Eletrónica de Sistemas de Comunicações

Communication Systems Electronics

Impedance Matching

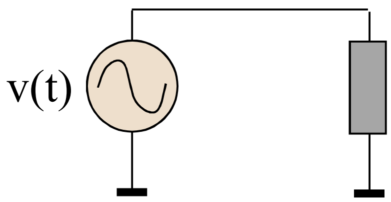
José Machado da Silva

Mestrado em Engenharia Eletrotécnica e de Computadores / Física

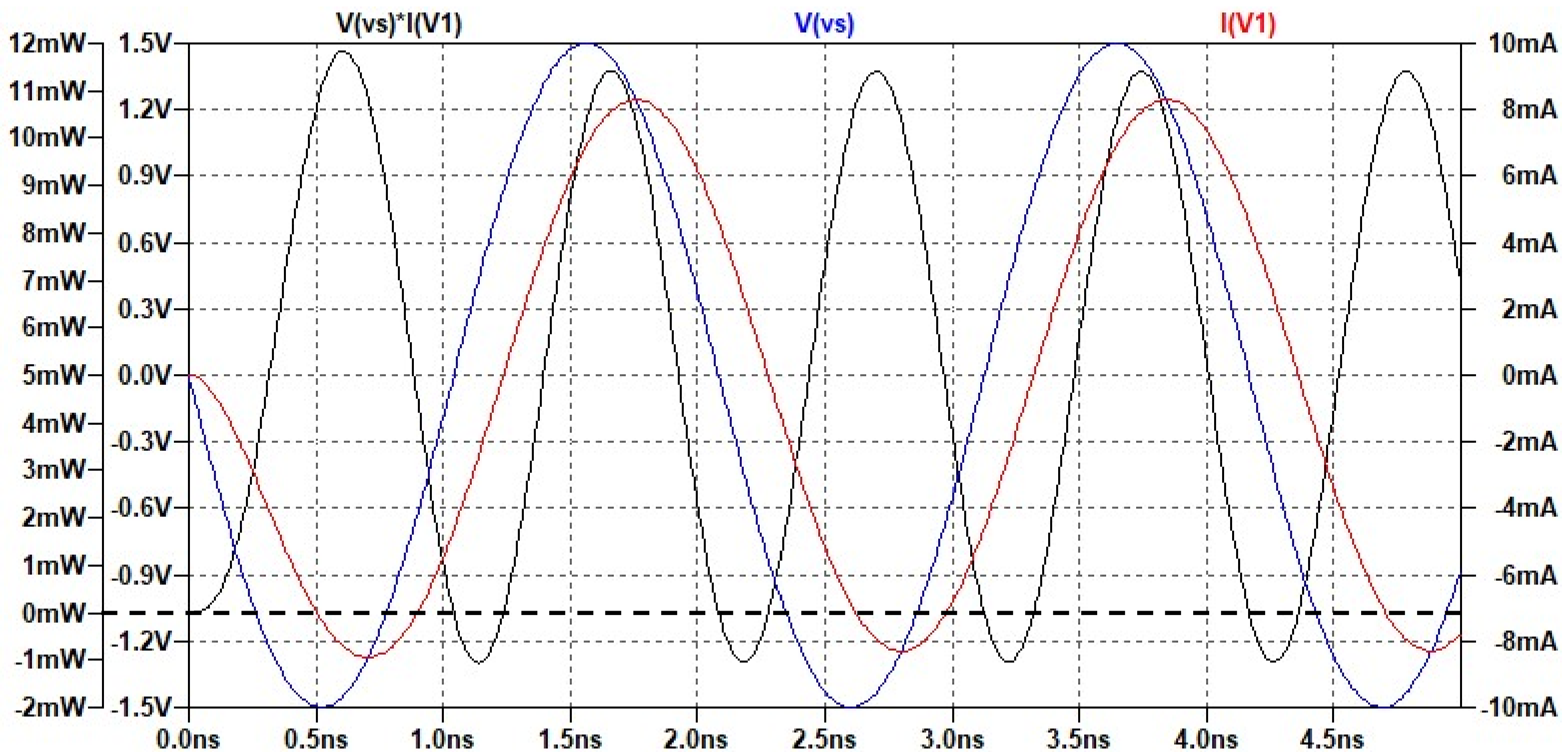
FEUP FACULDADE DE ENGENHARIA UNIVERSIDADE DO PORTO

U.PORTO

Basic concepts - Power



$$p(t) = \frac{V_{pk} I_{pk}}{2} \{\sin(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i)\}$$



Basic concepts - Power

Instantaneous power (watts) : the power at any instant of time

$$p(t) = v(t) \times i(t)$$

where:

$$v(t) = V_{pk} \sin(\omega t + \theta_v)$$

$$i(t) = I_{pk} \sin(\omega t + \theta_i)$$

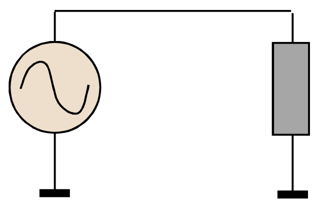
$$p(t) = \frac{V_{pk} I_{pk}}{2} \left\{ \cos(\theta_v - \theta_i) - \cos(2\omega t + \theta_v + \theta_i) \right\}$$

