Claim Counts Prediction Using Individual Data with `ReSurv`

Munir Hiabu[1], Emil Hofman[2], and Gabriele Pittarello[3]

[1]Department for Mathematical Sciences, University of Copenhagen

[2]Department for Mathematical Sciences, University of Copenhagen

[3]ESOMAS department, University of Turin

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Abstract

For non-life reserving, the industry typically relies on chain-ladder-type methods based on development triangles stemming from an aggregation of individual claims data. The more detailed databases that make up the development triangles are often held by insurers and contain information about claims at an individual level that could potentially improve reserving. In this manuscript we present `ReSurv`, an R package for estimating feature-dependent development factors using individual claims data. We show how the results of the statistical modeling tools included in the package are translated into the form of the commonly used development factors and how they are used to predict claim frequencies. The methodology implemented in the package was derived in Hiabu et al. (2023). This paper includes an informal presentation of that framework and a detailed, practitioner-oriented guide to the use of the `ReSurv` package.

Keywords: survival analysis; individual reserving; non-life insurance; proportional hazard; gradient boosting; feed-forward neural networks.

1 Introduction

In the non-life insurance practice, the term *reserves* is used to indicate the insurer's estimate of unpaid claims that incurred prior to the financial statement reporting date. (Friedland, 2010, p. 13). One part of the reserve are the so called Incurred But Not Reported (IBNR) claims. Since the number and average size of IBNR claims is not known at the time the reserve is computed, actuaries

must estimate it. One option is to model the number and size of IBNR claims separately. In this paper we present a method for estimating the number of claims. While not being discussed here further, one simple way of estimating the avergae size of IBNR claims is to take the average of the severity of historical claims; possibly depended on the reporting delay and further claim features, x. In future work we will look into ways to handle frequency and severity together.

Modelling of future reported claim counts usually utilises development triangles, an aggregate representation of the reserving data that is standard practice in the industry (Brown et al., 2023). A development triangle is a set of observations

$$\mathcal{O} = \{O_{kj} : k, j = 0, \dots, m; k + j \leqslant m\}, \quad m > 0,$$

where O_{kj} is the total number of reported claims in accident date k with reporting delay j; i.e. the claim has been reported in $k + j$. Here, $k = 0$ denotes the earliest accident date and $m > 0$ is the input dimension of the development triangle. A commonly used model calculate reserves from development triangles is the chain-ladder. Given the cumulative triangle

$$C_{kj} = \sum_{l \leqslant j} O_{kl}, \quad k, j = 0, \dots, m; k + j \leqslant m, \tag{1}$$

one calculates so-called age-to-age factors $\hat{f}_{kj}$,

$$\tilde{f}_{kj} = C_{kj}/C_{k,j-1}, \quad k = 0, \dots, m; j = 1, \dots, m; j + k \leqslant m. \tag{2}$$

These age-to-age factors cannot be directly used for prediction since they are only defined on the upper triangle $j + k \leqslant m$. Instead, they are averaged over accident dates k:

$$\hat{f}_j = \sum_{k=0}^{m-j} w_k \tilde{f}_{kj}. \qquad \text{(development factors)}$$

Various ways of averaging can be used, but the most common is the volume-weighted average $w_k = C_{k,j-1}/\sum_{l=0}^{m-j} C_{l,j-1}$. Lastly, future reported claims are derived via:

$$\widehat{C}_{kj} = \begin{cases} \widehat{f}_j C_{k,j-1} & \text{for } k+j = m+1, \\ \widehat{f}_j \widehat{C}_{k,j-1} & \text{for } k+j > m+1. \end{cases}$$

While reserving models for aggregate data have a long history in the industry, the advent of new advanced computing tools has encouraged actuarial professionals to explore reserving models based on individual data (Richman, 2021). Often the more advanced techniques are based on sound theory, but there is no open source implementation that makes the models directly applicable in practice. A notable exception is the hierarchical model in Crevecoeur, Antonio, et al. (2023), implemented in the R package `hirem` (Crevecoeur and Robben, 2024). In this manuscript we present a pipeline for individual claims reserving using the R package `ReSurv`. Our package calculates accident date and feature-dependent development factors that can be used to predict future Incurred But Not Reported (IBNR) claims. Indeed, instead of chain ladder's development factors $\hat{f}_j$, the `ReSurv` package calculates development factors

$$\hat{f}_{kj}(x), \quad k = 0, \ldots, m; j = 1, \ldots, m$$

that may additionally depend on accident date k and some features x. It is important to note here that the development factors the `ReSurv` packages derives are defined on the whole square including the lower triangle, $k + j > m$, and can therefore be directly used for prediction. This is different to the age-to-age factors that are only defined on the upper triangle $k + j \leqslant m$, cf. equation (2). The features x can be seen as indexation of different development triangles (e.g. business line, cover type or claim type) which the `ReSurv` package can handle simultaneously. For example one could consider one feature $x =$ `claim_type` that can take either value `PD` for Property Damage claims or `BI` for Bodily Injured claims. In this case we would have two triangles:

$$\begin{aligned} \mathcal{O}(\texttt{PD}) &= \{O_{kj}(\texttt{PD}) : k, j = 0, \ldots, m; k + j \leqslant m; \texttt{claim\_type} = \texttt{PD}\}, \\ \mathcal{O}(\texttt{BI}) &= \{O_{kj}(\texttt{BI}) : k, j = 0, \ldots, m; k + j \leqslant m; \texttt{claim\_type} = \texttt{BI}\}, \quad m > 0. \end{aligned}$$

Given multiple cumulative triangles $C_{kj}(x)$, prediction is now done via

$$\widehat{C}_{kj}(x) = \begin{cases} \hat{f}_{kj}(x) C_{k,j-1}(x) & \text{for } k+j = m+1, \\ \hat{f}_{kj}(x) \widehat{C}_{k,j-1}(x) & \text{for } k+j > m+1. \end{cases} \tag{3}$$

Lastly, for reporting purposes one usually wishes to have an aggregation in e.g. quarters or years. However, the original input j, k for the `ReSurv` package is assumed to be as granular as possible, and may e.g. be in days. To this end the `ReSurv` allows to specify an output dimension M as a multiple of the input dimension$((m \mod M) = 0, M < m)$.

On the output scale M, we will work with a $j', k' = 0, \ldots, M$ and we will have the cumulative triangle

$$C'_{k'j'}(x) = \sum_{h,\nu=0,\ldots,m: \lfloor h \cdot \frac{M}{m} \rfloor \leqslant k'; \lfloor \nu \cdot \frac{M}{m} \rfloor \leqslant j'} O_{h\nu}(x).$$

The `ReSurv` package also calculates feature dependent development factors for the output granularity M:

$$\hat{f}'_{k'j'}(x), \quad k' = 0, \ldots, M; j' = 1, \ldots, M,$$

with

$$\widehat{C}'_{k'j'}(x) = \begin{cases} \hat{f}'_{k'j'}(x) C'_{k',j'-1}(x) & \text{for } k'+j' = M+1, \\ \hat{f}'_{k'j'}(x) \widehat{C}_{k',j'-1}(x) & \text{for } k'+j' > M+1. \end{cases} \tag{4}$$

The rest of the paper is organized as follows. In Section 1.1, we give a short overview on the machine learning methods employed and where to find further reading. In Section 1.2, we provide a simplified explanation of the methodology in Hiabu et al. (2023); for a full review, please refer to the main manuscript. After describing in detail how to install the package, we discuss a pipeline for individual reserving in Section 2. We show a data application on a simulated dataset in Section 3.

1.1 Machine learning foundation

`ReSurv` leverages three machine learning techniques in its implementation: splines, feed-forward

neural networks and gradient boosting. While familiarity with machine learning principles can enhance usability and help with modelling decisions, the interpretability of the results remains independent of the user's expertise in this area. Our package is designed to provide meaningful results regardless of the user's background in machine learning. For readers interested in an introduction to machine learning, we refer to Section 7 of James et al., 2023 for a gentle introduction to splines and to Section 10.2 in James et al., 2023 for an accessible introduction to feed-forward neural networks, Section 10.7 provides an overview of the optimisation task and the algorithm used to minimise the objective functions. Section 10 in Géron, 2019 provides a less mathematical introduction to neural networks with accompanying `Python` code. For further reading, Goodfellow et al., 2016 is a comprehensive handbook on neural networks. Sections 8.1 and 8.2 in James et al., 2023 will give a better understading of tree-based methods while Chen and Guestrin, 2016 introduces and explains the gradient boosting algorithm.

1.2 Hiabu et al., 2023 in a nutshell

The mathematical basis for `ReSurv` follows from the formulation of the reserving problem in a survival analysis setting. We model the reporting delay of individual claims by specifying a regression model for the corresponding hazard function.

Consider a set of reported claims. Each observed claim i is associated with a reporting delay t_i, an accident date u_i and p covariates $x_i \in \mathbb{R}^p$ that can be categorical or numerical. We specify the following regression model for the hazard function

$$\alpha(t|u,x) = \lim_{h \downarrow 0} P\left\{t_i \in [t, t+h) | t_i \geqslant t, u_i = u, x_i = x\right\} = \alpha_0(t) e^{\phi(x,u)}, \tag{5}$$

where $\alpha_0(t)$ is called the baseline hazard and $e^{\phi(x,u)}$ is the risk score; a component that depends on the features x_i, the accident date u_i. Note that the hazard function is defined in its own right as infinitesimal conditional probability. In particular it is not defined via the intensity of a stochastic process. The main use of the hazard function in the `ReSurv` package after estimating it is its transformation into development factors via (7) described below. In that sense the hazard function can be seen as intermediate object used in the reserving task.

Actuaries reading this manuscript will likely be familiar with the concept of GLM regression for modelling frequencies in insurance pricing. In pricing, one assumes that the number of claims Y_i is

Poisson distributed conditionally on some covariates X_i and an exposure W_i

$$Y_i|W_i, X_i \sim \text{Poisson}\left(W_i \exp\left(\theta_{\text{GLM}}^T X_i\right)\right). \tag{6}$$

In this case, it is possible to derive an analytic solution for an estimator of the parameter θ_{GLM} by minimising the model negative likelihood. In this manuscript, we perform regression on reserving data using the survival model in Equation (5). In our application we look for an estimate the risk score $\phi(x, u)$ and the baseline $\alpha_0(t)$. For a detailed comparison of hazard modelling and Poisson regression, we refer the reader to Carstensen and Diabetes (2004). It is important to note that, unlike the pricing case discussed therein, our approach does not rely on a distributional assumption and estimation of relies on the partial-likelihood theory described in Hiabu et al. (2023). Lastly, depending on the machine learning model that we choose, the risk score will have a different specification. Estimation of the risk score $\phi(x, u)$ can be performed in `ReSurv` with three different algorithms: the cox model (COX, Cox, 1972), neural networks (NN, Katzman et al., 2018) and gradient boosting (XGB, Chen and Guestrin, 2016).

