IBM Watson Studio Projects Community Services Manage Support

## Linear Regression Algorithms Demo





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LANGUAGE

21 Dec 2017, 2:30 AM Python 2.7

#### This notebook shows:

- Install SystemML Python package and jar file
  - pip
  - SystemML 'Hello World'
- Example 1: Matrix Multiplication
- Load diabetes dataset from scikit-learn
- Example 2: Implement three different algorithms to train linear regression model
  - Algorithm 1: Linear Regression Direct Solve (no regularization)
  - Algorithm 2: Linear Regression Batch Gradient Descent (no regularization)
  - Algorithm 3: Linear Regression Conjugate Gradient (no regularization)
- Example 3: Invoke existing SystemML algorithm script LinearRegDS.dml using MLContext API
- Example 4: Invoke existing SystemML algorithm using scikit-learn/SparkML pipeline like API
- Example 5: Invoking a Keras model with SystemML

### Install SystemML Python package and jar file

#### In [1]: !pip show systemml

Name: systemml Version: 1.1.0

Summary: Apache SystemML is a distributed and declarative machine learn

ing platform.

Home-page: http://systemml.apache.org/

Author: Apache SystemML

Author-email: dev@systemml.apache.org

License: Apache 2.0

Location: /gpfs/global\_fs01/sym\_shared/YPProdSpark/user/scf4-b69284e162

5908-5ca7710237a9/.local/lib/python2.7/site-packages Requires: Pillow, scikit-learn, pandas, scipy, numpy

#### Import SystemML API

```
In [2]: sc.version
Out[2]: u'2.1.0'
In [3]: from systemml import MLContext, dml
# Create a MLContext object
ml = MLContext(sc)
# And print the information of SystemML version
print(ml.info())
```

Archiver-Version: Plexus Archiver

```
Artifact-Id: systemml
        Build-Jdk: 1.8.0_111
        Build-Time: 2017-12-19 13:17:52 CST
        Built-By: biuser
        Created-By: Apache Maven 3.0.5
        Group-Id: org.apache.systemml
        Main-Class: org.apache.sysml.api.DMLScript
        Manifest-Version: 1.0
        Minimum-Recommended-Spark-Version: 2.1.0
        Version: 1.1.0-SNAPSHOT
In [4]: # Create a DML script for a Hello World' example and execute it using M
        LContext
        script = dml("""
        print('Hello World');
        ml.execute(script)
        Hello World
        SystemML Statistics:
        Total execution time:
                                      0.001 sec.
        Number of executed Spark inst: 0.
Out[4]: MLResults
In [5]: # Let's modify the above script to get the Hello World string
        script = dml("""
        s = 'Hello World'
        """).output("s")
        hello_world_str = ml.execute(script).get("s")
        print(hello_world_str)
        SystemML Statistics:
        Total execution time:
                                       0.000 sec.
        Number of executed Spark inst: 0.
        Hello World
```

#### Import numpy, sklearn, and define some helper functions

```
In [6]: import sys, os
   import matplotlib.pyplot as plt
   import numpy as np
   from sklearn import datasets
   plt.switch_backend('agg')
```

### **Example 1: Matrix Multiplication**

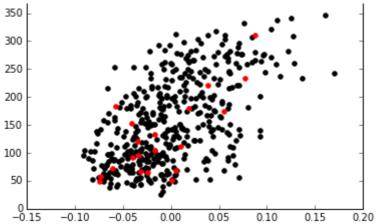
Createm MII coulet to concrete a random matrix northwest matrix

## Systemiw∟ script to generate a random matrix, perform matrix multiplication, and compute the sum of the output

```
In [7]: | script = """
          X = rand(rows=$nr, cols=1000, sparsity=0.5)
          A = t(X) %*% X
          s = sum(A)
       prog = dml(script).input('$nr', 1e6).output('s')
       s = ml.execute(prog).get('s')
       print s
       [Stage 0:>
                                                               (0 +
       0) / 59]
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       10) / 59]
       [Stage 0:========>>
                                                              (54 +
       5) / 59]
       SystemML Statistics:
                         14.134 sec.
       Total execution time:
       Number of executed Spark inst: 2.
```

