Dynamics Fall 12 Exam 1 Solns

1) a) T e.g. $\underline{v} = \dot{s}\,\underline{e}_t$

b) T e.g. $\underline{v} = \dot{x}\,\underline{E}_1 + \dot{y}\,\underline{E}_2 = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta$

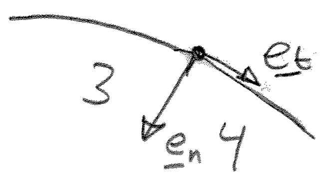
the vector is the same but can be represented in terms of different unit vectors

c) F velocity of a particle does not depend upon origin

(However, in some formulations (e.g. polar) position wrt origin is included but its location does not effect change of position)

d) F $\underline{a} = \ddot{s}\,\underline{e}_t + \frac{\dot{s}^2}{\rho}\,\underline{e}_n$

e) F



$\frac{\dot{s}^2}{\rho} > 0$ $\underline{e}_n$ always pts. concave inward

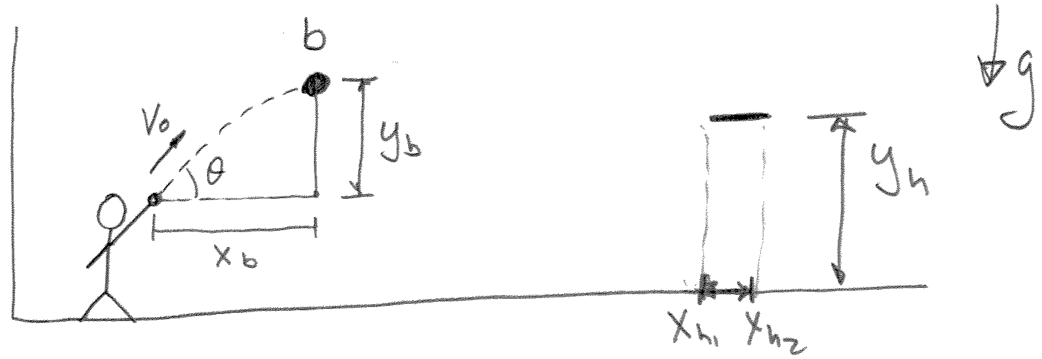
$\therefore$ $\underline{a}$ can only pt. in 3 or 4 quad.

f) F $\underline{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\,\underline{e}_r + (2\dot{r}\dot{\theta} + \cancel{r\ddot{\theta}})\,\underline{e}_\theta$

$\qquad\qquad 0 \qquad\qquad\qquad\qquad 0$

$= -r\dot{\theta}^2\,\underline{e}_r + 2\dot{r}\dot{\theta}\,\underline{e}_\theta$ in general $\neq 0$

2)



Estimate from photo: $x_b, y_b, \theta, x_{h_1}-x_{h_2}, y_h$

- take origin of coord. system at man's hand
- assume planar (x-y) motion
- no drag

Determine $|\underline{V}_0|$ use projectile eqns.

$$\begin{cases} y_b = V_0 \sin\theta\, t_b - \frac{1}{2} g t_b^2 \\ x_b = V_0 \cos\theta\, t_b \end{cases} \Rightarrow \quad V_0 t_b = \frac{x_b}{\cos\theta}$$

$$y_b = \sin\theta\left(\frac{x_b}{\cos\theta}\right) - \frac{1}{2} g t_b^2$$

Solve for t_b = time for ball to reach position in photo

then get $\boxed{V_0}$

Now use projectile eqns

$$y = V_0 \sin\theta\, t - \frac{1}{2} g t^2 \qquad x = V_0 \cos\theta\, t$$

Plot (x,y) for range of t. Check if ball passes through hoop.

