ENGR2340 EXAM 1 2012-09-30

- You are taking this exam under the honor code.
- You must do your own work. You may not discuss the exam with anyone except Aaron or Chris. There is one exception-if you have trouble using MATLAB, you may also ask Ben for help.
- There is no time limit. However, please estimate how long you spend on it.
- This exam is due Tues 10/9/12 at 5pm. Turn in what you have completed at that time. - No late exams. Extensions for unusual circumstances will be considered.
- You may use class-related material only: textbooks, reserve books in the library, notes taken in class, all files in the course folder, including assignments solutions. No other materials may be used.
- You may only use your computer for MATLAB simulations and calculations.
- Please write neatly and show the details of your work. We cant give you credit if we cant understand or read what you wrote.
- There are 6 questions for a total of 100 points.

Name: ______________________________

1. (10 points) Answer the following questions. You must explain your answers to receive full credit.

   True or False

   (a) The velocity of a particle, $\mathbf{v}_p$, is always tangent to its path.

   (b) The expression for $\mathbf{v}_p$ depends on the coordinate system chosen to represent the position of the particle.

   (c) The expression for $\mathbf{v}_p$ depends on the origin of the coordinate system.

   (d) The acceleration of the particle always has a nonzero component normal to its path.

   (e) The acceleration vector, $\mathbf{a}_p$, at the point on the path shown in figure 1 can lie in any of the four quadrants.

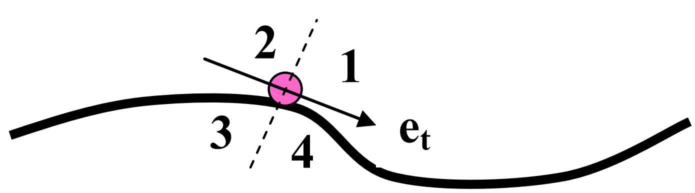

Figure 1: Path coordinates

(f) The acceleration of a particle moving in a plane with constant values of $\dot{r}$ and $\dot{\theta}$ is always zero.

2. (15 points) Examine the photo in figure 2. Does the basketball go through the hoop? Discuss your reasoning and provide quantitative evidence to support your case. Clearly specify the assumptions that you make. You should assume that the ball was released at the angle of the mans arms with respect to the horizontal and that the man is 6 feet tall.

   You will need to take direct measurements on the photograph and use your projectile simulation codes. There will, of course, be uncertainty in your measurements and you should discuss how you handled them. Also, determine some bounding values for your calculations.

CP12_03.jpg

Figure 2: Projectiles

3. (15 points) If you become an engineer working with systems shaped like a torus (e.g., Tokamak-style fusion reactors or, perhaps, bagels) you may use toroidal coordinate systems like the one shown in figure 3. Such a (orthonormal) coordinate system is defined by the unit vectors $\{\mathbf{e}_\rho, \mathbf{e}_\theta, \mathbf{e}_\phi\}$ with respect to a reference circle of (constant) radius, R. The reference circle lies in a horizontal plane (X-Y). Toroidal and Cartesian (origin at the center of the reference circle) coordinate systems are related by:

$$x = (R + \rho\cos\phi)\cos\theta$$
$$y = (R + \rho\cos\phi)\sin\theta$$
$$z = \rho\sin\phi$$

The toroidal basis vectors with respect to the Cartesian system are related by

$$\mathbf{e}_\rho = \cos\phi\cos\theta\,\mathbf{E}_1 + \cos\phi\sin\theta\,\mathbf{E}_2 + \sin\phi\,\mathbf{E}_3 \quad (1)$$
$$\mathbf{e}_\theta = -\sin\theta\,\mathbf{E}_1 + \cos\theta\,\mathbf{E}_2 \quad (2)$$
$$\mathbf{e}_\phi = \sin\phi\cos\theta\,\mathbf{E}_1 - \sin\phi\sin\theta\,\mathbf{E}_2 + \cos\phi\,\mathbf{E}_3 \quad (3)$$

(a) Write the Cartesian basis vectors $\{\mathbf{E}_1, \mathbf{E}_2, \mathbf{E}_3\}$ in terms of $\{\mathbf{e}_\rho, \mathbf{e}_\theta, \mathbf{e}_\phi\}$.

