

Problem 6. The sum of the squares of the first ten natural numbers is,

$$1^2 + 2^2 + \dots + 10^2 = 385$$

The square of the sum of the first ten natural numbers is,

$$(1 + 2 + \dots + 10)^2 = 55^2 = 3025$$

Hence the difference between the sum of the squares of the first ten natural numbers and the square of the sum is $3025 - 385 = 2640$. Find the difference between the sum of the squares of the first one hundred natural numbers and the square of the sum.

Knowledge Required: Closed form equation for sum of n natural numbers and sum of squares of n natural numbers.

Solution Outline: This problem is asking the difference between sum of the squares of first one hundred natural numbers and the square of the sum.

We will have two functions `sum_n` which returns the sum of n natural numbers, and `sum_square_n` which returns the sum of squares of n natural numbers. The internal implementations of each function can have a *for* loop, this implementation has time complexity $O(n)$. We can improve this by using the closed form equations taught to us in high school resulting in $O(1)$ time complexity.

Python Solution

```
1 def sum_n(n):
2     return (n * (n + 1)) // 2
3
4 def sum_square_n(n):
5     return (n * (n + 1) * (2*n + 1)) // 6
6
7 print(sum_n(100)**2 - sum_square_n(100))
```