- Could account for drag → ball will not travel as far
- Vary measurements on photo to determine sensitivity of (x,y)

3)

$$x = (R + \rho\cos\phi)\cos\theta$$
$$y = (R + \rho\cos\phi)\sin\theta$$
$$z = \rho\sin\phi$$

$$\begin{Bmatrix} \underline{e}_\rho \\ \underline{e}_\theta \\ \underline{e}_\phi \end{Bmatrix} = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta & \sin\phi \\ -\sin\theta & \cos\theta & 0 \\ -\sin\phi\cos\theta & -\sin\phi\sin\theta & \cos\phi \end{bmatrix} \begin{Bmatrix} \underline{E}_1 \\ \underline{E}_2 \\ \underline{E}_3 \end{Bmatrix}$$

a) invert $[R] \Rightarrow [R]^{-1} = [R]^T$ since $\underline{e}_\rho, \underline{e}_\theta, \underline{e}_\phi$ are orthornormal

$$\boxed{\begin{Bmatrix} \underline{E}_1 \\ \underline{E}_2 \\ \underline{E}_3 \end{Bmatrix} = \begin{bmatrix} \cos\phi\cos\theta & -\sin\theta & -\sin\phi\cos\theta \\ \cos\phi\sin\theta & \cos\theta & -\sin\phi\sin\theta \\ \sin\phi & 0 & \cos\phi \end{bmatrix} \begin{Bmatrix} \underline{e}_\rho \\ \underline{e}_\theta \\ \underline{e}_\phi \end{Bmatrix}}$$

b) $\underline{r} = (R + \rho\cos\phi)\cos\theta\, \underline{E}_1 + (R + \rho\cos\phi)\sin\theta\, \underline{E}_2 + \rho\sin\phi\, \underline{E}_3$

$$\left.\frac{d}{dt}\right|_{0_1} \underline{r} = \left[R\cos\theta + \dot{\rho}\cos\phi\cos\theta\right]^{\cdot} \underline{E}_1 + \left[R\sin\theta + \dot{\rho}\cos\phi\sin\theta\right]^{\cdot} \underline{E}_2 + [\rho\sin\phi]^{\cdot}\underline{E}_3$$

$$= \left[-R\dot{\theta}\sin\theta + \dot{\rho}\cos\phi\cos\theta - \rho\dot{\phi}\sin\phi\cos\theta - \rho\cos\phi\sin\theta\dot{\theta}\right]\underline{E}_1$$
$$+ \left[R\dot{\theta}\cos\theta + \dot{\rho}\cos\phi\sin\theta - \rho\dot{\phi}\sin\phi\sin\theta + \rho\cos\phi\dot{\theta}\cos\theta\right)\underline{E}_2$$
$$+ \left[\dot{\rho}\sin\phi + \rho\dot{\phi}\cos\phi\right]\underline{E}_3$$

(*)

$$= \left[-R\dot{\theta}\sin\theta + \dot{\rho}\cos\phi\cos\theta - \rho\dot{\phi}\sin\phi\cos\theta - \rho\dot{\theta}\cos\phi\sin\theta\right]\left(\cos\phi\cos\theta\, \underline{e}_\rho - \sin\theta\, \underline{e}_\theta - \sin\phi\cos\theta\, \underline{e}_\phi\right)$$
$$+ \left[R\dot{\theta}\cos\theta + \dot{\rho}\cos\phi\sin\theta - \rho\dot{\phi}\sin\phi\sin\theta + \rho\dot{\theta}\cos\phi\cos\theta\right]\left(\cos\phi\sin\theta\, \underline{e}_\rho + \cos\theta\, \underline{e}_\theta - \sin\phi\sin\theta\, \underline{e}_\phi\right)$$
$$+ \left[\dot{\rho}\sin\phi + \rho\dot{\phi}\cos\phi\right]\left(\sin\phi\, \underline{e}_\rho + \cos\phi\, \underline{e}_\phi\right)$$

3) Cont.

