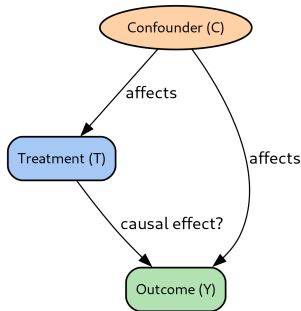




Estimating Causal Effects

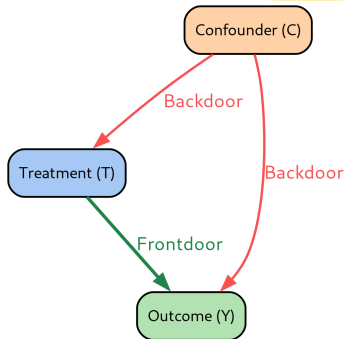
- **Goal:** Determine the causal effect of a treatment (aka intervention) variable T on an outcome Y
- **Example:**
 - T = “takes drug”
 - Y = “recovers”
 - C = “overall health”
 - Healthier people may both take medicine and recover faster \implies correlation without causation
- In **observational data**
 - Confounding variable C affects both treatment T and outcome Y
 - C creates *spurious correlation* between T and Y
- **Problem**
 - There is a “backdoor path” from $Treatment \rightarrow Confounder \rightarrow Outcome$



Frontdoor and Backdoor Paths: Intuition

- **Backdoor path**

- A backdoor path is any path from T to Y starting with an arrow into T
 - E.g., backdoor path:
 $T \leftarrow C \rightarrow Y$
- Interpretation:
 - C is a common cause of both T and Y and it confounds the relationship between T and Y
 - Controlling for (condition) C blocks the backdoor path, allowing identification of the causal effect of T on Y



- **Frontdoor path**

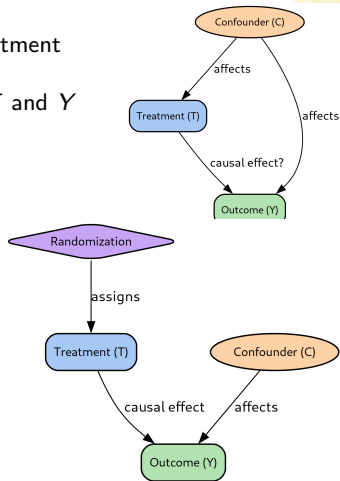
- A frontdoor path goes directly or indirectly from T to Y through mediators, following the direction of causal flow
 - E.g., front-door path: $T \rightarrow Y$
- Interpretation:
 - This is the direct causal path of interest
 - No mediators here, so the front-door path is the direct causal effect of T on Y

Randomized Controlled Trials (RCTs)

- **Randomized Controlled Trial (RCT)** is an experimental study to assess the causal effect of an intervention or treatment
 - Determine whether an intervention causes an effect, not just associated with it
 - Eliminate selection bias and confounding variables through randomization
- **Key Components**
 - *Randomization*: ensures groups are statistically equivalent at baseline
 - *Control Group*: receives a placebo or standard treatment
 - *Blinding*: participants and/or researchers do not know the assignment to avoid bias
 - *Outcome Measurement*: pre-defined metrics assess the intervention's effect
- **Example**: testing a new drug
 - Treatment group receives the new drug
 - Control group receives a placebo
 - Compare recovery rates after a fixed period
- **Pros**
 - Provides clear causal inference due to randomization
- **Cons**
 - Expensive and time-consuming
 - Ethical or practical constraints may prevent randomization

RCTs Solve the Problem of Confounders

- In **observational data**
 - Confounding variable C affects both treatment T and outcome Y
 - C creates *spurious correlation* between T and Y
- In **experimental settings**
 - Randomization (R) breaks the link between C and T
 - Everyone is assigned randomly, so nothing can influence both treatment and outcome
 - Now T is independent of C : $T \perp C$
 - The only open path between T and Y is the causal path $T \rightarrow Y$



