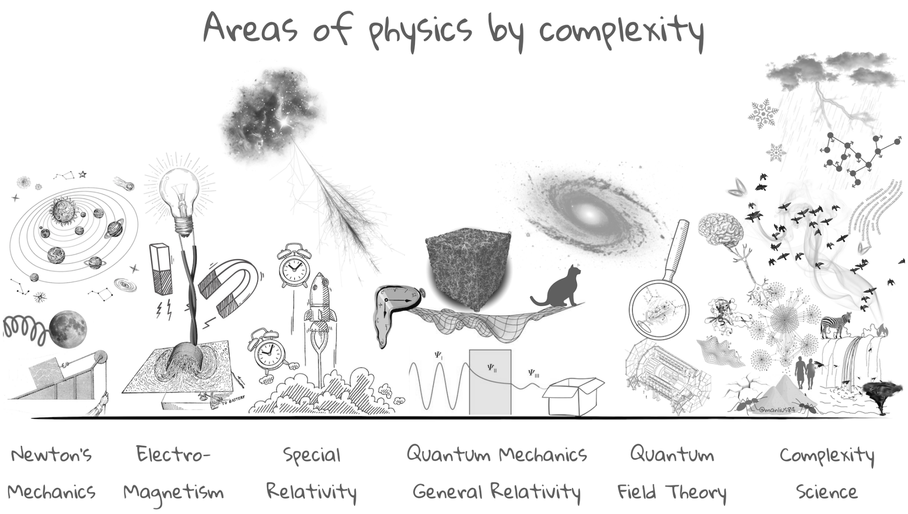
Università degli Studi di Padova



Project 20: Swarmalators

Poccianti Gabriele

Last update: September 14, 2024

Final Report

Physics of Complex Networks: Structure and Dynamics

Contents

1 | Swarmalators on a complete network

1.1 | Model presentation

This model attempts to describe the simultaneous swarming (self-organization of positions in space) and synchronization (self-organization of internal states in time) observed in groups of animals. However, the resulting conclusions may also be applicable to other types of systems, such as colloidal suspensions of magnetic particles [1, 2, 3]. In many frameworks, those two behaviors—swarming and synchronization—interact. For instance some population of myxobacteria appear to have a biochemical degree of freedom (d.o.f.), which can be modeled as a phase. Evidence suggests the presence of a bidirectional coupling between spatial and phase dynamics [4]. Another example can be chemotactic oscillators, whose movements in space are mediated by the diffusion of a background chemical [5, 6].

In our treatment we considered a generalization of the Kuramoto model [7] on a network [8], where we ignored alignment (which would have required an additional d.o.f.), and we associated to each node at time t a phase and a position in the xy plane, as in [9, 10]:

$$\frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i + \frac{1}{k_i}\sum_{j=1}^{N} A_{ij}\left[\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|}\left(A + J\cos(\theta_j - \theta_i)\right) - B\frac{\mathbf{x}_j - \mathbf{x}_i}{|\mathbf{x}_j - \mathbf{x}_i|^2}\right] \tag{1.1}$$

$$\frac{d\theta_i}{dt} = \omega_i + \frac{K}{k_i}\sum_{j=1}^{N} A_{ij}\frac{\sin(\theta_j - \theta_i)}{|\mathbf{x}_j - \mathbf{x}_i|} \tag{1.2}$$

$\mathbf{x}_i$: Position vector of the ith swarmalator in space
θ_i : Phase of the ith swarmalator
$\mathbf{v}_i$: Self-propulsion velocity vector
ω_i : Natural frequency
A_{ij} : Element of the adjacency matrix of the network
k_i : Degree of node i
N : Total number of swarmalators in the system
J : Position-phase coupling constant
K : Synchronization coupling constant
A : Amplitude of the simple attraction term
B : Amplitude of the repulsion term

From now on we set

$$A = B = 1 \qquad \mathbf{v}_i = \mathbf{0} \qquad \omega_i = 0 \ \forall i$$

We firstly considered a complete network and, by means of numerical simulations, we reproduced the different states of the system (described in [9, 10]) as J and K vary.

1.2 | Numerical simulations

For our simulation we relied on the R numerical implementation of the Kuramoto model showed in the third laboratory of this course (which used the deSolve package for numerical handling of ODEs). We extended the model including the equations of motion paying attention not to have singular initial conditions. Luckily [11] ensures that, given initial non-collisional data (initial phases and positions were uniformly generated in $[0, 2\pi[\times [0, 1]^2$, with the previous presciption), a minimal inter-particle distance among the swarmalators is guaranteed. Even so we still checked the distance matrix at every step to prevent numerical divergence in eq. (1.1)-(1.2).