Basic concepts - Power

- Average power in an impedance

$$P = \frac{V_{pk} I_{pk}}{2}\cos(\theta_v - \theta_i)$$

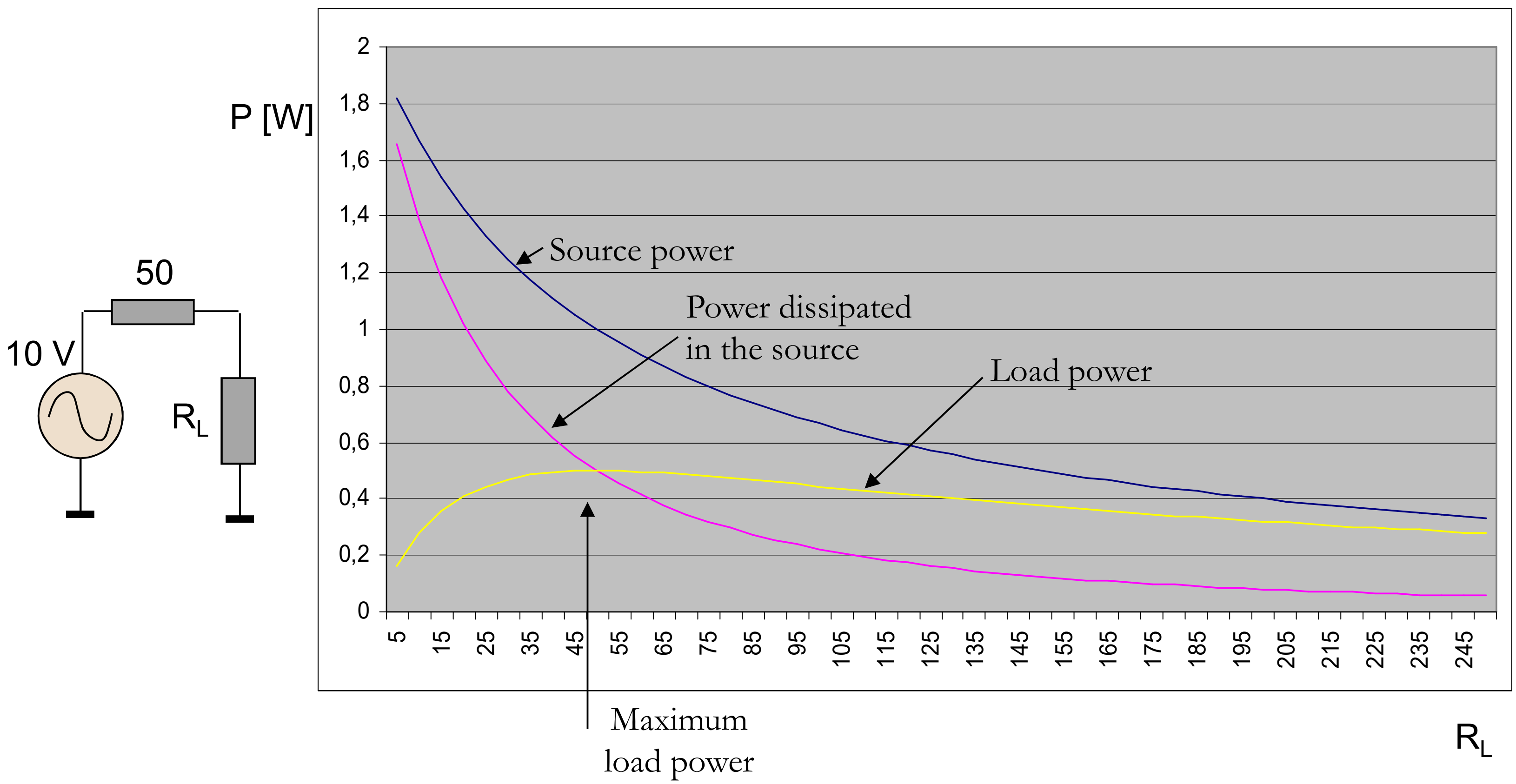
$$\mathrm{v(t)} = \mathrm{V_p}\sin(\omega \mathrm{t})$$



$$\mathrm{i(t)} = \frac{\mathrm{V_p}}{|Z|}\sin(\omega \mathrm{t} - \phi)$$

$$\mathrm{P} = \frac{1}{\mathrm{T}}\int_{\mathrm{T}} \mathrm{v(t)}\, i(t)\, dt = \frac{\mathrm{V_p^2}}{2|Z|}\cos(\phi)$$

Basic concepts - Power transfer

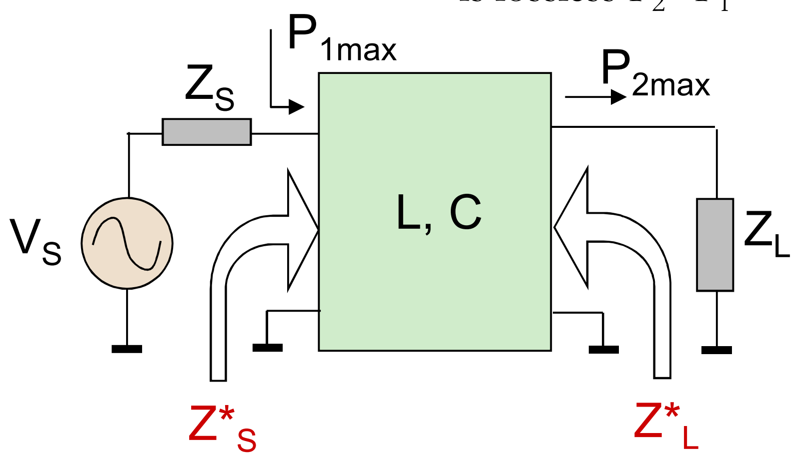


Maximizing power transfer is especially required for frequencies above 10 MHz.

Basic concepts – impedance matching

- Impedance matching (maximization of power transmitted to a load) is carried-out using a LC network which synthesises the conjugate impedances of source and load.

