A holographic look at topologically disconnected black hole remnants

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Topologically disconnected black hole remnants, i.e. disconnected regions of spacetime formed during black hole evaporation, are considered in a holographic framework. In this setting, these remnants consist of disconnected $d-1$ dimensional boundaries connected through a d dimensional bulk spacetime and are isometric to the BTZ black hole, a specific type of AdS wormhole. We use the Ryu-Takayanagi formula in order to establish a relationship between the entanglement entropy of the dual CFTs on each boundary and the connectivity of the bulk. We conjecture that this relationship holds for all types of entanglement entropy, not just the entanglement entropy of the dual CFT. We conclude that remnants in this setting are not a viable resolution to the information paradox.

I. INTRODUCTION

The discovery due to Hawking [1] that black holes emit thermal radiation has raised perplexing questions about the ultimate fate of matter that falls into a black hole. Information that crosses the horizon appears to be lost to external observers, violating some basic notions of quantum mechanics. This conflict between general relativity and quantum mechanics is known as the *black hole information paradox* and remains an open problem in the study of black holes. One of many proposed resolutions is that a topology change occurs during the evaporation process, and the "lost" information is in fact stored in a topologically disconnected region of spacetime [2].

The topology change occurs deep within the black hole, and is shielded from probing by external observers by the black hole horizon. This fact makes the disconnected remnant proposal somewhat intractable, as the implications and validity of the topology change are hard to discern when the topology change itself is shielded from observation. Therefore, it would be desirable to formulate an analogous scenario in which statements *can* be made about the effect of topology change on the resulting spacetime.

One possible route of inquiry arises via the holographic principle, and in particular the Anti-deSitter/Conformal Field Theory (AdS/CFT) correspondence. This correspondence states that a CFT defined on a fixed d-dimensional spacetime non-perturbatively defines a gravitational theory in one higher dimension higher. By studying the dual CFT to a gravitational theory, we can begin to study regions in the spacetime that are otherwise inaccessible. In the present work, we construct an AdS spacetime that shares important features with the spacetimes featured in the disconnected remnant proposal. Using results from holography, we make claims about the effects that a topology change would have on the entanglement structure between the two disconnected regions. In particular, we conjecture that topology change destroys the entanglement between regions, resulting in a scenario similar to black hole firewalls [3]. Therefore, disconnected remnants do not seem to be a viable solution to the information paradox.

The paper is organized as follows. The remainder of Section I goes into further detail regarding the black hole information paradox. Section II discusses the details of the topological remnants scenario, with particular focus placed on the qualitative features of the spacetime that are important for the information paradox. Section III proposes an analog to the remnants scenario in the language of the AdS/CFT correspondence. Section IV discusses what new information might be gained from such a formulation. Finally, Section V offers concluding remarks and routes for future work.

A. The paradox at a glance

The information paradox arises out of well understood physics. In order to see this, it is first necessary to review some basic notions regarding quantum information. Consider an arbitrary partition of a quantum system into a subsystems A and B. The Hilbert space of the initial system can be decomposed

$$\mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$

where $\mathcal{H}_{A,B}$ are the Hilbert spaces of the respective subsystems. Given the total state $|\Phi\rangle \in \mathcal{H}$, we are interested in the state of the subsystem A. In some cases, this state can simply be written as $|\phi\rangle_A \in \mathcal{H}_A$. This means that the state of the subsystem is completely specified by an element of the Hilbert space corresponding to that subsystem, and thus the state of the total, original system is written as $|\Phi\rangle = |\phi\rangle_A \otimes |\phi\rangle_B$. However, this is not the case in general. The general form for the state $|\Phi\rangle$ is

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$$|\Psi\rangle = \sum_{i,j} a_{ij} |\alpha_i\rangle \otimes |\beta_j\rangle \tag{1}$$

which cannot always be factored into a product of states $|\phi\rangle_A \otimes |\phi\rangle_B$. Those states $|\Phi\rangle$ which cannot be factored into a product state $|\phi\rangle_A \otimes |\phi\rangle_B$ are called *entangled states.* All of the information pertaining to a system is contained in the density matrix, defined as

$$\rho = |\Phi\rangle \langle\Phi| \tag{2}$$

For example, the expectation value $\langle O\rangle$ of an observable O is given by $\langle O\rangle = \mathrm{Tr}(O\rho)$. If the density matrix of a state has the form (2) the state is called pure, and states that are not pure are called mixed. The density matrix can also be defined as

$$\rho = \sum_i p_i |\psi_i\rangle \langle\psi_i| \tag{3}$$

for an ensemble of states $|\psi_i\rangle$, each with classical probability p_i. Pure states correspond to those states where $p_i = 0$ for all but one i. These two definitions are equivalent.