We chose the number of swarmalators $N = 300$, step of integration $dt = 0.05$ and $T = 2000$ timesteps (in fig.1.5 more steps were necessary to reach the final configuration). We reproduced the noise-free states illustratated in fig 2 and fig 5 of [9]:

1. Static synchrony (fig. 1.2): the swarmalators form a cirulary simmetric, crystal-like distribution in space, and are fully synchronized in phase.
2. Static asynchrony (fig. 1.3): at any given location $\mathbf{x}$ all phases θ can occur.
3. Static phase wave (fig. 1.4): since in this special case $K = 0$ swarmalators' phases are frozen at their initial values. Still, since $J > 0$, units want to settle near others with similar phase.
4. Splintered phase wave (fig. 1.5): moving to $K < 0$ static phase wave spinters into disconnected clusters of distinct phases where nodes execute small ampltiude oscillations in position and phase.
5. Active phase wave (fig. 1.6): as K decreases oscillations increase in amplitude until swarmalators start to execute regular cycles in both space and phase [1].

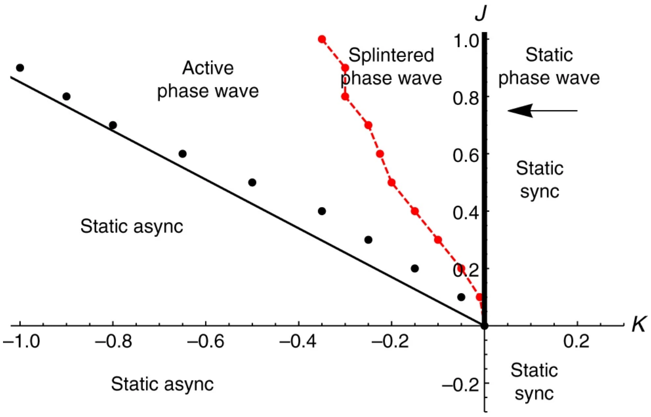


Figure 1.1: Phase diagram from [9]. The straight line separating the static async and active phase wave states is a semi-analytic approximation, while dots are derived from simulation data

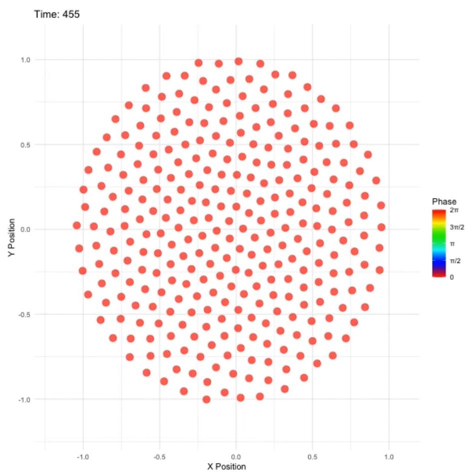


Figure 1.2: Static synchronous state ($K = 1,\ J = 0.1$)

[1]This final state is similar to the double milling states found in biological swarms [12]

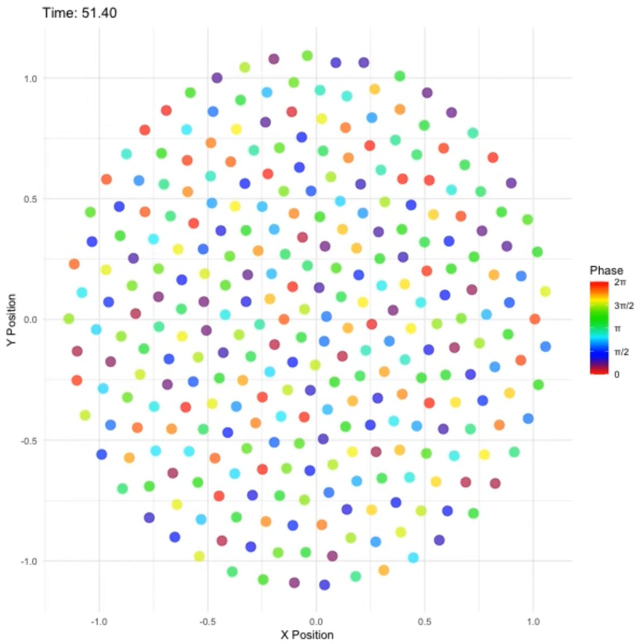


Figure 1.3: Static asynchronous state ($K = -1,\ J = 0.1$)