If the LC network is lossless P_2=P_1



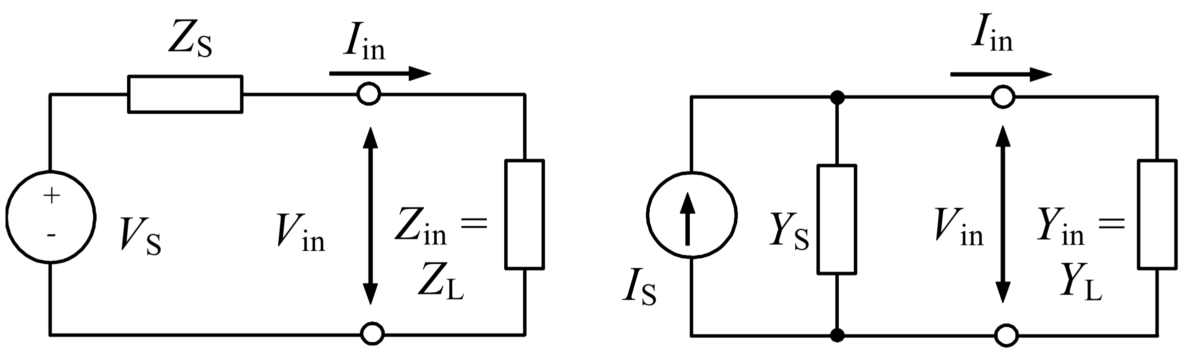
$$P_L = \mathrm{Re}\left[\frac{V_S Z_L}{Z_S + Z_L}\left(\frac{V_S}{Z_S + Z_L}\right)^*\right] =$$

$$= \mathrm{Re}[Z_L] \cdot \frac{|V_S|^2}{[\mathrm{Re}(Z_S + Z_L)]^2 + [\mathrm{Im}(Z_S + Z_L)]^2}$$

- Impedance matching is obtained in a narrow frequency band.
- The higher the quality factor the narrower the frequency bandwidth.
- Maximum Z_L voltage is obtained when impedances match.

Basic concepts – impedance matching

Impedance matching provides maximum source to load power delivery



$Z_S = R_S + jX_S$ - source impedance

$Z_L = R_L + jX_L$ - load impedance

$$P_L = \frac{1}{2} V_{in}^2 \,\mathrm{Re}\left(\frac{1}{Z_L}\right) = \frac{1}{2} V_S^2 \left|\frac{Z_L}{Z_S + Z_L}\right|^2 \mathrm{Re}\left(\frac{1}{Z_L}\right)$$

- power delivered to the load

(substitution of real and imaginary parts of source and load impedances)

$$P_L = \frac{1}{2} V_S^2 \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

- power delivered to the load as a function of circuit parameters

Basic concepts – impedance matching

$$\frac{\partial P}{\partial R_L} = 0 \qquad \frac{\partial P}{\partial X_L} = 0 \quad \Rightarrow \quad \begin{cases} R_S^2 - R_L^2 + (X_L + X_S)^2 = 0 \\ X_L(X_L + X_S) = 0. \end{cases}$$

To maximize the output power (@ constant source impedance Z_S)

$$P_L = \frac{1}{2} V_S^2 \frac{R_L}{(R_S + R_L)^2 + (X_S + X_L)^2}$$

$$\begin{cases} R_S = R_L \\ X_L = -X_S \end{cases} \quad \text{or} \quad Z_L = Z_S^*$$

- impedance conjugate matching conditions

$$P_L = \frac{V_S^2}{8R_S}$$

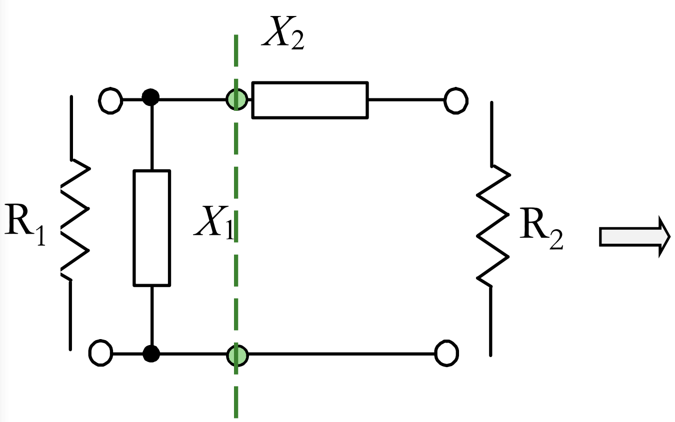
- maximum power delivered to load

$$\begin{cases} G_S = G_L \\ B_L = -B_S \end{cases} \quad \text{or} \quad Y_L = Y_S^*$$

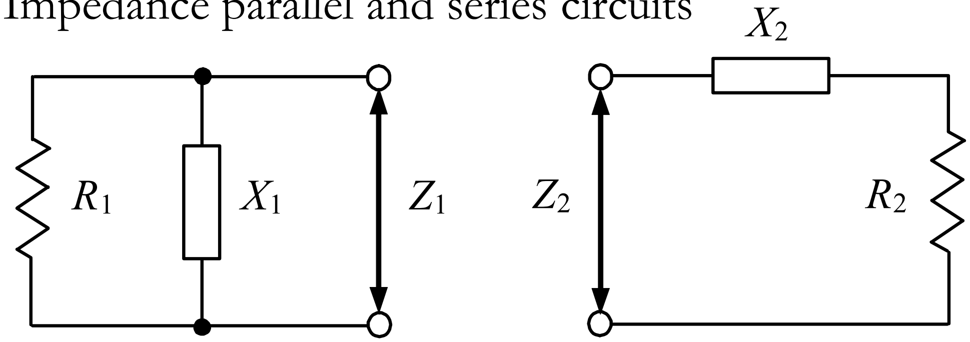
- admittance conjugate matching conditions

Impedance matching with lumped elements

- Usually applied for frequencies below 10 GHz
- L-transformer



Impedance parallel and series circuits



If $Z_1 = Z_2$ $\quad R_2 + jX_2 = R_1 // jX_1 = \dfrac{R_1 X_1^2}{R_1^2 + X_1^2} + j\dfrac{R_1^2 X_1}{R_1^2 + X_1^2}$

