**Problem 15.** Starting in the top left corner of a  $2\times2$  grid, and only being able to move to the right and down, there are exactly 6 routes to the bottom right corner.



How many such routes are there through a  $20 \times 20$  grid?

Knowledge required Dynamic Programming.

**Solution Outline** This is a classic and popular Dynamic Programming problem. Here are the transition state equations:-

$$ways[i][j] = \begin{cases} 1 & \text{if i = 0 or j = 0} \\ ways[i-1][j] + ways[i][j-1] & \text{otherwise} \end{cases}$$

Here ways[i][j], means the number ways to reach a grid of size  $i \times j$ .

## **Python Solution**

```
N = 21
ways = [[0] * N for i in range(N)]

for i in range(N):
    for j in range(N):
        if i == 0 or j == 0: ways[i][j] = 1
        else:
        ways[i][j] = ways[i - 1][j] + ways[i][j - 1]

# we need to give the result of ways[20][20]
print(ways[20][20])
```