62608781691.5

### Load diabetes dataset from scikit-learn



```
In [10]: diabetes_y_train
Out[10]: matrix([[ 151.],
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[ 131.], [ 174.], [ 257.], [ 55.], [ 84.], [ 42.], [ 146.], [ 212.]])

# Example 2: Implement three different algorithms to train linear regression model

# Algorithm 1: Linear Regression - Direct Solve (no regularization)

#### **Preliminaries**

 The builtin function solve(A, b) computes the least squares solution for system of linear equations

$$Ax = b$$

for the vector x such that

$$||Ax-b||$$

is minimized. It is important to note that this function can operate only on small-to-medium sized input matrix that can fit in the driver memory. See the <u>DML language reference</u> (<a href="http://apache.github.io/systemml/dml-language-reference.html">http://apache.github.io/systemml/dml-language-reference.html</a>) for more details.

2. Linear regression model assumes that relationship between input explanatory (feature) variables X and numerical response variable y is linear. The goal is to estimate regression coefficient w (and residual variable) such that

$$y = ext{Normal}(Xw, \sigma^2)$$
 $ext{Cost function, } J(w) = rac{1}{2}(Xw - y)^2$ 

Differentiating with respect to w,

$$egin{aligned} dw &= rac{\partial}{\partial w} rac{1}{2} (Xw - y)^2 \ &= rac{1}{2} 2X^T (Xw - y) \ &= (X^T X) w - X^T y \end{aligned}$$

#### Setting the gradient

To find minima, we set the derivative with respect to w to zero,

$$(X^TX)w - (X^Ty) = 0$$

$$w = (X^TX)^{-1}(X^Ty)$$
Let  $A = X^TX$ 
and  $b = X^Ty$ 
Therefore,  $w = solve(A, b)$ 

```
בוו [בב]. | סכו בער -
              # add constant feature to X to model intercept
             ones = matrix(1, rows=nrow(X), cols=1)
             X = cbind(X, ones)
             A = t(X) %*% X
             b = t(X) %*% y
             w = solve(A, b)
             bias = as.scalar(w[nrow(w),1])
             w = w[1:nrow(w)-1,]
In [13]: prog = dml(script).input(X=diabetes_X_train, y=diabetes_y_train).output
          ('w', 'bias')
          w, bias = ml.execute(prog).get('w','bias')
         w = w.toNumPy()
         SystemML Statistics:
         Total execution time:
                                          0.028 sec.
         Number of executed Spark inst: 2.
In [14]: plt.scatter(diabetes_X_train, diabetes_y_train, color='black')
         plt.scatter(diabetes_X_test, diabetes_y_test, color='red')
          plt.plot(diabetes_X_test, (w*diabetes_X_test)+bias, color='blue', lines
          tyle ='dotted')
Out[14]: [<matplotlib.lines.Line2D at 0x7f6348328d10>]
           400
           350
           300
           250
           200
          150
          100
           50
```

## Algorithm 2: Linear Regression - Batch Gradient Descent (no regularization)

0.05

0.10

0.15

#### **Algorithm**

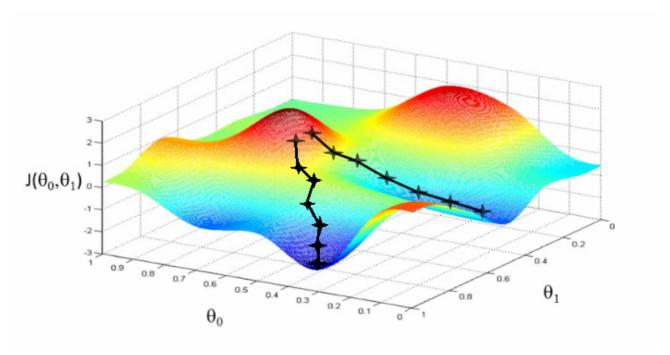
```
Step 1: Start with an initial point
while(not converged) {
  Step 2: Compute gradient dw.
  Step 3: Compute stepsize alpha.
  Step 4: Update: w_new = w_old - alpha*dw
```

-0.10

-0.05

0.00





#### **Gradient formula**

$$dw = r = (X^T X)w - (X^T y)$$

#### Step size formula

We perform a line search to choose the step size alpha to minimize the cost function J(w). From basic calculus, alpha minimizes the function J(w) when the directional derivative with respect to alpha is zero.

$$alpha = rac{r^T r}{r^T X^T X r}$$

SystemML Statistics:

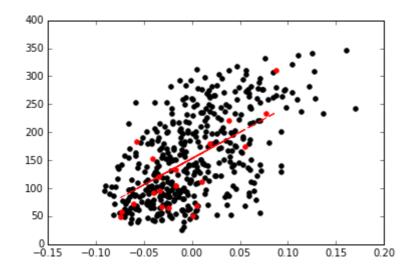
Total execution time: 0.081 sec.

Number of executed Spark inst: 2.