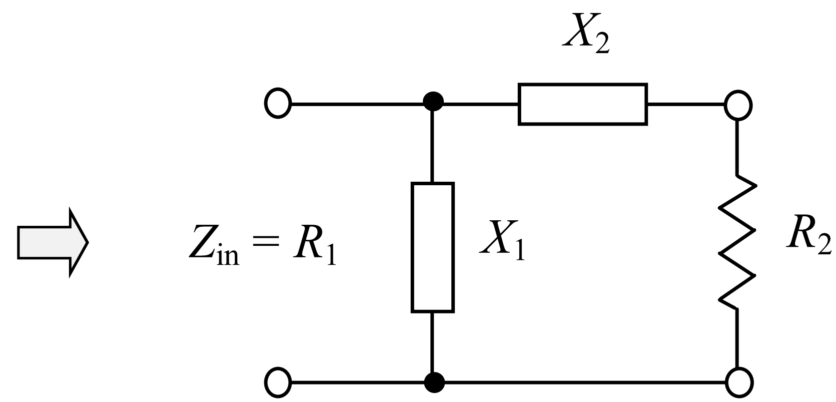
$$R_1 = R_2\left(1 + Q^2\right) \qquad X_1 = X_2\left(1 + Q^{-2}\right)$$

where $\mathbf{Q = R_1/|X_1| = |X_2|/R_2}$ is the quality factor; same value for series and parallel circuits

Impedance matching with lumped elements

- Conjugate matching with reactance compensation

$$R_1 = R_2\left(1 + Q^2\right)$$
$$X_1 = -X_2\left(1 + Q^{-2}\right)$$



The input impedance, Zin, is resistive and equal to R1 if :

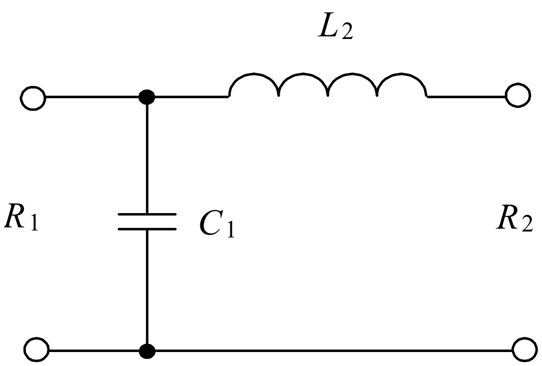
$$\begin{cases} |X_1| = R_1 / Q \\ |X_2| = R_2\, Q \\ Q = \sqrt{R_1 / R_2 - 1}\,. \end{cases}$$

where Q = R1/ | X1 | = | X2 | /R2

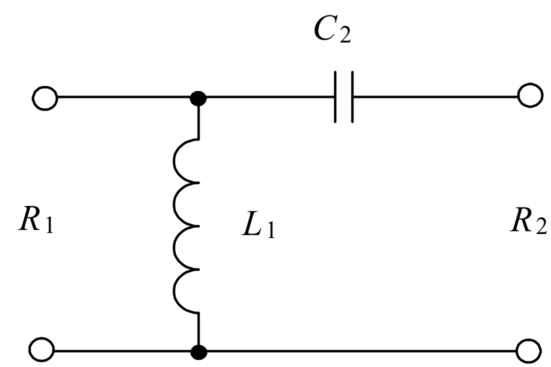
Impedance matching with lumped elements

- L-type matching circuits

$$R_1 > R_2$$



a).



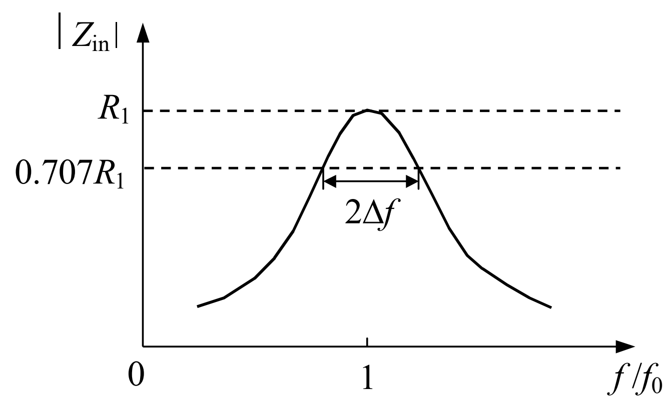
b).

$$\left.\begin{aligned} \omega C_1 &= Q / R_1 \\ \omega L_2 &= Q R_2 \end{aligned}\right\} \quad Q = \sqrt{\frac{R_1}{R_2} - 1} \quad \left\{\begin{aligned} \omega L_1 &= R_1 / Q \\ \omega C_2 &= 1/(Q R_2) \end{aligned}\right.$$

Bandwidth properties

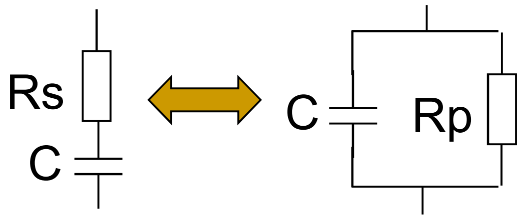
$$\begin{cases} Q \cong f_0 / 2\Delta f \\ F_n \cong Q^2\left(h_n^{\;2} - 1\right) \end{cases}$$