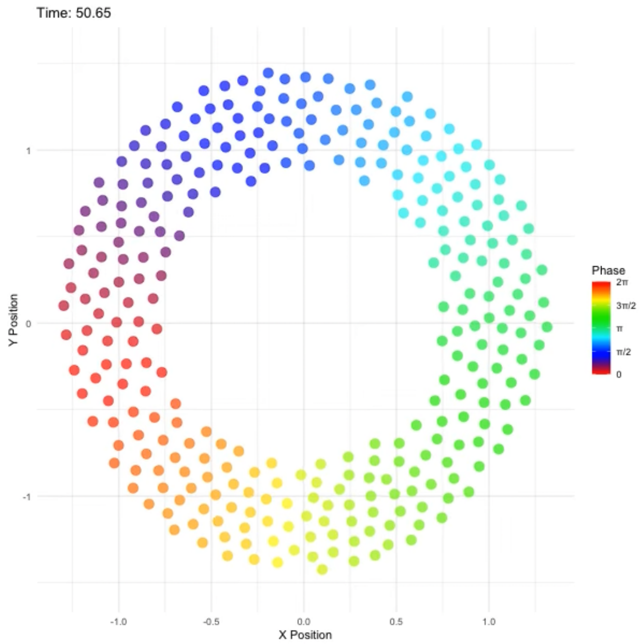


Figure 1.4: Static phase wave state ($K = 0,\ J = 1$)

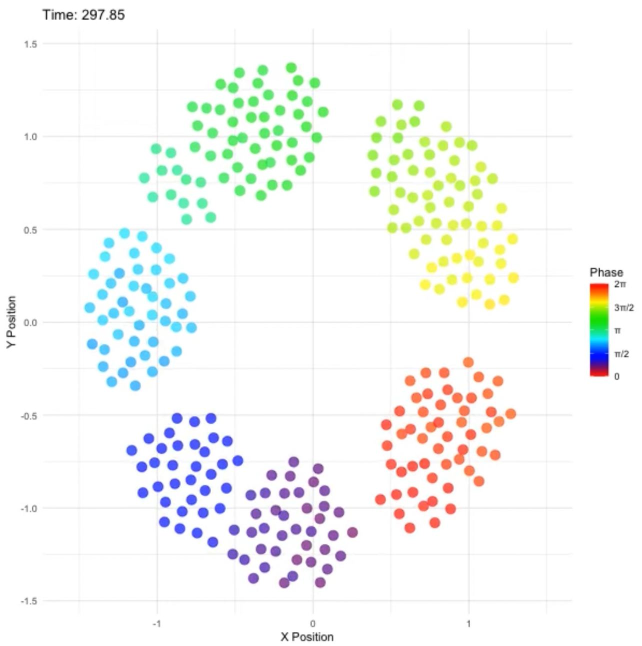


Figure 1.5: Splinter phase wave state ($K = -0.1,\ J = 1$)

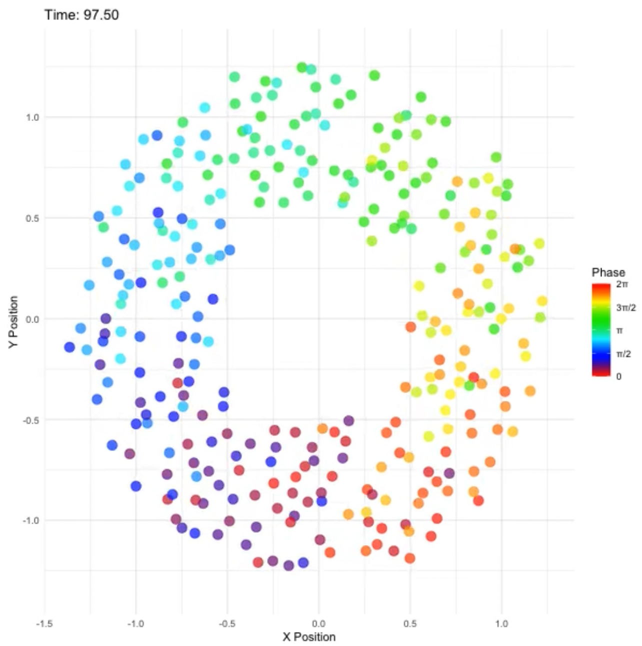


Figure 1.6: Active phase wave state ($K = -0.6,\ J = 0.9$)

2 | Swarmalators on an ER Network

We now explore how the active phase wave state ($K = -0.6$, $J = 0.9$) behaves on an Erdős-Rényi (ER) network as the connection probability p increases.