(b) Write the velocity of a particle in terms of (ρ, θ, ϕ) and $\{\mathbf{e}_\rho, \mathbf{e}_\theta, \mathbf{e}_\phi\}$. Start with the position of the particle in 3D space defined by $\mathbf{r} = x\,\mathbf{E}_1 + y\,\mathbf{E}_2 + z\,\mathbf{E}_3$.

(c) What are the steps, in detail, needed to write out the acceleration of P in terms of $\{\mathbf{e}_\rho, \mathbf{e}_\theta, \mathbf{e}_\phi\}$. You do not have to determine the expression for acceleration.

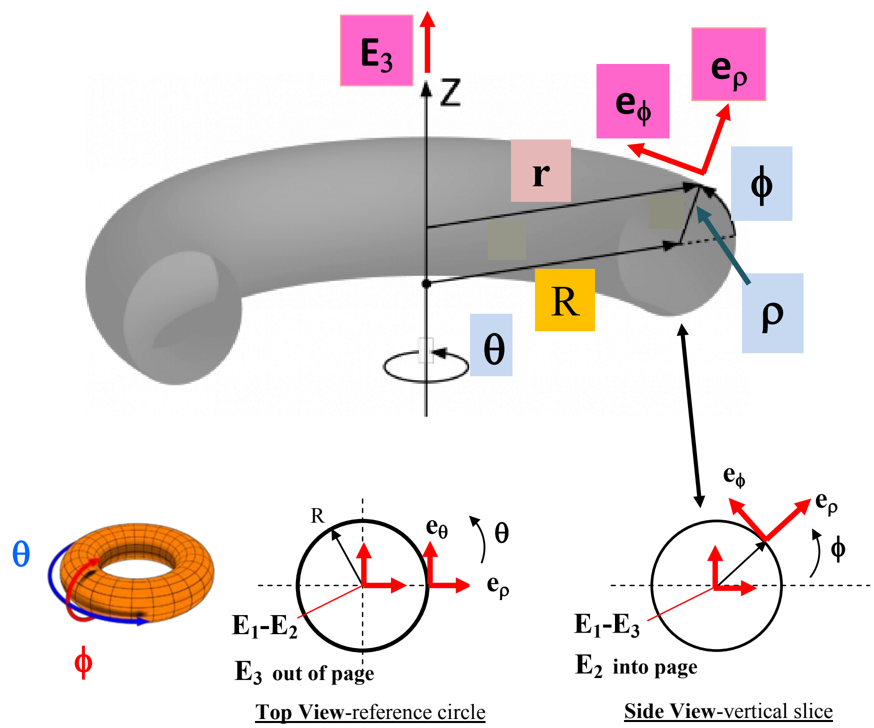

Figure 3: Toroidal Coordinates

4. (15 points) Figure 4 shows a collar (m=0.2 kg) sliding along a circular guide. The coefficient of friction between the collar and the guide is 0.4. A cable connected to the collar is pulled through a hole at a constant rate of 25 m/s, in turn pulling the collar along the guide. The guide is aligned in a vertical plane parallel to the (downward) direction of gravity.

(a) What is the velocity and acceleration of the collar at the instant it passes through point A ("3 o'clock position")?

(b) What is the tension in the cable at this instant?

