Problem 16. $2^{15} = 32768$ and the sum of its digits is $3 + 2 + 7 + 6 + 8 = 26$. What is the sum of the digits of the number 2^{1000}?

Knowledge required None

Solution Outline This problem can be solved easily if the programming language supports Big Integer library. I still do not know how to solve in languages that does not support such library. Instead of directly using the in-built `pow` function, we implement function `fast_power` which takes in two numbers a, b and calculates a^b in $O(\log b)$ time. After computing power we will implement a function which returns the digit sum of a number.

Python Solution

```python
def fast_power(a, b):
    # calculates a^b in O(log n) time
    res = 1
    while b:
        if b & 1:
            res *= a
        a *= a
        b >>= 1

    return res


def digit_sum(num):
    res = 0
    while num:
        res += num % 10
        num //= 10
    return res


N = 1000
pow2 = fast_power(2, N)
dig_sum = digit_sum(pow2)

print(dig_sum)
```