- In Cox (1972), the risk score function is assumed to be linear, $\phi(x, u) = \theta^T x + \theta_u u$, with $\theta \in \mathbb{R}^p$ and $\theta_u \in \mathbb{R}$. Our package provides the option to include splines for modelling continuous features.
- In NN, $\phi(x, u)$ is a feed-forward neural network.
- In XGB, $\phi(x, u)$ is an ensemble of decision trees, i.e., functions that are piecewise constant on rectangles.

After an estimator $\hat{\phi}(x, u)$ is derived, the baselines $\alpha_0(t)$ is estimated using the full-likelihood where it is assumed that claim reports are uniformly distributed within a tie. Putting the estimators together we derive an estimator of the hazard function

$$\hat{\alpha}(t|u, x) = \hat{\alpha}_0(t) e^{\hat{\phi}(x,u)},$$

Finally, we model the development factor from reporting delay $j-1$ to j and accident date k as

$$\tilde{f}_{kj}(x) = \frac{2 + \hat{\alpha}(j|k, x)}{2 - \hat{\alpha}(j|k, x)}. \tag{7}$$

For a derivation and discussion of formula (7), we refer to Pittarello (2024). From here, these development factors can be applied using the chain-ladder rule for forecasting, cf. equation (3). By introducing feature and accident date dependency in the hazard estimation, we allow the same properties in the development factors.

Installation

The `ReSurv` package can be installed from The Comprehensive R Archive Network (CRAN).

```
> install.packages("ReSurv")
```

The developer version of `ReSurv` can be installed from GitHub.

```
> library(devtools)
> devtools::install_github("edhofman/resurv")
```

The package can then be imported in R using the command

```
> library(ReSurv)
```

Additional resources on our project can be found at https://github.com/edhofman/ReSurv. We remark that this manuscript refers to version `1.0.0` of our package:

```
> packageVersion("ReSurv")
  1.0.0
```

The main page of the help guide of our package can be accessed with the following command:

```
> help(package="ReSurv")
```

1.3 Handling the Python dependency

In this Section we illustrate the approach we use to handle the Python dependency in our package. The `ReSurv` package interfaces with a Python implementation to implement the NN models. The reader who is not interested in using the NN models can disregard this Section.

The interaction with Python is handled by creating an isolated virtual environment through the `install_pyresurv` function

```
> install_pyresurv()
```

The default name of the virtual environment is `"pyresurv"`.

We then suggest to refresh the `R` session and to import the `ReSurv` package in R using

```
> library(ReSurv)
> reticulate::use_virtualenv("pyresurv")
```

This approach uses the `reticulate` package implementation for interfacing R and Python (Ushey et al., 2024).

Managing Multiple Package Dependencies

In case the user is working with other libraries that use isolated package specific environments, the most straightforward solution would be installing a dedicated environment for both. Below, an example using the `R` package `pysparklyr`.

```
> envname <- "./venv"
> ReSurv::install_pyresurv(envname = envname)
> pysparklyr::install_pyspark(envname = envname)
```

The interested reader can read more about this topics in the vignette Managing an R Package's Python Dependencies.

2 IBNR modelling using ReSurv

In this section, we illustrate individual claims reserving in six steps that simulate the steps that an actuary can take to perform individual reserving using our software.

2.1 A pipeline for modelling expected counts

1. **Start from the data**. In Section 2.2, we discuss the required structure of the input data. In Section 2.2.1 we will use the simulator embedded in our package to generate a synthetic reserving data set to show the usage of `ReSurv`.
2. **Pre-process the data**. Before using a reserving model, individual data must be elaborated in a format that is ready for using the individual model. This is done in Section 2.3

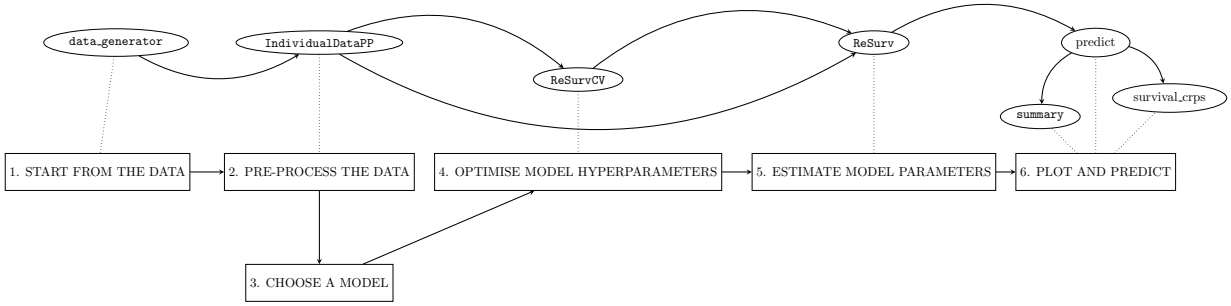


Figure 1: At the bottom, we show the six steps of individual reserving illustrated in Section 2. The steps are numbered and connected by solid arrows. The six steps can be performed using the `ReSurv` package tools that we show in the upper part of the figure. The package tools we are showing are described in detail in Section 3. Each package tool is linked to the corresponding step by a dotted arrow. The directional arrows in the diagram indicate the flow of information or processes, where the output of one step serves as the input for the next.

3. **Choose a model**. As mentioned earlier, our package allows us to model the log-risk function using three different models (COX, NN and XGB). The aim of Section 3 is to compare the three available approaches performances according to some performance metrics that we will define.

4. **Optimise the model hyperparamters**. After a brief introduction on hyperparameters in machine learning, the objective of Section 2.4 is to describe hyperparameters tuning. Section 2.5 explains how Baysian hyperparamter tuning can be performed utilizing the `ParBayesianOptimizatior` package.

5. **Estimate the model parameters.** Once we have estimated the hyperparameters of the models, we can fit our models using the `ReSurv` method, as described in Section 2.6.

6. **Visualize claim development and predict future IBNR.** The optimised model can be used to plot feature dependent development factors as well as predict IBNR claims for different data granularities. For example, if actuaries have daily data available for fitting, our approach is flexible enough to report and visualize future claims on a monthly, quarterly or annual scale. This is shown in Section 2.7.

2.2 How my input data should look like?

We begin this Section considering the **least** number of columns that should be included in the input data:

- Accident date of the claim, k (in calendar time).
- Reporting date of the claim, $k + j$ (in calendar time).

As we will show in the next two examples, the input dates can be both in `Date` and `numeric` format.

Optionally, the data can also include a set of static covariates, categorical and/or continuous, that can be used for modelling the feature dependent development factors, x, in Equation (2).

The individual input data can then be of two types. The first option is working with a data set where each row corresponds to a reporting time. An example is the data that we will simulate in Section 2.2.1. The first ten records of our simulated data are shown in Table 1. The data set is comprehensive of one categorical feature x =`claim_type` and the relevant dates: accident date (k=`AP`) and reporting date ($k + j$=`RP`). In this example, `AP` and `RP` are in `Date` format.

claim_number	claim_type	AP	RP
1	0	2017-01-01	2017-01-19
2	0	2017-01-01	2017-07-20
3	0	2017-01-01	2019-06-14
4	0	2017-01-01	2019-06-02
5	0	2017-01-01	2017-02-05
6	0	2017-01-01	2017-01-22
7	0	2017-01-01	2017-01-23
8	0	2017-01-01	2018-10-24
9	0	2017-01-01	2017-01-24
10	0	2017-01-01	2020-10-06

Table 1: The first ten records of the data simulated in Section 2.2.1 and described in Table 3. Each row corresponds to a reported claim. The `AP` and `RP` columns are converted to a `Date` format for illustrative purposes, assuming that our time series starts on the First of January 2017.

Alternatively, in the input data there could be multiple records for each claim. As an example, we consider the data set `test_transaction_dataset`, built-in in the `SynthETIC` package. The data set is displayed in Table 2. `test_transaction_dataset` is the case of an individual reserving data set where multiple events for each individual are recorded (e.g. multiple payments) and each individual is associated with a unique identifier (`claim_no`). In this example, the reporting date (`RP`) and the accident date (`AP`) are in `numeric` format.

Since the objective of the `ReSurv` package is modeling the reporting delay which we assume unique for each individual claim, these multiple records will not be part of our modeling but the pre-

claim_no	pmt_no	...	payment_period	payment_size	...	AP	RP
1	1	...	5	25104.78	...	1	2
1	2	...	8	26176.62	...	1	2
1	3	...	12	26333.19	...	1	2
1	4	...	15	26341.10	...	1	2
1	5	...	19	592456.91	...	1	2
1	6	...	19	89458.19	...	1	2
2	1	...	3	2005.48	...	1	3
2	2	...	3	2124.63	...	1	3
2	3	...	4	15986.06	...	1	3
2	4	...	4	2446.12	...	1	3

Table 2: The first ten records of the `test_transaction_dataset` data set from the `SynthETIC` package. The columns `AP` and `RP` are the accident date and the calendar date of reporting, we derived from the data and added them to the Table in our manuscript notation. For each claimant the reporting date is unique (`RP`). Some of the columns in the data where replaced by ... to ease the visualisation.

preprocessing of this type of input can be performed internally by our software.

2.2.1 Data simulation

In this Section, we generate a synthetic data set to show the usage of the `ReSurv` package. This section may be ignored by a reader who already has a dataset in the input format described in Section 2.2.

Let us consider the `data_generator` function.

```
> input_data <- data_generator(random_seed = 7,
                               scenario = "alpha",
                               time_unit = 1 / 360,
                               years = 4,
                               period_exposure = 200)
```

This function contains 5 different parameters:

- The `random_seed`, `integer`. It guarantees full replicable code.
- Our package offers 5 different simulated scenarios that can be used to replicate our manuscript analysis. The scenarios are described below and selected with the parameter `scenario`. Here, we choose to simulate from the so-called scenario Alpha. The user can input one of the following `character`: `'alpha'`, `'beta'`, `'gamma'`, `'delta'`, `'epsilon'`.

