Chapter 10

Parametric Equations and Polar Coordinates

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Section 10.1

Parametrizations of Plane Curves

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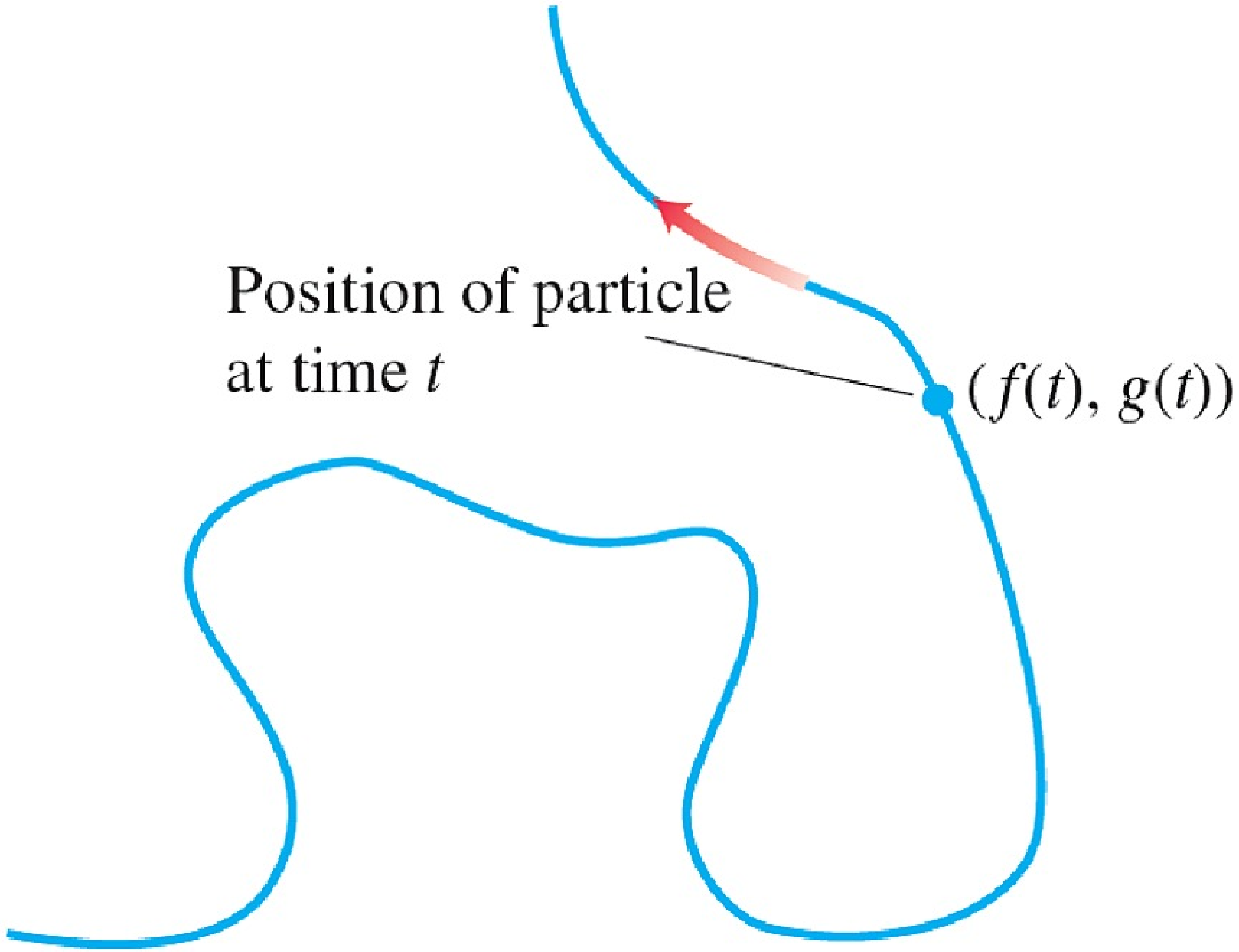

FIGURE 10.1 The curve or path traced by a particle moving in the xy-plane is not always the graph of a function or single equation.

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DEFINITION If x and y are given as functions

$$x = f(t), \qquad y = g(t)$$

over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a **parametric curve**. The equations are **parametric equations** for the curve.

