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A Search for New Physics With Tau Leptons at the CMS Experiment

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Abstract

The Standard Model of Particle Physics is currently the best model of the fundamental particles and their interactions. However, there are still significant theoretical issues and recently seen experimental tensions with the model. The theoretical issues include the hierarchy problem which forecasts the breakdown of the Standard Model when looking at the size of corrections needed to calculate the mass of the newest found member of the theory, the Higgs boson particle. The current experimental tensions include the B-anomalies and the g-2 measurement. These results, although they do not yet sit at the required 5σ deviation for a discovery, offer the most prominent leads into where new physics may be hiding. Looking for signatures of theoretical explanations of these anomalies offers excellent search options for new fundamental particles. This thesis describes the search for new physics that can explain both the theoretical problems and experimental tensions. This is done using tau leptons seen during Run-2 of the Large Hadron Collider (LHC) at the Compact Muon Solenoid (CMS) experiment. The Beyond Standard Model theories searched for range from Supersymmetry, leptoquarks, to type-X two Higgs doublet models. Each theory is separately studied and an analysis is tailored to find its most sensitive signature. In the process of optimisation, data-driven background modelling is improved to aid the reliability of the results. The results are currently blinded however the expected limits offer some of the largest constraints that are placed on these prominent Beyond Standard Model Theories.

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1 Introduction

2 The Standard Model of Particle Physics

- 2.1 Quantum Field Theories
- 2.2 Electroweak Unification
- 2.3 The Higgs Mechanism
- 2.4 Quantum Chromodynamics
- 2.5 Experimental Evidence for the Standard Model
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3 The LHC and CMS Experiment

3.1 The LHC

3.2 The CMS Detector

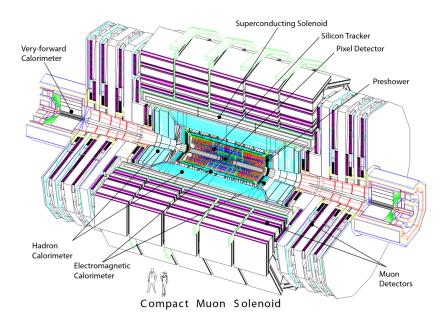


Figure 1: A perspective view of the CMS detector [?].

3.3 Event Reconstruction

4 Tau Identification

Decay Mode	Branching Fraction
Leptonic Decay (e, μ)	35.2%
$e^-ar{ u}_e u_ au$	17.8%
$\mu^-ar u_\mu u_ au$	17.4%
	64.8%
$h^-\pi^0 u_ au$	25.9%
$h^- u_ au$	11.5%
$h^-2\pi^0 u_ au$	9.3%
$\pi^-\pi^-\pi^+ u_ au$	9.0%
$\pi^-\pi^-\pi^+\pi^0 u_ au$	2.7%
other	6.4%

Table 1: Measured branching fractions, that are greater than 2%, for the tau lepton. h represents a charged hadron either a pion or a kaon.

5 BSM H/A $ightarrow au^+ au^-$ Analysis

5.1 Theoretical Motivation

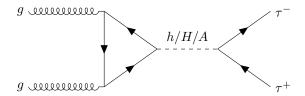


Figure 2

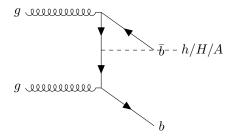


Figure 3

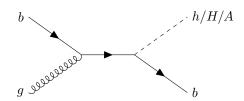


Figure 4

5.2 Event Selection

Channel	Branching Fraction
$\tau_h \tau_h$	42.0%
$e au_h$	23.1%
μau_h	22.6%
$e\mu$	6.2%
ee	3.2%
$\mu\mu$	3.0%

Table 2

- 5.3 Background Modelling
- 5.4 Fake Factor Method
- 5.4.1 Future Improvements to the Fake Factor Method
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- 6 Vector Leptoquark Reinterpretation of the BSM H/A $\to \tau^+\tau^-$ Analysis
- 6.1 Theoretical Motivation

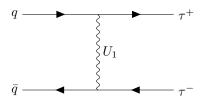
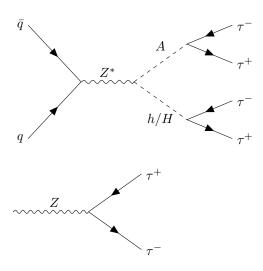


Figure 5: Feynman diagram showing the vector leptoquark t-channel interaction that produces a tau pair from a pair of bottom quarks.