Consider an observer Alice who can only perform measurements on the subsystem A. If Alice wants to make predictions on the subsystem A, she traces over the B degrees of freedom in $|\Phi\rangle$ to get a reduced density matrix describing the subsystem A:

$$\rho_A = \mathrm{Tr}_B(|\Phi\rangle \langle\Phi|) \tag{4}$$

where Tr_B represents the trace over the basis vectors of B. Essentially, the degrees of freedom of the B portion of the system are averaged out in order to make measurements on A. A useful measure of the degree to which a given state is entangled is the entanglement entropy of the subsystem A

$$S_A = -\mathrm{Tr}(\rho_A \log \rho_A) \tag{5}$$

$S_A = 0$ only for non-entangled states, while for entangled states we have that $S_A > 0$. Finally, if $B = \bar{A}$, we have that $S_A = S_{\bar{A}}$.

The black hole information paradox arises when we consider entanglement in spacetimes where a black hole forms and subsequently evaporates. In such spacetimes, some initial configuration of matter gravitationally collapses to form a black hole. These black holes emit Hawking radiation [1], which escapes to infinity resulting in a positive energy flux away from the black hole. This radiation is thermal, meaning that it carries no informational content, and is entangled with the matter that collapsed to form the black hole. After long time scales (i.e. exponential in the mass of the black hole), the black hole evaporates completely. The Penrose diagram for such a spacetime is presented in Figure 1.

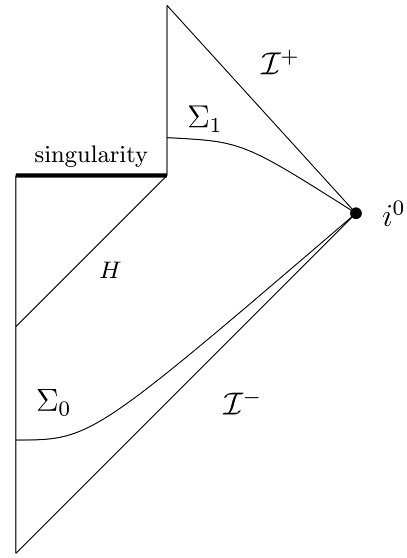

FIG. 1. The Penrose diagram of an evaporating black hole spacetime. The important feature of this spacetime is that the state of the system on the Cauchy surface Σ_1 would be mixed, despite the fact that the initial state on Σ_0 was pure. This implies evolution from a pure to mixed state, which can never be unitary. This pathology arises from the fact that there is no smooth foliation of the spacetime between Σ_0 and Σ_1. It is emphasized that the singularity acts as an additional future boundary, in that at late times matter is either on the singularity or on $\mathcal{I}^+$, and the information on one is inaccessible to the other.

If the initial state that collapsed to form the black hole was pure, this evaporation process raises challenging questions. Consider an observer Alice at future infinity $\mathcal{I}^+$ for a non-evaporating black hole. In addition to $\mathcal{I}^+$, the singularity acts as an second future boundary. This means that Alice cannot reconstruct early time states given only the information available to her on $\mathcal{I}^+$: some of the information from these early states is trapped at the singularity. When she performs measurements on A, she traces over the entangled subsystem B that corresponds to the matter inside the black hole at the additional boundary (see Figure 1). Her resulting reduced density matrix ρ_A will correspond to a mixed state, while the density matrix of the total system ρ_{AB} remains pure.

Turning on evaporation, we find that the black hole completely evaporates leaving only the $\mathcal{I}^+$ boundary. The total state of the system at late times is thus is characterized by ρ_A, which was established to be mixed. The information trapped at the singularity is lost when the black hole evaporates since the Hawking radiation reaching $\mathcal{I}^+$ carries with it no information about the matter that fell into the black hole. This process is an example of pure to mixed state evolution, which is forbidden in quantum mechanics so long as the time evolution is unitary. In general, unitary evolution requires the spacetime to be smoothly foliated (in a technical sense given in [4]) by Cauchy surfaces, which are the closest surface to "all of space at an instant of time" in a relativistic setting. If no such smooth foliation exists, unitary evolution is not possible. Black hole evaporation thus prevents such a smooth foliation, as highlighted in Figure 1.

This breakdown of unitary evolution is the black hole information paradox. Relatively straightforward quantum mechanical calculations in curved spacetimes imply a dramatic breakdown of quantum reversibility. It has been noted [5] that these calculations occur in regions where we should not expect modifications to physics, i.e. regions arbitrarily low curvature. In the decades since this paradox was brought to light, multiple resolutions have been postulated. We briefly review the main ones here. A more comprehensive review of these proposals, as well as of the paradox in general, can be found in [5].

The first is the *fuzzball* scenario [6], which proposes that quantum mechanical effects actually prohibit black hole formation in the first place. This solves the paradox by replacing black holes with structures that lack the troublesome features such as horizons that give rise to the paradox. However, there are serious issues to be resolved with proposals of this type. Perhaps the most pressing is the fact that black holes can form in regimes that are well understood semiclassically, such as regions of low curvature. The fuzzball scenario would have to modify physics in regimes like these these where we shouldn't expect modifications to be needed [5].

More recently, the AMPS proposal [3] suggests that the final state of black hole evaporation is not the highly entangled state that we expect from semiclassical calculations. Entanglement breaks down at the horizon during evaporation, forming a sort of "firewall" that separates the interior and exterior of the black hole. This proposal, too, runs into the difficulties. As noted in [5], the modifications to physics required for this type of proposal would be non-local in nature in order to account for the fact that the firewall coincides with the horizon.