Causal Graphs and Interventions

- Simply **observing correlations** between variables *does not reveal causality*
 - $\Pr(Y|T)$ confounds direct and indirect influences
- **Randomized Controlled Trials (RCTs)** provide the *gold standard* for causal inference
 - Randomization breaks all back-door (confounding) paths
 - But RCTs are expensive, slow, or ethically impossible
- **Alternative solution**
 - Can we estimate the *causal effect* from *observational data alone*?
 - Under *what conditions* and using *which variables*?
- **Idea**: If we identify and condition on the right *confounders*, you can
 - Block spurious associations between T and Y
 - Recover the true causal effect $P(Y|do(T))$

Intervention in Structural Equations

- **Purpose of Structural Equations**
 - Capture causal mechanisms among variables
 - Go beyond correlations: predict the impact of external interventions
- **Effect of Intervention** $do(X_j = x_j)$
 - Original equation: $X_j = f_j(Parents(X_j), U_j)$
 - Modified by intervention: $X_j = x_j$ (fixed value)
 - “Mutilate” the causal network by *removing incoming edges* to X_j
 - Use the modified structure to recompute the joint distribution of all variables
- **Intuition**
 - The *do*-operator enforces a variable’s value externally, breaking causal dependencies
 - Enables reasoning about “what would happen if...?” scenarios

Adjustment Formula in Causal Networks

- **Goal**

- Estimate causal effect of intervention $do(X_j = x_{jk})$ on another variable X_i

- **The Adjustment Formula**

- Derived from the post-intervention joint distribution:

$$\Pr(X_i = x_i | do(X_j = x_{jk})) = \sum_{Parents(X_j)} \Pr(x_i | x_{jk}, Parents(X_j)) \Pr(Parents(X_j))$$

- The mechanism for X_j is *removed*: it is treated as a fixed cause, not a random variable

- **Interpretation**

- Computes a **weighted average** of effects of X_j and its parents on X_i
- Weights come from prior probabilities of the parents' values

- **Back-Door Criterion**

- A set Z is a valid adjustment set if it blocks *all back-door paths* from X_j to X_i
- Ensures $X_i \perp\!\!\!\perp Parents(X_j) \mid X_j, Z$

- **Why It Matters**

- Enables causal inference from observational data
- Forms the basis for estimating treatment and policy effects *without*



Intervention: Sprinkler Example

- “Intervene” by turning the sprinkler on, i.e., in do-calculus $do(Sprinkler = T)$
 - Now the sprinkler variable s is not dependent on whether it’s a cloudy day c
- The structural equations after the intervention become:

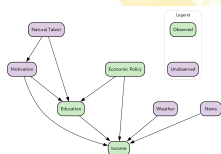
$$C = f_C(U_C)$$

$$R = f_R(C, U_R)$$

$$S = True$$

$$W = f_W(R, S, U_W)$$

$$G = f_G(W, U_G)$$



- Then the $\Pr(s|c) = 1$ and $\Pr(w|r, s) = \Pr(w|r, s = T)$ and the joint probability becomes:

$$\Pr(c, r, w, g|do(S = True)) = \Pr(c) \Pr(r|c) \Pr(w|r, s = True) \Pr(g|w)$$

- The variables that are affected are only the descendants of the manipulated variable *Sprinkler*

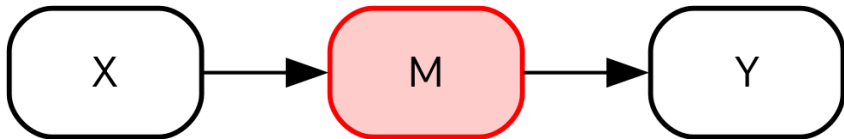
Intervention vs Observation

- **Intervention** “breaks” normal causal link between *Weather* and *Sprinkler*
 - Causal graph shows no influence of *Sprinkler* on *Weather*
- Difference between $do(\textit{Sprinkler} = T)$ (intervention) and $\textit{Sprinkler} = T$ (observation)
 - Observing sprinkler on:
 - Less likely weather is cloudy
 - Turning on sprinkler:
 - Weather unaffected, probability of cloudy unchanged

