

# Lab Report 3

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## Problem Statement 1:

Methanol (A) is evaporated into a stream of dry air (B) in a cylindrical tube at 328.5 K. The distance from the tube inlet to the liquid surface is  $z_2 - z_1 = 0.238$  m. At  $T = 328.5$  K, the vapor pressure of methanol is  $P_{A0} = 68.4$  kPa and the total pressure is  $P = 99.4$  kPa. The binary molecular diffusion coefficient of methanol in air under these conditions is  $D_{AB} = 1.991 \times 10^{-5}$  m<sup>2</sup>/sec. The temperature profile in the tube exhibits a linear characteristic from the liquid surface ( $T = 328.5$  K) to the tube inlet ( $T = 295$  K). Calculate the molar flux of methanol within the tube and plot the mole fraction profile of methanol from the liquid surface to the flowing air stream.

### Multicomponent Diffusion

- For multicomponent diffusion in gases at low density to the positive  $z$  direction, the Maxwell–Stefan equation can be expressed as

$$\frac{dC_i}{dz} = \sum_{j=1}^n \frac{x_i N_j - x_j N_i}{D_{ij}}$$

where

$C_i$ , (kg mol/m<sup>3</sup>) is the concentration of species  $i$

$x_i$  is the mole fraction of species  $i$

$N_i$ , (kg mol/(m<sup>2</sup> sec)) is the mole flux of species  $i$

$D_{ij}$  (m<sup>2</sup>/sec) is the molecular diffusivity of  $i$  in  $j$

$n$  is the number of components

For gas mixture of three components (A, B, C) the Maxwell-Stefan equation yields

$$\frac{dC_A}{dz} = \sum_{i=1}^n \frac{x_A N_B - x_B N_A}{D_{AB}} + \frac{x_A N_C - x_C N_A}{D_{AC}}$$

$$\frac{dC_B}{dz} = \sum_{i=1}^n \frac{x_B N_A - x_A N_B}{D_{AB}} + \frac{x_B N_C - x_C N_B}{D_{BC}}$$

$$\frac{dC_C}{dz} = \sum_{i=1}^n \frac{x_C N_A - x_A N_C}{D_{AC}} + \frac{x_C N_B - x_B N_C}{D_{BC}}$$

where  $D_{ij} = D_{ji}$  (i,j=A,B,C)

## Solution:

### Algorithm/strategy-

At  $z = z_1$ ,  $x = P/P_{A0}$ . We guess  $N_A$ , solve the differential equation  $dx_A/dz = -(1-x_A)N_A/D_{AB}C$ , and check whether the condition  $x_{A2} = 0$  is satisfied. If  $x_{A2} \neq 0$ , the calculation procedure is repeated by using another initial guess of  $N_A$ . The bisection method can be effectively used in the iterative calculation procedure.

The built-in function `ode45` is used to solve the ode and the obtained value is compared wrt the convergence criteria which is set to  $1e-11$ , if the solution hasn't converged yet a new guess value is obtained by the bisection method which divides the range of solution by half at every iteration.

### Code-

```
1. Dab = 1.991e-5; T = 328.5; R = 8.314; P = 99.4; Pa0 = 68.4; Tf = 295;
2. z2 = 0.238; z1 = 0; zv = [z1 z2];
3. C = P/(R*T); x0 = Pa0/P; critN = 1e-11; errN = 1;
4. Na1 = 3.5e-6; Na2 = 3.6e-6;
5. while errN > critN
6.     Nam = (Na1+Na2)/2;
7.     [z x1] = ode45(@xaz,zv,x0,[],Na1,Dab,C);
8.     [z x2] = ode45(@xaz,zv,x0,[],Na2,Dab,C);
9.     [z xm] = ode45(@xaz,zv,x0,[],Nam,Dab,C);
10.    if x1(end)*xm(end) < 0
11.        Na2 = Nam;
12.    else
13.        Na1 = Nam;
14.    end
15.    errN = abs(Na1 - Na2);
16. end
17. xblm = x0/(log(1/(1-x0)));
18. Nanal = Dab*C*x0/((z2-z1)*xblm);
19. fprintf('Estimated Nab = %e, xA = %8.6f\n',Nam, xm(end));
```

```

20. fprintf('Analytic Nab = %e\n',Nanal);
21. figure(1)
22. plot(z,xm), xlabel('z(m)'), ylabel('x_A'), axis tight
23. function dxdz = xaz(z,x,Na,Dab,C)
24. dxdz = -(1-x)*Na/Dab/C;
25. end

