INDIAN INSTITUTE OF TECHNOLOGY KHARAGPUR
Time: 3 hours, Full Marks: 100, Department of E & ECE
No. of students 120, End-Autumn Semester Examination, 2015
EC 60011, Data Structure and Object Representation
B.Tech., M.Tech (Dual) and 1st year M.Tech

Answer any five questions

1. (a) Prove from definition that (2 + 2 = 4)

   I. $100n^2$ is $O(n^3)$

   II. $n^3 + \log_2(n)^{10}$ is $O(n^3)$

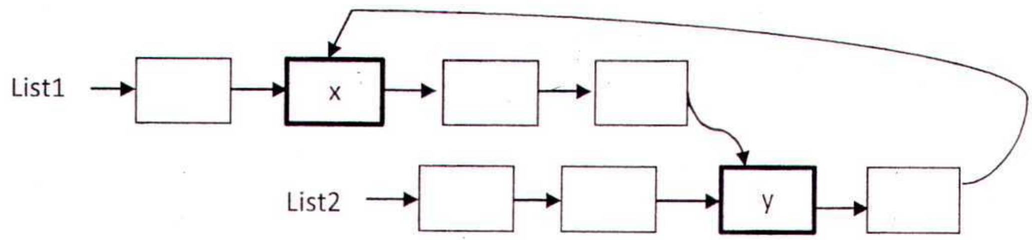
   (b) Let there be *'little **o**'* whose definition is as follows: A function $f(n)$ is $o(g(n))$ if for every real number $b > 0$ there exists an integer $a > 0$ such that for all integers $n \geq a$, $f(n) < b\, g(n)$.

   Now prove that (4 + 4 = 8)

   I. n is not $o(2n)$ but n is $o(n^2)$

   II. $\log_{10}(n)$ is $O(\log_2(n))$ but not $o(\log_2(n))$

   (c)



   Consider the example linked lists shown above. List1 ends at one of the nodes (here a node y) of List2 whereas List2 ends at one of the nodes (here a node x) of List1. From Node x to x or from node y to y it is a complete loop. Let an application needs to traverse the nodes starting from either List1 or List2. Write a generalised C function such that, while traversing such linked lists, whenever the function is invoked, the application will be able to determine whether the current node is inside or outside the loop. (8)

2. (a) Consider the pseudo code **Action** (i, j) given below. If i = A[m] and j = A[n]; m << n, explain what the algorithm does. Discuss each line and show it with an example. Name it appropriately. (5+1 = 6)

```
Action (i, j)
x := i + 1
y := j
while x ≤ y do
        while x ≤ y and A[ x ] < A[ i ] do x := x + 1
        while x ≤ y and A[ y ] ≥ A[ i ] do y := y –1
        if x < y then begin exchange A[ x ] and A[ y ]; x := x + 1; y := y –1 end
        end
Exchange A[ i ] and A[ y ].
return y –1 and y + 1
end
```

(b) Let there be a forest (orchard) of trees where any node of a tree can have zero, one, two or maximum three chidren. The nodes of the trees are represented by linked lists with four pointers, one for each children and one extra (for example if a node has one child, the rest three pointers are put to NULL). Write a C code to convert the whole of such a forest (orchard) into a binary tree to facilitate preorder traversal. Let the function be called *ConvertoBinary(p, q, r, s, t, ...)* where *p, q, r. s, t, ...* are the pointers to the root of the trees in the forest (Note: preorder traversal routine is not required). (10)

(c) Demonstrate what happens when we insert the keys 5, 28, 19, 15, 20, 33, 12, 17, 10 into a hash table with collisions resolved by chaining. Let the hash table have 9 slots and let the hash function be $h(k) = k \bmod 9$. (no program is necessary) (4)

3. (a) A *Priority Queue* is implemented with a binary tree where the only condition is that the *info* of a node is smaller than the *info* of any of its children (Heap). Therefore to take out the minimum valued info (*say* the highest priority), the root node has to be deleted and replaced by an appropriate node. If the binary tree is implemented with linked list, write a C function to carry out this (*deletemin*) operation. (The replacement technique of the root node is at your own discretion. Also the caller function is not necessary). (8)

(b) A binary tree is called the *AVL tree if every non-leaf node except the root has a sibling, and for every pair of siblings, their height differs by at most 1*. Now give an example of an AVL tree where the height of the left subtree of the root is greater than that of the right subtree of the root, but the min-height of the left subtree of the root is smaller than the min-height of the right subtree of the root. (Give the sketch of the tree. No code is necessary) (3)