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TABLE 10.1 Values of $x = \sin \pi t/2$ and $y = t$ for selected values of t.

t	x	y
0	0	0
1	1	1
2	0	2
3	-1	3
4	0	4
5	1	5
6	0	6

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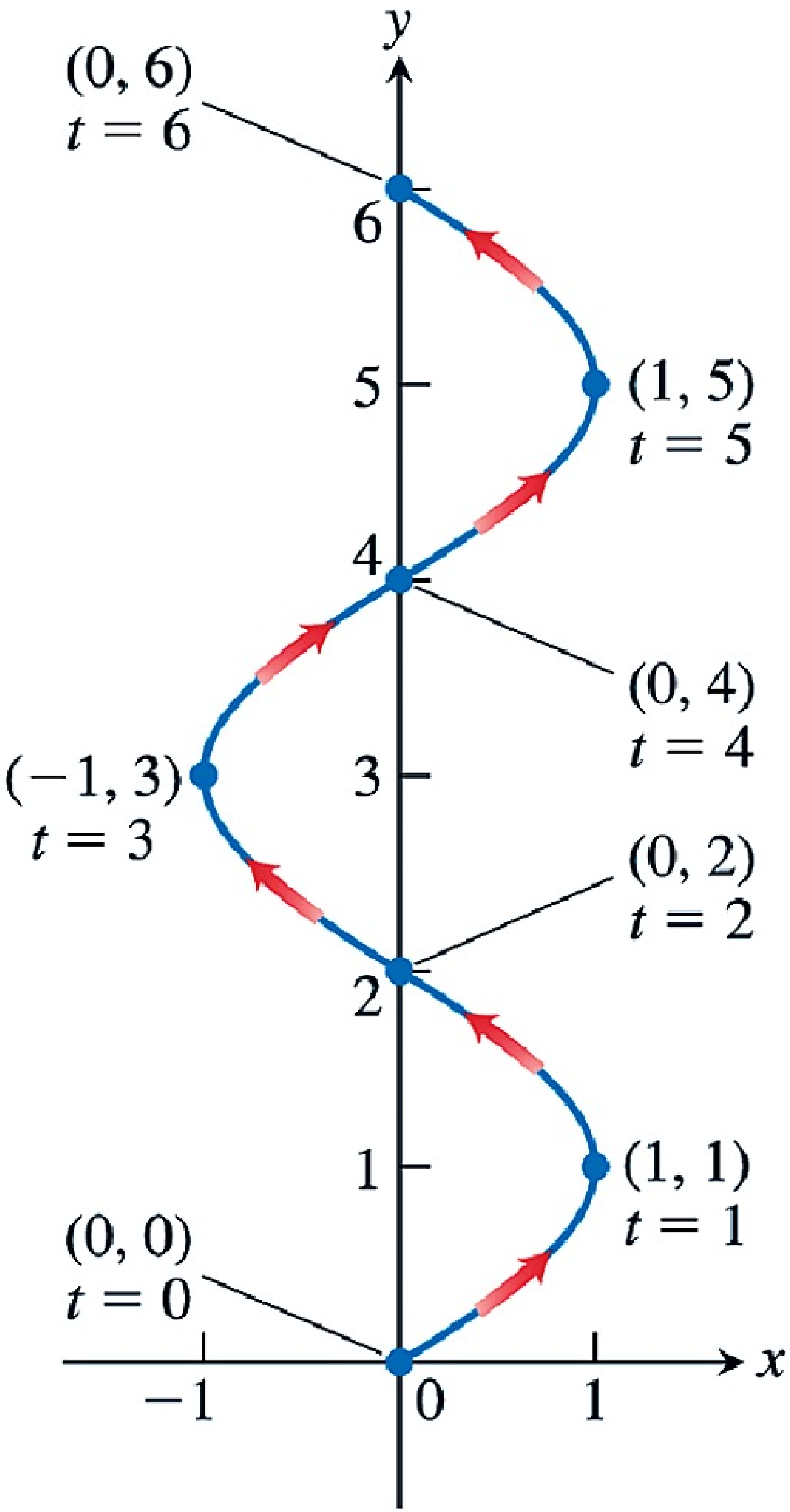

FIGURE 10.2 The curve given by the parametric equations $x = \sin \pi t/2$ and $y = t$ (Example 1).

