Problem Set 2

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Due: Sunday June 23rd at 11:59pm

Linear Algebra Questions

Question 1 We sometimes want to think about a matrix a partition of submatrices. Accordingly, we might define:

$$A = \left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array} \right]$$

Using this definition, it can be proven that:

$$A^{-1} = \left[\begin{array}{cc} A_{11} & A_{12} \\ A_{21} & A_{22} \end{array} \right]^{-1} = \left[\begin{array}{cc} \left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)^{-1} & -\left(A_{11} - A_{12}A_{22}^{-1}A_{21}\right)^{-1} A_{12}A_{22}^{-1} \\ -\left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1} A_{21}A_{11}^{-1} & \left(A_{22} - A_{21}A_{11}^{-1}A_{12}\right)^{-1} \end{array} \right]$$

Consider a linear model

$$y = X\beta_x + Z\beta_z + e$$

Using the partition matrix inverse, find formulas for $\hat{\beta}_x$ and $\hat{\beta}_z$.

Programming Questions

Question 2 Consider the following data generating process (DGP). $\{y_t\}$ is defined by

$$y_t = y_{t-1} + e_t, \quad e_t \sim iid\mathcal{N}\left(0, \sigma^2\right)$$

for $t = 1, \ldots, T$ and $\{x_{ti}\}$ is defined by

$$x_{ti} = x_{t-1,i} + u_{ti}, \; u_{ti} \sim iid\mathcal{N}\left(0, \sigma^2\right)$$

where $t = 1, \ldots, T$ and $i = 1, \ldots, n$. Clearly, x_{ti} and y_t are completely independent. Construct monte carlo simulations where you generate $\{y_t\}$ and $\{x_t\}$ using this DGP. In the simulation, run the following model using ordinary least squares for different input values and compute the R^2 statistic:

$$y_t = \beta_0 + \beta_1 x_{t1} + \cdots + \beta_r x_{tr} + \varepsilon_t$$

Find the limiting values of R^2 as $T \to \infty$ for the cases $n = 1, 2, 3, 5, 10$. Once you have completed that, repeat this simulation with the following DGP and compare your findings:

$$\begin{array}{rcl} y_t & = & e_t, \quad e_t \sim iid\mathcal{N}\left(0, \sigma^2\right) \\ x_{ti} & = & u_{ti}, \; u_{ti} \sim iid\mathcal{N}\left(0, \sigma^2\right). \end{array}$$

Question 3 Consider the following DGP for $\{y_t\}$:

$$y_t = \rho y_{t-1} + e_t, \; e_t \sim iid\mathcal{N}\left(0, \sigma^2\right)$$

for $t = 1, \ldots, T$. When $|\rho| < 1$, $\{y_t\}$ is stationary. We are only concerned here with the stationary case. Run the following two OLS regressions:

$$\begin{array}{rcll} y_t & = & \beta_1 y_{t-1} + \varepsilon_t & (1) \\ y_t & = & \beta_0 + \beta_1 y_{t-1} + \varepsilon_t & (2) \end{array}$$

Find the bias and variance of the $\hat{\beta}_1$ values in the above equations by using Monte Carlo simulation.