Back Door

- A causal network can predict the effect of an intervention using the adjustment formula
- The problem is that it requires accurate knowledge of the conditional distributions of the model
- E.g., in the Sprinkler example, why does someone turn on the sprinkler? Maybe they check the weather, but how do they make their decision?
- Besides the direct route, we need to take in account the “back door” route

- We want to estimate the effect of *Sprinkler* on *GreenerGrass*
 - We intervene and set *Sprinkler* = *True*
- Besides the direct route, we need to take in account the “back door” route *Cloudy* \rightarrow *Rain*



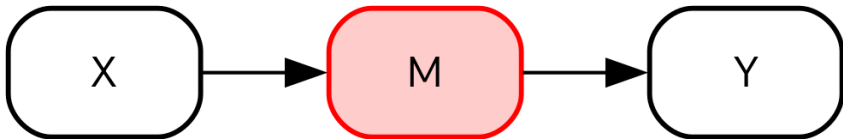
- If we knew the value of *Rain*, this back door path would be blocked and we could condition on *Rain* instead of *Cloudy*
- In formal way we need to find a set of Z variables such that X_i is conditionally independent of $Parents(X_j)$ given X_j and Z
 - TODO: ?

Backdoor Path

- Z blocks all backdoor paths from X to Y
- A backdoor path is any path from X to Y that starts with an arrow pointing into X . These paths create confounding relationships that can bias the estimate of X 's effect on Y

Backdoor Criteria: Condition

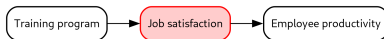
- Variables Z satisfy the backdoor criterion for X and Y in a causal network if:
 1. No element of Z is a descendant of X
 - Z do not capture the effect of X on Y through any causal pathway
 2. Z “blocks” every path from X to Y
- The idea is to estimate the causal effect of X on Y without confounding relationships, by controlling variables Z satisfying the backdoor criterion
- Then we can use non-experimental (i.e., observational data), assigning X randomly



- *Type of Variables in Causal AI*

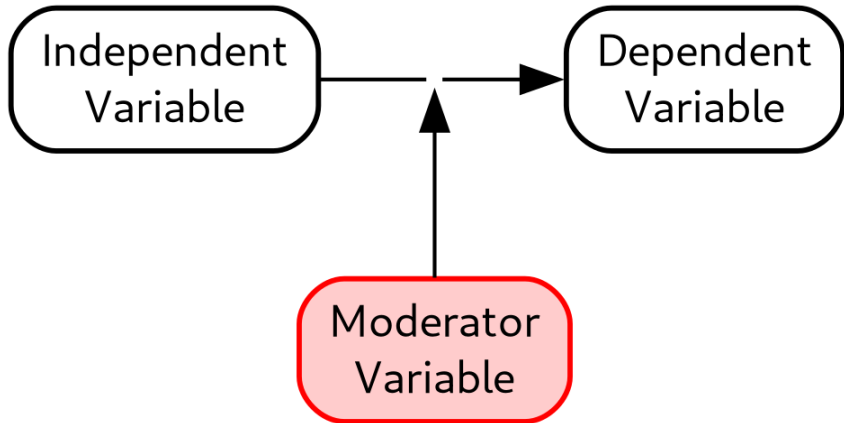
Mediator Variable

- A **mediator variable** M lies on the causal path between a treatment X and an outcome Y



Mediator Variable: Example

- Does a Training Program increase Employee Productivity?
- Training Program may not directly increase productivity
 - It might increase job satisfaction (mediator), leading to greater productivity



Moderator variable

- **Moderator variable** affects the strength or direction between two other variables
- **E.g.,**
 - Study the relationship between stress X and job performance Y
 - The level of social support an individual receives M could be a moderator
 - If social support is high, the negative effect of stress on job performance might be weaker
 - If social support is low, the negative effect might be stronger