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TABLE 10.2 Values of $x = t^2$ and $y = t + 1$ for selected values of t.

t	x	y
-3	9	-2
-2	4	-1
-1	1	0
0	0	1
1	1	2
2	4	3
3	9	4

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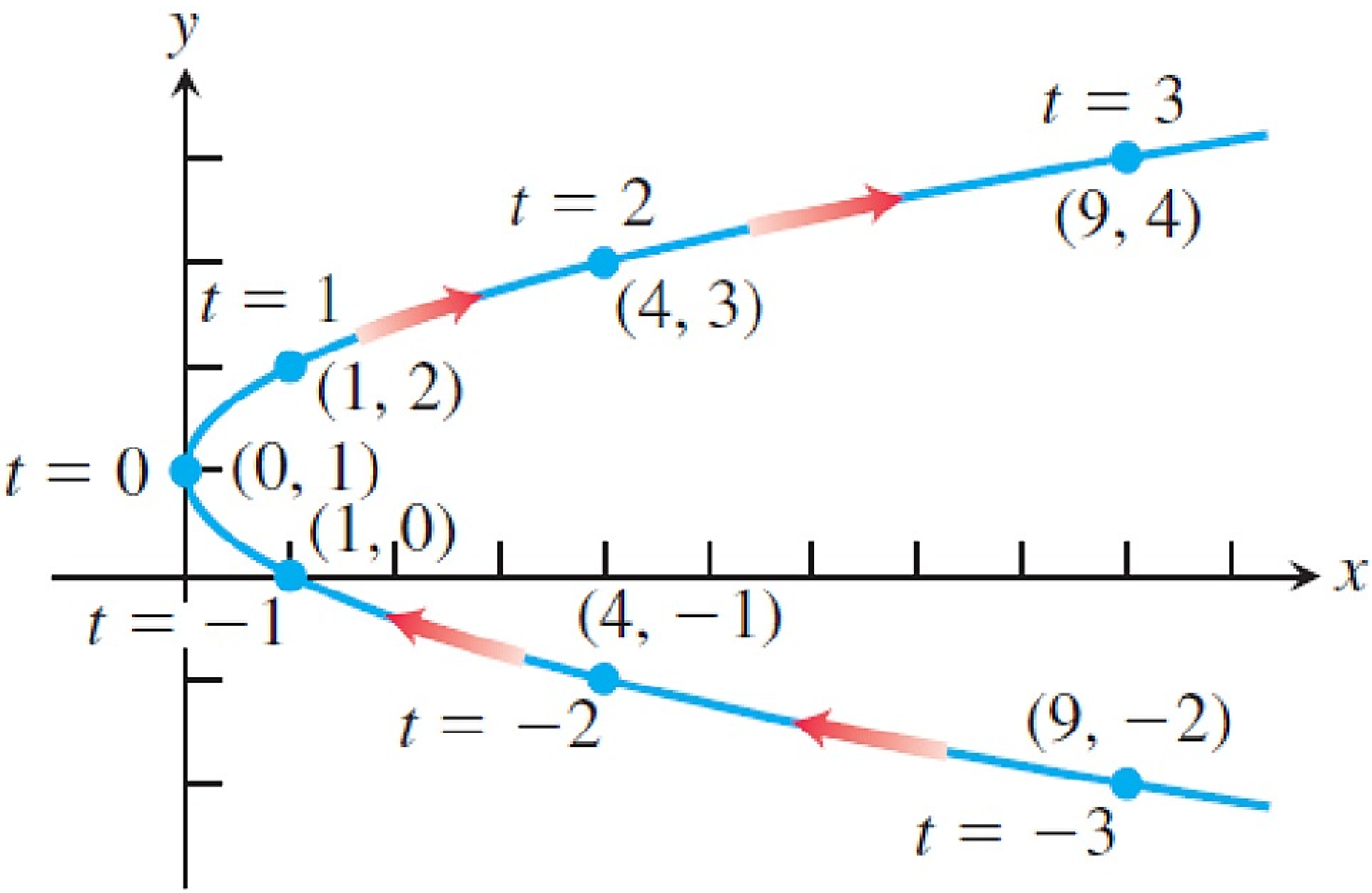

FIGURE 10.3 The curve given by the parametric equations $x = t^2$ and $y = t + 1$ (Example 2).