2.1 | Order Parameters

A useful way to assess the system's state is through three order parameters [9], independent of the network topology:

- r measures the degree of phase synchronization, ranging from 0 (desynchronized) to 1 (fully synchronized), as introduced in the Kuramoto model [7]:

$$r = \left| \frac{1}{N} \sum_{j=1}^{N} e^{i\theta_j} \right| \tag{2.1}$$

- S quantifies the correlation between phases θ_j and spatial angles ϕ_j:

$$S = \max \left\{ \left| \frac{1}{N} \sum_{j=1}^{N} e^{i(\phi_j + \theta_j)} \right|, \left| \frac{1}{N} \sum_{j=1}^{N} e^{i(\phi_j - \theta_j)} \right| \right\} \tag{2.2}$$

- U measures the fraction of swarmalators that complete at least one full cycle in both space and phase:

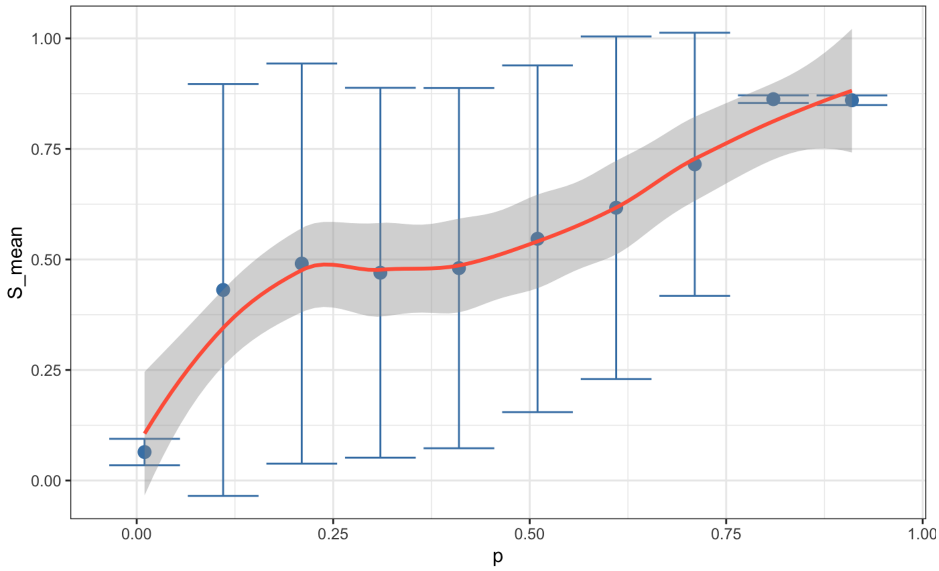
$$U = \frac{N_{rot}}{N} \tag{2.3}$$

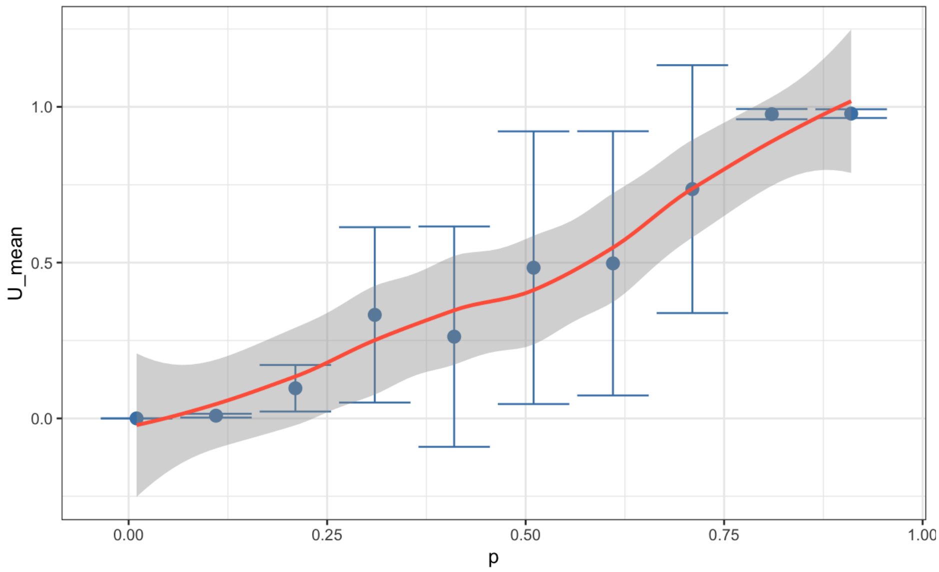
S is positive in both splintered and active phase wave states, while U is non-zero only in the active phase wave.

