1) Given an example of

a) **Single Variable Unconstrained Optimization Problem :**

A square tank of capacity 500 m^3 has to be dug out. The cost of land is Rs. 50/m^2 . The cost of digging increases with the depth and for the whole tank is 400 $(depth)^2$ rupees. We have to find the dimensions of the tank for the least cost.

That is, we have to minimize cost : $f'(x) = -\frac{2500}{\boxed{x^2}} + 800x$

b) **Multi Variable Unconstrained Optimization Problem :**

Minimize $f(x1,x2) = x_1 - x_2 + 2x_1^2 + 2x_1x_2 + x_2^2$ By taking the starting point at as $X_1 \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

c) **Single Variable constrained Optimization Problem :**

$f(x) = x_1 + 2x_2^2 + 3x_3^3$
$x_1 + x_2 = 0$
$x_3 - x_2 + x_1 \le 6$
$x_1^2 + x_2^2 + x_3^2 \le 1024$
Using constraints this can be transformed into single variable optimization problem

d) **Multi variable constrained optimization problem**

$f(x) = x_1+x_2+x_3+x_4$

$x_1^2 + x_2^2 - x_3 < 200$

$5x_1 + x_2^2 + x_4 > 6$

$2x_4 + x_1 > 5$

$x1, x4 \in \mathbb{Z}$

2) Give examples of (a) LP, (b) MILP, (c) NLP, (d) MINLP. Each of the problem should involve at least four variables and three constraint. Each constraint should involve at least two variables.

Ans:

a) **Linear Programming (LP) Example with atlease 4 Variables and 3 constraints :**

A publisher has orders for 600 copies of a certain text from Mumbai and 400 copies from Kolkata. The company has 700 copies in a warehouse in New Delhi and 800 copies in a warehouse in Bangalore. It costs Rs 50 to ship a text from New Delhi to Kolkata, but it costs Rs 10 to ship it to Mumbai. It costs Rs150 to ship a text from Lodi to Kolkata, but it costs Rs 40 to ship it from Bangalore to Mumbai. How many copies should be assigned from each warehouse to Mumbai and Kolkata to fill the order at the least cost?

x = the number of books from New Delhi to Mumbai
y = the number of books from New Delhi to Kolkata
z = the number of books from Bangalore to Mumbai
w = the number of books from Bangalore to Kolkata.

OBJECTIVE Function: cost $C = 5x + 10y + 15z + 4w$

CONSTRAINTS :

- For Mumbai : $x + z = 600 \Rightarrow z = 600 - x$
- For Kolkata : $y + w = 400 \Rightarrow w = 400 - y$
- There are only 700 books in New Delhi: $x + y \leq 700$
- There are only 800 books in Bangalore: $z + w \leq 700$
- $x \geq 0$, $y \geq 0$, $z \geq 0$, $w \geq 0$
- $x + y \leq 700$
- $x + y \geq 200$
- $x \leq 600$
- $y \leq 400$

b) Mixed Integer Linear Programming (MILP) Example :

Maximize the sum of perimeter of two rectangles with a, b and c, d as their length and breadth respectively, with such constraints

OBJECTIVE FUNCTION : Maximize $\{ 2 (a + b) + 2 (c + d) \}$

Constraints

- $a+b+c-14d=0$
- $b+2c-8d=0$
- $2c-d=0$
- $a-b-c \geq 0$

c) Non Linear Programming (NLP) Example :

Maximize the area of two rectangles with x_1, x_2 and x_3, x_4 as their length and breadth respectively, with such constraints

OBJECTIVE FUNCTION: Maximize $\{ f(x) = x_1x_2 + x_3x_4 \}$

where $x = (x_1, x_2, x_3, x_4)$.

Constraints

- $x_1^2 - x_2^2 - x_3^2 + x_4^2 \leq 5$
- $x_1^2 + x_2^2 + x_3^2 + x_4^2 \leq 20$
- $x_1 + x_2 + x_4 \leq 20$
- $x_4 + x_2 - x_3 \geq 0$
- $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$

d) Mixed Integer Non Linear Programming (MILP) Example :

Maximize sum of volume of a cylinder (with r, h), perimeter of rectangle (l , b) , volume of a cube with side x and with certain constraints

Minimize { $2 (l + b) + 2 \pi h r^2 + x^3$}

<u>Constraints</u>

- $x_1 x_2 + \sin(x_3) + x_4 \leq 4$
- $x_1 + x_2 \in [-4, 40] \cap Z$
- $x_1 + x_3 + x_4 \leq 20$
- $x_4 + x_2 - x_3 \geq 0$
- $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0$

3) Specify a non-linear, unconstrained, single variable optimization problem. Determine its stationary points and identify whether the function has a minima or maxima at this point or is it a saddle point.