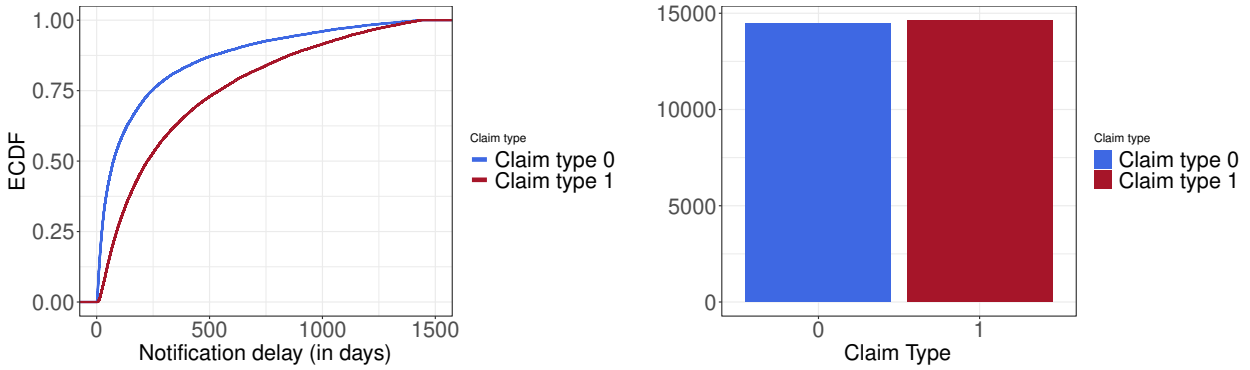
Feature	Description
`claim_number`	Policy identifier.
`claim_type`	Type of claim.
`AP`	Accident Date.
`RP`	Calendar Date of reporting.

Table 3: Description of the information in the simulated data.

- `time_unit`, `numeric`. It controls the data granularity (time unit) as a fraction of a calendar year. In our manuscript we generate daily data and set `time_unit`= 1/360. Implicitly, we are assuming that a year is made of 360 days.
- `years`, `integer`. This is the total number of accident years in the simulation. In the manuscript we use a four calendar year time horizon for daily data.
- The `period_exposure` consists of the total underwriting volume for time unit. In our manuscript we have a daily volume of 200 contracts.

Our package allows to simulate reserving data under 5 scenarios using the function `data_generator`. The simulator is based on the `SynthETIC` package (Avanzi et al., 2021). We named the 5 scenarios Alpha, Beta, Gamma, Delta, Epsilon. The 5 scenarios contain the information described in Table 3. Our simulated data consist of a mix of short tail claims (`claim_type 0`) and claims with longer resolution (`claim_type 1`). We chose the parameter of the simulator to resemble a mix of property damage (PD, `claim_type 0`) and bodily injuries (BI, `claim_type 1`).

However, each scenario has distinctive characteristics. Scenario Alpha is a mix of `claim_type 0` and `claim_type 1` with the proportion of type 0 and 1 claims held constant across the accident days. Chain ladder is expected to work well in this case such that we cannot expect `ReSurv` to provide improved prediction performance. In scenario Beta the proportion of `claim_type 1` claims are decreasing in the most recent accident dates. In scenario Gamma we add an interaction between `claim_type 1` and accident date: in a real world setting this can be motivated by a change in consumer behavior or company policies resulted in different reporting patterns over time. In scenario Delta, we introduce a seasonality effect period (spring, summer, winter, fall) dependent on the accident date for the proportion of `claim_type 0` versus `claim_type 1`. In the real word, the scenario Delta resembles seasonal changes in the workforce composition. Chain ladder is expected to have problems to handle scenarios Beta, Gamma and Delta accurately and the `ReSurv` package is expected to provide significantly better predictions in those cases. The object `input_data` is a



(a) Empirical Cumulative Density Function, notification delay (RT). (b) Data distribution by claim type (`claim_type`).

Figure 2: Empirical simulated cumulative distribution function (left-hand side) and data distribution by claim type (right-hand side).

`data.frame` frame (each row is a claim observation with data elements attached) and its structure can be displayed with the following command.

```
> str(input_data)
  'data.frame':        29088 obs. of  4 variables:
   $ claim_number: int  1 2 3 4 5 6 7 8 9 10 ...
   $ claim_type  : num  0 0 0 0 0 0 0 0 0 0 ...
   $ AP          : num  1 1 1 1 1 1 1 1 1 1 ...
   $ RP          : num  19 201 895 883 36 ...
```

We show the empirical cumulative density function of the simulated notification delay (Figure 2a) and the data distribution by claim type (Figure 2b). We can notice quicker notification for `claim_type 0`. The two claim types have similar volumes.

2.3 Data pre-processing

Before fitting our model on the data, the individual data must be pre-processed. In our software, this can be achieved with the built-in function `IndividualDataPP`. This step involves the dummy encoding of categorical features using the function `dummy_cols` from the `fastDummies` package (Kaplan, 2020) and scaling of continuous features using the MinMax Scaler approach (Wüthrich, 2018). Dummy encoding is used to convert categorical variables into numerical format. It creates separate columns for each category (or level) of the variable, and assigns a 1 if the observation belongs to that category, and a 0 if it does not. MinMax scaling is a normalization technique that

transforms data to a specific range, typically [-1, 1]. In the MinMax Scaler approach, we consider a continuous covariate x_i^ν with $x_i = \left(x_i^1, \ldots, x_i^\nu, \ldots, x_i^p\right)$ and apply the transformation

$$x_i^\nu \mapsto \bar{x}_i^\nu : \bar{x}_i^\nu = 2(x_i^\nu - \min(x_i^\nu))/(\max(x_i^\nu) - \min(x_i^\nu)) - 1,$$

to bring all features to a comparable scale.

`IndividualDataPP` creates an individual data set that is ready to be modelled.

```
> individual_data <- IndividualDataPP(data = input_data,
                                      categorical_features = "claim_type",
                                      continuous_features = "AP",
                                      accident_period = "AP",
                                      calendar_period = "RP",
                                      input_time_granularity = "days",
                                      output_time_granularity = "quarters",
                                      years = 4)
```

- The input `data`. E.g. the simulated output of `data_generator` in Section 2.2.1.
- `id`, `character`. This is the data column that contains the policy identifier. Default is `NULL`. When `NULL`, we assume that each row in the data is a distinct claim. If a column name is specified, the first record in `calendar_period` for the `id` is treated as the reporting delay for the claimant.
- `categorical_features` and `continuous_features` allow us to specify which columns to handle as categorical, and which as numeric. The input should be a `character`. This can be important in a NN setting when preprocessing the data. We only have one categorical feature in our simulated data, but multiple can be specified.
- `accident_period` and `calender_period` tells us the name of the accident date and reporting date columns in the `data`. This should be input as a `character`.
- The `input_time_granularity` tells us how granular our input is, and `output_time_granularity` allows the user to specify the granularity of the output development factors. The user

can choose between one of the following `character` input: `'days'`, `'months'`, `'quarters'`, `'semesters'`, `'years'`.

- `years`, `numeric` and optional. Number of different accident years in the data. If not provided it is calculated internally.

The output of `IndividualDataPP` is a list containing:

- `full.data`, `data.frame`: the input data after pre-processing.
- `starting.data`, `data.frame`: the input data as they were provided from the user.
- `string_formula_i`, `character`: string of the `survival` formula to model the data in input granularity.
- `training.data`: the input data pre-processed for training.
- `conversion_factor`, `numeric`: the conversion factor for going from input granularity to output granularity. E.g, the conversion factor for going from months to quarters is $1/3$.
- `string_formula_o`, `character`: string of the `survival` formula to model the in data output granularity.
- `continuous_features`, `character`: the continuous features names as provided from the user.
- `categorical_features`, `character`: the categorical features names as provided from the user.

After pre-processing, we provide a standard encoding for the time components. This regards the output in `training.data` and `full.data`. In the `ReSurv` notation, the main information is:

- `AP_i`: input accident date.
- `AP_o`: output accident date.
- `DP_i`: input reporting delay.

Additional information contained in this `data.frame` structures is included for internal modelling purposes and it is described in the `ReSurv::IndividualDataPP` documentation.

2.4 Selection of the hyper-parameters

Machine learning models are characterized by two distinct sets of elements: hyperparameters, which govern the learning process, and parameters, which are learned from the data during training. Machine learning algorithms are sensitive to the hyperparameters chosen. While there exists several routines for hyperparameters selection, `ReSurv` offers a built-in implementation of a standard K-Fold cross-validation (Hastie et al., 2009): the `ReSurvCV` method of an `IndividualDataPP` object. We show an illustrative example for XGB and NN below.

In Section 2.5, we will show that our K-Fold cross-validation implementation (`ReSurvCV`) can be combined with the methods from Snoek et al., 2012 implemented in the R package `ParBayesianOptimization` (Wilson, 2022).

Table 4 includes a concise yet descriptive list of our machine learning models hyperparameters. For a more thorough explanation of their utilizaiton we refer to `xgboost` reference guide and `.pytorch` reference guide.

2.4.1 XGB: K-Fold cross-validation

`ReSurvCV` requires three inputs:

- The input `IndividualDataPP`. This is usually the output from the data pre-processing step.
- Model to tune. One can choose from three different `models` to estimate the hazard function. Here we chose XGB, the other possibilities include NN (deep survival neural network) and COX.
- To perform the grid search, we need to specify the `hyperparameter_grid` on which we optimize. This will be dependent on the chosen model, and in this example, we have given values for some parameters for XGB.

We remark that our XGB implementation extends for left-truncation and tied data the implementation from Chen and Guestrin, 2016. For a more detailed description of the model parameters that can be cross-validated for XGB, please visit `xgboost` reference guide.

```
> resurv_cv_xgboost <- ReSurvCV(IndividualDataPP = individual_data,
                                model = "XGB",
```

```
                    hparameters_grid = list(booster = "gbtree",
                                            eta = c(.001),
                                            max_depth = c(3),
                                            subsample = c(1),
                                            alpha = c(0),
                                            lambda = c(0),
                                            min_child_weight = c(.5)),
                    print_every_n = 1L,
                    nrounds = 1,
                    verbose = FALSE,
                    verbose.cv = TRUE,
                    early_stopping_rounds = 1,
                    folds = 5,
                    parallel = TRUE,
                    ncores = 2,
                    random_seed = 1)
```

For XGB, the output of `ReSurvCV` consists of two items which are `data.frame` objects: `out.cv` and `out.cv.best.oos`. The two outputs contain the hyperparameters `booster`, `eta`, `max_depth`, `subsample`, `alpha`, `lambda`, `min_child_weight`. They also contain the metrics `train.lkh` (in-sample likelihood), `test.lkh` (out-of-sample likelihood), and the computational time `time`. `out.cv` contains the output of the cross-validation (all the input parameters combinations). `out.cv.best.oos` contains the combination with the best out of sample likelihood.