Of particular interest for this paper is the *remnants* proposal [7, 8]. In this picture, the end state of the evaporation is a small remnant object that appears as a point-like particle to outside observers. However, these remnants can have potentially very large interior geometries so that they can store the arbitrarily large amounts of information required to resolve the paradox. Certain versions of this proposal suggest that remnants take the form of topologically disconnected regions of spacetime, often called "child universes". By allowing for disconnected topologies some serious issues with the remnant scenario are avoided. We discuss this in further detail in Section II.

II. TOPOLOGICAL REMNANTS

The black hole remnants proposal [8] modifies the end state of the evaporation process. Evaporation turns off at late times and a long lived planck-sized remnant is the final state of the black hole. This solves the paradox by postulating that the second boundary does not actually disappear. Instead of the information being lost, it instead trapped within the remnant. If we are careful to consider both the subsystem A of the Hawking radiation on $\mathcal{I}^+$ as well as the subsystem B contained within the remnant, the entire state remains pure. Thus unitary evolution is preserved.

As noted in [5], the remnants proposal has some serious flaws. There are arbitrarily many initial configurations of matter that could have collapsed to form the black hole, so the remnants that result from the evaporation process would need to have an equally large number of states in order to be entangled with the Hawking radiation on $\mathcal{I}^+$. This amounts to an essentially infinite number of microstates for each remnant. If remnants interact thermodynamically with the outside world, we should expect them to be spontaneously produced at extremely high rates. A lack of observational evidence makes such a scenario unlikely.

However, it is possible that remnants do not interact thermodynamically with the outside world. While this scenario is unappealing as it constitutes a practical if not theoretical loss of information (i.e. if the information is trapped in the remnant, the outside observer still must trace over it when performing measurements), it is nevertheless a possibility. One form of non-interacting remnants are topologically disconnected black hole remnants. These take the form of pockets of spacetime that form within the black hole that completely pinch off during the evaporation process. We examine this proposal in more detail.

Multiple authors [2, 7, 9] note that the interior of a Schwarzchild black hole resembles an infinitely long throat. An observer falling into the black hole sees the end-point of this throat recede superluminally, and as such it is never actually reached. At the bottom of this throat is classically understood to be the Schwarzchild singularity, a region of spacetime where both the density and curvature approach Planckian scales. This is precisely regime in which we expect classical theories to break down and quantum gravitational effects to dominate. In particular, analyses such as [9, 10] suggest that quantum gravitational effects actually replace the singularity with a non-singular region of spacetime and effectively stabilize the throat of the interior. In such a scenario, an observer falling into the black hole passes through this throat in a finite proper time.

The result of such a proposal is that the black hole is essentially replaced with a wormhole into a "child universe", an arbitrarily large spatial region within the black hole that is reached by matter tunneling through this stabilized throat. This is illustrated in the Penrose diagram of Figure 2. Region I is the parent universe, with future and past null infinity in their usual location. The dotted line is the quantum gravitationally dominated nonsingular region where the singularity used to be. Region

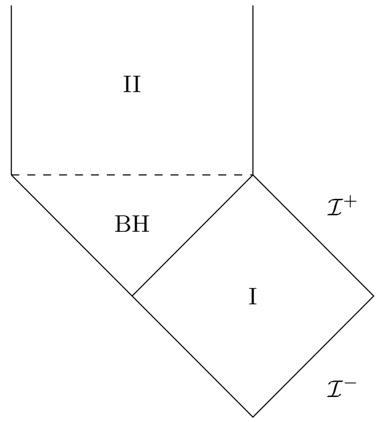

FIG. 2. A Penrose diagram for a non-singular black hole with the child universe attached. At the dotted line, spatial and temporal directions are interchanged. In general, matter that falls into the black hole from the parent universe is able to reach the entirety of the interior universe. Note that this is not the only possible diagram. Notably, the Kerr black hole possesses a similar structure and allows for tunneling into child universes. See [9] for details.

II is the child universe, turned on its side so that the space and time coordinates are flipped. It should be noted that this is not the only such possible diagram. In fact, the Kerr black hole has a similar internal structure and allows for tunneling to child universes.

The topologically disconnected remnants [2] proposal conjectures that that this throat actually pinches off to form a topologically disconnected region that acts as a repository for the lost information. That is, at late times there is no classical path from region I to region II — the original spacetime is disconnected into two regions. The information that has fallen into the black hole is therefore trapped, with no way to interact with the parent universe. This modification solves the thermodynamic issues already discussed at the cost of sealing away the information. Still, information is not actually lost and evolution is still unitary, provided we are careful to consider the state on both $\mathcal{I}^+$ and $\tilde{\mathcal{I}}^+$, the future boundary of the child universe. Namely, the final state is still pure. The information has not been destroyed, but rather is just "somewhere else". See [2] for further details, including possible difficulties with this picture and resolutions thereof.