```
In [17]: plt.scatter(diabetes_X_train, diabetes_y_train, color='black')
    plt.scatter(diabetes_X_test, diabetes_y_test, color='red')

plt.plot(diabetes_X_test, (w*diabetes_X_test)+bias, color='red', linest
    yle ='dashed')
```

Out[17]: [<matplotlib.lines.Line2D at 0x7f6348296710>]



# Algorithm 3: Linear Regression - Conjugate Gradient (no regularization)

Problem with gradient descent: Takes very similar directions many times

Solution: Enforce conjugacy

```
Step 1: Start with an initial point
while(not converged) {
   Step 2: Compute gradient dw.
   Step 3: Compute stepsize alpha.
   Step 4: Compute next direction p by enforcing conjugacy with previous direction.
   Step 4: Update: w_new = w_old + alpha*p
}
```

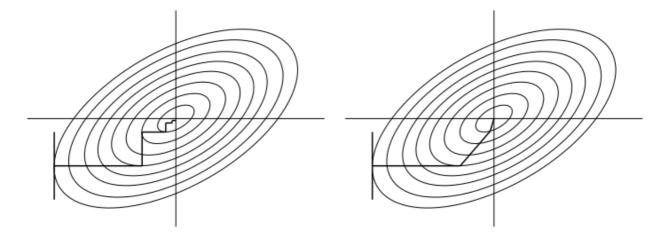


Figure 6.14: Steepest descent vs. conjugate gradient.

```
In [18]: | script = """
             # add constant feature to X to model intercepts
             X = cbind(X, matrix(1, rows=nrow(X), cols=1))
             m = ncol(X); i = 1;
             max_iter = 20;
             w = matrix (0, rows = m, cols = 1); # initialize weights to 0
             dw = - t(X) %*% y; p = - dw;
                                              \# dw = (X'X)w - (X'y)
             norm_r2 = sum (dw ^ 2);
              for(i in 1:max_iter) {
                  q = t(X) %*% (X %*% p)
                  alpha = norm_r2 / sum (p * q); # Minimizes f(w - alpha*r)
                 w = w + alpha * p;
                                                  # update weights
                  dw = dw + alpha * q;
                  old_norm_r2 = norm_r2; norm_r2 = sum (dw ^ 2);
                  p = -dw + (norm_r2 / old_norm_r2) * p; # next direction - conju
         gacy to previous direction
                  i = i + 1;
             bias = as.scalar(w[nrow(w),1])
             w = w[1:nrow(w)-1,]
In [19]: | prog = dml(script).input(X=diabetes_X_train, y=diabetes_y_train).output
         ('w').output('bias')
         w, bias = ml.execute(prog).get('w','bias')
         w = w.toNumPy()
         SystemML Statistics:
         Total execution time:
                                          0.007 sec.
         Number of executed Spark inst: 2.
         plt.scatter(diabetes_X_train, diabetes_y_train, color='black')
In [20]:
         plt.scatter(diabetes_X_test, diabetes_y_test, color='red')
         plt.plot(diabetes_X_test, (w*diabetes_X_test)+bias, color='red', linest
         yle ='dashed')
Out[20]: [<matplotlib.lines.Line2D at 0x7f634816a550>]
          400
          350
          300
          250
          200
          150
          100
           50
                  -0.10
                        -0.05
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                                      0.05
                                            0.10
                                                   0.15
```

### Example 3: Invoke existing SystemML algorithm

## script LinearRegDS.dml using MLContext API

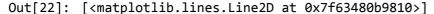
```
In [21]: from systemml import dmlFromResource
    prog = dmlFromResource('scripts/algorithms/LinearRegDS.dml').input(X=di
    abetes_X_train, y=diabetes_y_train).input('$icpt',1.0).output('beta_ou
    t')
    w = ml.execute(prog).get('beta_out')
    w = w.toNumPy()
    bias=w[1]