$$= \Big[\big(-R\dot\theta \cancel{\cos\phi \sin\theta\cos\theta} + \dot\rho\cos^2\phi\cos^2\theta - \rho\dot\phi\cancel{\sin\phi\cos\phi}\cos^2\theta - \rho\dot\theta\cancel{\cos^2\phi\sin\theta\cos\theta}\big)$$

$$+\big(R\dot\theta\cancel{\cos\theta\cos\phi\sin\theta} + \dot\rho\cos^2\phi\sin^2\theta - \rho\dot\phi\cancel{\sin\phi\cos\phi}\sin^2\theta + \rho\dot\theta\cancel{\cos^2\phi\cos\theta\sin\theta}\big)$$

$$+\big(\dot\rho\sin^2\phi + \rho\dot\phi\cancel{\sin\phi\cos\phi}\big)\Big]\,\underline{e}_\rho$$

$$+\Big[-\big(-R\dot\theta\sin^2\theta + \dot\rho\cancel{\cos\phi\cos\theta\sin\theta} - \rho\dot\phi\cancel{\sin\phi\cos\theta\sin\theta} - \rho\dot\theta\cos\phi\sin^2\theta\big)$$

$$+\big(R\dot\theta\cos^2\theta + \dot\rho\cancel{\cos\phi\sin\theta\cos\theta} - \rho\dot\phi\cancel{\sin\phi\sin\theta\cos\theta} + \rho\dot\theta\cos\phi\cos^2\theta\big)\Big]\,\underline{e}_\theta$$

$$+\Big[-\big(-R\dot\theta\cancel{\sin\theta\sin\phi\cos\theta} + \dot\rho\cos\phi\sin\phi\cos^2\theta - \rho\dot\phi\sin^2\phi\cos^2\theta - \rho\dot\theta\cancel{\cos\phi\sin\theta\sin\phi\cos\theta}\big)$$

$$+\big(-R\dot\theta\cancel{\cos\theta\sin\phi\sin\theta} - \dot\rho\cos\phi\sin^2\theta\sin\phi + \rho\dot\phi\sin^2\phi\sin^2\theta - \rho\dot\theta\cancel{\cos\phi\cos\theta\sin\phi\sin\theta}\big)$$

$$+\big(\dot\rho\sin\phi\cos\phi + \rho\dot\phi\cos^2\phi\big)\Big]\,\underline{e}_\phi$$

$$= \Big[\dot\rho\cos^2\phi(\cos^2\theta+\sin^2\theta) + \dot\rho\sin^2\phi\Big]\,\underline{e}_\rho$$

$$+\Big[R\dot\theta(\sin^2+\cos^2\theta) + \rho\dot\theta\cos\phi(\sin^2+\cos^2\theta)\Big]\,\underline{e}_\theta$$

$$+\Big[\cancel{-\dot\rho\cos\phi\sin\phi(\cos^2\theta+\sin^2\theta)} + \rho\dot\phi\sin^2\phi(\cos^2\theta+\sin^2\theta) + \cancel{\dot\rho\sin\phi\cos\phi} + \rho\dot\phi\cos^2\phi\Big]\,\underline{e}_\phi$$

$$= [\dot\rho]\,\underline{e}_\rho + [R\dot\theta + \rho\dot\theta\cos\phi]\,\underline{e}_\theta + [\rho\dot\phi]\,\underline{e}_\phi$$

$$\therefore\ \boxed{\underline{V} = \frac{d}{dt}(\underline{r}) = \dot\rho\,\underline{e}_\rho + \dot\theta(R+\rho\cos\phi)\,\underline{e}_\theta + \rho\dot\phi\,\underline{e}_\phi}$$

units chk ✓

3) Cont.

How to Find $\left.\frac{d^2}{dt^2}\right|_{O_1} \underline{r} = \underline{a}$

Step 1) Determine $\left.\frac{d^2}{dt^2}\right|_{O_1} \underline{r} = \ddot{x}\,\underline{E}_1 + \ddot{y}\,\underline{E}_2 + \ddot{z}\,\underline{E}_3$ from ⊛

Step 2) Substitute $\underline{e}$'s for $\underline{E}$'s (same as $\underline{v}$)

Step 3) Simplify

$\Rightarrow$ Could do by MATLAB symbolic tool box

Outline Procedure to get $\underline{v}$

Given $x, y, z = f(\rho, \theta, \phi)$ and $\underline{e} = [R]\,\underline{E}$ a) $[R]^{-1}\underline{e} = \underline{E}$

