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November 9th, 2020  
Math 5610

## Project Report

We began the project by tackling the exercises on an individual level and then comparing answers. Personally, I initially got caught up on translating from transforming from cartesian to DMS and DMS to cartesian in general time. What I was originally confused on was the idea that latitude and longitude always stay the same no matter the time, whereas xyz coordinates change with time. I initially botched my attempts to formulate the conversion from cartesian to DMS since I couldn't figure out a way to determine the initial DMS data, but then I realized once we convert cartesian coordinates to DMS coordinates we can account for the time offset by tracing the longitude backwards according to the earth's movement. Once we finished the first 7 problems we compared answers and found that we approached the problems very differently. For example, my partner accounted for shifting the phi angle from the vertical by using the sine function whereas I merely tacked on  $\pi/2$  to the radians. However, all of our approaches yielded the same results.

We then examined problems 8 through 12. On problem 8, we figured that we could determine the satellites that could see our given point by constructing a plane tangent to that point of the earth. Any satellites above that plane were able to see the given point and any below that plane were not visible. Problem 9 was a bit more complex, but we decided that we could expand  $X_s$  using a Taylor polynomial and then we could solve for  $t_s$  using Newton's method. From there we could plug  $t_s$  back into  $X_s$  in order to determine  $X_s$ .

The rest of the problems discussed the two cases that could yield a solution for  $t_v$  and  $X_v$ . The first case was the proper case in which we only have 4 satellites giving us data. Here, we just use Newton's method to solve  $F \cdot \text{solutionVec} = 0$ . We can use the information received from the vehicle to make our first guess.

The second and more complex case is the overdetermined case in which we have data from more than 4 satellites. In this case, we take the partial derivatives of  $E$ , which is merely  $F^T \cdot F$ , with respect to the  $x$  coordinate,  $y$  coordinate,  $z$  coordinate, and the  $t$ . Then, we solve the partial derivatives of  $E$  set equal to 0 which yields our  $x$  coordinate,  $y$  coordinate,  $z$  coordinate, and the  $t$  coordinate.

Our code was simply based on our exercises. The toughest part of this section was figuring out how exactly the satellite, vehicle, and receiver were designed to interact as objects. Once we figured out the information that was supposed to be written from the vehicle and the satellite, the problem became much simpler. Most of our

methods were based on the algorithms described by our exercises, so that was not too difficult. The only method that was a bigger hurdle was designing the receiver to be capable of examining information given by  $n$  satellites in general. After building the satellite and receiver we put together a pipeline to allow us to determine the data output by the interactions given from the satellite, vehicle and receiver. All that data gets written into a file when the program is launched.

While working on this project, I learned how to compile different mathematical tools learned in a large assortment of classes in order to problem solve. I also learned how to effectively collaborate with regards to mathematics. I got to see how powerful numerical approximations can be for problem solving when we don't have all the necessary information. Finally, I learned how to appropriately bridge the gap between mathematics and code when working on a project. Thank you for the wonderful opportunity.