2.4.2 NN: K-Fold cross-validation

The `ReSurv` NN implementation uses `reticulate` to interface R Studio to Python and it is based on a similar approach to Katzman et al. (2018), corrected to account for left-truncation and ties in the data. Similarly to the original implementation we relied on the Python library `pytorch` (Paszke et al., 2019). The syntax of our NN is then the syntax of `pytorch`. See the reference guide for further information on the NN parametrization.

```
> resurv_cv_nn <- ReSurvCV(IndividualDataPP = individual_data,
                           model = "NN",
```

```
hparameters_grid = list(num_layers = c(1, 2),
                        num_nodes = c(2, 4),
                        optim = "Adam",
                        activation = "ReLU",
                        lr = .5,
                        xi = .5,
                        eps = .5,
                        tie = "Efron",
                        batch_size = as.integer(5000),
                        early_stopping = "TRUE",
                        patience = 20),
epochs = as.integer(300),
num_workers = 0,
verbose = FALSE,
verbose.cv = TRUE,
folds = 3,
parallel = FALSE,
random_seed = as.integer(Sys.time()))
```

For NN models, the columns in `out.cv` and `out.cv.best.oos` are the hyperparameters `num_layers`, `optim`, `activation`, `lr`, `xi`, `eps`, `tie`, `batch_size`, `early_stopping`, `patience`, `node` train.lkh test.lkh. They also contain the metrics `train.lkh`, `test.lkh`, and the computational time `time`.

2.5 `ReSurv` and Bayesian Parameters Optimisation

Our methods can be easily combined with those from the `ParBayesianOptimization` package. While we refer to Snoek et al., 2012 for a mathematical explanation of the Bayesian Optimisation method that we use. We show a code example below.

2.5.1 NN: Bayesian Parameters Optimisation

We specify the bounds for the hyper-parameters search in the NN case.

```
> bounds <- list(num_layers = c(2L, 10L),
                 num_nodes = c(2L, 10L),
```

```
            optim = c(1L, 2L),
            activation = c(1L, 2L),
            lr = c(0.005, 0.5),
            xi = c(0, 0.5),
            eps = c(0, 0.5))
```

Secondly, we need to specify an objective function to be optimized with the Bayesian approach. The score metric we inspect is the negative (partial) likelihood. The likelihood is returned with negative sign as Wilson (2022) is maximizing the objective function.

```
>  obj_func <- function(num_layers,
                        num_nodes,
                        optim,
                        activation,
                        lr,
                        xi,
                        eps) {

    optim <- switch(optim,
                    "Adam",
                    "SGD")

    activation <- switch(activation, "LeakyReLU", "SELU")
    batch_size <- as.integer(5000L)
    number_layers <- as.integer(num_layers)
    num_nodes <- as.integer(num_nodes)

    deepsurv_cv <- ReSurvCV(IndividualData = individual_data,
                            model = "NN",
                            hparameters_grid = list(num_layers = number_layers,
                                                    num_nodes = num_nodes,
                                                    optim = optim,
                                                    activation = activation,
```

```
                                                lr = lr,
                                                xi = xi,
                                                eps = eps,
                                                tie = "Efron",
                                                batch_size = batch_size,
                                                early_stopping = "TRUE",
                                                patience  = 20),
                                    epochs = as.integer(300),
                                    num_workers = 0,
                                    verbose = FALSE,
                                    verbose.cv = TRUE,
                                    folds = 3,
                                    parallel = FALSE,
                                    random_seed = as.integer(Sys.time()))


    lst <- list(Score = -deepsurv_cv$out.cv.best.oos$test.lkh,
                train.lkh = deepsurv_cv$out.cv.best.oos$train.lkh)

    return(lst)
  }
```

As a last step, we use the `bayesOpt` function to perform the optimization.

```
> bayes_out <- bayesOpt(FUN = obj_func,
                        bounds = bounds,
                        initPoints = 50,
                        iters.n = 1000,
                        iters.k = 50,
                        otherHalting = list(timeLimit = 18000))
```

To select the optimal hyperparameters we inspect `bayes_out$scoreSummary` output in Table 5. Below, we print the first five rows of one of our runs. Observe `scoreSummary` is a `data.table` that

also contains some parameters specific of the original implementation. While we refer to Wilson (2022) for more details on the complete output, below is the information that an actuary using this optimisation routine should focus on. Each row in the Table contains one of the combinations of hyperparameters tested: `num_layers`, `num_nodes`, `optim`, `activation`, `lr`, `xi`, `eps`, `batch_size`. We select the final combination that minimizes the negative (partial) likelihood, displayed in the `Score` column.

XGB: Bayesian Parameters Optimisation

Similarly to the NN case, we specify the bounds of our parameters search.

```
> bounds <- list(eta = c(0, 1),
               max_depth = c(1L, 25L),
               min_child_weight = c(0, 50),
               subsample = c(0.51, 1),
               lambda = c(0, 15),
               alpha = c(0, 15))
```

We then define an objective function.

```
> obj_func <- function(eta,
                     max_depth,
                     min_child_weight,
                     subsample,
                     lambda,
                     alpha) {


    xgbcv <- ReSurvCV(
      IndividualDataPP = individual_data,
      model = "XGB",
      hparameters_grid = list(
        booster = "gbtree",
        eta = eta,
        max_depth = max_depth,
```

```
      subsample = subsample,
      alpha = lambda,
      lambda = alpha,
      min_child_weight = min_child_weight
    ),
    print_every_n = 1L,
    nrounds = 500,
    verbose = FALSE,
    verbose.cv = TRUE,
    early_stopping_rounds = 30,
    folds = 3,
    parallel = FALSE,
    random_seed = as.integer(Sys.time())
  )

  lst <- list(Score = -xgbcv$out.cv.best.oos$test.lkh,
              train.lkh = xgbcv$out.cv.best.oos$train.lkh)

  return(lst)
}
```

The optimisation is then performed with the `bayesOpt` function as follows.

```
> bayes_out <- bayesOpt(FUN = obj_func,
                        bounds = bounds,
                        initPoints = 50,
                        iters.n = 1000,
                        iters.k = 50,
                        otherHalting = list(timeLimit = 18000))
```

2.6 Estimation

Once we estimated the best set of hyper-parameters for NN and XGB we invoke our algorithms to estimate the hazard function.

COX

For fitting the COX model the required input of the `ReSurv` method is simply the pre-processed data `individual_data` (`IndividualDataPP` class) and the chosen `hazard_model` as character (`'COX'`).

```
> resurv_fit_cox <- ReSurv(individual_data,
                           hazard_model = "COX")
```

The `ReSurv` fit output is a list containing

- `model.out`: `list` containing the pre-processed covariates data for the fit (`data`) and the basic model output (COX, XGB or NN).
- `is_lkh`: `numeric` Training negative log likelihood.
- `os_lkh`: `numeric` Validation negative log likelihood. Not available for COX.
- `hazard_frame`: `data.frame` containing the fitted model results, we will describe this output below.
- `IndividualDataPP`: starting `IndividualDataPP` object.

The `hazard_frame` data set contains our features (`claim_type` and `AP_i`) and the main information that we write below.

- `expg`: fitted risk score $\phi(x; u)$.
- `baseline`: fitted baseline $\alpha_0(t)$.
- `hazard`: fitted hazard rate (`expg`*`baseline`).
- `f_i`: fitted development factors for the input granularity.
- `cum_f_i`: fitted cumulative development factors for the input granularity.
- `S_i`:fitted survival function.

XGB

After selecting the hyper parameters we can finally fit our models to our pre-processed data. The optimised hyper-parameters are saved in `hparameters_xgb` as a list.

```
> hparameters_xgb <- list(
    params = list(
      booster = "gbtree",
      eta = 0.9611239,
      subsample = 0.62851,
      alpha = 5.836211,
      lambda = 15,
      min_child_weight = 29.18158,
      max_depth = 1
    ),
    print_every_n = 0,
    nrounds = 3000,
    verbose = FALSE,
    early_stopping_rounds = 500
  )
```

In the `ReSurv` package the fitting can be performed using the homonymous `ReSurv` method.

```
> resurv_fit_xgb <- ReSurv(individual_data,
                           hazard_model = "XGB",
                           hparameters = hparameters_xgb)
```

The `ReSurv` function, simply requires to specify the pre-processed individual data, the selected model for the hazard (`hazard_model` argument) and the necessary hyperparameters (`hparameters` argument).

NN

In the NN case we find the following hyper-parameters.

```
> hparameters_nn <- list(
    num_layers = 2,
    early_stopping = TRUE,
    patience = 350,
    verbose = FALSE,
    network_structure = NULL,
    num_nodes = 10,
    activation = "LeakyReLU",
    optim = "SGD",
    lr = 0.02226655,
    xi = 0.4678993,
    epsilon = 0,
    batch_size = 5000L,
    epochs = 5500L,
    num_workers = 0,
    tie = "Efron"
  )
```

We can fit our NN model as follows.

```
> resurv_fit_nn <- ReSurv(individual_data,
                          hazard_model = "NN",
                          hparameters = hparameters_nn)
```

2.7 Prediction

We use the method `predict` to predict the future claim frequencies. The method can be used by simply specifying a `ReSurv` model. Below, we only show an example for the COX model but a similar routine can be used for NN and XGB.

```
> resurv_fit_predict_q <- predict(resurv_fit_cox)
```

Our software also allows to predict IBNR counts for different granularities. In order to do so, it is sufficient to pre-process the input data using the `IndividualDataPP` class and changing the

output_time_granularity argument. In the example below, we process our data for yearly predictions.

```
> individual_data_y <- IndividualDataPP(input_data,
                                        id = "claim_number",
                                        continuous_features = "AP_i",
                                        categorical_features = "claim_type",
                                        accident_period = "AP",
                                        calendar_period = "RP",
                                        input_time_granularity = "days",
                                        output_time_granularity = "years",
                                        years = 4,
                                        continuous_features_spline = NULL,
                                        calendar_period_extrapolation = FALSE)
```

We then use the predict method on the new data for forecasting.

```
> resurv_fit_predict_y <- predict(resurv_fit_cox,
                                  newdata = individual_data_y,
                                  grouping_method = "probability")
```

The same routine is applied monthly in the next code.

```
> individual_data_m <- IndividualDataPP(input_data,
                                        id = "claim_number",
                                        continuous_features = "AP_i",
                                        categorical_features = "claim_type",
                                        accident_period = "AP",
                                        calendar_period = "RP",
                                        input_time_granularity = "days",
                                        output_time_granularity = "months",
                                        years = 4,
                                        continuous_features_spline = NULL,
                                        calendar_period_extrapolation = FALSE)
```

```
> resurv_fit_predict_m <- predict(resurv_fit_cox,
                                  newdata = individual_data_m,
                                  grouping_method = "probability")
```

The `predict` method creates our models output both in long triangle format in triangle shape. In particular, `predict` output contains:

- `long_triangle_format_out`: `list`. Predicted development factors and IBNR claim counts for each feature combination in long format.
  - `input_granularity`: `data.frame`. Predictions for each feature combination in long format for `input_time_granularity`. Below, we print a head of the data for the input granularity from our quarterly COX model.