Ans. : Let the function be f(x) = $\frac{x^3}{3} + \frac{x^2}{2} - 6x + 1$

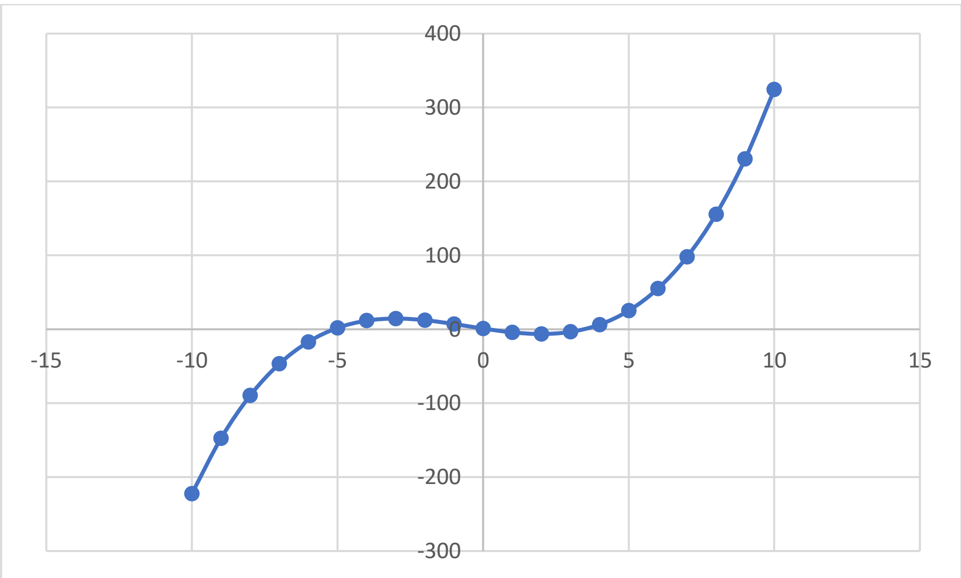


Fig of f(x)

J = $\frac{df(x)}{dx}$ = $x^2+x-6 = 0$ (for minima or maxima slope is 0)

= (x-2)(x+3) => x=2,-3 Stationary points

$\frac{df^2(x)}{dx^2}$ = 2x +1

At x=2, $\frac{df^2(x)}{dx^2} = 5 \geq 0$ Minima at this point

At x=-3 , $\frac{df^2(x)}{dx^2} = -5 \leq 0$ Maxima at this point

4) Provide a three variable optimization problem with at least three constraints. Each constraint should at least involve two variables. Specify (a) two feasible solution and (b) two infeasible solution. Provide a correct and incorrect value of the penalty coefficient such that the three rules of selecting a solution are demonstrated.

Ans.

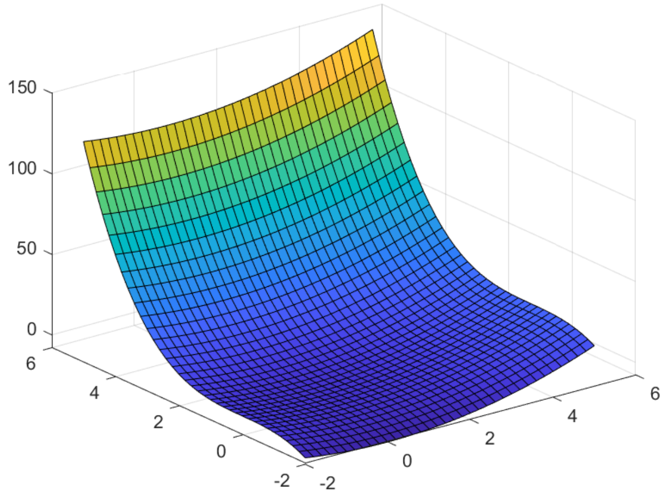
5) Choose a non-linear, two variable function and plot at least two contours. Generate at least five well spaced points for each of the contour.