There are certain qualitative features of this spacetime that we would like to highlight. Foremost is the addition of a secondary future boundary $\tilde{\mathcal{I}}^+$ that is disconnected from the first. In this paper, "disconnected boundaries" means only that the boundaries are disconnected as subsets of the entire conformal boundary of the spacetime. They may still be connected through the bulk of the spacetime, as is the case here before the topology change has occurred. This boundary acts as a sink that prevents information from reaching $\mathcal{I}^+$. In the standard picture, this boundary vanishes along with the black hole and takes its information along with it. Here, however, the boundary does not vanish and instead disconnects from the original spacetime through the bulk at the end of the evaporation process .

Another key feature of these spacetimes is the fact that the boundaries are shielded from each other by horizons. For the parent universe, this is the normal black hole horizon, whereas for the child universe it is a white hole horizon that acts as a source for matter. External observers in both universes are shielded from the topology change by the presence of these horizons, in accordance with well know topological censorship theorems. Finally, the end result after topology change is two entirely disconnected regions, each with their own future boundary. At all stages, the two boundaries are highly entangled with each other, due to the fact that outgoing Hawking radiation is entangled with matter inside the black hole. Our aim is to construct an AdS_3 spacetime that shares these features.

III. HOLOGRAPHIC FORMULATION

The AdS/CFT correspondence [11] has proven to be a promising route for developing a non-perturbative gravitational theory in very high curvature regions. The central conjecture of this correspondence is that a conformal field theory defined on a fixed spacetime of dimension d is dual to a $d+1$ dimensional gravitational theory that is asymptotically AdS. Certain states of the boundary CFT correspond to certain bulk geometries. However, the exact conditions required for a field theory to be holographic (i.e. to admit a gravitational dual) are not well understood, and neither are the actual mechanisms that allow the bulk theories to emerge from the CFT boundary theories. A more comprehensive dictionary between boundary states and bulk geometries is likely required before the conjecture can be fully evaluated.

Nonetheless, there is a large class of $2+1$ dimensional *AdS wormholes* that are well studied. These wormholes consist of $n \geq 2$ disconnected boundaries and asymptotically AdS regions. General discussion of these wormholes can be found in more detail in [12–15]. In the general case, the wormhole consists of n asymptotic regions, each of which is a genus g Riemann surface S. As such, the overall topology of these wormholes will be $S \times \mathbb{R}$. For this analysis we will restrict our attention to the case where $g = 0$ and $n = 2$. The local AdS structure of these geometries makes them an appealing target for holographic studies.

As we shall see, the $n = 2$ wormhole also shares certain important qualities with the topological remnant spacetimes above. First, it is characterized by two disconnected asymptotic regions, analogous to the parent and child universes discussed above. Further, each of these regions has its own conformal boundary which is disconnected (in the boundary sense mentioned above) from each other. These asymptotic regions are shielded from each other by horizons which serve as holographic screens for observers located in one region. Finally, the entire spacetime is dual to an entangled state of the CFTs on each boundary, which is suggestive of the fact that fields on $\mathcal{I}^+$ are in general entangled with fields on $\tilde{\mathcal{I}^+}$. Additionally, as we will see, there are certain processes which can be interpreted as these asymptotic regions becoming disconnected from each other. Based on these similarities, we postulate that these wormholes serve as suitable analogs in a holographic context to the remnant spacetimes of Section II. Below, we formulate this in explicit detail.

A. Construction of the spacetime

This construction adapts the process presented in [12]. Given a universal covering space $\tilde{M} \to S$ of a connected and complete Riemann surface S, we can write $S = \tilde{M}/\pi_1(S)$. It is a known fact that every complete, connected, and simply connected Riemann manifold of constant negative curvature is isometric to $\mathbb{H}$. By definition the universal cover is simply connected and thus satisfies this condition. We can write $S \simeq \mathbb{H}/\pi_1(S)$. We also note that $\pi_1(S)$ acts on $\mathbb{H}$ isometrically with respect to the Poincaré metric.

If we were interested in general wormholes, we might start with $\mathbb{H}$ and a group Γ of discrete isometries and take the quotient to form a Riemann surface $S = \mathbb{H}/\Gamma$. This surface will represent a spatial slice of the total wormhole. This method requires special care that only certain subgroups of the isometry group $\mathrm{PSL}(2,\mathbb{R})$ are chosen so that the action of Γ on $\mathbb{H}$ has no fixed points, as this prevent S from having conical singularities. Since are interested in a specific wormhole, it is easier to start by assuming the Riemann surface S is given and explicitly construct it as $\mathbb{H}/\Gamma$ where $\Gamma = \pi_1(S)$. This allows us to control what properties the resulting wormhole has. We are also guaranteed that $\Gamma = \pi_1(S)$ is fixed point free, as otherwise S would not be a manifold.