- 6.2 Analysis Strategy
- 6.3 Signal Modelling
- 6.4 Results
- 6.5 Future Improvements to Vector Leptoquark Search



- 7 BSM $Z^* \to HA \to \tau^+ \tau^- \tau^+ \tau^-$
- 7.1 Theoretical Motivation
- 7.2 Event Selection

Channel	Branching Fraction
$e\tau_h\tau_h\tau_h$	19.4%
$\mu \tau_h \tau_h \tau_h$	18.9%
$\tau_h \tau_h \tau_h \tau_h$	17.6%
$e\mu\tau_h\tau_h$	15.6%
$ee au_h au_h$	8.0%
$\mu\mu\tau_h\tau_h$	7.6%
$ee\mu\tau_h$	4.3%
$e\mu\mu\tau_h$	4.2%
$eee au_h$	1.5%
$\mu\mu\mu\tau_h$	1.4%
$eee\mu$	1.4%
$ee\mu\mu$	0.6%
$eee\mu$	0.4%
$e\mu\mu\mu$	0.4%
eeee	0.1%
$\mu\mu\mu\mu$	0.1%

Table 3

Channel	Branching Fraction
	17.6%
$ \begin{array}{c} \tau_h \cdot \tau_h \cdot \tau_h \cdot \tau_h \\ \tau_h^+ \tau_h^- \tau_h^- e^+ \\ \tau_h^+ \tau_h^+ \tau_h^- e^- \\ \mu^- \tau_h^+ \tau_h^+ \tau_h^- \\ \mu^+ \tau_h^+ \tau_h^- \tau_h^- \end{array} $	9.7%
$\tau_{h}^{+}\tau_{h}^{+}\tau_{h}^{-}e^{-}$	9.7%
$\mu^- \tau_h^+ \tau_h^+ \tau_h^-$	9.5%
$ \begin{array}{c} \mu^{-} \tau_{h}^{+} \tau_{h}^{+} \tau_{h}^{-} \\ \mu^{-} \tau_{h}^{+} \tau_{h}^{+} \tau_{h}^{-} \\ \mu^{+} \tau_{h}^{+} \tau_{h}^{-} \tau_{h}^{-} \\ \tau_{h}^{+} \tau_{h}^{-} e^{+} e^{-} \end{array} $	9.5%
$\tau_h^+ \tau_h^- e^+ e^-$	5.3%
$\mu^- \tau_h^+ \tau_h^- e^+$	5.2%
$\mu^{-} \tau_{h}^{+} \tau_{h}^{-} e^{+}$ $\mu^{+} \tau_{h}^{+} \tau_{h}^{-} e^{-}$ $\mu^{+} \tau_{h}^{+} \tau_{h}^{-} e^{-}$	5.2%
$\mu \mu \mu \tau_b \tau_b$	5.1%
$\mu \tau_h \tau_h e$	2.6%
μ ' $ au_h$ $ au_h$ e '	2.6%
$\mu^{-}\tau_{h}^{+}e^{+}e^{-}$	1.4%
$\mu^{+}\tau_{h}^{-}e^{+}e^{-}$	1.4%
$\mu^{+}\mu^{-}\tau_{h}^{-}e^{+}$	1.4%
$\mu^{+}\mu^{-}\tau_{h}^{+}e^{-}$	1.4%
$\tau_{h}^{-}\tau_{h}^{-}e^{+}e^{+}$	1.3%
$\tau_h^+ \tau_h^+ e^- e^-$	1.3%
$\mu^-\mu^-\tau_h^+\tau_h^+$	1.3%
$\mu^+\mu^+ au_h^- au_h^-$	1.3%
$\tau_{h}^{-}e^{+}e^{+}e^{-}$	0.7%
$\tau_h^+ e^+ e^- e^-$	0.7%
$\mu^{-}\tau_{h}^{-}e^{+}e^{+}$	0.7%
$\mu^+ \tau_h^+ e^- e^-$	0.7%
$\mu^{-}\mu^{-}\tau_{h}^{+}e^{+}$	0.7%
$\mu^{+}\mu^{+}\tau_{h}^{-}e^{-}$	0.7%
$\mu^{+}\mu^{-}\mu^{-}\tau_{h}^{+}$	0.7%
$\mu^{+}\mu^{+}\mu^{-}\tau_{h}^{-}$	0.7%
$\mu^{+}\mu^{-}e^{+}e^{-}$	$0.4\% \\ 0.2\%$
$\begin{array}{ c c c c c c c c c c c c c c c c c c c$	0.2%
$\mu^{+}e^{-}e^{-}e^{+}$	0.2%
$\mu^{+}\mu^{-}\mu^{-}e^{-}$	0.2%
$e^{+}e^{+}e^{-}e^{-}$	0.1%
$\mu^{-}\mu^{-}e^{+}e^{+}$	0.1%
$\mu^{+}\mu^{+}e^{-}e^{-}$	0.1%
$\mu^{+}\mu^{+}\mu^{-}\mu^{-}$	0.1%
r. r. r. r.	0.270