Ans: Lets take a NLP function as $Z = x_1^2 + y_1^3$

It has two variable functions x_1 and y_1

3D plot can be made using surf function in MATLAB

surf(X,Y,Z)



Using MATLAB we can plot contour

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contour(X,Y,Z,'ShowText','on')
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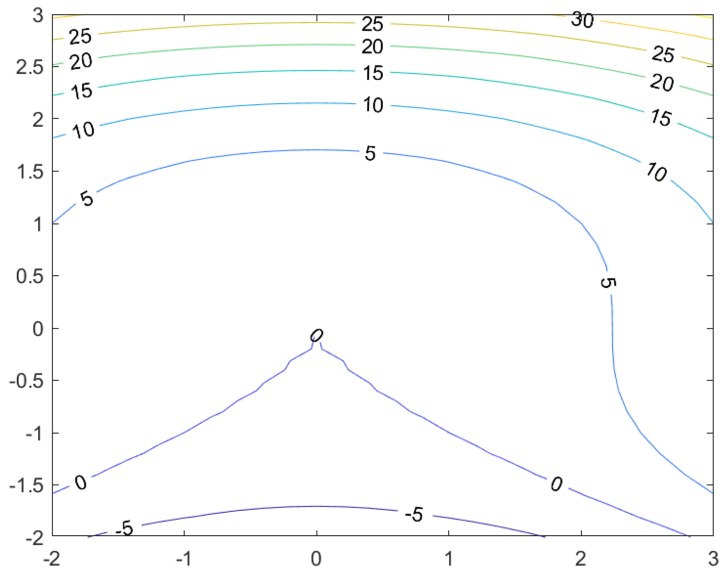


6) Specify a two variable feasible linear programming problem with four constraints. Plot the constraints and mark the feasible region. Draw at least three isocost lines. Each constraint should involve both the variables.

Ans : Let us find the feasible solution for the problem of a decorative item dealer whose LPP is to maximize profit function.

maximize Z = 100x + 100y.

Subject to the constraints

since x ≥0, y ≥ 0, consider only the first quadrant of the plane graph the following straight lines on a graph

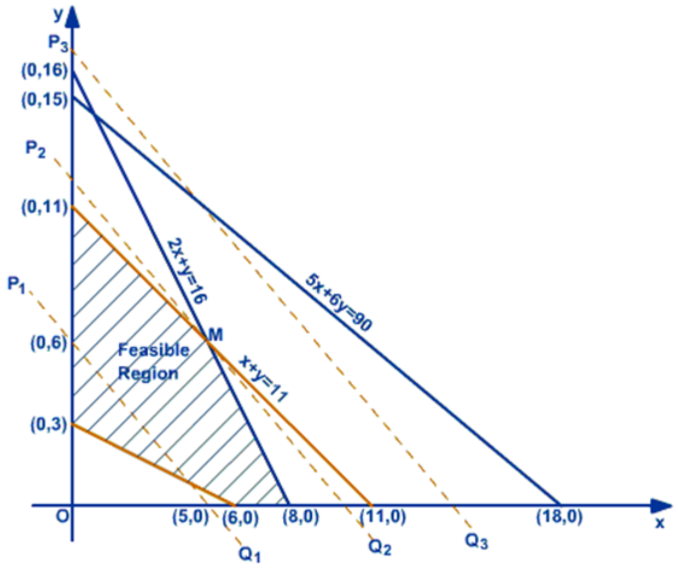
10x + 5y ≤ 80 or 2x+y ≤16

6x + 6y ≤ 66 or x +y ≤11

$4x + 8y \le 24$ or $x + 2y \le 6$

$5x + 6y \le 90$

Identify all the half planes of the constraints. The intersection of all these half planes is the feasible region as shown in the figure.



Give a constant value 600 to Z in the objective function, then we have an equation of the line

$120x + 100y = 600$ or $6x + 5y = 30$

P_1Q_1 is the line corresponding to the equation $6x + 5y = 30$. We give a constant 1200 to Z then the P_2Q_2 represents the line.

$120x + 100y = 1200$ or $6x + 5y = 60$

P_2Q_2 is a line parallel to P_1Q_1 and has one point 'M' which belongs to feasible region and farthest from the origin. If we take any line P_3Q_3 parallel to P_2Q_2 away from the origin, it does not touch any point of the feasible region.

The co-ordinates of the point M can be obtained by solving the equation $2x + y = 16$

$x + y = 11$ which give $x = 5$ and $y = 6$

The optimal solution for the objective function is $x = 5$ and $y = 6$

The optimal value of Z

$120 (5) + 100 (6) = 600 + 600 = 1200$

7) Specify a two variable infeasible linear programming problem with three constraints. Plot the constraints and show that the problem is infeasible. Each constraint should involve both the variables.