where F_n - out-of-band suppression factor
h_n - harmonic number



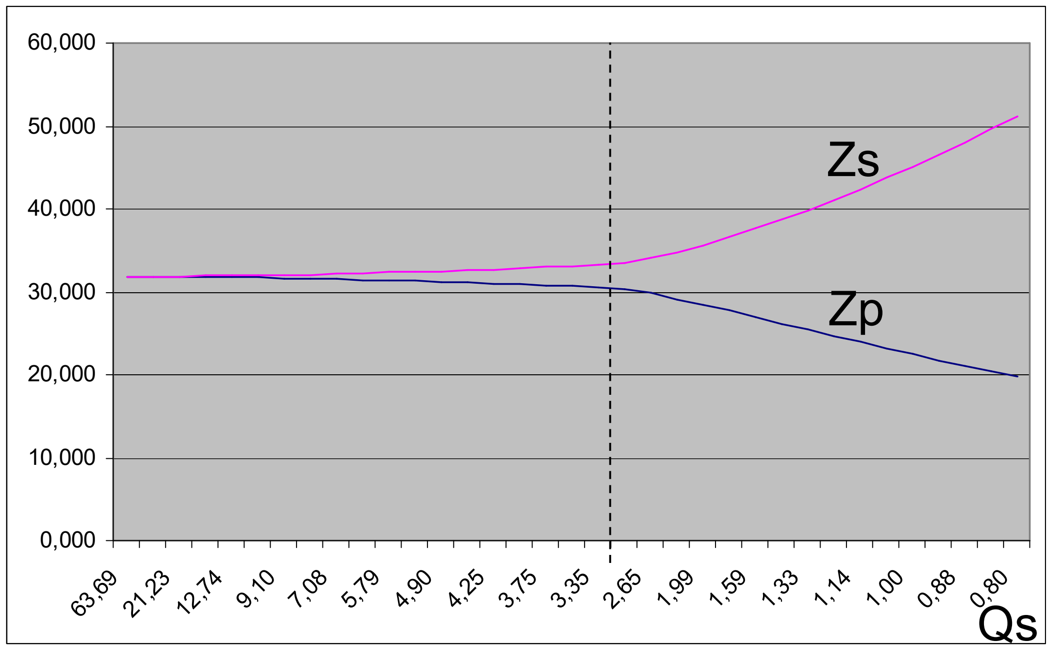
Impedance transformation

- Narrow band series/parallel transformation (ω_0)



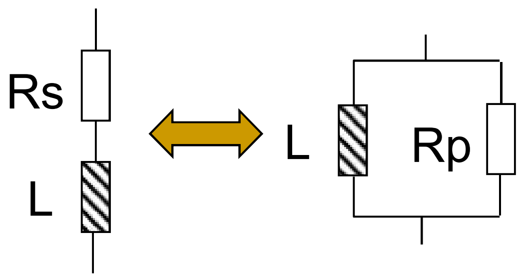
$$R_p = \left(Q^2+1\right)R_S \text{ and}$$
$$Q_p = R_p C\omega_o$$

$$\text{If } Q_s = \frac{1}{R_S C\omega_o} > 3$$



Impedance transformation

- Narrow band series/parallel transformation (ω_0)

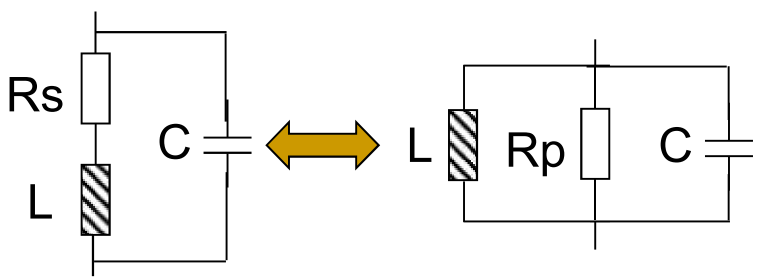


$$R_p = \left(1+Q^2\right)R_S \textit{ and}$$

$$Q_p = \frac{R_p}{\omega_o L}$$

$$\text{If } Q_s = \frac{\omega_o L}{R_S} > 3$$

- RLC resonant network ($LC\omega_0^{2}=1$)



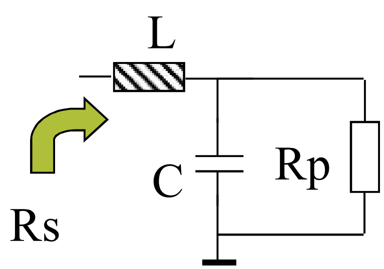
$$R_p = \left(Q^2+1\right)R_S \text{ and}$$

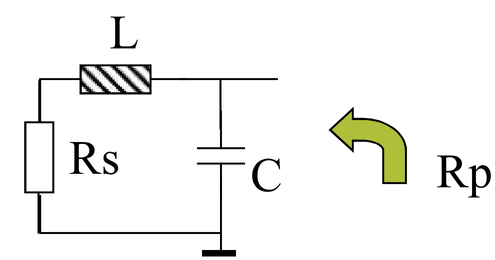
$$Q_p = \frac{R_p}{\omega_o L} = R_p \omega_o C$$