Table 4

$$\mathcal{L}_{\text{yukawa}}^{\text{2HDM}} = -\sum_{f=u,d,l} \left(\frac{m_f}{\nu} \xi_h^f \bar{f} f h + \frac{m_f}{\nu} \xi_H^f \bar{f} f H - i \frac{m_f}{\nu} \xi_A^f \bar{f} \gamma_5 f A \right) - \left[\frac{\sqrt{2} V_{ud}}{\nu} \bar{u} (m_u \xi_A^u P_L + m_d \xi_A^d P_R) dH^+ + \frac{\sqrt{2} m_l \xi_A^d}{\nu} \bar{\nu}_L l_R H^+ + h.c. \right]$$

$$\tag{1}$$

	g_h^u	g_h^d	g_h^l	g_H^u	g_H^d	g_H^l	g_A^u	g_A^d	g_A^l
Type I	c_{α}/s_{β}	c_{α}/s_{β}	c_{α}/s_{β}	s_{α}/s_{β}	s_{lpha}/s_{eta}	s_{lpha}/s_{eta}	$1/t_{\beta}$	$-1/t_{\beta}$	$-1/t_{\beta}$
Type II	c_{α}/s_{β}	$-s_{\alpha}/c_{\beta}$	$-s_{\alpha}/c_{\beta}$	s_{lpha}/s_{eta}	c_{lpha}/c_{eta}	c_{lpha}/c_{eta}	$1/t_{\beta}$	t_{eta}	t_{eta}
Type X	c_{α}/s_{β}	c_{lpha}/s_{eta}	$-s_{\alpha}/c_{\beta}$	s_{lpha}/s_{eta}	s_{lpha}/s_{eta}	c_{lpha}/c_{eta}	$1/t_{\beta}$	$-1/t_{\beta}$	t_{eta}
Type Y	c_{α}/s_{β}	$-s_{\alpha}/c_{\beta}$	c_{lpha}/s_{eta}	s_{lpha}/s_{eta}	c_{lpha}/c_{eta}	s_{lpha}/s_{eta}	$1/t_{\beta}$	t_{eta}	$-1/t_{\beta}$

Table 6

In the alignment limit, for the normal scenario $h_{SM}=h, \sin(\beta-\alpha)=1, \implies c_{\alpha}/s_{\beta}=1, s_{\alpha}/c_{\beta}=-1, s_{\alpha}/s_{\beta}=-1/t_{\beta}, c_{\alpha}/c_{\beta}=t_{\beta}.$

Channel	Triggers
$e\tau_h\tau_h\tau_h$	EleTau, singleEle, diTau
$\mu \tau_h \tau_h \tau_h$	MuonTau, singleMuon, diTau
$\tau_h \tau_h \tau_h \tau_h$	diTau
$e\mu\tau_h\tau_h$	MuonEle, diTau
$ee au_h au_h$	diEle, diTau
$\mu\mu\tau_h\tau_h$	diMuon, diTau

Table 5

	g_h^u	g_h^d	g_h^l	g_H^u	g_H^d	g_H^l	g_A^u	g_A^d	g_A^l
Type I	1	1	1	$-1/t_{\beta}$	$-1/t_{\beta}$	$-1/t_{\beta}$	$1/t_{\beta}$	$-1/t_{\beta}$	$-1/t_{\beta}$
Type II	1	1	1	$-1/t_{\beta}$	t_{eta}	t_{eta}	$1/t_{\beta}$	t_{eta}	t_{eta}
Type X	1	1	1	$-1/t_{\beta}$	$-1/t_{\beta}$	t_{eta}	$1/t_{\beta}$	$-1/t_{\beta}$	t_{eta}
Type Y	1	1	1	$-1/t_{\beta}$	t_{eta}	$-1/t_{\beta}$	$1/t_{\beta}$	t_{eta}	$-1/t_{\beta}$