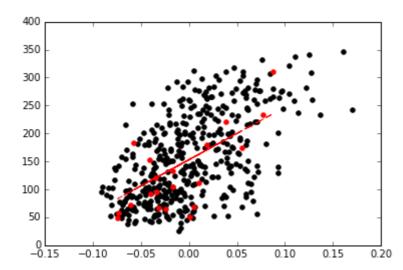
BEGIN LINEAR REGRESSION SCRIPT
Reading X and Y...
```

Calling the Direct Solver... Computing the statistics... AVG\_TOT\_Y,153.36255924170615 STDEV\_TOT\_Y,77.21853383600028 AVG\_RES\_Y,3.633533705616816E-14 STDEV\_RES\_Y,63.038506337610244 DISPERSION, 3973.853281276927 R2,0.3351312506863875 ADJUSTED\_R2,0.33354822985468835 R2\_NOBIAS, 0.3351312506863875 ADJUSTED\_R2\_NOBIAS, 0.33354822985468835 Writing the output matrix... END LINEAR REGRESSION SCRIPT SystemML Statistics: 0.008 sec. Total execution time: Number of executed Spark inst: 2.

```
In [22]: plt.scatter(diabetes_X_train, diabetes_y_train, color='black')
    plt.scatter(diabetes_X_test, diabetes_y_test, color='red')

plt.plot(diabetes_X_test, (w[0]*diabetes_X_test)+bias, color='red', lin estyle ='dashed')
```

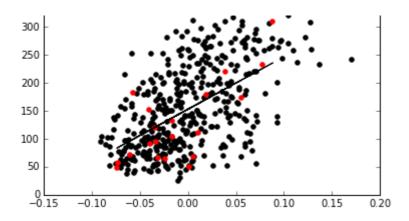




# Example 4: Invoke existing SystemML algorithm using scikit-learn/SparkML pipeline like API

mllearn API allows a Python programmer to invoke SystemML's algorithms using scikit-learn like API as well as Spark's MLPipeline API.

