INDIAN INSTITUTE OF TECHNOLOGY MADRAS
Department of Chemical Engineering

CH5115: Parameter and State Estimation

Assignment 5

Due: Friday, January 22, 2021

---

1. A linear time-varying dynamical system is known to be switching between the following two models (depending on the operating conditions)

   Model 1: $y[k] = a_1^{(1)} y[k-1] + b_1 u[k-1] + b_2 u[k-2] + e[k], \qquad e[k] \sim \mathcal{N}(0, \sigma_e^2)$

   Model 2: $y[k] = a_1^{(2)} y[k-1] + a_2 y[k-2] + b_2 u[k-2] + b_3 u[k-3] + e[k], \quad e[k] \sim \mathcal{N}(0, \sigma_e^2)$

   Kautuhalya performs an experiment that supposedly excites the process in both regimes. The data is provided in `ltvdata1.mat`

   (a) Formulate the identification of switching times and the models as that of a recursive LS.

   (b) Using the `recursiveLS` routine, estimate the switching times and model parameters for each regime by optimizing the forgetting factor $\lambda \in [0.95, 1]$.

   (c) Solve the same problem using a sliding window method (the so-called *finite history* approach) using the `recursiveLS` routine. Optimize the window length (in number of observations) for a reasonable trade-off between variance of parameter estimates and identification of switching times.

   (d) Which among the methods in (1b) and (1c) are better suited for the given data?

2. Given the SS description of a discrete-time LTI system

$$\mathbf{A} = \begin{bmatrix} 0.9 & 0 & 0 \\ 1 & 1.2 & -0.5916 \\ 0 & 0.5916 & 0 \end{bmatrix}; \; \mathbf{B} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}; \; \mathbf{C} = \begin{bmatrix} 2 & 0.8 & -0.6761 \end{bmatrix}; \; \mathbf{D} = 0$$

   do the following:

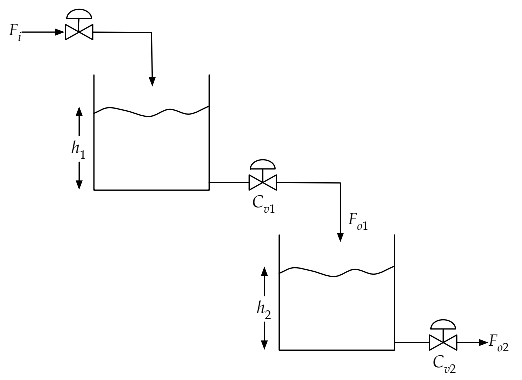
   (a) Determine if the given description is a minimal realization. If yes, proceed to the next task. Else, construct a minimal realization of the maximum order.

   (b) Design an observer for the given SS / minimal realization such that the eigenvalues are placed in the interval $[0.05, 0.3]$.

   (c) Implement the designed observer on the given system and verify its performance. For this purpose, you may use the `estim` routine while setting `sensors` and `known` to 1, respectively.

   (d) Repeat (2c) under noisy conditions, with variances of process and measurement noise set to 0.1 and 1, respectively. What would be the effect (on your state estimation) of choosing very small vs. high eigenvalues for the observer? Support your answer with arguments and also preferably with simulation results.

3. Consider a two tank system as shown in the figure below.



Parameter	Value
A_1	1.2
A_2	1.2
C_{v1}	0.8
C_{v2}	0.8
$F_{i,s}$	2
T_s	2

where it is assumed that the tanks are cylindrical with cross-section areas A_1 and A_2. Perform / answer the following.

(a) Develop a continuous-time state-space model with liquid levels as states and the input as F_i, the inlet flow rate. Assume that the outlet flow rates are *linearly* proportional to the heads with proportionality constants as C_{v1} and C_{v2}.

(b) Discretize the SS model in (3a) at a sampling interval $T_s = 0.1$ units. For this purpose, first create a continuous-time SS object in MATLAB and use the c2d routine.

(c) Suppose that only the outlet flow rate F_{o2} is sensed. Determine theoretically if the liquid levels in both tanks can be uniquely determined under noise-free conditions.

(d) Generate the input-output data with process and measurement noise variances set to $\mathbf{Q} = \text{diag}(0.2, 0.1)$ and $\mathbf{R} = 0.1$, respectively. Use a PRBS input sequence of $N = 1275$ length with *deviations* between -1 and +1, and containing frequencies in the band $[0 \ 0.2]$.

(e) If your answer to (3c) is yes, design and implement a Kalman filter (in MATLAB) for estimating h_1 and h_2 using the data generated in (3d), assuming that $\mathbf{R}$ is known. Tune $\mathbf{Q}$ for optimal trade-off between goodness (variance) of state and output estimates. Does this value agree with the value used in the simulation?

4. Suppose the outlet flow rates of each tank are proportional to the square root of respective liquid levels. Develop a non-linear continuous-time state-space model and implement the extended Kalman filter. Use $T_s = 0.1$ and all other settings for input generation, process and measurement noise variances as in Q.3. For implementation, make suitable modifications to the SIMULINK model developed in the class.

5. Consider a scalar-valued RV X with mean μ and variance σ_X^2. Suppose we have $(N+1)$ noisy observations of X,

$$y[k] = x[k] + v[k]$$

where $v[k] \sim \text{WN}(0, \sigma_v^2)$ and $\text{corr}(v[k], x[k]) = 0, \ \forall k$. Show that (using the Kalman filter equations) the MMSE of $x[N|N]$ is

$$\hat{\mathbf{x}}[N|N] = \alpha(N)\left(\frac{1}{N+1}\sum_{k=1}^{N} y[k]\right) + \beta(N)\bar{x}$$

where $\bar{x}$ is known before. Determine $\alpha(N)$ and $\beta(N)$. What happens when $N \to \infty$?