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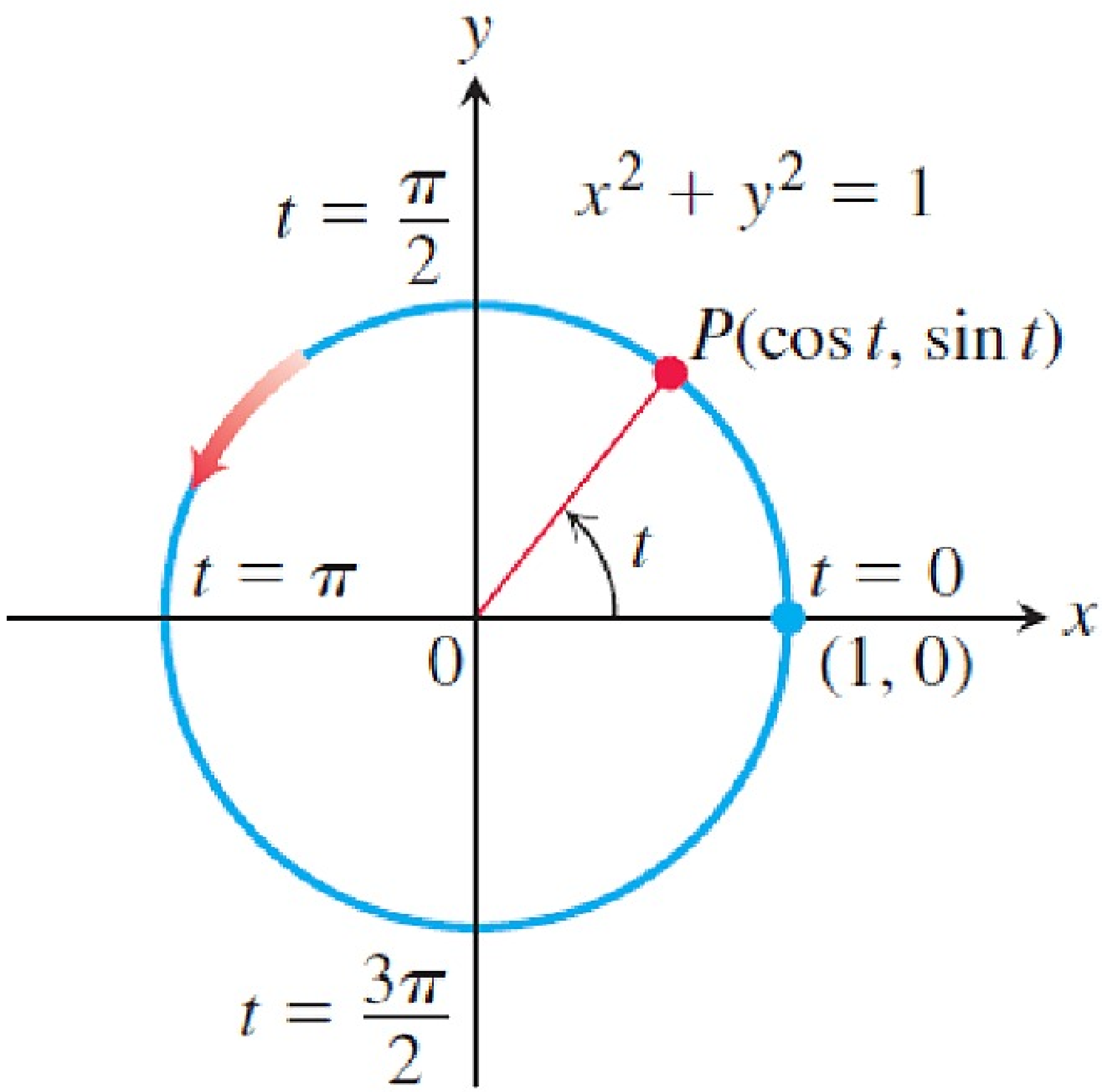

FIGURE 10.4 The equations $x = \cos t$ and $y = \sin t$ describe motion on the circle $x^2 + y^2 = 1$. The arrow shows the direction of increasing t (Example 3).

