higher than 12.5 in our case).



# P-value

Let's say the p-value is 0.11, but how do we interpret it? A p-value of 0.11 means that we are 89% sure of the results. In other words, there is an 11% chance that the results are due to random chance. Similarly, a p-value of 0.05 means that there is a 5% chance that the results are due to random chance.

**Note:** Lower p-values show more certainty in the result.

If the average of sample means from the random variable B turns out to be 15, which is a more extreme value, the p-value will be lower than 0.11.

## 8. Expected value of random variables

The expected value of a random variable is the weighted average of all possible values of the variable. The weight here means the probability of the random variable taking a specific value.

The expected value is calculated differently for discrete and continuous random variables.

- Discrete random variables take finitely many or countably infinitely many values. The number of rainy days in a year is a discrete random variable.
- Continuous random variables take uncountably infinitely many values. For instance, the time it takes from your home to the office is a continuous random variable. Depending on how you measure it (minutes, seconds, nanoseconds, and so on), it takes uncountably infinitely many values.

# Expected value of random variables

The formula for the expected value of a discrete random variable is:

$$E[X] = \sum x_i p(x_i)$$

$x_i$  = The values that  $X$  takes

$p(x_i)$  = The probability that  $X$  takes the value  $x_i$

The expected value of a continuous random variable is calculated with the same logic but using different methods. Since continuous random variables can take uncountably infinitely many values, we cannot talk about a variable taking a specific value. We rather focus on value ranges.

## Expected value of random variables

In order to calculate the probability of value ranges, probability density functions (PDF) are used. PDF is a function that specifies the probability of a random variable taking value within a particular range.

$$E[X] = \int_{xmin}^{xmax} xf(x)dx$$

$f(x)$  is the PDF of X

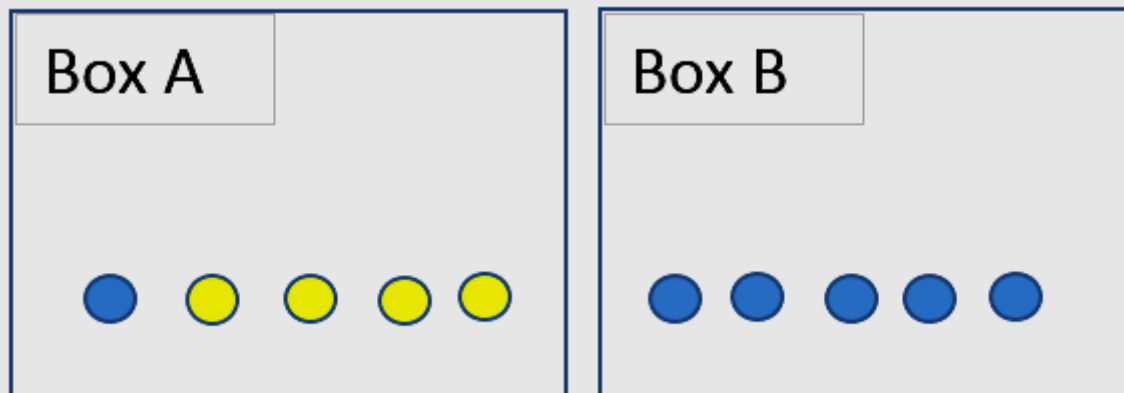
## 9. Conditional probability

Probability simply means the likelihood of an event to occur and always takes a value between 0 and 1 (0 and 1 inclusive). The probability of event A is denoted as  $p(A)$  and calculated as the number of the desired outcome divided by the number of all outcomes. For example, when you roll a die, the probability of getting a number less than three is  $2/6$ . The number of desired outcomes is 2 (1 and 2); the number of total outcomes is 6.

Conditional probability is the likelihood of an event A to occur, given that another event that has a relation with event A has already occurred.

# Conditional probability

Suppose that we have 6 blue balls and 4 yellows placed in two boxes, as seen below. I ask you to randomly pick a ball. The probability of getting a blue ball is  $6/10 = 0.6$ . What if I ask you to pick a ball from box A? The probability of picking a blue ball clearly decreases. The condition here is to pick from box A which clearly changes the probability of the event (picking a blue ball). The probability of event A given that event B has occurred is denoted as  $p(A|B)$ .



# 10. Bayes' theorem

According to Bayes' theorem, the probability of event A given that event B has already occurred can be calculated using the probabilities of event A and event B and probability of event B given that A has already occurred.

$$p(A|B) = \frac{p(A) \cdot p(B|A)}{p(B)} \quad (\textit{Bayes' Theorem})$$

Bayes' theorem is so fundamental and ubiquitous that a field called “Bayesian statistics” exists. In Bayesian statistics, the probability of an event or hypothesis as evidence comes into play. Therefore, prior probabilities and posterior probabilities differ depending on the evidence.



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