2.2 ER Network

Using tools from the third laboratory, we implemented functions to compute the order parameters for each simulation. After reaching the steady state ($t_f = 300$), we computed r, S (discarding the transient dynamics), and U. Each value of p was averaged over ten different realizations of the system to obtain means and standard deviations.

The results for S and U are shown below.





Both S and U increase monotonically with p. As connectivity increases, more swarmalators interact, leading to greater coordination in phase and spatial rotation. However, these results, especially for S, may be approximate due to the limited simulation time.

Remarks

Part of the code was generated with the assistance of ChatGPT (v4), though it was never used blindly. For instance this tool helped build functions for the calculation of the U order parameter and for video generation. Additionally, this tool was helpful in checking grammar and making minor adjustments to improve the readability of this text.

3 | Appendix 1: synchronization

Here are shown the phases oscillations corresponding to fig.1.2-1.6:

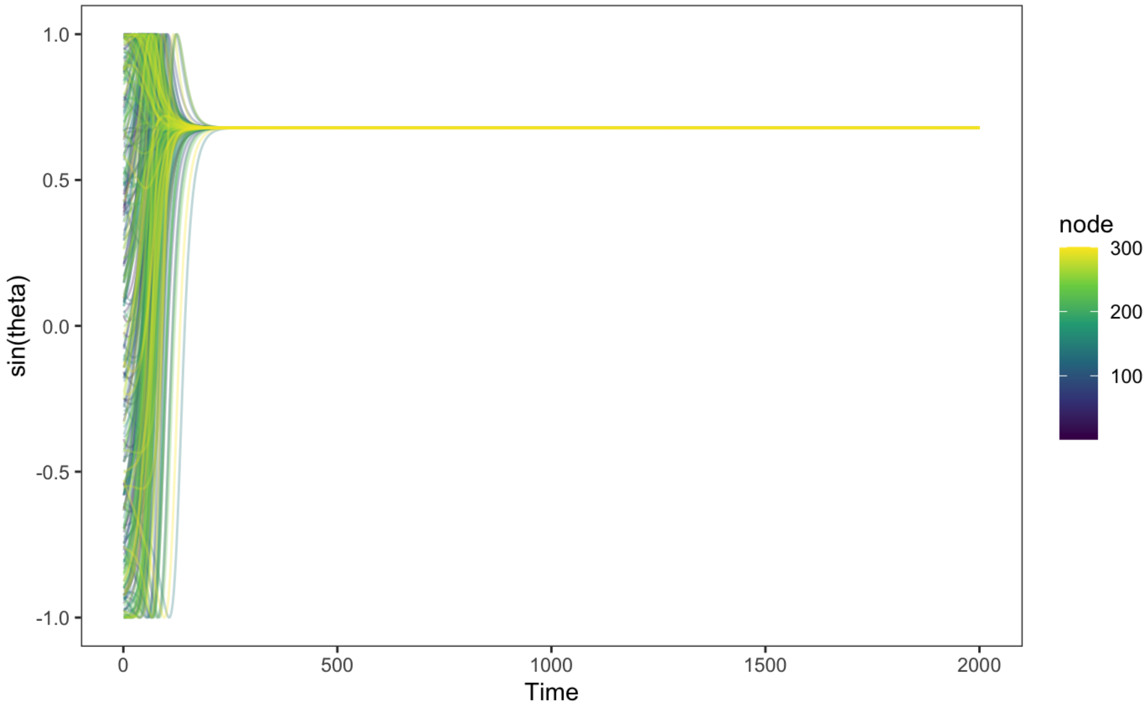


Figure 3.1: Static synchronous state
$(K = 1,\ J = 0.1)$

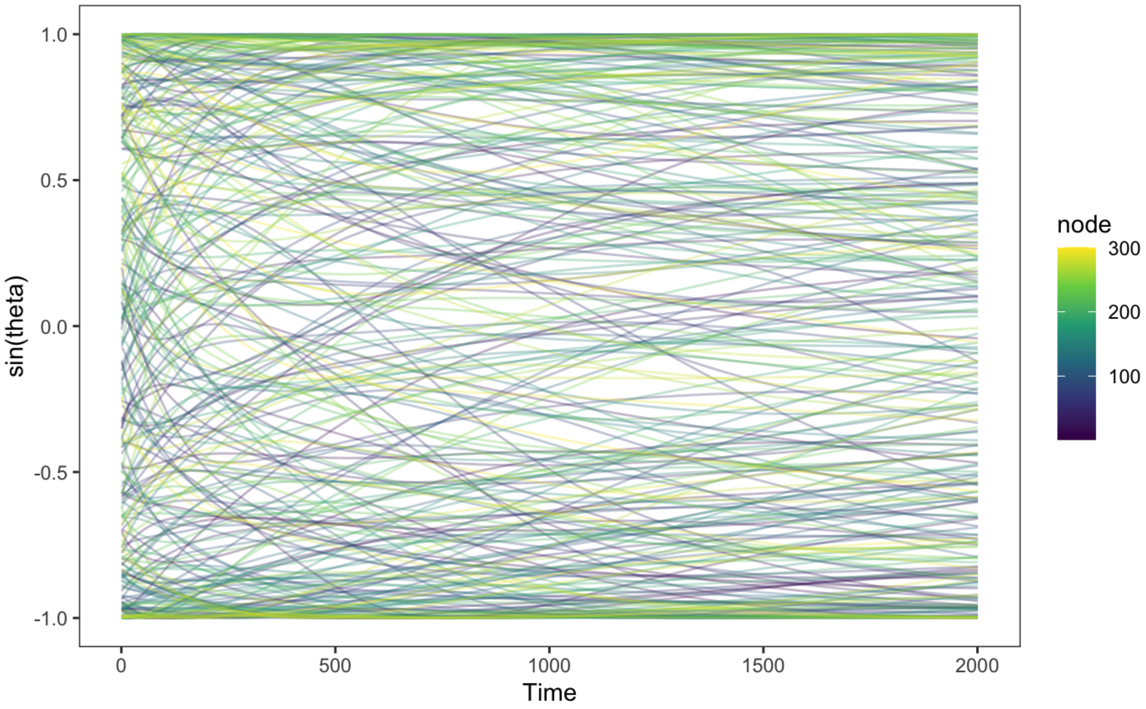


Figure 3.2: Static asynchronous state
($K = -1,\ J = 0.1$)

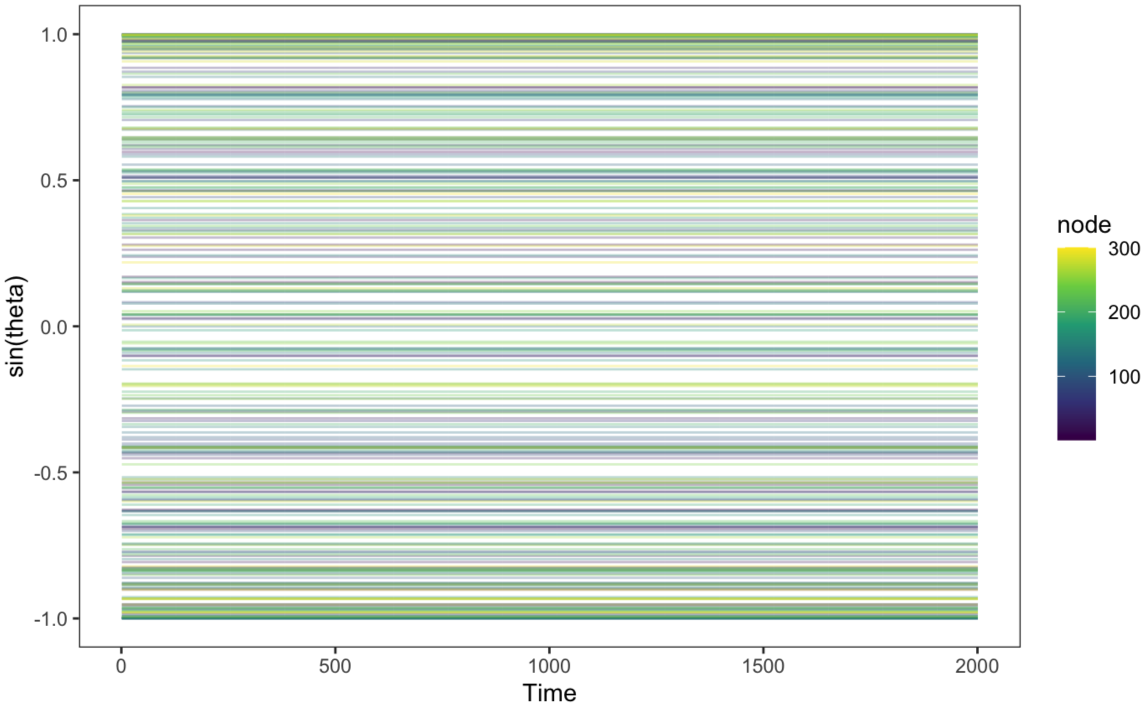


Figure 3.3: Static phase wave state
($K = 0,\ J = 1$)