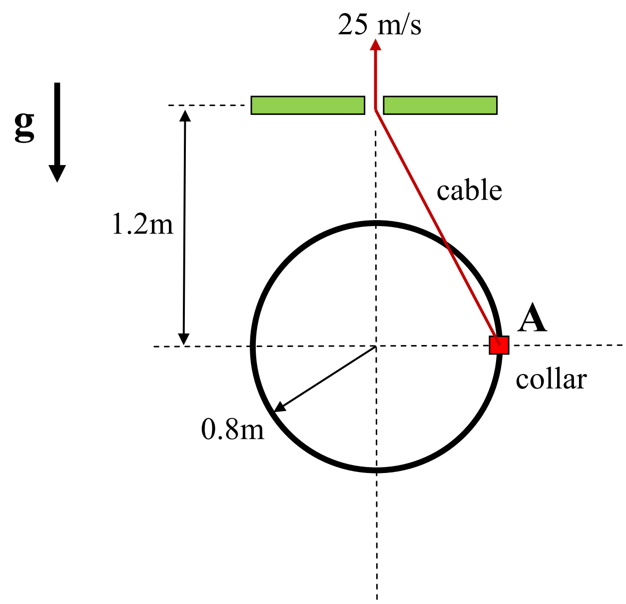

Figure 4: Kinetics of a collar on a guide

5. (35 points) **The Spring-Loaded Inverted Pendulum -** Fig. 5 shows an illustration of a simple planar model used to investigate legged locomotion. The model consists of a point mass, m, atop a massless leg consisting of a spring with linear stiffness, k, and relaxed length, L_0. The mass travels with an initial velocity, $\mathbf{v}_0$, and the leg touches down at an initial angle β. During stance, the friction force is assumed to be high enough that the foot does not slip (ie. the leg rotates about a virtual pin joint at the ground). At the end of the stance phase the leg lifts off the ground and begins the flight phase. Assume that at the end of the flight phase (ie. beginning of the next stance phase), the leg touchdown angle can be automatically reset to β.

Note that ψ is measured from the vertical and (unlike previous examples from class) increases in the clockwise direction. Adjust your coordinates accordingly.

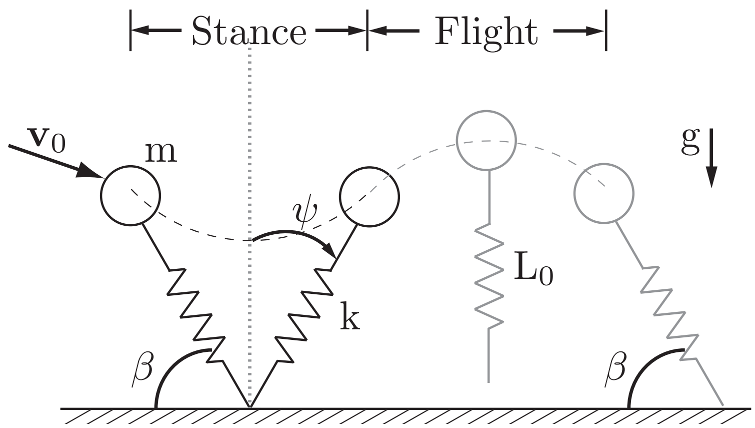

Figure 5: Schematic illustration of the planar Spring-Loaded Inverted Pendulum (SLIP) model of legged running.

(a) Draw a free body diagram of for the mass during the stance phase

(b) Write out the vector expressions for the position, velocity, and acceleration of the mass during the stance phase.

(c) Derive the two scalar equations that result from applying Newton's Second Law during the stance phase.

(d) What are the state variables you would use to numerically integrate the equations of motion during the stance phase?

(e) Draw a free body diagram for the mass during the flight phase.

(f) Write out the vector expressions for the position, velocity, and acceleration of the mass during the flight phase.

(g) Derive the two scalar equations that result from applying Newton's Second Law during the flight phase.

(h) What are the state variables you would use to numerically integrate the equations of motion during the flight phase?

6. (10 points) Copy the `integrateSlip.m` and `run_slip.m` files in the "Exam1" directory of the course folder on Public to your hard drive. Use the code in `integrateSlip.m` to numerically solve the equations of motion for the system. The code in `run_slip.m` is a wrapper for calling `integrateSlip` with the parameters and initial conditions from Table 1 known to produce stable periodic locomotion (ie. the system doesn't fall over). **You will need to fill in gaps in the code with your equations of motion and the appropriate coordinate transformations when the model transitions from stance to flight and back.** Use your results to answer the following questions:

   (a) Show that energy is conserved during both phases of locomotion.

   (b) Plot the trajectory of the mass for a minimum of 15 strides (one stride is defined as one stance period plus one flight period). On a subplot within the same figure, plot and individually label the magnitudes of each of the forces acting on the mass.

   (c) Show that linear momentum is conserved during flight.

   (d) Show that angular momentum about the foot is conserved during stance.

m (kg)	$\mathbf{L}_0$ (m)	**k** (N/m)	**g** (m/s^2)	β (rad)	$\mathbf{q}_0$	foot	$\mathbf{y_0}$
1	1.5	600	9.81	$7\,\pi$ / 16	2	0	[-$L_0 \cos\beta$ ($L_0 \sin\beta$ + 0.1) 5 -0.2]

Table 1: One set of stable parameters and initial conditions for the SLIP model.

(e) Find at least one additional set of parameters and initial conditions that produces stable periodic locomotion, and describe how you verified that the model runs stably.

(f) Find at least one set of parameters and initial conditions for which the model fails to run. Plot the trajectory of center of mass for this failure.