    ```
    > head(resurv.fit.predict.Q$long_triangle_format_out$input_granularity)
    ```

	claim_type	AP_i	DP_i	f_i	group_i	expected_counts	IBNR
1	0	1	1388	1.000581	1	0.006389218	NA
2	1	1	1388	1.001328	2	0.019890757	NA
3	0	2	1388	1.000581	3	0.004065945	NA
4	1	2	1388	1.001328	4	0.015912916	NA
5	0	3	1388	1.000581	5	0.008132050	NA
6	1	3	1388	1.001328	6	0.002652204	NA

    The data contains the following columns
    - `AP_i`: Accident date, `input_time_granularity`.
    - `DP_i`: Reporting delay, `input_time_granularity`.
    - `f_i`: Predicted development factors, `input_time_granularity`.
    - `group_i`: Group code, input_time_granularity. This associates to each feature indexation an identifier. In the example, `claim_type 0` and `AP_i 1` is group 1.
    - `expected_counts`: Expected counts, `input_time_granularity`.
    - `IBNR`: Predicted IBNR claim counts, `input_time_granularity`. This is equal to expected counts if $j + k > m$, otherwise NA.
  - `output_granularity`: data.frame. Predictions for each feature combination in long format for `output_time_granularity`.

The columns for the output granularity include the following information:

* `AP_o`: Accident date, `output_time_granularity`.
* `DP_o`: Reporting delay, `output_time_granularity`.
* `f_o`: Predicted development factors, `output_time_granularity`.
* `group_o`: Group code, `output_time_granularity`. This associates to each feature combination an indexation. In the example, `claim_type 0` and `AP_o 1` is output group 1.
* `expected_counts`: Expected counts, `output_time_granularity`.
* `IBNR`: Predicted IBNR claim counts, `output_time_granularity`.

- `lower_triangle`: Predicted lower triangle. All features x are added together into one single triangle.
  - `input_tg`: `data.frame`. Predicted lower triangle for `input_time_granularity`.
  - `output_tg`: `data.frame`. Predicted lower triangle for `output_time_granularity`. For example, we show below the lower triangle of predicted counts for the yearly output.

```
> resurv.fit.predict.Y$lower_triangle$output_granularity
     1        2        3        4        5
1 NA       NA       NA       NA 130.5229
2 NA       NA       NA 444.6290 133.6801
3 NA       NA 747.1519 433.4708 129.6334
4 NA 2150.727 749.9761 436.6663 124.0186
```

- `predicted_counts`: `numeric`. Total predicted frequencies.

A summary of the predictions total output can be displayed with a print of the predictions `summary`.

```
> model_s <- summary(resurv_fit_predict_y)
> print(model_s)
   Hazard model:
  "COX"


  Categorical Features:
```

```
claim_type
Continuous Features:
AP_i
Total IBNR level:
[1]  5480
```

Using our approach we can produce, for each combination of features, the feature-dependent development factors in Equation (7). In Figure 3 we produce an example for the COX output illustrated in this Section and compare it with the chain-ladder model. We show for monthly, quarterly and yearly data the development factors fitted with the COX model for some combinations of features (rows two and three). Differently from the chain-ladder development factors (row one), we can catch data heterogeneity by feature.

The feature dependent development factors Figure 3 can be plot with the `plot` method. For example, in Figure 3a we show the output for `AP_o 15` and `claim_type 1`. As described in Section 2.3, we used indeed `AP` and `claim_type` as features.

As we will see below, we need to select the output group code for using the plot method, and use it as `group_code` parameter of the `plot` method. The output `group_code` can be found in the `group_o` column of the `long_triangle_format_out$output_granularity` output of the `predict.ReSurv` method. Similarly, the input `group_code` would be in the `group_i` column of the `long_triangle_format_out$input_granularity` output of the `predict.ReSurv` method.

```
> plot(resurv_fit_predict_q,
       granularity = "output",
       title_par = "COX: Accident Quarter 15 Claim Type 1",
       group_code = 30)
```

The other main parameters include the `granularity` of the development factors (either `"input"` for the `input_time_granularity` or `"output"` for the `output_time_granularity`).

The code for the other feature indexations in Figure 3 can be found in Appendix B.

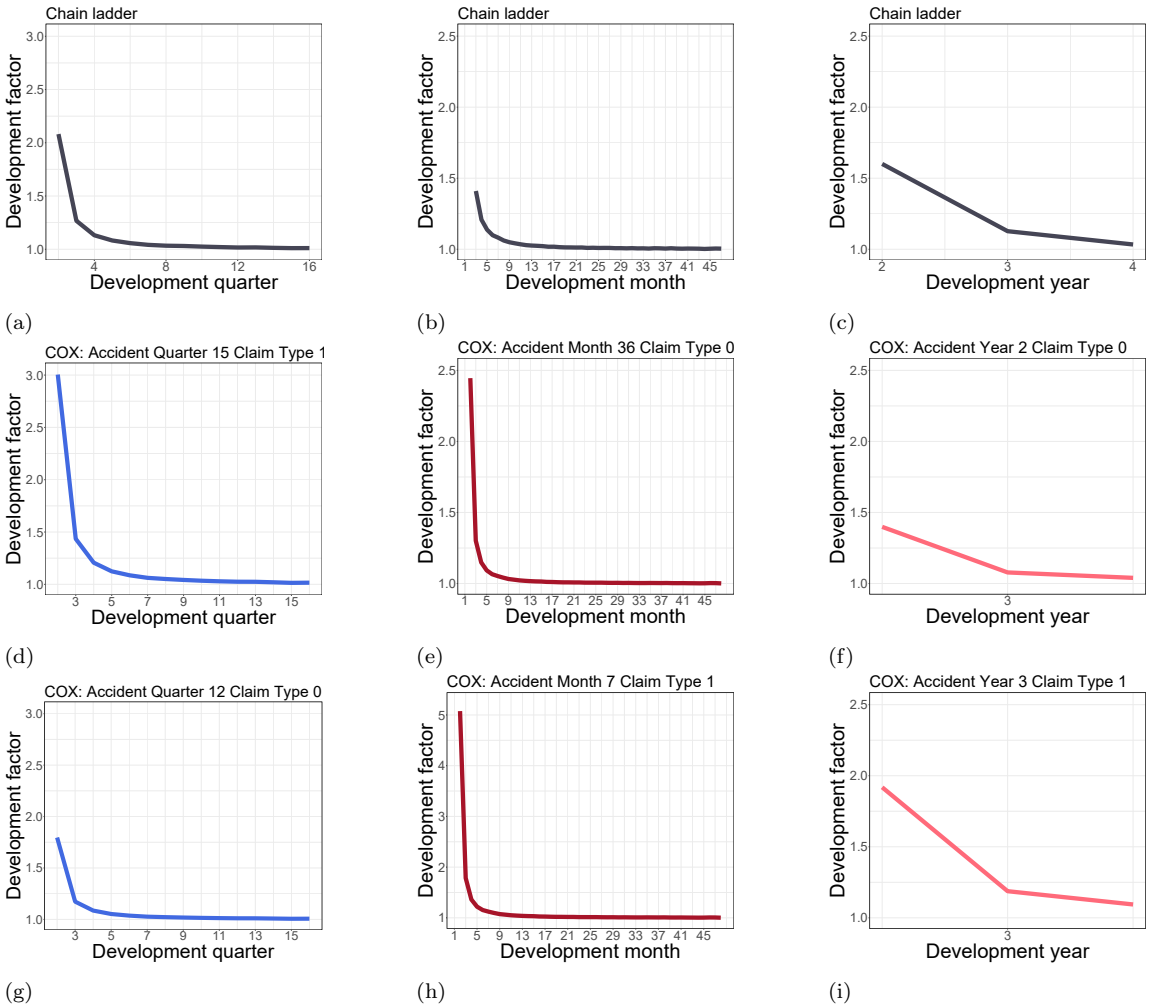


Figure 3: The first column, shows quarterly development factors for the chain ladder (top panel) and the COX model for feature combinations Accident Quarter 15 and `Claim_type 0` (center panel) and Accident Quarter 12 and `Claim_type 1` (bottom panel). The second column shows monthly development factors for the chain ladder (top panel) and the COX model for the feature combinations Accident Month 36 and `Claim_type 1` (top panel) and Accident Month 7 and `Claim_type 1` (bottom panel). The third column shows yearly development factors for the chain ladder (top panel) and the COX model for the feature combinations Accident Year 2 and `Claim_type 0` (central panel), and Accident Quarter 3 and `Claim_type 1` (bottom panel).

3 Data application

In this Section, we show a small data application that compares our fitted models on the data from Section 2.2.1 using three different performance metrics defined below. The code is shown for the COX model but similar compuations can be performed with XGB and NN. The three metrics are the Total Absolute Relative Error ($\texttt{ARE}^{\texttt{TOT}}$), Calendar Absolute Relative Error ($\texttt{ARE}^{\texttt{CAL}}$), and the Continuously Ranked Probability Score (CRPS; Gneiting, A. Raftery, et al., 2004). While the code for the first two metrics is computed in Appendix C, we show here the usage of the built-in function

`survival_crps` for the computation of the CRPS of `ReSurv` models.