```
In [23]: | from pyspark.sql import SQLContext
         from systemml.mllearn import LinearRegression
         sqlCtx = SQLContext(sc)
In [24]: regr = LinearRegression(sqlCtx)
         # Train the model using the training sets
         regr.fit(diabetes_X_train, diabetes_y_train)
         BEGIN LINEAR REGRESSION SCRIPT
         Reading X and Y...
         Running the CG algorithm...
         ||r|| initial value = 64725.64237405237, target value = 0.064725642374
         05237
         Iteration 1: ||r|| / ||r| init|| = 0.013822097283108787
         Iteration 2: ||r|| / ||r| init|| = 5.369915930350396E-14
         The CG algorithm is done.
         Computing the statistics...
         AVG_TOT_Y,153.36255924170615
         STDEV_TOT_Y,77.21853383600028
         AVG_RES_Y,-8.227243004822623E-12
         STDEV_RES_Y,63.03850633759284
         DISPERSION, 3973.853281274733
         R2,0.33513125068675453
         ADJUSTED R2,0.3335482298550564
         R2_NOBIAS, 0.33513125068675453
         ADJUSTED_R2_NOBIAS, 0.3335482298550564
         Writing the output matrix...
         END LINEAR REGRESSION SCRIPT
         SystemML Statistics:
                                         0.003 sec.
         Total execution time:
         Number of executed Spark inst: 2.
Out[24]: 1r
In [25]: predictions = regr.predict(diabetes_X_test)
         SystemML Statistics:
         Total execution time:
                                          0.000 sec.
         Number of executed Spark inst: 1.
In [26]:
         # Use the trained model to perform prediction
         %matplotlib inline
         plt.scatter(diabetes_X_train, diabetes_y_train, color='black')
         plt.scatter(diabetes X test, diabetes y test, color='red')
         plt.plot(diabetes_X_test, predictions, color='black')
Out[26]: [<matplotlib.lines.Line2D at 0x7f632e6e5b90>]
```



### (Optional) Install OpenBLAS

!wget https://github.com/xianyi/OpenBLAS/archive/v0.2.20.tar.gz !tar -xzf v0.2.20.tar.gz !cd OpenBLAS-0.2.20/ && make clean !cd OpenBLAS-0.2.20/ && make USE\_OPENMP=1

# Example 5: Invoking a Keras model with SystemML

See <u>SystemML's deep learning documentation (http://apache.github.io/systemml/deep-learning)</u> for more detail.

```
In [ ]: | from mlxtend.data import mnist_data
        import numpy as np
        from sklearn.utils import shuffle
        # DownLoad the MNIST dataset
        X, y = mnist_data()
        X, y = shuffle(X, y)
        # Split the data into training and test
        n_samples = len(X)
        X_train = X[:int(.9 * n_samples)]
        y_train = y[:int(.9 * n_samples)]
        X test = X[int(.9 * n samples):]
        y_test = y[int(.9 * n_samples):]
        from keras.models import Sequential
        from keras.layers import Input, Dense, Conv2D, MaxPooling2D, Dropout,Fl
        atten
        from keras import backend as K
        from keras.models import Model
        input shape = (1,28,28) if K.image data format() == 'channels first' el
        se (28,28, 1)
        keras model = Sequential()
        keras_model.add(Conv2D(32, kernel_size=(5, 5), activation='relu', input
         shape=input shape, padding='same'))
        keras_model.add(MaxPooling2D(pool_size=(2, 2)))
        keras_model.add(Conv2D(64, (5, 5), activation='relu', padding='same'))
        keras_model.add(MaxPooling2D(pool_size=(2, 2)))
        keras_model.add(Flatten())
        keras_model.add(Dense(512, activation='relu'))
        keras model.add(Dropout(0.5))
        keras model.add(Dense(10, activation='softmax'))
        # Scale the input features
        scale = 0.00390625
        V +nain - V +nain*ccale
```

```
%_CLGIN = %_CLGIN SCALE
X_test = X_test*scale

from systemml.mllearn import Keras2DML
sysml_model = Keras2DML(spark, keras_model, input_shape=(1,28,28), weig
hts='weights_dir')
sysml_model.setConfigProperty('sysml.native.blas', 'openblas')
sysml_model.setConfigProperty('sysml.native.blas.directory', os.path.jo
in(os.getcwd(),'OpenBLAS-0.2.20/'))
# sysml_model.setGPU(True).setForceGPU(True)
sysml_model.summary()
sysml_model.fit(X_train, y_train)
```

Using TensorFlow backend.

Loading the model from weights\_dir...

SystemML Statistics:

Total execution time: 0.000 sec.

Number of executed Spark inst: 0.

```
[Stage 9:========>>
                                 (2
+ 1) / 3]
+-----
---+-----+
      Name| Type| Output| Weight|
Top| Bottom|Memory* (train/test)|
+-----
---+----+
               Data| (, 1, 28, 28)|
  conv2d_1_input
 |conv2d_1_input,label|
                              1/0
     conv2d_1| Convolution|(, 32, 28, 28)| [32 X 25]| [32 X
111
       conv2d 1 | conv2d 1 input|
```