```

## Problem Statement 2:

Gases A and B are diffusing through stagnant gas C at a temperature of 55°C and a pressure of 0.2 atmospheres from point 1 (z1) to point 2 (z2). The distance between these two points is 0.001 m. The molar flux of B was measured to be  $N_B = -4.143 \times 10^{-4} \text{ kg mol/(m}^2 \cdot \text{sec)}$  (i.e., gas B diffuses from z2 to z1). The gas mixture is assumed to be ideal gas. Estimate the molar flux of A ( $N_A$ ). Data:  $CA1 = 2.229 \times 10^{-4}$ ,  $CA2 = 0$ ,  $CB0 = 0$ ,  $CB2 = 2.701 \times 10^{-3}$ ,  $CC1 = 7.208 \times 10^{-3}$ ,  $CC2 = 4.730 \times 10^{-3}$ ,  $DAB = 1.47 \times 10^{-4}$ ,  $DAC = 1.075 \times 10^{-4}$ ,  $DBC = 1.245 \times 10^{-4}$

## Solution:

### Algorithm/strategy-

Since component C is stagnant,  $N_C = 0$ . A simple way to solve this problem is first to assume the value of  $N_A$ , solve the differential equations, and check whether the concentration conditions at  $z_2$  are satisfied. The initial guess for  $N_A$  can be determined by  $N_A = -D_{AC}(C_{A2} - C_{A1})/L$  assuming the gas is a binary mixture of A and C. This procedure is repeated using updated  $N_A$  until the concentration conditions at  $z_2$  are satisfied. The bisection method can be used to update  $N_A$ . The total concentration is given by  $C_t = n/V = P/RT$  from ideal gas law.

The system of 3 odes are solved using the built in ode45 function with initial guesses, the solution has at each iteration is compared for convergence wrt convergence criteria, i.e, concentration at  $z_2$ , here  $\text{critN} = 1e-10$ , ensures that. If the solutions have converged we stop else we take new initial guesses by bisection method which reduces the range of solutions at each iteration by half.

## Code-

```

1. Ca1=2.229e-4; Cb1=0; Cc1=7.208e-3; Ca2=0; Cb2=2.701e-3; Cc2=4.73e-3;
2. D1=1.075e-4; D2=1.245e-4; D3=1.47e-4;
3. P=0.2; T=328; R=82.057e-3; Ct = P/(R*T);
4. L=0.001; Nb=-4.143e-4; Nc=0; Na=-D3*(Ca2-Ca1)/L;
5. zspan = [0 L]; c0 = [Ca1 Cb1 Cc1]; critN = 1e-10; errA = 1;
6. Na1 = Na/2; Na2 = 2*Na; iter = 1;
7. while errA > critN
8.   Nam = (Na1+Na2)/2;
9.   [z c1] = ode45(@mdif,zspan,[Ca1,Cb1,Cc1],[],D1,D2,D3,Na1,Nb,Nc,Ct);
10.  [z c2] =
    ode45(@mdif,zspan,[Ca1,Cb1,Cc1],[],D1,D2,D3,Na2,Nb,Nc,Ct);

```

```

11. [z cm] =
    ode45(@mdif,zspan,[Ca1,Cb1,Cc1],[],D1,D2,D3,Nam,Nb,Nc,Ct);
12. if c1(end,1)*cm(end,1) < 0 % check whether Ca(z2)=0 is satisfied
13.     Na2 = Nam;
14. else
15.     Na1 = Nam;
16. end
17. errA = abs(Na1 - Na2); iter = iter+1;
18. end
19. c = cm; xa = c(:,1)/Ct; xb = c(:,2)/Ct; xc = c(:,3)/Ct;
20. figure(2)
21. plot(z,xa,z,xb,':',z,xc,'.-'), legend('x_A','x_B','x_C')
22. xlabel('Distance z(m)'), ylabel('Mole fraction')
23. iter, Na = Nam
24. function dcdz = mdif(z,c,D1,D2,D3,Na,Nb,Nc,Ct)
25. % c1=Ca, c2=Cb, c3=Cc, D1=Dab, D2=Dbc, D3=Dac
26. xa = c(1)/Ct; xb = c(2)/Ct; xc = c(3)/Ct;
27. dcdz = [(xa*Nb-xb*Na)/D1 + (xa*Nc-xc*Na)/D3;
28.          (xb*Na-xa*Nb)/D1 + (xb*Nc-xc*Nb)/D2;
29.          (xc*Na-xa*Nc)/D3 + (xc*Nb-xb*Nc)/D2];
30. end