Total Absolute Relative Error $\texttt{ARE}^{\texttt{TOT}}$

We first evaluate the Total (relative) Absolute Errors $\texttt{ARE}^{\texttt{TOT}}$ on the input grid. The $\texttt{ARE}^{\texttt{TOT}}$ is defined as

$$\texttt{ARE}^{\texttt{TOT}} = \frac{\sum_{j,k:k+j>m} |\sum_x O_{k,j}(x) - \sum_x \hat{O}_{k,j}(x)|}{\sum_{j,k:k+j>m} \sum_x O_{k,j}(x)}, \tag{8}$$

where $\hat{O}_{k,j}(x)$ denotes the estimated reportings in accident date k and reporting delay j.

The $\texttt{ARE}^{\texttt{TOT}}$ computation for the COX fit is displayed in Appendix C

Calendar Absolute Relative Error $\texttt{ARE}^{\texttt{CAL}}$

We then want to evaluate our models performance a second time diagonal-wise with a different performance metric. We considers the new information available at the end of each calendar date until the end of the duration for which we have data. We call this metric the Total (relative) Absolute Errors by Calendar time ($\texttt{ARE}^{\texttt{CAL}}$). The $\texttt{ARE}^{\texttt{CAL}}$ is defined as

$$\texttt{ARE}^{\texttt{CAL}} = \frac{\sum_{\tau=m+1}^{2m-1} \sum_{j,k:k+j=\tau} |\sum_x O_{k,j}(x) - \sum_x \hat{f}_{k,j}(x) O_{k,j-1}(x)|}{\sum_{j,k:k+j>m} \sum_x O_{k,j}(x)}. \tag{9}$$

The computation of the quarterly $\texttt{ARE}^{\texttt{CAL}}$ for the COX model is shown in Appendix C.

Continuously Ranked Probability Score (CRPS)

The Continuously Ranked Probability Score (CRPS) is defined in Gneiting and A. E. Raftery, 2007 as

$$\begin{aligned}
\text{CRPS}(\hat{S}(\cdot|x), y) &= \int_0^\infty \text{PS}(\hat{S}(z|x), \mathbb{I}\{y > z\}) \mathrm{d}z \\
&= \int_0^\infty (\hat{S}(z|x) - \mathbb{I}\{y > z\})^2 \mathrm{d}z \\
&= \int_0^y \left(1 - \hat{S}(z|x)\right)^2 dz + \int_y^{+\infty} \left(\hat{S}(z|x)\right)^2 dz,
\end{aligned}$$

with $\text{PS}(\hat{S}(z|x), \mathbb{I}\{y > z\}) = (\hat{S}(z|x) - \mathbb{I}\{y > z\})^2$ being the Brier Score (Selten, 1998; Gneiting and A. E. Raftery, 2007).

We can use correspondence between survival function and predicted development factors (Hiabu et al., 2023) to estimate

$$\hat{S}(j|x) = \frac{1}{\prod_{l=1}^{j} \hat{f}_{k,l}(x)}, \tag{10}$$

where $\hat{S}(j|x)$ is an estimator for the survival function $S(j|x) = P\left(T_i \geqslant j | X_i = x\right)$.

The CRPS can be computed with the built-in method `survival_crps`

```
> crps <- survival_crps(resurv.fit.cox)
```

The output of `survival_crps` is a `data.table` that contains the CRPS (`crps`) for each observation (`id`). For comparison among models, we take the average CRPS on the data.

```
> m_crps <- mean(crps$crps)
> m_crps
366.2639
```

Models comparison

The results on the data simulated in Section 2.2.1 are shown in Table 6. In our numerical application, we seem to prefer the NN model to COX and XGB according to the metrics defined in this section. The results are shown in bold for the NN model. We observe that the NN model shows the smallest $\texttt{ARE}^{\texttt{TOT}}$ and $\texttt{ARE}^{\texttt{CAL}}$, and smallest average CRPS. However, we observe that NN and XGB can be sensitive to hyper-parameters choice and they require the extensive cross-validation procedure that we illustrated in the previous sections. Our models are compared with the Chain-Ladder (column one). While the Chain-Ladder seems to work well, it is outperformed by the NN model.

Interestingly, in scenario Alpha the Chain-Ladder was expected to work well as the mix of the two claim types has a stable pattern. The performance differential between the Chain-Ladder approach and our proposed methodologies was relatively modest. However, the usage of our more sophisticated methods is motivated in more complex scenarios like scenario Delta. In Table 7, we show the results for one of the simulation of Scenario Delta. Scenario Delta was described in detail

in Section 2.2.1 and it resembles a real world scenario with seasonal patterns. Here, our models outperform the Chain-Ladder in terms of $\texttt{ARE}^{\texttt{TOT}}$ and $\texttt{ARE}^{\texttt{CAL}}$. The best the $\texttt{ARE}^{\texttt{TOT}}$ and $\texttt{ARE}^{\texttt{CAL}}$ are obtained with the XGB model, which also shows the best CRPS result.

The results in Table 7 are taken from the main manuscript replication material and were derived with the same procedure described in this paper.

Further Reading

The interested reader can find additional material and information on `ReSurv` at our website

https://edhofman.github.io/ReSurv/.

It is possible to track the package development at

https://github.com/edhofman/ReSurv.

The ArXiv version of the main manuscript has archive identifier

arxiv:2312.14549.

An `RMarkdown` version of the code included in this manuscript can be downloaded at

https://github.com/edhofman/ReSurv/blob/main/vignettes/cas_call.Rmd.

The code is also available in the `html` knitted version at articles/cas_call.html.

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A Code to replicate Figure 2

The code in this section relies on the libraries `dplyr` and `ggplot2`

```
> library(dplyr)
> library(ggplot2)
```

A.1 Code to replicate Figure 2a

```
> p1 <- input_data %>%
    as.data.frame() %>%
    mutate(claim_type = as.factor(claim_type)) %>%
    ggplot(aes(x = RT - AT, color = claim_type)) +
    stat_ecdf(size = 1) +
    labs(title = "", x = "Notification delay (in days)", y = "ECDF") +
```

```
    xlim(0.01, 1500) +
    scale_color_manual(
      values = c("royalblue", "#a71429"),
      labels = c("Claim type 0", "Claim type 1")
    ) +
    scale_linetype_manual(values = c(1, 3),
                          labels = c("Claim type 0", "Claim type 1")) +
    guides(
      color = guide_legend(
        title = "Claim type",
        override.aes = list(
          color = c("royalblue", "#a71429"),
          linewidth = 2
        )
      ),
      linetype = guide_legend(
        title = "Claim type",
        override.aes = list(linetype = c(1, 3), linewidth = 0.7)
      )
    ) +
    theme_bw() +
    theme(
      axis.text = element_text(size = 20),
      axis.title.y = element_text(size = 20),
      axis.title.x  = element_text(size = 20),
      legend.text = element_text(size = 20)
    )
> p1
```

A.2 Code to replicate Figure 2b

```
> p2 <- input_data %>%
as.data.frame() %>%
```

```
mutate(claim_type = as.factor(claim_type)) %>%
ggplot(aes(x = claim_type, fill = claim_type)) +
geom_bar() +
scale_fill_manual(
  values = c("royalblue", "#a71429"),
  labels = c("Claim type 0", "Claim type 1")
) +
guides(fill = guide_legend(title = "Claim type")) +
theme_bw() +
labs(title = "", x = "Claim Type", y = "") +
theme(
  axis.text = element_text(size = 20),
  axis.title.y = element_text(size = 20),
  axis.title.x  = element_text(size = 20),
  legend.text = element_text(size = 20)
)
> p2
```

B Code for plotting feature-dependent development factors

Chain ladder

```
> clmodel <- resurv_fit_cox$IndividualDataPP$training.data %>%
    mutate(DP_o = 16 -
             DP_rev_o + 1) %>%
    group_by(AP_o, DP_o) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(AP_o) %>%
    arrange(DP_o) %>%
    mutate(I_cum = cumsum(I), I_cum_lag = lag(I_cum, default = 0)) %>%
    ungroup() %>%
    group_by(DP_o) %>%
    reframe(df_o = sum(I_cum * (
```

```
      AP_o <= max(resurv_fit_cox$IndividualDataPP$training.data$AP_o) - DP_o +
        1
    )) /
      sum(I_cum_lag * (
        AP_o <= max(resurv_fit_cox$IndividualDataPP$training.data$AP_o) - DP_o +
          1
      )),
    I = sum(I * (
      AP_o <= max(resurv_fit_cox$IndividualDataPP$training.data$AP_o) - DP_o
    ))) %>%
    mutate(DP_o_join = DP_o - 1) %>%
    as.data.frame()



> clmodel %>%
    filter(DP_o > 1) %>%
    ggplot(aes(x = DP_o, y = df_o)) +
    geom_line(linewidth = 2.5,
              color = "#454555") +
    labs(title = "Chain ladder",
         x = "Development quarter",
         y = "Development factor") +
    ylim(1, 3. + .01) +
    theme_bw(base_size = rel(5)) +
    theme(plot.title = element_text(size = 20))

> clmodel_months <- individual_data_m$training.data %>%
    mutate(DP_o = 48 -
             DP_rev_o + 1) %>%
    group_by(AP_o, DP_o) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(AP_o) %>%
```

```
    arrange(DP_o) %>%
    mutate(I_cum = cumsum(I), I_cum_lag = lag(I_cum, default = 0)) %>%
    ungroup() %>%
    group_by(DP_o) %>%
    reframe(df_o = sum(I_cum * (
      AP_o <= max(individual_data_m$training.data$AP_o) - DP_o + 1
    )) /
      sum(I_cum_lag * (
        AP_o <= max(individual_data_m$training.data$AP_o) - DP_o + 1
      )),
    I = sum(I * (
      AP_o <= max(individual_data_m$training.data$AP_o) - DP_o
    ))) %>%
    mutate(DP_o_join = DP_o - 1) %>%
    as.data.frame()

> ticks_at <- seq(1, 48, 4)
> labels_as <- as.character(ticks_at)

> clmodel_months %>%
    filter(DP_o > 1) %>%
    ggplot(aes(x = DP_o,
               y = df_o)) +
    geom_line(linewidth = 2.5,
              color = "#454555") +
    labs(title = "Chain ladder",
         x = "Development month",
         y = "Development factor") +
    ylim(1, 2.5 + .01) +
    scale_x_continuous(breaks = ticks_at, labels = labels_as) +
    theme_bw(base_size = rel(5)) +
    theme(plot.title = element_text(size = 20))
```

```
> clmodel_years <- individual_data_y$training.data %>%
    mutate(DP_o = 4 -
             DP_rev_o + 1) %>%
    group_by(AP_o, DP_o) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(AP_o) %>%
    arrange(DP_o) %>%
    mutate(I_cum = cumsum(I), I_cum_lag = lag(I_cum, default = 0)) %>%
    ungroup() %>%
    group_by(DP_o) %>%
    reframe(df_o = sum(I_cum * (
      AP_o <= max(individual_data_m$training.data$AP_o) - DP_o + 1
    )) /
      sum(I_cum_lag * (
        AP_o <= max(individual_data_m$training.data$AP_o) - DP_o + 1
      )),
    I = sum(I * (
      AP_o <= max(individual_data_m$training.data$AP_o) - DP_o
    ))) %>%
    mutate(DP_o_join = DP_o - 1) %>%
    as.data.frame()

> ticks_at <- seq(1, 4, 1)
> labels_as <- as.character(ticks_at)

> clmodel_years %>%
    filter(DP_o > 1) %>%
    ggplot(aes(x = DP_o,
               y = df_o)) +
    geom_line(linewidth = 2.5,
              color = "#454555") +
```

```
    labs(title = "Chain ladder",
         x = "Development year",
         y = "Development factor") +
    ylim(1, 2.5 + .01) +
    scale_x_continuous(breaks = ticks_at, labels = labels_as) +
    theme_bw(base_size = rel(5)) +
    theme(plot.title = element_text(size = 20))
```

Quarterly Output

```
> ct <- 1
> ap <- 15

> resurv_fit_predict_q$long_triangle_format_out$output_granularity %>%
    filter(AP_o == ap & claim_type == ct) %>%
    filter(row_number() == 1) %>%
    select(group_o)

> plot(
    resurv_fit_predict_q,
    granularity = "output",
    title_par = "COX: Accident Quarter 15 Claim Type 1",
    group_code = 30)
```

Monthly Output

```
> ct <- 0
> ap <- 36

> resurv_fit_predict_m$long_triangle_format_out$output_granularity %>%
    filter(AP_o == ap & claim_type == ct) %>%
```

```
    filter(row_number() == 1) %>%
    select(group_o)

> plot(resurv_fit_predict_m,
       granularity = "output",
       color_par = "#a71429",
       title_par = "COX: Accident Month 36 Claim Type 0",
       group_code = 71)



> ct <- 1
> ap <- 7

> resurv_fit_predict_m$long_triangle_format_out$output_granularity %>%
    filter(AP_o == ap & claim_type == ct) %>%
    filter(row_number() == 1) %>%
    select(group_o)

> plot(
    resurv_fit_predict_m,
    granularity = "output",
    color_par = "#a71429",
    title_par = "COX: Accident Month 7 Claim Type 1",
    group_code = 14
  )
```

Yearly Output

```
> ct <- 0
> ap <- 2

> resurv_fit_predict_y$long_triangle_format_out$output_granularity %>%
```

```
    filter(AP_o == ap & claim_type == ct) %>%
    filter(row_number() == 1) %>%
    select(group_o)

> plot(
    resurv_fit_predict_y,
    granularity = "output",
    color_par = "#FF6A7A",
    title_par = "COX: Accident Year 2 Claim Type 0",
    group_code = 3
  )



> ct <- 1
> ap <- 3

> resurv_fit_predict_y$long_triangle_format_out$output_granularity %>%
    filter(AP_o == ap & claim_type == ct) %>%
    filter(row_number() == 1) %>%
    select(group_o)

> plot(
    resurv_fit_predict_y,
    granularity = "output",
    color_par = "#FF6A7A",
    title_par = "COX: Accident Year 3 Claim Type 1",
    group_code = 6
  )
```

C Code for computing $\texttt{ARE}^{\texttt{TOT}}$ and $\texttt{ARE}^{\texttt{CAL}}$

The computations in this Appendix rely on the `dplyr` and `tidyr` packages from the `tidyverse` (Wickham et al., 2019).

```
> library(dplyr)
> library(tidyr)
```

The predictions for XGB and NN are obtained with the following code.

```
> resurv_predict_xgb <- predict(resurv_fit_xgb)
> resurv_predict_nn <- predict(resurv_fit_nn)
```

Computation of $\texttt{ARE}^{\texttt{TOT}}$ and $\texttt{ARE}^{\texttt{CAL}}$

```
> conversion_factor <- individual_data$conversion_factor


> max_dp_i <- 1440


> true_output <- resurv_fit_cox$IndividualDataPP$full.data %>%
    mutate(
      DP_rev_o = floor(max_dp_i * conversion_factor) -
        ceiling(
                DP_i * conversion_factor +
                  ((AP_i - 1) %% (
                    1 / conversion_factor
                  )) * conversion_factor) + 1,
      AP_o = ceiling(AP_i * conversion_factor),
      TR_o = AP_o - 1
    ) %>%
    filter(DP_rev_o <= TR_o) %>%
    group_by(claim_type, AP_o, DP_rev_o) %>%
    mutate(claim_type = as.character(claim_type)) %>%
    summarize(I = sum(I), .groups = "drop") %>%
```

```
    filter(DP_rev_o > 0)

> out_list <- resurv_fit_predict_q$long_triangle_format_out
> out <- out_list$output_granularity

otal output
> score_total <- out[, c("claim_type", "AP_o", "DP_o", "expected_counts")] %>%
    mutate(DP_rev_o = 16 - DP_o + 1) %>%
    inner_join(true_output, by = c("claim_type", "AP_o", "DP_rev_o")) %>%
    mutate(ave = I - expected_counts, abs_ave = abs(ave)) %>%
    # from here it is reformulated for the are tot
    ungroup() %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave = abs(sum(ave)), I = sum(I))

> are_tot <- sum(score_total$abs_ave) / sum(score_total$I)



> dfs_output <- out %>%
    mutate(DP_rev_o = 16 - DP_o + 1) %>%
    select(AP_o, claim_type, DP_rev_o, f_o) %>%
    mutate(DP_rev_o = DP_rev_o) %>%
    distinct()

Cashflow on output scale.Etc quarterly cashflow development
> score_diagonal <- resurv_fit_cox$IndividualDataPP$full.data  %>%
    mutate(
      DP_rev_o = floor(max_dp_i * conversion_factor) -
        ceiling(
                DP_i * conversion_factor +
                  ((AP_i - 1) %% (
                    1 / conversion_factor
```

```
                )) * conversion_factor) + 1,
      AP_o = ceiling(AP_i * conversion_factor)
    ) %>%
    group_by(claim_type, AP_o, DP_rev_o) %>%
    mutate(claim_type = as.character(claim_type)) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(claim_type, AP_o) %>%
    arrange(desc(DP_rev_o)) %>%
    mutate(I_cum = cumsum(I)) %>%
    mutate(I_cum_lag = lag(I_cum, default = 0)) %>%
    left_join(dfs_output, by = c("AP_o", "claim_type", "DP_rev_o")) %>%
    mutate(I_cum_hat =  I_cum_lag * f_o,
           RP_o = max(DP_rev_o) - DP_rev_o + AP_o) %>%
    inner_join(true_output[, c("AP_o", "DP_rev_o")] %>%  distinct()
               , by = c("AP_o", "DP_rev_o")) %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave2_diag = abs(sum(I_cum_hat) - sum(I_cum)), I = sum(I))

> are_cal_q <- sum(score_diagonal$abs_ave2_diag) / sum(score_diagonal$I)


scoring XGB ----

> out_xgb <- resurv_predict_xgb$long_triangle_format_out$output_granularity

> score_total_xgb <- out_xgb[, c("claim_type",
                                 "AP_o",
                                 "DP_o",
                                 "expected_counts")] %>%
    mutate(DP_rev_o = 16 - DP_o + 1) %>%
    inner_join(true_output, by = c("claim_type", "AP_o", "DP_rev_o")) %>%
    mutate(ave = I - expected_counts, abs_ave = abs(ave)) %>%
```

```
    # from here it is reformulated for the are tot
    ungroup() %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave = abs(sum(ave)), I = sum(I))

> are_tot_xgb <- sum(score_total_xgb$abs_ave) / sum(score_total$I)



> dfs_output_xgb <- out_xgb %>%
    mutate(DP_rev_o = 16 - DP_o + 1) %>%
    select(AP_o, claim_type, DP_rev_o, f_o) %>%
    mutate(DP_rev_o = DP_rev_o) %>%
    distinct()

ashflow on output scale.Etc quarterly cashflow development
> score_diagonal_xgb <- resurv_fit_cox$IndividualDataPP$full.data  %>%
    mutate(
      DP_rev_o = floor(max_dp_i * conversion_factor) -
        ceiling(
                DP_i * conversion_factor +
                  ((AP_i - 1) %% (
                    1 / conversion_factor
                  )) * conversion_factor) + 1,
      AP_o = ceiling(AP_i * conversion_factor)
    ) %>%
    group_by(claim_type, AP_o, DP_rev_o) %>%
    mutate(claim_type = as.character(claim_type)) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(claim_type, AP_o) %>%
    arrange(desc(DP_rev_o)) %>%
    mutate(I_cum = cumsum(I)) %>%
    mutate(I_cum_lag = lag(I_cum, default = 0)) %>%
```

```
    left_join(dfs_output_xgb, by = c("AP_o", "claim_type", "DP_rev_o")) %>%
    mutate(I_cum_hat =  I_cum_lag * f_o,
           RP_o = max(DP_rev_o) - DP_rev_o + AP_o) %>%
    inner_join(true_output[, c("AP_o", "DP_rev_o")] %>%  distinct()
               , by = c("AP_o", "DP_rev_o")) %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave2_diag = abs(sum(I_cum_hat) - sum(I_cum)), I = sum(I))

> are_cal_q_xgb <- sum(score_diagonal_xgb$abs_ave2_diag) / sum(score_diagonal$I)

scoring NN ----

> out_nn <- resurv_predict_nn$long_triangle_format_out$output_granularity

> score_total_nn <- out_nn[, c("claim_type",
                               "AP_o",
                               "DP_o",
                               "expected_counts")] %>%
    mutate(DP_rev_o = 16 - DP_o + 1) %>%
    inner_join(true_output, by = c("claim_type", "AP_o", "DP_rev_o")) %>%
    mutate(ave = I - expected_counts, abs_ave = abs(ave)) %>%
    # from here it is reformulated for the are tot
    ungroup() %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave = abs(sum(ave)), I = sum(I))

> are_tot_nn <- sum(score_total_nn$abs_ave) / sum(score_total$I)


> dfs_output_nn <- out_nn %>%
    mutate(DP_rev_o = 16 - DP_o + 1) %>%
    select(AP_o, claim_type, DP_rev_o, f_o) %>%
```

```
    mutate(DP_rev_o = DP_rev_o) %>%
    distinct()

ashflow on output scale.Etc quarterly cashflow development
> score_diagonal_nn <- resurv_fit_cox$IndividualDataPP$full.data  %>%
    mutate(
      DP_rev_o = floor(max_dp_i * conversion_factor) -
        ceiling(
                DP_i * conversion_factor +
                  ((AP_i - 1) %% (
                    1 / conversion_factor
                  )) * conversion_factor) + 1,
      AP_o = ceiling(AP_i * conversion_factor)
    ) %>%
    group_by(claim_type, AP_o, DP_rev_o) %>%
    mutate(claim_type = as.character(claim_type)) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(claim_type, AP_o) %>%
    arrange(desc(DP_rev_o)) %>%
    mutate(I_cum = cumsum(I)) %>%
    mutate(I_cum_lag = lag(I_cum, default = 0)) %>%
    left_join(dfs_output_nn, by = c("AP_o", "claim_type", "DP_rev_o")) %>%
    mutate(I_cum_hat =  I_cum_lag * f_o,
           RP_o = max(DP_rev_o) - DP_rev_o + AP_o) %>%
    inner_join(true_output[, c("AP_o", "DP_rev_o")] %>%  distinct()
               , by = c("AP_o", "DP_rev_o")) %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave2_diag = abs(sum(I_cum_hat) - sum(I_cum)), I = sum(I))

> are_cal_q_nn <- sum(score_diagonal_nn$abs_ave2_diag) / sum(score_diagonal$I)

Scoring Chain-Ladder ----
```

```
> true_output_cl <- individual_data$full.data %>%
    mutate(
      DP_rev_o = floor(max_dp_i * conversion_factor) -
        ceiling(
                DP_i * conversion_factor +
                  ((AP_i - 1) %% (
                    1 / conversion_factor
                  )) * conversion_factor) + 1,
      AP_o = ceiling(AP_i * conversion_factor)
    ) %>%
    filter(DP_rev_o <= TR_o) %>%
    mutate(DP_o = max(individual_data$training.data$DP_rev_o) - DP_rev_o + 1) %>%
    group_by(AP_o, DP_o, DP_rev_o) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    filter(DP_rev_o > 0)
  latest_observed <- individual_data$training.data %>%
    filter(DP_rev_o >= TR_o) %>%
    mutate(DP_o = max(individual_data$training.data$DP_rev_o) - DP_rev_o + 1) %>%
    group_by(AP_o) %>%
    mutate(DP_max = max(DP_o)) %>%
    group_by(AP_o, DP_max) %>%
    summarize(I = sum(I), .groups = "drop")

> clmodel <- individual_data$training.data %>%
    mutate(DP_o = max(individual_data$training.data$DP_rev_o) - DP_rev_o + 1) %>%
    group_by(AP_o, DP_o) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(AP_o) %>%
    arrange(DP_o) %>%
    mutate(I_cum = cumsum(I), I_cum_lag = lag(I_cum, default = 0)) %>%
    ungroup() %>%
```

```
    group_by(DP_o) %>%
    reframe(df = sum(I_cum * (
      AP_o <= max(individual_data$training.data$AP_o) - DP_o + 1
    )) /
      sum(I_cum_lag * (
        AP_o <= max(individual_data$training.data$AP_o) - DP_o + 1
      )), I = sum(I * (
      AP_o <= max(individual_data$training.data$AP_o) - DP_o
    ))) %>%
    mutate(DP_o_join = DP_o) %>%
    mutate(DP_rev_o = max(DP_o) - DP_o + 1)

> predictions <- expand.grid(AP_o = latest_observed$AP_o,
                             DP_o = clmodel$DP_o_join) %>%
    left_join(clmodel[, c("DP_o_join", "df")], by = c("DP_o" = "DP_o_join")) %>%
    left_join(latest_observed, by = "AP_o") %>%
    rowwise() %>%
    filter(DP_o > DP_max) %>%
    ungroup() %>%
    group_by(AP_o) %>%
    arrange(DP_o) %>%
    mutate(df_cum = cumprod(df)) %>%
    mutate(I_expected = I * df_cum - I * lag(df_cum, default = 1)) %>%
    select(DP_o, AP_o, I_expected)

> conversion_factor <- individual_data$conversion_factor
> max_dp_i <- 1440
> score_total <- predictions  %>%
    inner_join(true_output_cl, by = c("AP_o", "DP_o")) %>%
    mutate(ave = I - I_expected, abs_ave = abs(ave)) %>%
    # from here it is reformulated for the are tot
    ungroup() %>%
```

```
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave = abs(sum(ave)), I = sum(I))

> are_tot <- sum(score_total$abs_ave) / sum(score_total$I)

> score_diagonal <- individual_data$full.data  %>%
    mutate(
      DP_rev_o = floor(max_dp_i * conversion_factor) -
        ceiling(
                DP_i * conversion_factor +
                  ((AP_i - 1) %% (
                    1 / conversion_factor
                  )) * conversion_factor) + 1,
      AP_o = ceiling(AP_i * conversion_factor)
    ) %>%
    group_by(claim_type, AP_o, DP_rev_o) %>%
    mutate(claim_type = as.character(claim_type)) %>%
    summarize(I = sum(I), .groups = "drop") %>%
    group_by(claim_type, AP_o) %>%
    arrange(desc(DP_rev_o)) %>%
    mutate(I_cum = cumsum(I)) %>%
    mutate(I_cum_lag = lag(I_cum, default = 0)) %>%
    mutate(DP_o = max(individual_data$training.data$DP_rev_o) - DP_rev_o + 1) %>%
    left_join(CL[, c("DP_o", "df")], by = c("DP_o")) %>%
    mutate(I_cum_hat =  I_cum_lag * df,
           RP_o = max(DP_rev_o) - DP_rev_o + AP_o) %>%
    inner_join(true_output_cl[, c("AP_o", "DP_rev_o")] %>%  distinct()
               , by = c("AP_o", "DP_rev_o")) %>%
    group_by(AP_o, DP_rev_o) %>%
    reframe(abs_ave2_diag = abs(sum(I_cum_hat) - sum(I_cum)), I = sum(I))

> are_cal_q <- sum(score_diagonal$abs_ave2_diag) / sum(score_diagonal$I)
```