The surface S is taken to be the $T = 0$ slice of the entire wormhole spacetime. After constructing such a surface as above, we have yet to actually promote it to the status of an AdS wormhole. To do so, we must embed $\mathbb{H} \hookrightarrow \mathrm{AdS}_3$, as well as extend the action of Γ to some discrete subgroup of the isometry group of AdS_3. AdS_3 is the maximally symmetric spacetime with constant negative curvature. We realize AdS_3 as a hyperboloid embedded in $\mathbb{R}^{2,2}$ subject to:

$$X_\alpha X^\alpha = -\ell^2 \tag{6}$$

Here, ℓ essentially sets the scale of curvature. Without loss of generality we set $\ell = -1$. Then the above parameterization can be written as

$$-X_0^2 + X_1^2 - X_2^2 + X_3^2 = -1 \tag{7}$$

with metric

$$dS^2 = -dX_0^2 + dX_1^2 - dX_2^2 + dX_3^2 \tag{8}$$

A note of caution is required here. We may write the space in coordinates (t, ρ, ϕ) as

$$X_0 = -\cosh(\rho)\cos(t) \quad X_1 = -\sinh(\rho)\sin(\phi)$$

$$X_2 = -\cosh(\rho)\sin(t) \quad X_3 = -\sinh(\rho)\cosh(\phi)$$

which yields the metric

$$ds^2 = -\cosh^2\rho dt^2 + d\rho^2 + \sinh^2\rho d\phi^2 \tag{9}$$

Examining the expressions for X_0 and X_2, we see that the coordinate t is periodic (i.e. going from $t \to t+2\pi$ will yield the same point on the hyperboloid). This gives rise to closed time like curves, which in general we would like to avoid. To do so, we "unwrap" the time coordinate and allow $t \in (-\infty, \infty)$, i.e. we no longer identify $t \sim t + 2\pi$. Formally, this is lifting to the universal covering space of the space constructed here. For this reason, we will use "AdS_3" to refer to this universal cover free of closed timelike curves. These are known as global coordinates, as the metric covers the whole of AdS_3 space.

From (8) as well as (7), we see that AdS_3 has a $\mathrm{SO}(2,2)$ symmetry. Let $G = \mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,\mathbb{R})$. We define a group action

$$\begin{aligned} p : G \times M_2(\mathbb{R}) &\to M_2(\mathbb{R}) \\ (g_l, g_R, M) &\mapsto g_L M g_R^T \end{aligned} \tag{10}$$

This is an isometry, so it induces a homomorphism

$$p : G \to \mathrm{SO}(2,2) \tag{11}$$

whose kernel is $\{(1_L, 1_R), (-1_L, -1_R)\} \simeq \mathbb{Z}_2$. As such, the following diagram commutes

$$\begin{tikzcd} G \arrow[r, "p"] \arrow[d, "\pi"] & \mathrm{SO}^+(2,2) \\ G/\mathbb{Z}_2 \arrow[ur, "\tilde{p}"] & \end{tikzcd}$$

where $\mathrm{SO}^+(2,2)$ is the connected identity component of $\mathrm{SO}(2,2)$. Namely, $\tilde{p}$ is an isomorphism so we have $G/\mathbb{Z}_2 \simeq \mathrm{SO}^+(2,2)$. We may also see this by noting that $G \to \mathrm{SO}^+(2,2)$ is a double cover with deck transformation $\mathbb{Z}_2$, and the result follows.

It is now clear how we should extend the action of Γ to AdS_3. We define a homomorphism from $\mathrm{PSL}(2,\mathbb{R}) \to (\mathrm{SL}(2,\mathbb{R}) \times \mathrm{SL}(2,R))/\mathbb{Z}_2$ which sends $\gamma \to (\gamma, \gamma)$. This homomorphism sends a discrete group Γ of isometries of $\mathbb{H}$ to a discrete group $\tilde{\Gamma}$ of AdS_3. We may then form our AdS wormhole by taking the quotient $\mathrm{AdS}_3/\tilde{\Gamma}$. Note that if we take the $X_0 = 0$ slice, we get our original Riemann surface S. For the analysis of topological black hole remnants, we would like our spatial slices to have two disconnected boundaries connected through a bulk interior. This corresponds to our $X_0 = 0$ slice being a Riemann surface S with cylindrical topology $S^1 \times \mathbb{R}$. Considering this, we choose $\Gamma = \mathbb{Z}$ so that our spacetime is given by $\mathrm{AdS}_3/\mathbb{Z}$. The $X_0 = 0$ slice is presented in Figure 3.

As noted in [12, 14], the two regions separated by the shaded surface are each isometric to the BTZ black hole. The entire spacetime is thus by construction the extended BTZ black hole, i.e. the BTZ black hole with two asymptotic regions. The metric for one of these regions is given in Schwarzchild coordinates by [16, 17]

$$ds^2 = -\left(-M + r^2\right) dt^2 + \left(-M + r^2\right)^{-1} dr^2 + r^2 d\phi^2 \quad (12)$$

The singularities at $r = \pm\sqrt{M}$ are coordinate singularities, much like the more familiar 3+1 Schwarzchild black hole, and thus may be removed by Kruskal-like extensions.