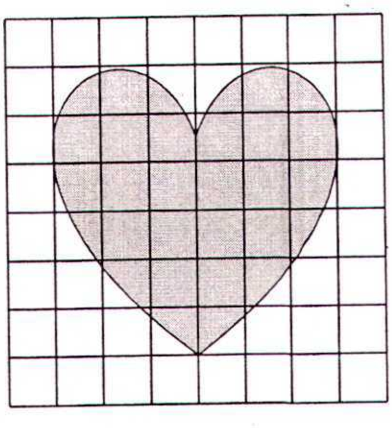
(c) Starting with an empty 2-3 tree, show the insertion of the following strings (in this order) using the usual English alphabetical ordering:

cat, hen, llama, aardvark, hog, donkey, rhino, hippo, tiger, lamb, lion, leopard, lynx, kitty, ant

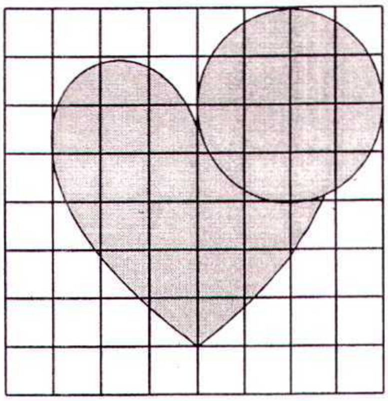
Write a pseudo code for the insertion operation. (C-code not necessary). (5+4)

4. (a) Show that it is possible to convert a *binary tree* to a *red-black tree* by working down from the root and coloring the vertex *red* only when it has a *black* parent and its virtual min-height (shortest length from that node to the *black* null leaf node) is larger than that of its sibling's (or it has no sibling). Write a C-function or a pseudo code, say *RB-Color(p)* starting to colour the root node (pointed to by *p*) *black* and then working down accordingly to convert a binary tree to RB-Tree. (8)

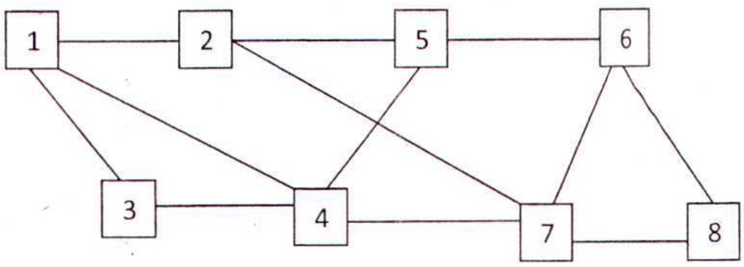
(b) Consider the image shown below. Assuming one smallest square as a pixel, represent it with help of a quad tree. No code is needed. (5)



(c) Now consider the circle below that is eating out a portion of the earlier figure. Represent the portion that is still left (not obstructed) of the earlier image with the help of a quad tree. (*Hint:* Heart – Circle). (7)



5. (a) From the undirected graph shown below, show how the algorithm proceeds and eventually forms the minimum weight spanning tree with the help of Prim's algorithm. Take vertex 5 as the initial vertex. No code is needed. (7)



Link cost: {1, 2} = 12, {1, 3} = 6, {1, 4} = 3, {2, 7} = 5, {2, 5} = 2, {3, 4} = 1, {4, 5} = 8, {4, 7} = 9, {5, 6} = 7, {6, 7} = 4, {6, 8} = 15, {7, 8} = 10 ;

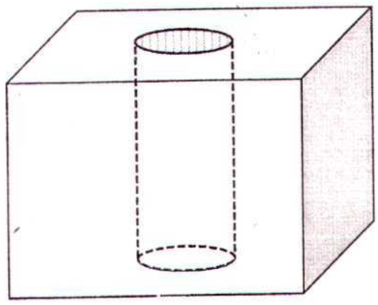
5. (b) Let a directed graph of 6 vertices be represented by a cost matrix as follows:

$$\begin{array}{c|cccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 \\
\hline
1 & 0 & 2 & 5 & 7 & \infty & \infty \\
2 & 3 & 0 & 1 & 8 & \infty & \infty \\
3 & \infty & \infty & 0 & 2 & 6 & \infty \\
4 & \infty & \infty & \infty & 0 & \infty & 10 \\
5 & 1 & \infty & \infty & 6 & 0 & 2 \\
6 & \infty & \infty & 3 & \infty & \infty & 0
\end{array}$$

Show the consecutive matrices while calculating shortest path between source node 1 and all other nodes using Dijkstra's algorithm. Write a C program for this algorithm. (6 + 7 =13)

6. (a) What do you mean by transitive closure of an adjacency matrix? What does it mean for a graph. Write a C-code that can compute the transitive closure of an adjacency matrix. (4 + 7 = 11)

(b) In the figure shown below, there is a cylindrical hole right through the cube. Show that the Euler-Poincare Relation holds true for this solid body. (5)



(c) Given a edge, show with a small pseudo-code, how a planar graph can be represented with its adjacent edges and faces. (4)

****************All the parts of a question should be in the same place ***************

************ DON'T ANSWER EXTRA QUESTIONS**********

*************** BEST OF LUCK ************