Input	Interpretation	Parameter name	Parameter range [default]	Parameter class
PyTorch (Neural Networks)				
Learning Rate	Controls step size during objective optimization.	`lr`	(0, ∞) [Depends on Optimizer]	`float`
Number of layers	Defines the depth of the network.	N/A (defined in model architecture)	[1, ∞) [N/A]	`integer`
Neurons per layer	Determines the width of each layer.	N/A (defined in model architecture)	[1, ∞) [N/A]	`integer`
Activation Function	Introduces non-linearity in the network.	e.g., `nn.ReLU()`	Any valid activation function [N/A]	`character`
Batch Size	Number of samples processed before the model is updated.	`batch_size` in DataLoader	[1, ∞) [1]	`integer`
Optimizer	Algorithm to update model weights based on the loss gradient.	e.g., `optim.Adam()`	Any valid optimizer[N/A]	`character`
Early Stopping	Stops training when performance on validation set stops improving.	Custom implementation	{True, False} [False]	`integer`
Patience	Number of epochs with no improvement after which training will be stopped.	Part of early stopping implementation	[1, ∞) [N/A]	`integer`
XGBoost				
Learning Rate	Controls step size during objective optimization.	`learning_rate` or `eta`	[0, 1] [0.3]	`float`
Max Depth	Maximum depth of trees. Controls model complexity.	`max_depth`	[0, ∞) [6]	`integer`
Subsample	Fraction of samples used for training each tree.	`subsample`	(0, 1] [1]	`float`
Colsample Bytree	Fraction of features used for training each tree.	`colsample_bytree`	(0, 1] [1]	`float`
Booster	The boosting algorithm to use.	`booster`	{'gbtree', 'gblinear', 'dart'} [gbtree]	`character`
Alpha	L1 regularization term on weights.	`alpha`	[0, ∞) [0]	`float`
Lambda	L2 regularization term on weights.	`lambda`	[0, ∞) [1]	`float`
Min Child Weight	Assures no final node is too small	`min_child_weight`	[0, ∞) [1]	`float`

Table 4: Main Inputs for PyTorch (Neural Networks, Block 1) and XGBoost (Block 2). In the table we show for each machine learning algorithm the hyperparameter name (column one), the description (column two), the respective notation in `ReSurv` (column three), and the full range of possible parameter values with default values in square brackets (column four). In column four, N/A indicates that there is no suggested default. The column parameter class indicates the object class of the input argument.

...	num_layers	num_nodes	optim	activation	lr	xi	eps	batch_size	...	Score	...
...	9	8	1	2	0.08	0.35	0.03	1226	...	-6.24	...
...	9	2	2	1	0.47	0.50	0.10	3915	...	-7.27	...
...	9	9	2	1	0.40	0.49	0.18	196	...	-5.98	...
...	8	8	1	2	0.03	0.23	0.01	4508	...	-7.39	...
...	9	7	2	1	0.13	0.13	0.12	900	...	-6.21	...

Table 5: Header of the output of `bayes_out$scoreSummary`. We select the hyperparameters that minimise the negative log-likelihood in the column `Score`.

Performance Metric	Chain-Ladder	**COX**	**NN**	**XGB**
$\texttt{ARE}^{\texttt{TOT}}$	0.115	0.116	**0.113**	0.124
$\texttt{ARE}^{\texttt{CAL}}$	0.111	0.107	**0.105**	0.111
CRPS (average)	-	366.264	**365.229**	367.950

Table 6: Results in terms of $\texttt{ARE}^{\texttt{TOT}}$, $\texttt{ARE}^{\texttt{CAL}}$ and CRPS (rows) for the different models (columns) on the simulated data set (Scenario Alpha).

Performance Metric	Chain-Ladder	**COX**	**NN**	**XGB**
$\texttt{ARE}^{\texttt{TOT}}$	0.290	0.173	0.199	**0.162**
$\texttt{ARE}^{\texttt{CAL}}$	0.229	0.181	0.199	**0.130**
CRPS (average)	-	388.752	389.334	**370.859**

Table 7: Results in terms of $\texttt{ARE}^{\texttt{TOT}}$, $\texttt{ARE}^{\texttt{CAL}}$ and CRPS (rows) for the different models (columns) on the simulated data set (Scenario Delta).