Table 7

In the alignment limit, for the inverted scenario $h_{SM}=H, \cos(\beta-\alpha)=1 \implies c_{\alpha}/s_{\beta}=1/t_{\beta}, s_{\alpha}/c_{\beta}=t_{\beta}, s_{\alpha}/s_{\beta}=1, c_{\alpha}/c_{\beta}=1.$

	g_h^u	g_h^d	g_h^l	g_H^u	g_H^d	g_H^l	g_A^u	g_A^d	g_A^l
Type I	$1/t_{\beta}$	$1/t_{\beta}$	$1/t_{\beta}$	1	1	1	$1/t_{\beta}$	$-1/t_{\beta}$	$-1/t_{\beta}$
Type II	$1/t_{\beta}$	$-t_{eta}$	$-t_{eta}$	1	1	1	$1/t_{\beta}$	t_{eta}	t_{eta}
Type X	$1/t_{\beta}$	$1/t_{\beta}$	$-t_{\beta}$	1	1	1	$1/t_{\beta}$	$-1/t_{\beta}$	t_{eta}
Type Y	$1/t_{\beta}$	$-t_{eta}$	$1/t_{\beta}$	1	1	1	$1/t_{\beta}$	t_{eta}	$-1/t_{\beta}$

Table 8

Trigger	HLT Path
SingleElectron	HLT_Ele25_eta2p1_WPTight_Gsf_v
DoubleElectron	HLT_Ele23_Ele12_CaloIdL_TrackIdL_IsoVL_DZ_v
TripleElectron	HLT_Ele16_Ele12_Ele8_CaloIdL_TrackIdL
SingleMuon	HLT_IsoMu24_v
	OR
	HLT_IsoTkMu24_v
DoubleMuon	HLT_Mu17_TrkIsoVVL_Mu8_TrkIsoVVL(_DZ)_v
	OR
	HLT_Mu17_TrkIsoVVL_TkMu8_TrkIsoVVL(_DZ)_v
TripleMuon	HLT_TripleMu_12_10_5
SingleTau	HLT_VLooseIsoPFTau120_Trk50_eta2p1_v
	OR
	$HLT_VLooseIsoPFTau140_Trk50_eta2p1_v$
DoubleTau	HLT_DoubleMediumIsoPFTau35_Trk1_eta2p1_Reg_v
Electron Tau Cross	HLT_Ele24_eta2p1_WPLoose_Gsf_LooseIsoPFTau20_SingleL1_v
Muon Tau Cross	HLT_IsoMu19_eta2p1_LooseIsoPFTau20_SingleL1_v

Table 9: 2016 HLT paths.

- 7.3 Background Modelling
- 7.4 Optimisation of Analysis
- 7.5 Signal Extraction
- 7.6 Results

$m_A \text{ (GeV)}$	60			100			150		
$m_{\phi} \; (\text{GeV})$	100	200	300	100	200	300	100	200	300
Cross Section (pb)	0.66	0.089	0.023	0.29	0.057	0.035	0.12	0.035	0.013

e	μ	$ au_h$
$p_T > 15 \text{ GeV}$	$p_T > 15 \text{ GeV}$	$p_T > 20 \text{ GeV}$
$ \eta < 2.5$	$ \eta < 2.4$	$ \eta < 2.3$
MVA 90% WP	Medium ID	HPS algorithm
$I_{rel} < 0.15$	$I_{rel} < 0.15$	deepTauVSjet VVVLoose WP
		deepTauVSele VLoose WP
		deepTauVSmu VLoose WP

Method/ Channel	$\tau_h \tau_h \tau_h \tau_h$	$\mu \tau_h \tau_h \tau_h$	$e\tau_h\tau_h\tau_h$	$\mu\mu\tau_h\tau_h$	$ee au_h au_h$	$e\mu\tau_h\tau_h$
Selected by HPS and loose isolation	0.53	0.26	0.23	0.12	0.15	0.12
Selected by DeepTau and tight isolation	0.10	0.19	0.19	0.31	0.29	0.35
Selected objects from Higgs decay	0.88	0.90	0.89	0.93	0.91	0.91