```

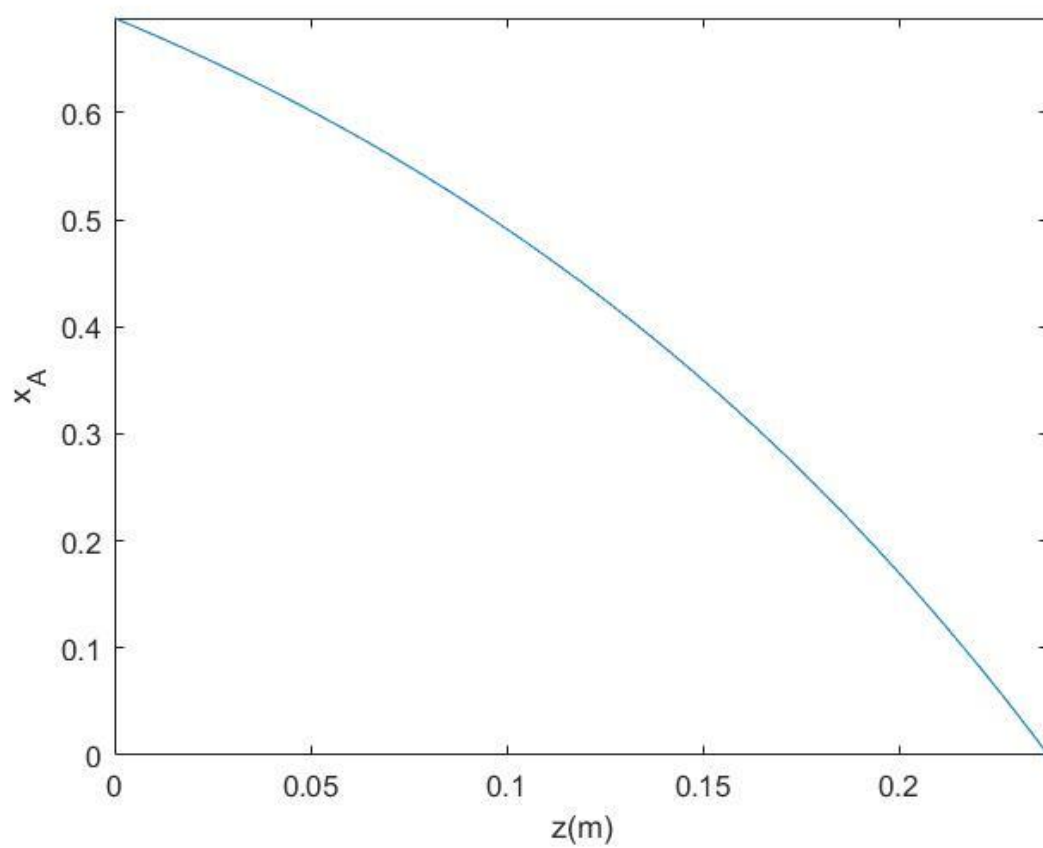
## Results:

For problem 1:-

Estimated  $N_{ab} = 3.547504e-06 \text{ m}^2/\text{s}$ ,  $x_A = -0.000000$

Analytic  $N_{ab} = 3.547502e-06 \text{ m}^2/\text{s}$ .

Plot of mole fraction profile of methanol from the liquid surface to the flowing air stream:



For problem 2:-

Estimated Molar flux of A ( $N_A$ ) =  $2.3471 \times 10^{-5}$  m<sup>2</sup>/s. after n=20 iterations of computation.

Plot of mole fractions of the components A, B and C v/s z:

