Synthesis of a 4th order Chebyshev Passive Filter

OBJECTIVE: Practice the procedures to design and implement a passive filter.

REPORT: Due the 30th October

1. FILTER SPECIFICATION (BAND-PASS).

Figure 1 illustrates the specification's diagram of a pass-band filter, where $f_1 = 8$ kHz, $f_2 = 12{,}5$ kHz, $f_3 = 1$ kHz, $f_4 = 40$ kHz, $A_{max} = 3$ dB and $A_{min} = 40$ dB.

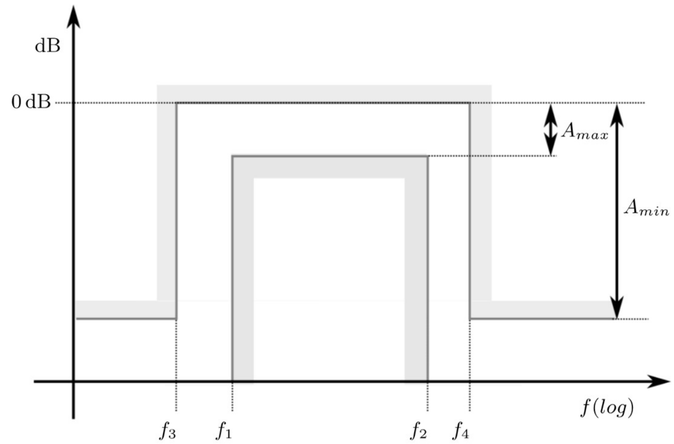

Figure 1 – Band-pass filter specification.

1.1 Show that a realization of this filter with a 4th order Chebyshev approximation function fulfils the specification's diagram. Note: check the symmetry conditions and, in any case, stick your design to the f_1 and f_2 frequencies.

1.2 Adopt the corresponding tabulated normalized Chebyshev polynomial and determine the two conjugated poles of the normalized low-pass function.

1.3 Determine the normalized Chebyshev transfer function and the corresponding real frequency filter transfer function.

2. SYNTHESIS OF THE FILTER

2.1 After obtaining the normalized transfer function, calculate the components values of the final passive LC filter. Consider a load resistance of 1 kΩ.

3. SIMULATION

3.1 Draw the schematic of the obtained filter and do an *.ac* simulation to confirm its characteristics.

3.2 Carry out now a transient simulation using for the input signal a voltage source comprising three frequencies, $v_i(t) = \sin(2\pi 1 \times 10^3) + 2 \cdot \sin(2\pi 10 \times 10^3) + \sin(2\pi 40 \times 10^3)$. Observe the input and output signals and draw your conclusions.

HOME WORK

4. SYNTHESIS WITH TWO 2ND-ORDER STAGES

Convert the obtained Chebyshev transfer function into the product of two 2nd order functions, $H(s) = H_{\sec 1}(s) \times H_{\sec 2}(s)$. For that purpose, resort to the Geffe algorithm (described in the annex), to achieve the factorization of 4th-order functions with complex poles.

Note: Following the annex, calculate $A_1 \times A_2 = 1/(|H_{sec1}(\omega_0)| \times |H_{sec2}(\omega_0)|)$, to ensure a 0 dB attenuation at the central frequency in the pass band. Suggestion: use Excel to perform these calculations.

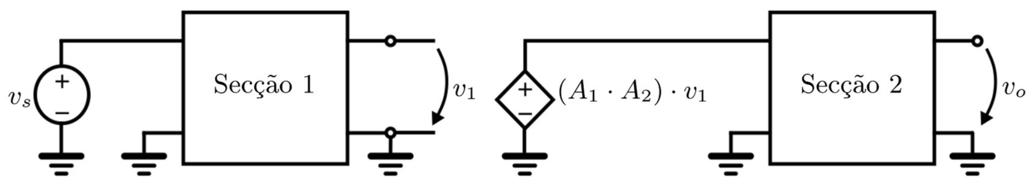

Figure 2 – Representation of the filter with two 2nd-order stages.

Calculate the new values of the passive components required to implement the filter after the two 2nd order stages, and draw the new simulation schematic according to the scheme depicted in figure 2. The voltage controlled – voltage source (E element in the LTSpice library) of gain A_1A_2 is used to avoid the load effect of the second stage on the first stage, which would change the overall performance of the final filter.

4.1 Repeat tasks 3.1 and 3.2 and characterize the achieved solution.

Annex 1: Geffe algorithm.

This algorithm allows finding the values for the factorization of the 4th-order denominator polynomial that results from the denormalization of the normalized filter, into the product of two 2nd-order functions, i. e.,

$$\left(s^2 + \frac{\omega_{o1}}{Q}s + \omega_{o1}^2\right) \times \left(s^2 + \frac{\omega_{o2}}{Q}s + \omega_{o2}^2\right) = s^4 + \alpha_1 \cdot s^3 + \alpha_2 \cdot s^2 + \alpha_3 \cdot s + \alpha_4$$

Note that the two 2nd-order polynomials show the same quality factor $\boldsymbol{Q}$.

First, let's define the starting point:

1- Define the conjugated poles of the normalized 2nd-order filter: : $-a \pm jb$ (taken form tables of the considered normalized approximation function – Butterworth, Chebyshev, or other) .
2- Find an auxiliary quality factor $Q_R = \omega_o/B$, with $B = \omega_2 - \omega_1$ and $\omega_o = \sqrt{\omega_1 \cdot \omega_2}$**,** after the values taken from the specifications diagram, assuming symmetry.

Considering these values, one can obtain the following auxiliary variables and the final parameters of the 2nd-order functions $\boldsymbol{Q}$, $\boldsymbol{\omega_{o1}}$ e $\boldsymbol{\omega_{o2}}$:

$C = a^2 + b^2$,

$D = 2 \cdot a/Q_R$,

$E = 4 + (C/Q_R^2)$,

$G = \sqrt{E^2 - 4 \cdot D^2}$,

$\boldsymbol{Q} = (1/D) \cdot \sqrt{(E+G)/2}$ (this will be the Quality factor of the two 2nd-order polynomials)

$M = a \cdot \boldsymbol{Q}/Q_R$,

$W = M + \sqrt{M^2 - 1}$,

$\boldsymbol{\omega_{o1}} = \omega_o/W$,

$\boldsymbol{\omega_{o2}} = \omega_o \cdot W$,

These parameters allow us to write the product of the two functions:

$$H_{P-B}(s) = H_{sec1}(s) \times H_{sec2}(s) = \frac{(\omega_{o1}/Q) \cdot s}{\left(s^2 + \frac{\omega_{o1}}{Q}s + \omega_{o1}^2\right)} \times \frac{(\omega_{o2}/Q) \cdot s}{\left(s^2 + \frac{\omega_{o2}}{Q}s + \omega_{o2}^2\right)}$$

It is necessary to correct the gain since, for $s = j\omega_o$ (central frequency), it is obtained $|H_{P-B}(j\omega_o)| \neq 1$. One can do an individual adjustment, such that $A_1 = A_{o1} \times |H_{sec1}(j\omega_o)| = 1$ and $A_2 = A_{o2} \times |H_{sec2}(j\omega_o)| = 1$, where A_{o1} and A_{o2} are the gains to be introduced in each individual filter so that each filter presents a unit gain at the central frequency ($s = j\omega_o$). Then:

$$H_{P-B}(s) = H_{sec1}(s) \times H_{sec2}(s) = A_1 \times A_2 \times \frac{(\omega_{o1}/Q) \cdot s}{\left(s^2 + \frac{\omega_{o1}}{Q}s + \omega_{o1}^2\right)} \times \frac{(\omega_{o2}/Q) \cdot s}{\left(s^2 + \frac{\omega_{o2}}{Q}s + \omega_{o2}^2\right)}$$

where:

$A_1 = \sqrt{1 + Q^2 \cdot \left(\frac{\omega_o}{\omega_{o1}} - \frac{\omega_{o1}}{\omega_o}\right)^2}$ and $A_2 = \sqrt{1 + Q^2 \cdot \left(\frac{\omega_o}{\omega_{o2}} - \frac{\omega_{o2}}{\omega_o}\right)^2}$.

Annex 2: Voltage-controlled voltage source

In Spice a voltage-controlled voltage source can be obtained with the E element. The general syntax of the linear version is (the descriptors between brackets are optional):

Exxx n+ n- <VCVS> in+ in- gain <MAX=val> <MIN=val> <SCALE=val>
+ <TC1=val> <TC2=val> <ABS=1> <IC=val>

Exemple: The following line descrives the implementation of an operational amplifier, in which the output voltage is obtained nodes out and 0 with a gain of 10, $V_{out-0} = (V_{in+} - V_{in-}) \times 10$.

Eoa out 0 in+ in- 10.0 MAX=+5 MIN=-5

ABS	Output is absolute value if ABS=1.
Exxx	Voltage controlled element name. The parameter must begin with an "E" followed by up to 1023 alphanumeric characters.
gain	Voltage gain
IC	Initial condition: the initial estimate of the value(s) of the controlling voltage(s). If IC is not specified, the default=0.0.
in +/-	Positive or negative controlling nodes. Specify one pair for each dimension.
MAX	Maximum output voltage value. The default is undefined and sets no maximum value.
MIN	Minimum output voltage value. The default is undefined and sets no minimum value.
n+/-	Positive or negative node of controlled element
SCALE	Element value multiplier
TC1,TC2	First and second order temperature coefficients. The SCALE is updated by temperature: $SCALEeff = SCALE \cdot (1 + TC1 \cdot \Delta t + TC2 \cdot \Delta t^2)$
VCVS	Keyword for voltage controlled voltage source. VCVS is a reserved word and should not be used as a node name.