$\boxed{[R]^T = [R]^{-1}}$

b) $\underline{r} = x\,\underline{E}_1 + y\,\underline{E}_2 + z\,\underline{E}_3$

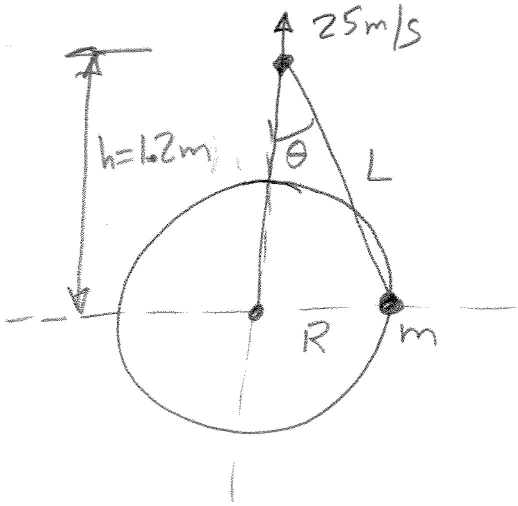
① take $\left.\frac{d}{dt}\right|_{O_1} \underline{r} \Rightarrow \dot{x}\,\underline{E}_1 + \dot{y}\,\underline{E}_2 + \dot{z}\,\underline{E}_3 \rightarrow \dot{\underline{E}}_i = 0$

② evaluate by chain / product rule, $\dot{x}, \dot{y}, \dot{z} = f(\rho, \theta, \phi)$

③ change basis $\underline{E} \rightarrow \underline{e}$ get

$$\left.\frac{d}{dt}\right|_{O_1} \underline{r} = \underline{v} = \{\dot{x} \quad \dot{y} \quad \dot{z}\}\,[R]^T \begin{Bmatrix} \underline{e}_\rho \\ \underline{e}_\theta \\ \underline{e}_\phi \end{Bmatrix}$$

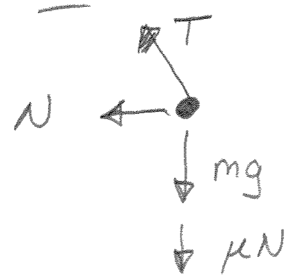
4)

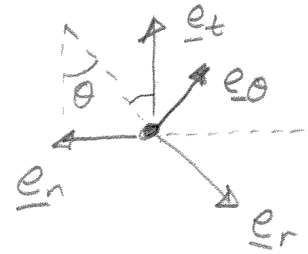


$m = 0.2\,kg,\ \mu = 0.4,\ h = 1.2\,m$

$\dot{L} = 25\,m/s \qquad R = 0.8\,m$

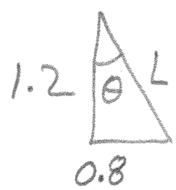
FBD





use 2 coordinate systems

From geometry at given pt.



$L = \sqrt{1.2^2 + 0.8^2} = 1.44\,m$

$\theta = \tan^{-1} \frac{0.8}{1.2} = 33.7°$

Transformation btw coor. sys.

$$\begin{Bmatrix} \underline{e}_t \\ \underline{e}_n \end{Bmatrix} = \begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix} \begin{Bmatrix} \underline{e}_r \\ \underline{e}_\theta \end{Bmatrix}$$

$$\underline{V} = \dot{s}\,\underline{e}_t = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta$$

$$\underline{a} = \ddot{s}\,\underline{e}_t + \frac{\dot{s}^2}{\rho}\,\underline{e}_n = (\ddot{r} - r\dot{\theta}^2)\,\underline{e}_r + (2\dot{r}\dot{\theta} + r\ddot{\theta})\,\underline{e}_\theta$$

Choose $\underline{e}_t$ - $\underline{e}_n$ for force components

$\underline{e}_t$: $\Sigma F_t = T\cos\theta - \mu N - mg \qquad = m\ddot{s}$

$\underline{e}_n$: $\Sigma F_n = T\sin\theta + N \qquad = m\frac{\dot{s}^2}{R}$

$\underline{V} \Rightarrow \quad \dot{s}\,\underline{e}_t = \dot{s}(-\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta) = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta$