Table 10

8 Conclusion

- 8.1 Global Interpretations of Results
- 8.2 $\,$ Projections to Run-3 and HL-LHC

Method/ Channel	$\tau_h \tau_h \tau_h \tau_h$	$\mu \tau_h \tau_h \tau_h$	$e\tau_h\tau_h\tau_h$	$\mu\mu\tau_h\tau_h$	$ee au_h au_h$	$e\mu\tau_h\tau_h$
Minimum single ΔR	0.84	0.84	0.84	0.84	0.83	0.84
Minimum summed ΔR	0.87	0.87	0.86	0.86	0.87	0.87
Minimum single $\Delta \phi$	0.82	0.84	0.83	0.83	0.83	0.82
Minimum summed $\Delta \phi$	0.85	0.87	0.85	0.85	0.85	0.86

Table 11

Channel	Triggers	$(m_{\phi}, m_A) = (100, 60)$	$(m_{\phi},m_A)=(300,60)$	$(m_{\phi}, m_A) = (100, 100)$	$(m_{\phi} , m_A) = (300, 100)$
$e\tau_h\tau_h\tau_h$	$\tau_h^1 \tau_h^2$	0.38	0.51	0.48	0.61

Table 12

Channel	Triggers	$m_A \; (\text{GeV})$	60		100			150			
Channel		$m_{\phi} \; (\text{GeV})$	100	200	300	100	200	300	100	200	300
$\tau_h \tau_h \tau_h \tau_h$	$ au_h au_h^1 au_h^2 \ au_h^1 au_h^2 ext{ or } au_h^1 au_h^3 \ au_h^3 au_h^3 ext{ or } au_h^1 au_h^3 \ au_h^3 au_h^3 $		0.41	0.5	0.52	0.5	0.6	0.62	0.56	0.65	0.7
	$\tau_h^1 \tau_h^2$ or $\tau_h^1 \tau_h^3$		0.47 0.48	0.59	0.64	0.57	0.7	0.73	0.64	0.74	0.79
	$\begin{array}{c} \tau_{h}^{1}\tau_{h}^{2} \text{ or } \tau_{h}^{1}\tau_{h}^{3} \\ \tau_{h}^{1}\tau_{h}^{2} \text{ or } \tau_{h}^{1}\tau_{h}^{3} \text{ or } \tau_{h}^{1}\tau_{h}^{4} \\ \tau_{h}^{1}\tau_{h}^{2} \text{ or } \tau_{h}^{1}\tau_{h}^{3} \text{ or } \tau_{h}^{1}\tau_{h}^{4} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \\ \tau_{h}^{1}\tau_{h}^{2} \text{ or } \tau_{h}^{1}\tau_{h}^{3} \text{ or } \tau_{h}^{1}\tau_{h}^{4} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \\ \tau_{h}^{1}\tau_{h}^{2} \text{ or } \tau_{h}^{1}\tau_{h}^{3} \text{ or } \tau_{h}^{1}\tau_{h}^{4} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \\ \tau_{h}^{1}\tau_{h}^{2} \text{ or } \tau_{h}^{1}\tau_{h}^{3} \text{ or } \tau_{h}^{1}\tau_{h}^{4} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \\ \end{array}$			0.62	0.67	0.59	0.72	0.75	0.66	0.76	0.8
				0.67	0.73	0.65	0.78	0.82	0.72	0.82	0.87
				0.68	0.75	0.65	0.79	0.83	0.72	0.83	0.88
	$\tau_h^1 \tau_h^2$ or $\tau_h^1 \tau_h^3$ or $\tau_h^1 \tau_h^4$ or $\tau_h^2 \tau_h^3$ or $\tau_h^2 \tau_h^4$ or $\tau_h^2 \tau_h^4$	$_{h}^{-3} au_{h}^{4}$	0.54	0.69	0.76	0.66	0.79	0.84	0.73	0.84	0.88
$\mu \tau_h \tau_h \tau_h$			0.64	0.71	0.76	0.69	0.75	0.78	0.7	0.77	0.8
	μ^1 or $\tau_h^2 \tau_h^3$		0.75	0.85	0.89	0.82	0.88	0.91	0.84	0.9	0.93