B. Boundary metric and dual CFT

A given spatial slice of our wormhole has two boundaries with S^1 topology. Given the metric of the total space, we would like to write down a metric for an observer on the boundary. It will first be useful to express the metric in terms of Fefferman-Graham coordinates [18, 19] which have the general form

$$ds^2 = \frac{1}{z^2}(dz^2 + g_{\mu\nu}dx^\mu dx^\nu) \quad (13)$$

where $g_{\mu\nu}$ is a 2-dimensional metric which can be expanded as $g_{\mu\nu} = g^{(0)}_{\mu\nu} + z^2 g^{(2)}_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}$. Metrics of this form are useful as they are particularly suited to express the geometry around the boundary: $g^{(0)}_{\mu\nu}$ is taken to be the boundary metric, and higher order terms are needed as we move away from the boundary. We may bring (12) into this form [20] by setting

$$-dr/\sqrt{r^2 - M} = dz/z \quad (14)$$

Solving this differential eqution, we see that $z = (2/M)(r + \sqrt{r^2 - M})^{-1}$, where the factor of $2/M$ was chosen so that z behaves as $1/r$ as $r \to \infty$. Solving for r, we get the relation

$$r = \frac{1}{z} + \frac{Mz}{4} \quad (15)$$

Under this transformation (12) takes the form

$$ds^2 = \frac{1}{z^2}\left(dz^2 - \left(1 - \frac{M}{4}z^2\right)^2 dt^2 + \left(1 + \frac{M}{4}z^2\right)^2 d\phi^2\right) \quad (16)$$

Note that now the metrics (13) and (16) have the same form. The z coordinate covers the region outside of the horizon with values $z \in [0, 2/\sqrt{M}]$, corresponding to $r > \sqrt{M}$ in the Schwarzchild coordinates. We just as easily made the transformation $+dr/\sqrt{r^2 - M} = dz/z$ in place of (14), which corresponds to $z = (2/M)(r - \sqrt{r^2 - M})^{-1}$ and yields the same metric. Under this transformation, z takes on values $z \in [2/\sqrt{M}, \infty]$ to cover the external region. Therefore, if we allow z to range from $[0, \infty]$, it will cover two regions that are both external to the horizon. This is analogous to coordinates of the typical Schwarzchild black hole which cover the two external regions of the Schwarzchild black hole. For a fixed time slice, the spatial topology takes the form of two asymptotic regions connected by a narrow throat. From (16), we can also read off the values of $g_{\mu\nu}$:

$$\begin{array}{ll} g^{(0)}_{00} = -1 & g^{(0)}_{11} = 1 \\ g^{(2)}_{00} = M/2 & g^{(2)}_{11} = M/2 \\ g^{(4)}_{00} = -M^2/16 & g^{(4)}_{11} = M^2/16 \end{array}$$

Notably, $g^{(0)}_{\mu\nu} = \eta_{\mu\nu}$ which indicates that the $z \to 0$ boundary geometry of our wormhole is that of flat Minkowski space. Notice that under the transformation $z \to 1/z$, the metric (16) retains its form. Therefore, we have another boundary at $z \to \infty$ with metric $g^{(4)}_{\mu\nu}$ [20]. Therefore, this form of the metric contains data regarding the geometry at both boundaries.

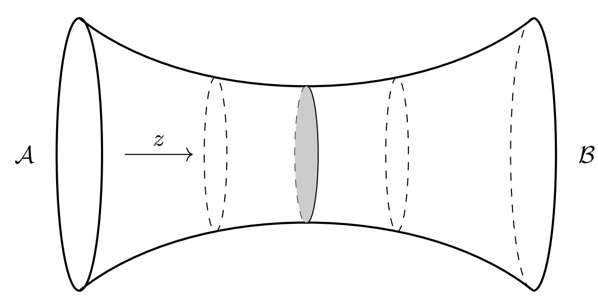

FIG. 3. The $X_0 = 0$ slice of our constructed wormhole covered by the coordinates (14), which is isometric to the extended BTZ black hole. The total spacetime is had by evolving this surface forward and backward in time. In both cases, as for all Lorentzian signature wormholes, the end result are singularities. In Fefferman-Graham coordinates, the boundaries are are at $z \to 0$ and $z \to \infty$ with flat metrics $g^{(0)}_{\mu\nu}$ and $g^{(4)}_{\mu\nu}$. The shaded surface is the horizon separating the two regions, which is located at $z = 2/\sqrt{M}$ and has area $2\pi\sqrt{M}$.

We would like to compute the Hawking temperature of this black hole. We first put the metric (12) into Euclidean form by making the transformation $t \to i\tau$. The Euclidean time coordinate is periodic with period β, and we have have a contractible cycle in the time coordinate. Such cycles gives rise to conical singularities, which we would like to remove. The condition removing the conical singularity is precisely when $\beta = 2\pi/\sqrt{M}$ [20]. The Hawking temperature of the black hole is defined as $T_H = 1/\beta$. It is thought that the CFT state dual to the BTZ black hole is a thermal CFT at temperature T_H defined on a flat boundary [20]. For the extended BTZ black hole, it is argued that the dual CFT of the total spacetime is obtained by entangling the thermal CFTs on each boundary [12, 14, 19, 21]. This state is known as the thermofield double state, and takes the general form

$$|\Phi\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_{\mathcal{A}} \otimes |E_n\rangle_{\mathcal{B}} \tag{17}$$

Here, $Z(\beta)$ is the partition function that corresponds to a thermal CFT at a temperature given by the Hawking temperature above.