$$\text{If } Q_s = \frac{\omega_o L}{R_S} = \\ = \frac{1}{R_s \omega_o C} > 3$$

Impedance matching with lumped elements

- Transformation of resistance values

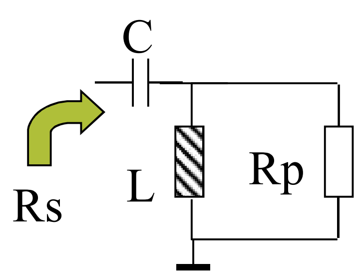


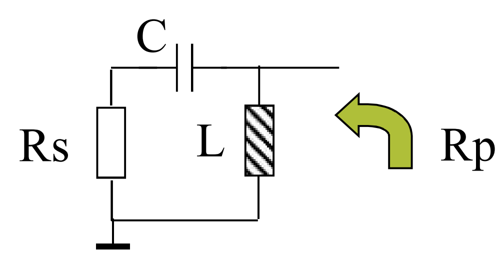


$$LC\omega_0^2 = 1$$

and

$$R_p=(Q^2+1)R_s$$



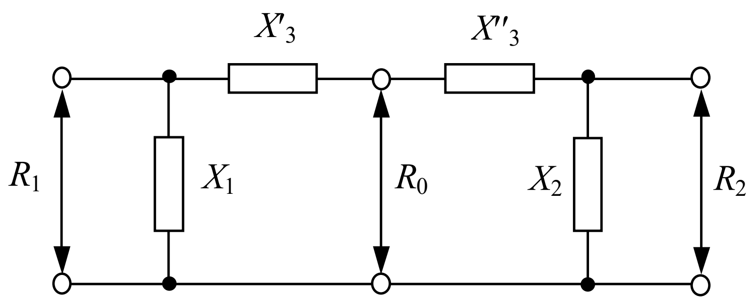


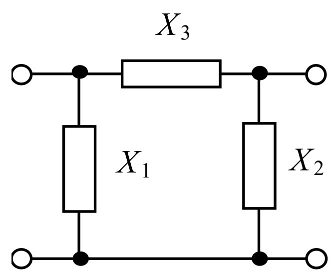
Impedance reducer network

Impedance increaser network

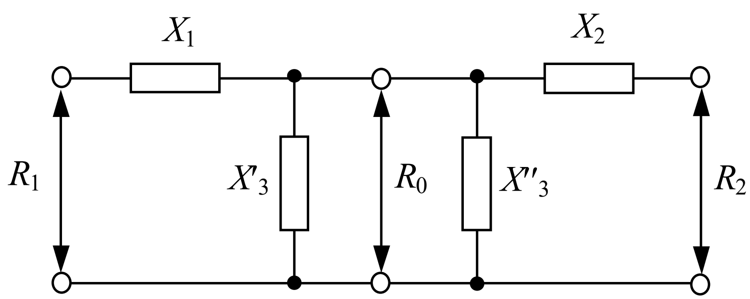
Impedance matching with lumped elements

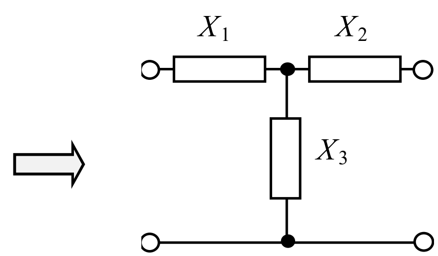
- Connection of two L-transformers





π - transformer



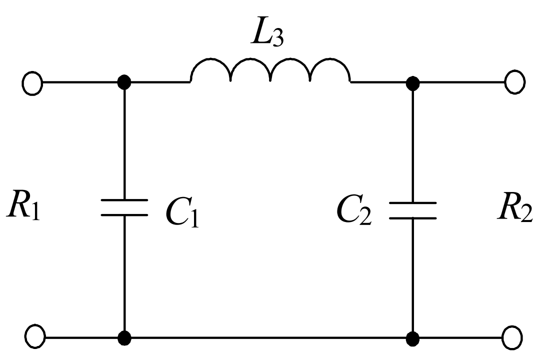


T- transformer

- Each L-transformer, transforms resistances R1 and R2 into an intermediate resistance of value R0 < (R1, R2)
- If $R_1 = R_2$, T- and π-transformers have better filtering properties, but narrower bandwidth compared with single L-transformer

Impedance matching with lumped elements

- π -type matching circuits



$$\omega C_1 = Q_1 / R_1 \qquad \omega C_2 = Q_2 / R_2$$

$$\omega L_3 = R_1 (Q_1 + Q_2) / \left(1 + Q_1^2\right)$$

$$Q_2 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_1^2\right) - 1} \qquad Q_1^2 > \frac{R_1}{R_2} - 1$$

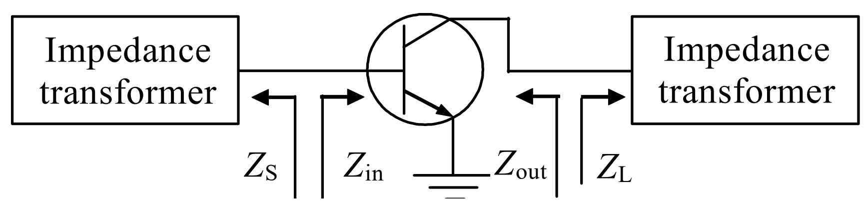
$$R_0 \approx \frac{\left(\sqrt{R_1} + \sqrt{R_2}\right)^2}{Q^2}$$

Q of the passing band

- Useful for interstage matching when the active devices' input and output capacitances can be easily incorporated within the matching circuit (C1 and C2)

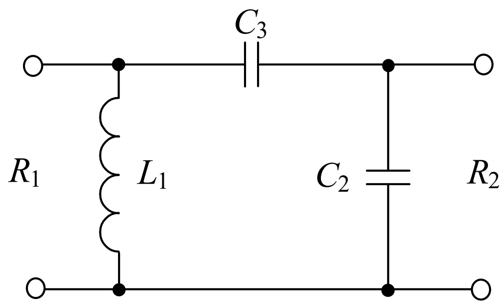
- Narrow band provides significant level of harmonic suppression

- Widely used as output matching circuit in power amplifiers to provide operation with sinusoidal collector voltage.