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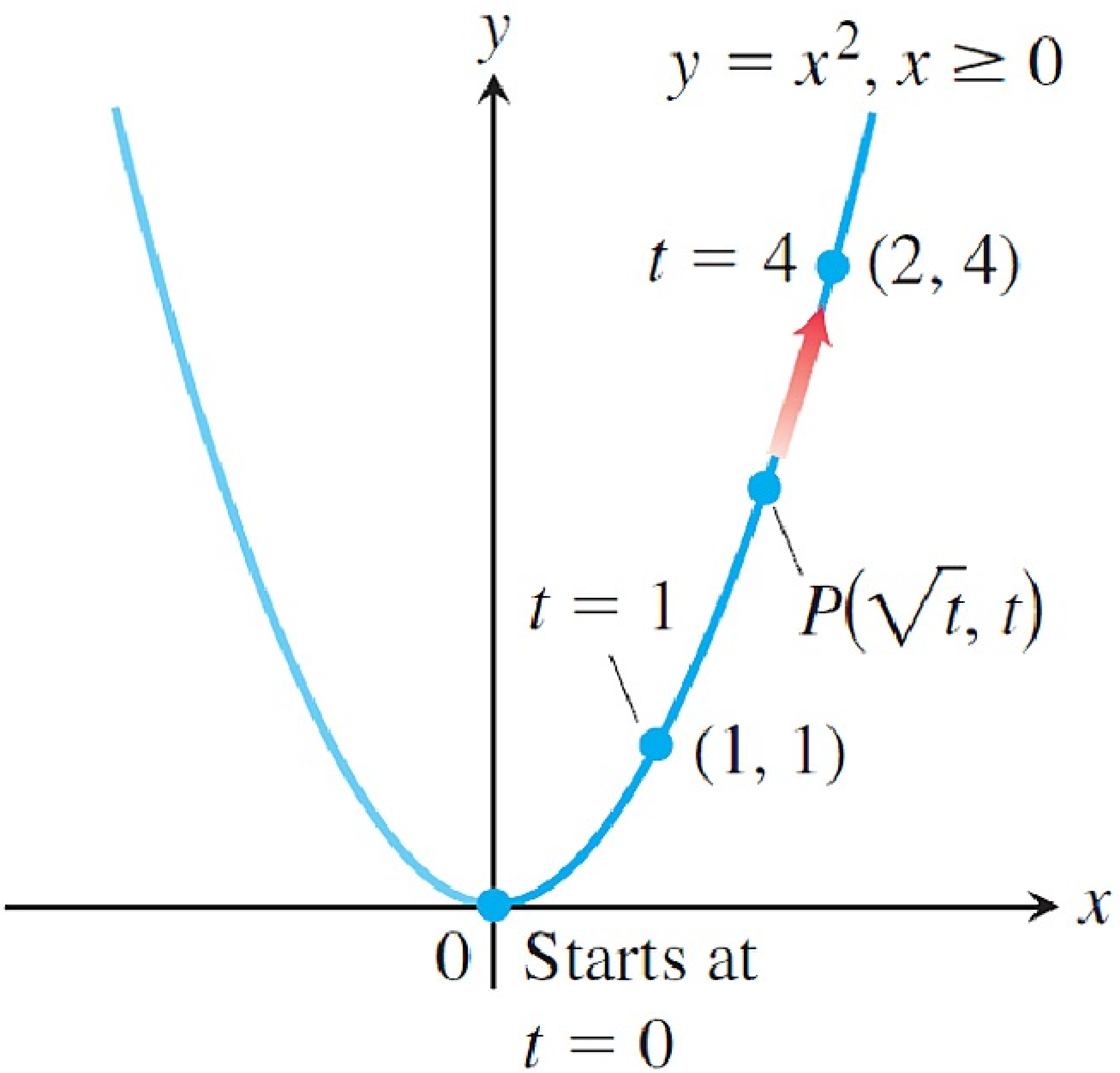

FIGURE 10.5 The equations $x = \sqrt{t}$ and $y = t$ and the interval $t \geq 0$ describe the path of a particle that traces the right-hand half of the parabola $y = x^2$ (Example 4).

