INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH5115: Parameter and State Estimation

Assignment 1

Due: Thursday, October 22, 2020

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1. (a) Determine if the random process $v[k] = A\cos^2(2\pi fk + \phi)$, where ϕ is a constant but A is a random variable with zero mean and unit variance, is covariance stationary.

   (b) The random walk process $v[k] = v[k-1] + e[k]$ is known to be variance non-stationary. Assuming $v[0] = 0$, prove this result. Verify your finding numerically using MATLAB.

2. A process evolves as $y[k] = y^\star[k] + e[k]$, where $y^\star[k] = \dfrac{b_2^0 q^{-2}}{1 + f_1^0 q^{-1}} u[k]$, $u[k]$ is a known signal and $y[k]$ is the measured version of $y^\star[k]$. The measurement noise is $e[k] \sim \text{WN}(0, \sigma_e^2)$ and $u[k] \sim \text{WN}(0, \sigma_u^2)$. Assume $\sigma_{eu}[l] = 0,\ \forall l$.

   (a) Develop expressions for σ_y^2, $\sigma_{yy}[1]$, $\sigma_{yu}[1]$, and $\sigma_{yu}[2]$ in terms of the variances of $u[k]$ and the white-noise sequences, i.e., σ_u^2 and σ_e^2 respectively.

   (b) Generate $N = 500$ observations of $y[k]$ with $\sigma_u^2 = 2$. Adjust σ_e^2 such that the SNR $\sigma_{y^\star}^2/\sigma_e^2$ is set to 10. Estimate the quantities (variance, auto-covariance and cross-covariance) in (2a) and compare their closeness with the theoretical answers in (2a).

3. For the series given in `a2_q3.mat`,

   (a) Determine the presence of any integrating effects.

   (b) Fit a suitable ARIMA model. Report all the necessary steps and the final model.

4. (a) For a GWN process $y[k] \sim \mathcal{N}(\mu, \sigma^2)$, where $0 \le \mu < \infty$, derive the ML estimate and Fisher information of μ given N observations and known σ^2.

   (b) Consider the linear regression problem $Y = aX + b + \varepsilon$. Determine the Fisher information of parameters a and b contained in N observations $\{(y[k], x[k])\}_{k=1}^N$ assuming X is free of randomness and $\varepsilon \sim \mathcal{N}(0, \sigma_e^2)$. Verify your analytical answer (for the ML estimate) with simulation in MATLABby plotting the likelihood functions and locating the maximum. Choose $N = 100$, $\sigma_e^2 = 1$, $a = 2, b = 3$ and $\mu_0 = 1$ (true value).

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