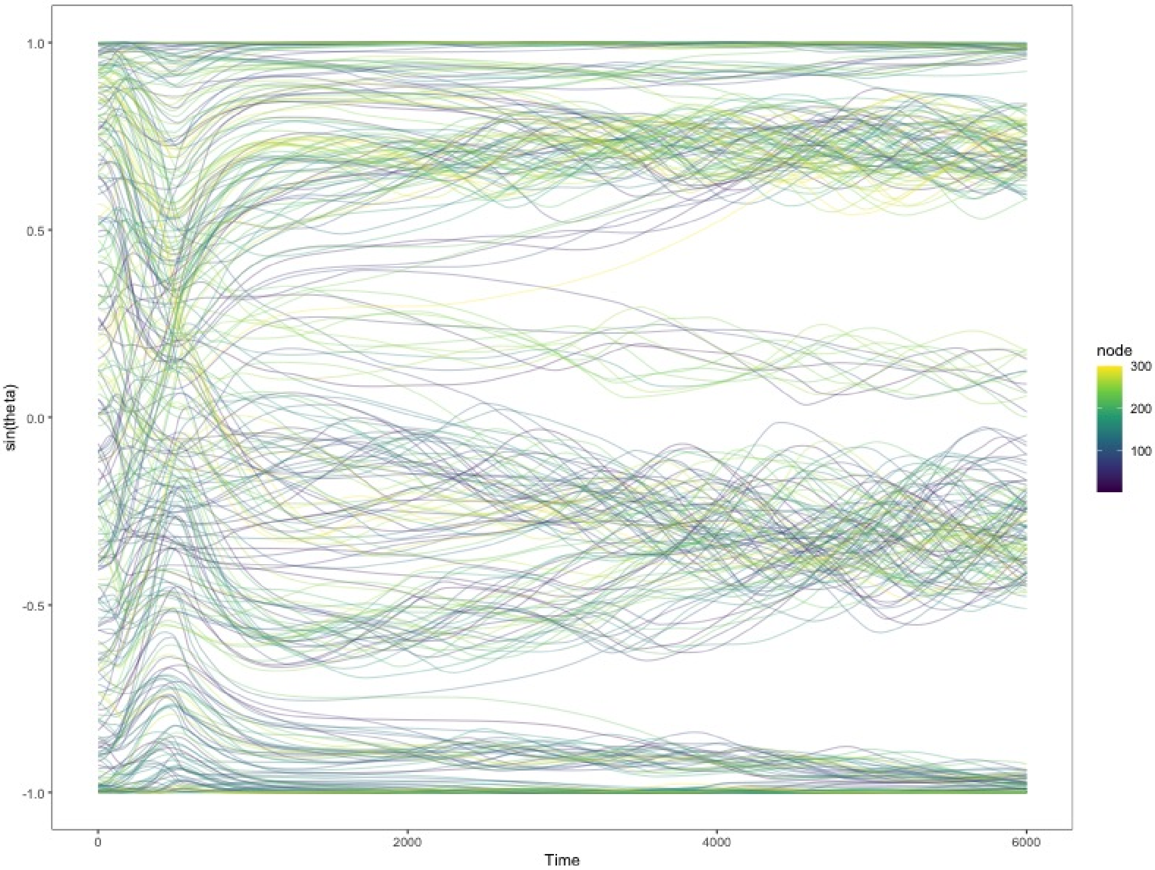


Figure 3.4: Splinter phase wave state
$(K = -0.1,\ J = 1)$

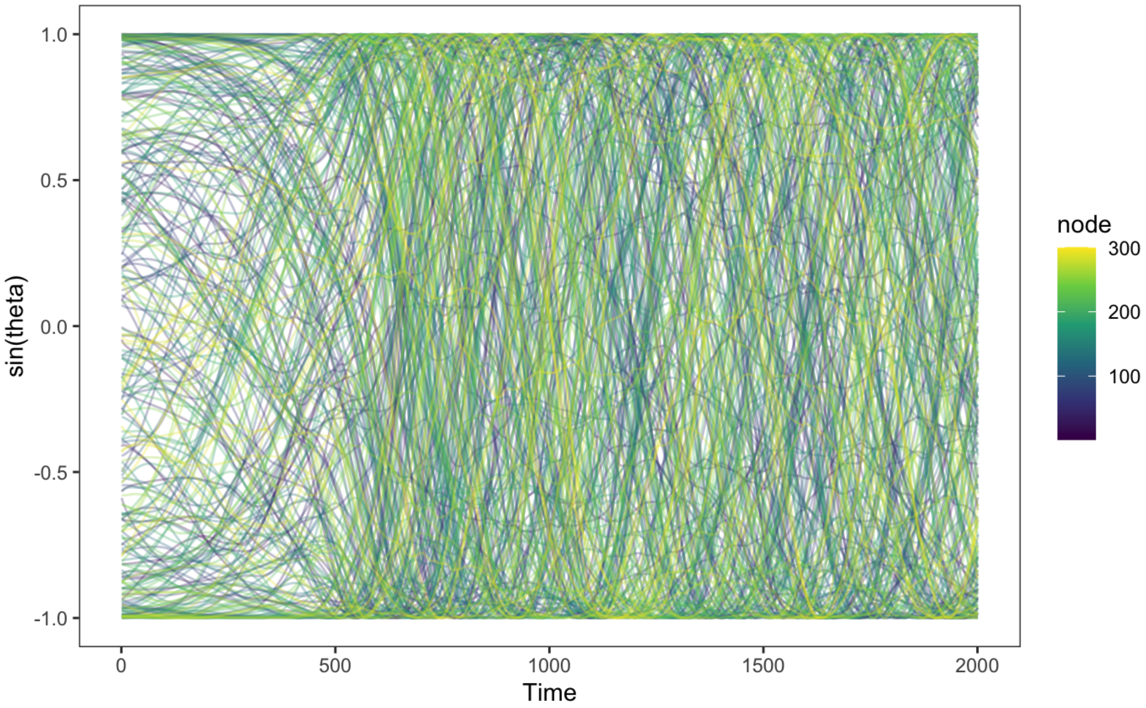


Figure 3.5: Active phase wave state
($K = -0.6$, $J = 0.9$)

4 | Bibliography

[1] James E. Martin and Alexey Snezhko. Driving self-assembly and emergent dynamics in colloidal suspensions by time-dependent magnetic fields. *Reports on Progress in Physics*, 76(12), 11 2013.

[2] Alexey Snezhko and Igor S. Aranson. Magnetic manipulation of self-assembled colloidal asters. *Nature materials*, 10 9:698–703, 2011.

[3] Jing Yan, Moses Bloom, Sung Chul Bae, Erik Luijten, and Steve Granick. Linking synchronization to self-assembly using magnetic janus colloids. *Nature*, 491:578–581, 2012.

[4] O. A. Igoshin, A. Mogilner, R. D. Welch, D. Kaiser, and G. Oster. Pattern formation and traveling waves in myxobacteria: theory and modeling. *Proceedings of the National Academy of Sciences*, 98:14913–14918, 2001.

[5] M. Iwasa, K. Iida, and D. Tanaka. Hierarchical cluster structures in a one-dimensional swarm oscillator model. *Physical Review E*, 81:046220, 2010.