Impedance matching with lumped elements

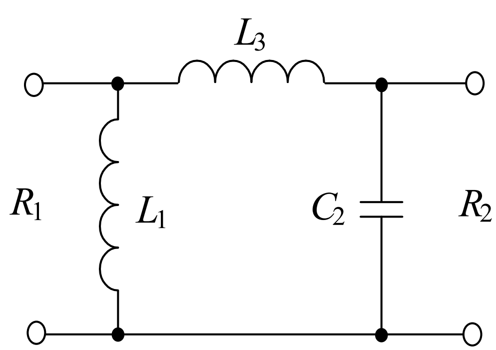
- π -type matching circuits



$$\omega L_1 = R_1 / Q_1 \qquad \omega C_2 = Q_2 / R_2$$

$$\omega C_3 = \left(1 + Q_2^2\right) / \left[R_2 \left(Q_1 - Q_2\right)\right]$$

$$Q_2 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_1^2\right) - 1} \qquad Q_1^2 > \frac{R_1}{R_2} - 1$$



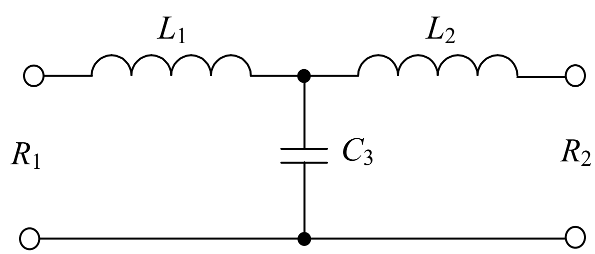
$$\omega L_1 = R_1 / Q_1 \qquad \omega C_2 = Q_2 / R_2$$

$$\omega L_3 = R_2 \left(Q_2 - Q_1\right) / \left(1 + Q_2^2\right),$$

$$Q_1 = \sqrt{\frac{R_1}{R_2}\left(1 + Q_2^2\right) - 1} \qquad Q_2^2 > \frac{R_2}{R_1} - 1$$

Impedance matching with lumped elements

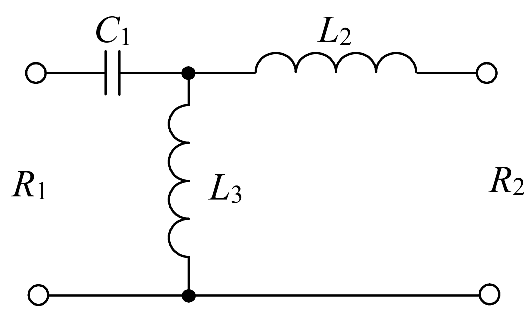
- T-type matching circuits



$$\omega L_1 = Q_1 R_1 \qquad \omega L_2 = Q_2 R_2$$

$$\omega C_3 = (Q_1 + Q_2)/\left[R_2\left(1 + Q_2^2\right)\right]$$

$$Q_1 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_2^2\right) - 1} \qquad Q_2^2 > \frac{R_1}{R_2} - 1$$



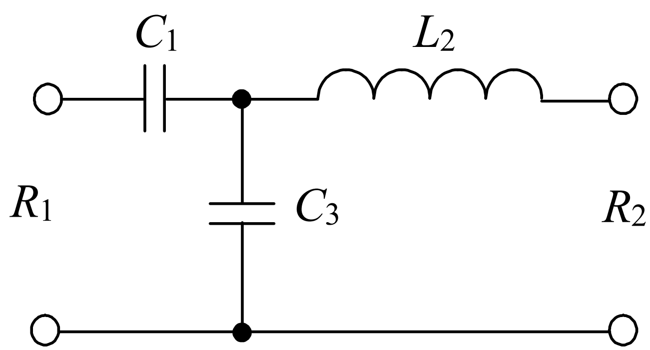
$$\omega C_1 = 1/\left(R_1 Q_1\right) \qquad \omega L_2 = Q_2 R_2$$

$$\omega L_3 = R_2\left(1 + Q_2^2\right)/(Q_1 - Q_2),$$

$$Q_2 = \sqrt{\frac{R_1}{R_2}\left(1 + Q_1^2\right) - 1} \qquad Q_1^2 > \frac{R_2}{R_1} - 1$$

Impedance matching with lumped elements

- T-type matching circuits



$$\omega C_1 = 1/(R_1 Q_1) \qquad \omega L_2 = Q_2 R_2$$

$$\omega C_3 = (Q_2 - Q_1)/\left[R_2\left(1 + Q_2^2\right)\right]$$

$$Q_1 = \sqrt{\frac{R_2}{R_1}\left(1 + Q_2^2\right) - 1} \qquad Q_2^2 > \frac{R_1}{R_2} - 1$$

- Can incorporate active device lead and bondwire inductances within matching circuit
- Provides significant level of harmonic suppression
- Widely used as input, interstage, and output matching circuits in high power amplifiers

Impedance matching with lumped elements

- **π-type**
  - High impedance matching: typically used for matching high impedance sources to lower impedance loads.
  - High Q-factor: they are suitable for applications requiring a high quality factor (Q), which helps in achieving narrow bandwidth filtering.
- **T-type**
  - Low impedance matching: preferred for matching lower impedance sources to higher impedance loads.
  - Broadband matching: they are more suitable for broadband applications due to their ability to provide a wider bandwidth.

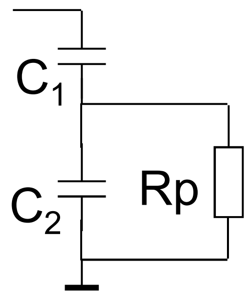
- General Considerations
  - Frequency Range: both types can be designed for specific frequency ranges; the choice depends on the required bandwidth and impedance transformation ratio.
  - Power handling: ensure that the components used can handle the power levels in your application.