IV. IMPLICATIONS FOR REMNANTS

The AdS/CFT correspondence provides various area-entropy results that can provide us details about spacetimes like those constructed in Section III. In particular, the Ryu-Takayanagi formula [22] provides a direct relationship between entanglement entropy of certain subsystems of the boundary and the area of specific surfaces in the bulk. This has implications in the case of disconnected remnants. Recall that the disconnected remnant proposal suggest that remnants remain highly entangled with the Hawking radiation emitted from the black hole, even while they are topologically disconnected. The Ryu-Takayanagi formula suggests that this may not be a valid scenario: entanglement is in fact a necessary condition for connectivity. Indeed, other proposals such as ER=EPR claim that entanglement is in fact a *sufficient* condition for connectivity — any two entangled systems must also be connected [23].

These proposals suggest that we should be able to draw conclusions regarding the entanglement entropy of the black hole remnant from information regarding the connectivity of the spacetime. Consider an observer localized on one of the conformal boundaries of Figure 1, such as $\mathcal{I}^+$, in a non-evaporating black hole spacetime. As discussed in Section I, an observer on this boundary does not have access to all of the degrees of freedom of the initial system, and instead must trace over the portion of the system on the other boundary (in this case, the boundary of the inner region of the black hole remnant). In the AdS scenario, we propose that such an observer corresponds to an observer on one of the conformal boundaries of a spatial slice of the spacetime. We would like to see this explicitly.

We will consider an observer on the $z \to 0$ boundary $\mathcal{A}$. This boundary is a spatial subsystem of the total CFT system of states which have the form (17). An observer localized in $\mathcal{A}$ does not have information regarding the subsystem of $|E_n\rangle_{\mathcal{B}}$ states. This portion of the state is traced out, and the result is a reduced density matrix $\rho_{\mathcal{A}} = \mathrm{Tr}_{\mathcal{B}} |\Phi\rangle \langle\Phi|$. From this, we can compute the entanglement entropy of $\mathcal{A}$ with $\mathcal{B}$ by $S_{\mathcal{A}} = -\mathrm{Tr}(\rho_{\mathcal{A}} \log \rho_{\mathcal{A}})$. Now, the Ryu-Takayanagi formula states that

$$S_{\mathcal{A}} = \frac{A_\Sigma}{4G} \tag{18}$$

where A_Σ is the area of the minimal surface Σ associated with $\mathcal{A}$, and G is understood to be Newton's constant in 3 dimensions. For a static geometry, Σ is the surface with extremal area in the bulk that both shares a boundary with and is homologous to $\mathcal{A}$ [19, 24]. Note that this result is conjectured to hold for any arbitrary division of the CFT into subsystems A and B, not just subdivisions across disconnected boundaries. In all cases, the surface Σ acts as a holographic screen for the observer.

When A and B are on disconnected boundaries, as in this example, we expect that the role of Σ is filled by the horizon at $z = 2/\sqrt{M}$. Heuristically, this is explained as follows. Recall that z does not cover the entire $X_0 = 0$ slice, but rather just the two regions external to the horizon. The CFT localized on the boundary $\mathcal{A}$ is therefore not determined by the portion of the bulk behind the horizon: for an observer on this boundary, the space time looks just like the non-extended BTZ black hole. For an observer on $\mathcal{B}$, the situation is analogous. The horizon can thus be viewed as actually causing the loss of information that results in the entanglement entropy $S_{\mathcal{A}}$. More explicitly, the entanglement entropy of the CFT on $\mathcal{A}$ with the CFT on $\mathcal{B}$ is proportional to the area of the horizon. This is exactly what is suggested by (18). This can be made more explicit by actually finding the minimal surface Σ.

Consider some subsystem A of $\mathcal{A}$. If there were one boundary, the minimal surface Σ would be the same for both A and its complement $\bar{A}$. Via Ryu-Takayanagi, this means that

$$S_A = S_{\bar{A}} \tag{19}$$

Since A and $\bar{A}$ share the same Σ, and both are mutually homologous to Σ. This implies that A is homologous to $\bar{A}$. The presence of horizons, and by extension disconnected boundaries, complicates the matter [19, 22, 24]. In such a scenario, $\bar{A}$ consists of segments on two disconnected boundaries and in general the surface $\Sigma_{\bar{A}}$ corresponding to $\bar{A}$ will be different than the surface Σ_A corresponding to A. Furthermore, in order to remain homologous to the entirety of $\bar{A}$ on both boundaries, $\Sigma_{\bar{A}}$

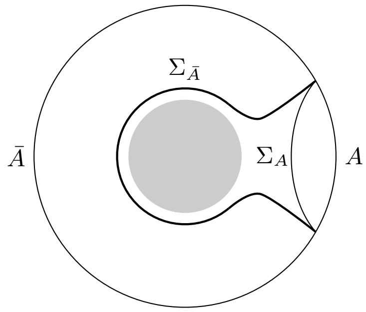

FIG. 4. A schematic diagram of the minimal surfaces in the presence of a horizon, represented here by the shaded area. The thick line $\Sigma_{\bar{A}}$ remains homologous to $\bar{A}$ as A is varied in length. As $A \to 0$, the two endpoints of the thick line will be brought together and it will completely encircle the horizon. For a diagram with the surfaces drawn directly on the spatial slice in Figure 3 see [19].

must envelop the horizon (see Figure 4). As the area of $A \to 0$, we see that $\bar{A} \to \mathcal{A}$, $S_A \to 0$, and $\Sigma_{\bar{A}} \to H$, where H is the area of the horizon. The surface $\Sigma_{\mathcal{A}}$ corresponding to $\mathcal{A}$ wraps completely around the horizon. This recovers the desired result of

$$S_{\mathcal{A}} = \frac{H}{4G} = \frac{\pi\sqrt{M}}{2G} \tag{20}$$

which is the 2+1 dimension equivalent of the Bekenstein-Hawking area law.