$\Rightarrow$ for $\underline{e}_r$-$\underline{e}_\theta$ $\quad \dot{r} = \dot{L} = -25\,m/s$

$\therefore \quad -\dot{s}\cos\theta = -\dot{r} \qquad \boxed{\dot{s}} = \frac{\dot{r}}{\cos\theta} = \frac{25}{\cos(33.7°)} = 30\,\frac{m}{s}$

$\therefore \quad \boxed{\underline{V} = 30\,\frac{m}{s}\,\underline{e}_t}$

4) Cont.

$$\Rightarrow \quad \dot{s}\sin\theta = r\dot{\theta} \quad \therefore \boxed{\dot{\theta}} = \frac{\dot{s}\sin\theta}{r} = \frac{(30)(\sin 33.7)}{1.44}$$

$$= \frac{30 \cdot \frac{0.8}{1.44}}{1.44} = \underline{11.57 \ \frac{rad}{s}}$$

also $\boxed{\underline{V}} = 30\,\underline{e}_t = 30(-\cos\,\underline{e}_r + \sin\theta\,\underline{e}_r) = 30\left(-\frac{1.2}{1.44}\,\underline{e}_r + \frac{0.8}{1.44}\,\underline{e}_\theta\right)$

$$\boxed{= -25\,\underline{e}_r + 16.67\,\underline{e}_\theta}$$

✓ Chk $\quad \underline{V} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta = -25\,\underline{e}_r + (1.44)(+11.57)\,\underline{e}_\theta = -25\,\underline{e}_r + 16.66$

from $\underline{a}$:

$$\ddot{s}\,\underline{e}_t + \frac{\dot{s}^2}{R}\,\underline{e}_n = \ddot{s}(-\cos\theta\,\underline{e}_r + \sin\theta\,\underline{e}_\theta) + \frac{\dot{s}^2}{R}(-\sin\theta\,\underline{e}_r - \cos\theta\,\underline{e}_\theta)$$

$$\Rightarrow \quad -\ddot{s}\cos\theta - \frac{\dot{s}^2}{R}\sin\theta = \ddot{r} - r\dot{\theta}^2$$

$$= \underbrace{\cancel{\ddot{L}}}_{0} - L\dot{\theta}^2 = -L\dot{\theta}^2$$

$$\Rightarrow \quad \underline{\ddot{s}} = \frac{-\frac{\dot{s}^2}{R}\sin\theta + L\dot{\theta}^2}{\cos\theta} = \frac{-\frac{(30)^2}{0.8}\,\frac{0.8}{1.44} + 1.44(11.57)^2}{\frac{1.2}{1.44}}$$

$$= \underline{-518.68 \ m/s^2}$$

$$\therefore \boxed{\underline{a}} = (-518.68)\,\underline{e}_t + \frac{(30)^2}{0.8}\,\underline{e}_n \ \boxed{= -518.68\,\underline{e}_t + 1125\,\underline{e}_n \ \frac{m}{s^2}}$$

Find T : eliminate N $\quad N = m\frac{\dot{s}^2}{R} - T\sin\theta$

$$T\cos\theta - \mu\left(m\frac{\dot{s}^2}{R} - T\sin\theta\right) - mg = m\ddot{s}$$

$$\Rightarrow \quad T\left(\cos\theta + \underline{\underline{\mu}}\sin\theta\right) = m\ddot{s} + mg + m\underline{\underline{\mu}}\frac{\dot{s}^2}{R}$$

$$T\left(\frac{1.2}{1.44} + 0.4\cdot\frac{0.8}{1.44}\right) = 0.2\left[(-518.68) + (9.81) + 0.4\frac{(30)^2}{0.8}\right]$$

$$T(1.05) = -11.8$$

$T < 0$!! Friction needs to increase to maintain tension

$\mu \to 0.5$