	μ^1 or $\tau_h^2 \tau_h^3$ or $\tau_h^2 \tau_h^4$		0.76	0.86	0.9	0.83	0.9	0.93	0.86	0.92	0.95
	$ \begin{vmatrix} \mu^1 \text{ or } \tau_h^2 \tau_h^3 \text{ or } \tau_h^2 \tau_h^4 \text{ or } \tau_h^3 \tau_h^4 \text{ or } \mu^1 \tau_h^2 \text{ or } \mu^1 \tau_h^3 \text{ or } \mu^1 \tau_h^4 \\ \tau_h^2 \tau_h^3 \text{ or } \tau_h^2 \tau_h^4 \text{ or } \tau_h^3 \tau_h^4 \end{vmatrix} $		0.76	0.87	0.91	0.84	0.9	0.94	0.87	0.92	0.95
			0.82	0.9	0.93	0.87	0.93	0.95	0.91	0.95	0.96
			0.39	0.53	0.58	0.5	0.61	0.68	0.58	0.68	0.74
	$\mu_1^1 \tau_h^2 \text{ or } \mu_1^1 \tau_h^3 \text{ or } \mu_1^1 \tau_h^4$	3 4	0.67	0.73	0.77	0.7	0.77	0.79	0.74	0.79	0.81
	$\mu^1 \tau_h^2$ or $\mu^1 \tau_h^3$ or $\mu^1 \tau_h^4$ or $\tau_h^2 \tau_h^3$ or $\tau_h^2 \tau_h^4$ or	$\tau_h^3 \tau_h^4$	0.78	0.85	0.89	0.83	0.89	0.92	0.88	0.92	0.94
$e\tau_h\tau_h\tau_h$	$\begin{bmatrix} e^1 \\ 1 \end{bmatrix}$		0.41	0.5	0.57	0.46	0.54	0.6	0.49	0.56	0.62
	e^1 or $\tau_h^2 \tau_h^3$		0.6	0.7	0.77	0.67	0.77	0.82	0.74	0.8	0.85
	e^1 or $\tau_h^2 \tau_h^3$ or $\tau_h^2 \tau_h^4$		0.63	0.72	0.81	0.69	0.8	0.85	0.77	0.83	0.88
	e^1 or $\tau_h^2 \tau_h^3$ or $\tau_h^2 \tau_h^4$ or $\tau_h^3 \tau_h^4$	9 1 4	0.64	0.75	0.83	0.71	0.81	0.87	0.78	0.85	0.9
	$\begin{bmatrix} e^1 \text{ or } \tau_h^2 \tau_h^3 \text{ or } \tau_h^2 \tau_h^4 \text{ or } \tau_h^3 \tau_h^4 \text{ or } e^1 \tau_h^2 \text{ or } e^1 \tau_h^2 \end{bmatrix}$	or $e^{1}\tau_{h}^{4}$	0.71	0.8	0.86	0.76	0.85	0.89	0.82	0.87	0.91
	$\tau_h^2 \tau_h^3$ or $\tau_h^2 \tau_h^4$ or $\tau_h^3 \tau_h^4$		0.41	0.48	0.57	0.48	0.6	0.68	0.56	0.66	0.73
	$\begin{bmatrix} e^{1}\tau_{h}^{2} \text{ or } e^{1}\tau_{h}^{3} \text{ or } e^{1}\tau_{h}^{4} \\ 1 & 2 & 1 & 3 & 1 & 4 & 2 & 3 & 2 & 4 \end{bmatrix}$	3 1	0.49	0.54	0.59	0.5	0.59	0.62	0.54	0.6	0.65
	$e^{i}\tau_{h}^{2}$ or $e^{i}\tau_{h}^{3}$ or $e^{i}\tau_{h}^{4}$ or $\tau_{h}^{2}\tau_{h}^{3}$ or $\tau_{h}^{2}\tau_{h}^{4}$ or τ_{h}^{2}	$\tau_h^4 \tau_h^4$	0.66	0.74	0.8	0.7	0.81	0.86	0.78	0.84	0.89
$e\mu\tau_h\tau_h$	$\begin{array}{c} e^{1} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \\ e^{1} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \\ e^{1} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \text{ or } \tau_{h}^{3}\tau_{h}^{4} \\ e^{1} \text{ or } \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \text{ or } \tau_{h}^{3}\tau_{h}^{4} \text{ or } e^{1}\tau_{h}^{2} \text{ or } e^{1}\tau_{h}^{2} \\ \tau_{h}^{2}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \text{ or } \tau_{h}^{3}\tau_{h}^{4} \text{ or } e^{1}\tau_{h}^{2} \text{ or } e^{1}\tau_{h}^{3} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \text{ or } \tau_{h}^{2}\tau_{h}^{2} \text{ or } \tau_{h}^{2}\tau_{h}^{4} \text{ or } \tau_{h}^{2}\tau_{h}^{2} or $		0.78	0.86	0.9	0.82	0.86	0.92	0.86	0.91	0.93
	$e^{1} \text{ or } \mu^{2} \text{ or } \tau_{h}^{3} \tau_{h}^{4}$ $e^{1} \text{ or } \mu^{2} \text{ or } \tau_{h}^{3} \tau_{h}^{4} \text{ or } e^{1} \mu^{1}$		0.82	0.91	0.94	0.86	0.91	0.95	0.9	0.94	0.96
	$\begin{bmatrix} e^1 \text{ or } \mu^2 \text{ or } \tau_h^2 \tau_h^2 \text{ or } e^1 \mu^2 \end{bmatrix}$		0.84	0.92	0.94	0.88	0.92	0.96	0.91	0.95	0.97
	$e^1\mu^1$		0.62	0.7	0.75	0.66	0.72	0.76	0.69	0.75	0.77
	$\begin{bmatrix} e^-\mu^- & \text{or } \tau_h^* \tau_h^- \\ -3 & -4 \end{bmatrix}$		0.69	0.78	0.83	0.74	0.82	0.86	0.78	0.85	0.88
	$\begin{array}{c} e^{1} {\mu^{1}} \text{ or } \tau_{h}^{3} \tau_{h}^{4} \\ \tau_{h}^{3} \tau_{h}^{4} \\ e^{1} \end{array}$		0.19	0.27	0.31	0.26	0.35	0.4	0.3	0.38	0.45
$ee\tau_h\tau_h$	$\begin{bmatrix} e^1 \\ e^1 \text{ or } e^2 \end{bmatrix}$		0.56	0.69		0.68				0.75	
	e or e		0.61	0.73	0.81	0.71	0.79	0.84	0.76	0.81	0.86
	$ \begin{array}{ c c c c c c c c c c c c c c c c c c c$	n o ² -4	0.69 0.78	0.8	0.87	0.76 0.81	0.86	$0.91 \\ 0.93$	0.83 0.88	0.89 0.92	0.92
	$\begin{bmatrix} e & \text{or } e & \text{or } \tau_h \tau_h & \text{or } e & \tau_h & \text{or } e & \tau_h & \text{or } e^- \tau_h^* & $	$T \in T_h$		0.80 0.24	$0.9 \\ 0.28$	0.81		0.93 0.37	0.88 0.32		
L	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.18	0.24	0.28	0.24	0.34	0.37		0.4	0.45
$\mu\mu\tau_h\tau_h$			0.82	!	0.9	0.84	0.89	0.91	$0.89 \\ 0.93$	0.9	0.91
	$\mu^1 \text{ or } \mu^2$			0.9				0.96		0.94	0.98
	$\begin{array}{c} \mu^1 \text{ or } \mu^2 \text{ or } \tau_h^3 \tau_h^4 \\ \mu^1 \text{ or } \mu^2 \text{ or } \tau_h^3 \tau_h^4 \text{ or } \mu^1 \tau_h^3 \text{ or } \mu^1 \tau_h^4 \text{ or } \mu^2 \tau_h^3 \end{array}$	on u2-4	0.86	0.93	0.96	0.9 0.94	$0.95 \\ 0.97$	0.98	0.95	0.96	
	μ or μ or $\tau_h^* \tau_h^*$ or $\mu^- \tau_h^*$ or $\mu^- \tau_h^*$ or $\mu^- \tau_h^*$	or μ - τ_h	0.9	0.96	0.98 0.34	0.94	0.97		0.97 0.29	0.98	0.98
	$ au_h^3 au_h^4$		0.21	0.3	0.34	0.20	0.34	0.4	0.29	0.39	0.44