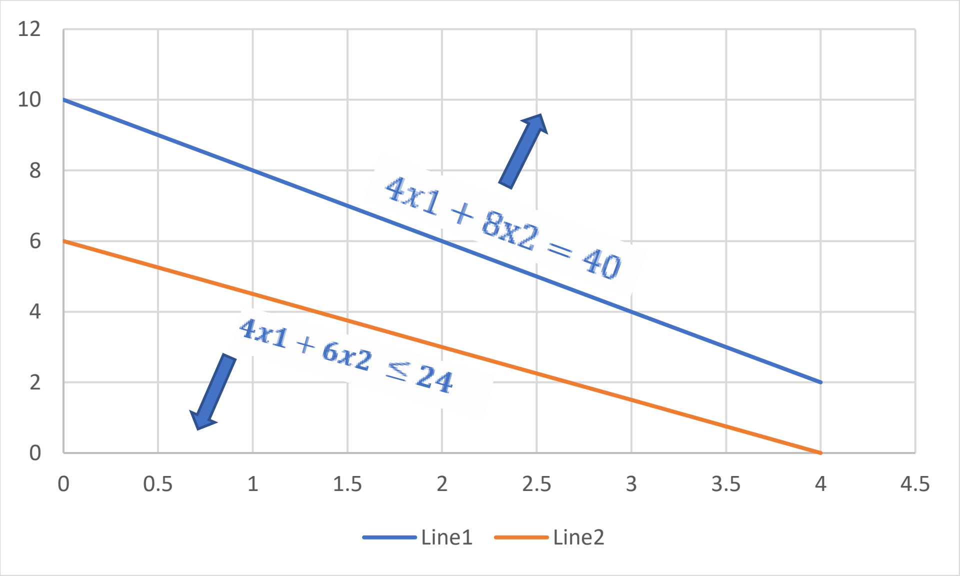
Ans: Max. $Z = 5 X_1 + 8 X_2$

Subject to constraints

$4 X_1 + 6 X_2 \le 24$

$4 X_1 + 8 X_2 \le 40$

$X_1, X_2 \ge 0$



There is no common feasible region for line AB and CD. Hence, solution is infeasible

8) Specify a non-linear constrained problem (involving at least two variables in each of the two constraints, the constraints should be of the form <= as well as >=), calculate the objective function value as well as the fitness function value with the procedure discussed in the class. Calculate it for at least three solutions with at least two infeasible solutions.

Minimize $f(x) = \frac{1 + x_2^2}{x_1}$

Given g(x) such that:

$g_1(x) = 9x_1 + x_2 \ge 6$

$g_2(x) = 9x_2 - x_1 \ge 1$

$g_3(x) = x_2^2 + x_1 \le 50$

R_m = 20

$0.1 \le x_1 \le 1$, $0 \le x_2 \le 5$

Practically it can be simply thought to be like this with some number of iterations

i.e if for some random values of x_1, x_2

if $g_1(x_1,x_2) \ge 6$:

$p_1=0$

else:

$p_1 = (g_1(x_1,x_2) - 6)^2$

if $g_2(x_1,x_2) \ge 1$:

$p_2=0$

else:

$p_2 = (g_2(x_1,x_2) - 1)^2$

if $g_3(x_1,x_2) \le 50$:

$p_3=0$

else:

$p_3 = (g_3(x_1,x_2) - 50)^2$

Fitness Function = $-\frac{1+ x_2{}^2}{x_1} + 10^{10}(p_1+p_2+p_3)$

$W_j(x)=\{|g_j(x)|$ if $g_j(x) < 0$ else 0

$\Omega = \sum w_j(x)$

or

or fitness function can be simply written as

$F(x) = f(x) + Rm\ \Omega$

$g_1 \ge 6$, $g_2 \ge 1$, $g_3 \le 50$

Solution	x_1	x_2	f	g_1	g_2	g_3	w_1	w_2	w_3	Ω	F
S1	0.31	-0.89	5.780968	-3.68 ✗	-8.01 ✗	-1.1021 ✓	3.6	8.01	1.1021	5.0079	-94.37
S2	0.38	2.73	22.24447	6.15 ✓	23.68 ✓	7.8329 ✓	0	0	0	0	811.1025
S3	0.22	-0.56	5.970909	-2.54 ✗	-2.31 ✗	-0.5336 ✓	2.54	2.31	0.533	5.3836	-110.357
S4	0.59	3.63	24.0286	8.94 ✓	32.11 ✓	13.7669 ✓	0	0	0	0	1142.767
S5	0.46	-2.9	20.45652	-7.04 ✓	-22.4 ✗	-8.87 ✓	7.04	22.4	8.87	38.38	-371.943
S6	0.66	4.11	1.769848	6.35 ✓	0.79 ✗	0.8281 ✓	0	0	0	0	277.1318