We propose that the topology change disconnecting the two regions is a process in which the area of the minimal surface separating the two asymptotic regions vanishes. We expect this to be the case for two reasons. Following the discussion of Section II, we expect the topology change to occur during the late stages of the evaporation process, when the mass of the black hole is very small. This relationship is reflected in the form of (20), where the $H \to 0$ only when $M \to 0$. In addition to this, the notion of the wormhole throat pinching off corresponds precisely to the area of the minimal surface shrinking — the shaded surface of Figure 3 shrinks and eventually vanishes. The result is two bulk disconnected regions and boundaries.

The key departure between the two scenarios (the original remnant scenario and the one presented here) has to do with the effect that disconnection has on the entanglement entropy of each boundary. As the topology change occurs and the area of the minimal surface shrinks, and observer located on one boundary sees a corresponding decrease in the entanglement entropy of the two horizons, as indicated by (20). The local degrees of freedom of the CFT on the observer's boundary $\mathcal{A}$ are no longer highly entangled with those of the other boundary. We propose that this relationship holds not only for the dual CFTs on each boundary, but in fact is a general result applicable to any two quantum theories defined on separated regions of spacetime. To be explicit, we postulate along with [23] that entanglement between subsystems in two separated regions of spacetime necessitates that the two regions be topologically connected in the bulk.

If such a conjecture holds, then topologically disconnected black hole remnants are not a valid resolution to the information paradox. Topology change resulting in disconnected remnants would destroy the entanglement between the matter that fell into the remnant and the outgoing Hawking radiation. The disconnected remnants proposal is then essentially reduced to that of firewalls [3], and the difficulties of both proposals remain. Such a scenario requires modifications to physics in regions we understand well so that the final state is not the highly entangled state that we should expect [5]. The information paradox would fail to be clearly resolved in such a scenario.

V. CONCLUSION

One possible resolution to the black hole information paradox is the topologically disconnected black hole remnants scenario, which postulates that matter falling into a black hole ends up in a topologically disconnected region of spacetime. We highlight important qualitative features of spacetimes with topologically disconnected remnants, and construct an AdS_3 spacetime that shares these features. This construction is done by quotienting AdS_3 by a subgroup of its isometry group, where the subgroup is chosen so that the resulting spatial slices of the spacetime have certain desired characteristics. The resulting Ads_3 wormhole is isometric to the extended BTZ black hole, a black hole solution in 2+1 coordinates.

Certain area-entropy results from the AdS/CFT correspondence are applied to this new space time. Specifically, the Ryu-Takayanagi formula is used to relate the entanglement entropy of the two boundaries. This entanglement entropy is found to be directly proportional to the area of the horizon separating the two regions. We conjecture that the topology change process corresponds to the shrinking of this area, and thus a decrease in the entanglement entropy of the CFTs on the boundaries. We further postulate that this topology change process affects all entanglement between the two regions, no just the entanglement between the dual CFTs. The result is two disconnected regions which are not highly entangled, in a sense combining the difficulties of the disconnected remnants and the firewalls proposals.

While the AdS spacetimes were constructed as to share qualitative features with the disconnected remnant spacetimes discussed in Section II, there are key differences. Namely, the AdS wormholes constructed in Section III have a different causal structure than the remnant spacetimes. In the remnant spacetimes, the future boundary of the interior region lies in the causal future of $\mathcal{I}^-$. In other words, the wormholes of Section II are traversable so that infalling matter can pass through.

The case for the extended BTZ black hole is closer to that of an Einstein-Rosen bridge. However, causal structure did not factor heavily into the arguments above, only the entanglement configuration. Furthermore, in certain settings wormholes of the type in Section III can be converted into traversable wormholes, which would restore the appropriate causal structure [25]. Considering this, we do not expect the difference in causal structure of the spacetime to affect the above arguments.

These results bear certain similarities to proposals such as the ER=EPR conjecture, which postulates that any entangled systems are connected via a small wormhole (literally, that Einstein-Rosen (bridges) = Einstein-Podolsky-Rosen (states)). We expect that an analysis of disconnected remnants in the context of ER=EPR would yield similar results to the one presented above. Our results also complement well certain connectivity theorems [26] which place limits on the types of bulk disconnections that can occur given the entanglement structure of the dual CFTs. Our central conjecture that the topology change process destroys all types of entanglement, not only entanglement between the dual CFTS, is motivated primarily by these results. Further formulation and justification of this conjecture is left for future work.

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