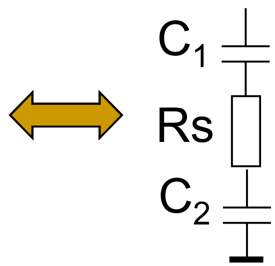
Impedance transformation

- Narrow band series/parallel transformation (ω_0)

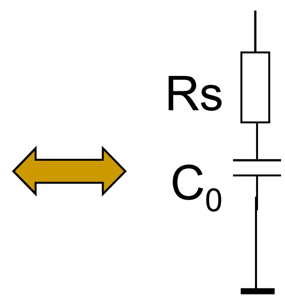
If $Q_p = R_p C_2 \omega_0 > 3$

If $Q_s = \dfrac{1}{R_s C_0 \omega_0} > 3$

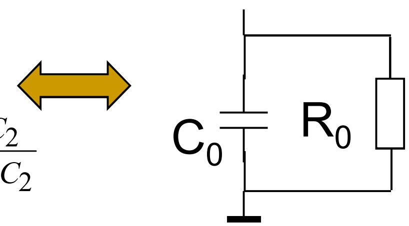




$$R_s = \frac{R_p}{Q_p^2}$$

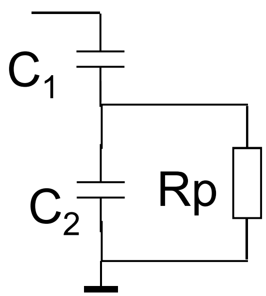


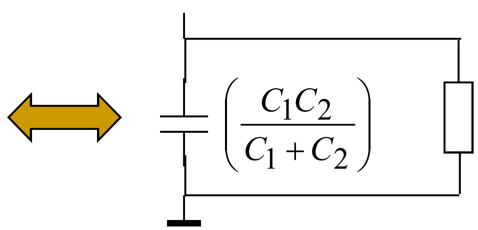
$$C_0 = \frac{C_1 C_2}{C_1 + C_2}$$



$$R_0 = R_s Q_s^2 = \frac{1}{R_s (C_0 \omega_0)^2} =$$

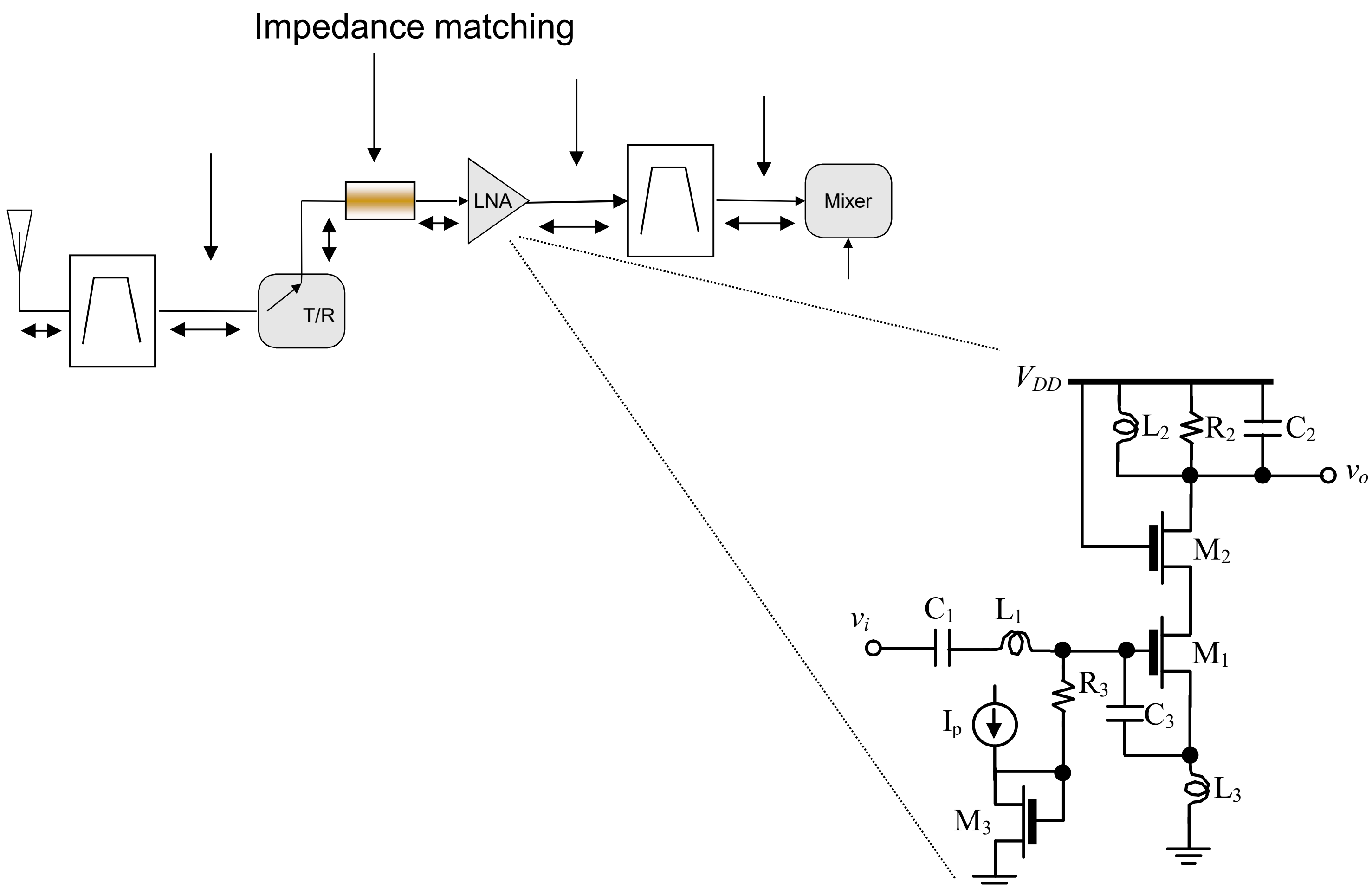
$$\frac{Q_p^2}{R_p (C_0 \omega_0)^2} = \frac{R_p^2 (C_2 \omega_0)^2}{R_p (C_0 \omega_0)^2}$$





$$\left(\frac{C_1 C_2}{C_1 + C_2}\right) \qquad R_p \left(1 + \frac{C_2}{C_1}\right)^2$$

Basic concepts – impedance matching

- 

References

- Lee, T. H. (2003). *The Design of CMOS Radio-Frequency Integrated Circuits* (2nd ed.). Cambridge University Press.