[6] D. Tanaka. General chemotactic model of oscillators. *Physical Review Letters*, 99:134103, 2007.

[7] Juan Acebron, Luis Bonilla, Conrad Vicente, Felix Ritort, and Renato Spigler. The kuramoto model: A simple paradigm for synchronization phenomena. *Reviews of Modern Physics*, 77:137, 01 2005.

[8] Francisco A. Rodrigues, Thomas K. DM. Peron, Peng Ji, and Jürgen Kurths. The kuramoto model in complex networks. *Physics Reports*, 610:1–98, January 2016.

[9] Kevin P. O'Keeffe, Hyunsuk Hong, and Steven H. Strogatz. Oscillators that sync and swarm. *Nature Communications*, 8(1), November 2017.

[10] Gourab Kumar Sar and Dibakar Ghosh. Dynamics of swarmalators: A pedagogical review. *Europhysics Letters*, 139(5):53001, August 2022.

[11] Jeongho Kim Jinyeong Park Seung-Yeal Ha, Jinwook Jung and Xiongtao Zhang. Emergent behaviors of the swarmalator model for position-phase aggregation. *Mathematical Models and Methods in Applied Sciences*, 29(12):2225, 2019.

[12] J. Carrillo, Maria R. D'Orsogna, and Vladislav Panferov. Double milling in self-propelled swarms from kinetic theory, preprint uab. *Kinet. Relat. Models*, 2:363–378, 06 2009.