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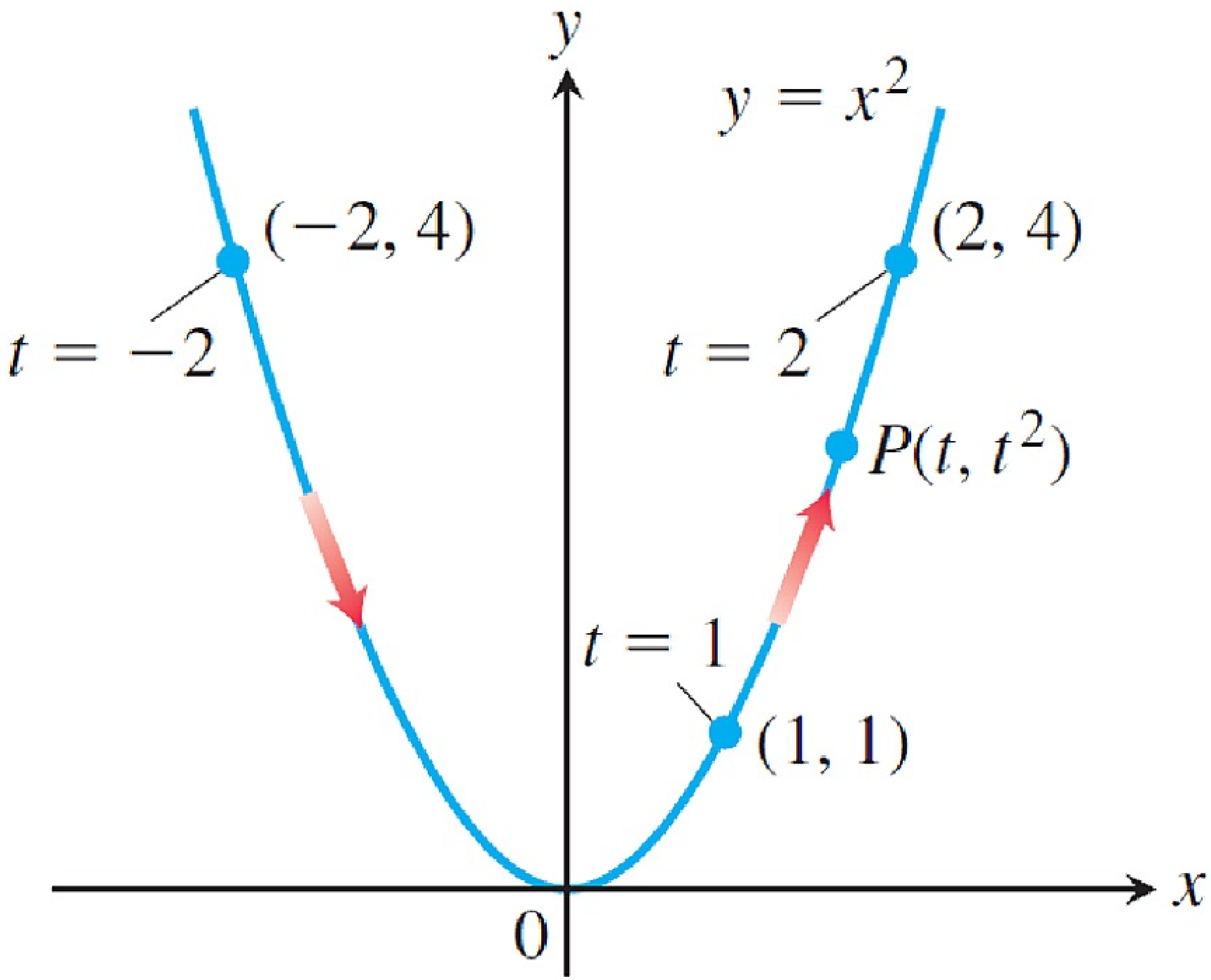

FIGURE 10.6 The path defined by $x = t, y = t^2, -\infty < t < \infty$ is the entire parabola $y = x^2$ (Example 5).

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TABLE 10.3 Values of $x = t + (1/t)$ and $y = t - (1/t)$ for selected values of t.

t	$1/t$	x	y
0.1	10.0	10.1	−9.9
0.2	5.0	5.2	−4.8
0.4	2.5	2.9	−2.1
1.0	1.0	2.0	0.0
2.0	0.5	2.5	1.5
5.0	0.2	5.2	4.8
10.0	0.1	10.1	9.9

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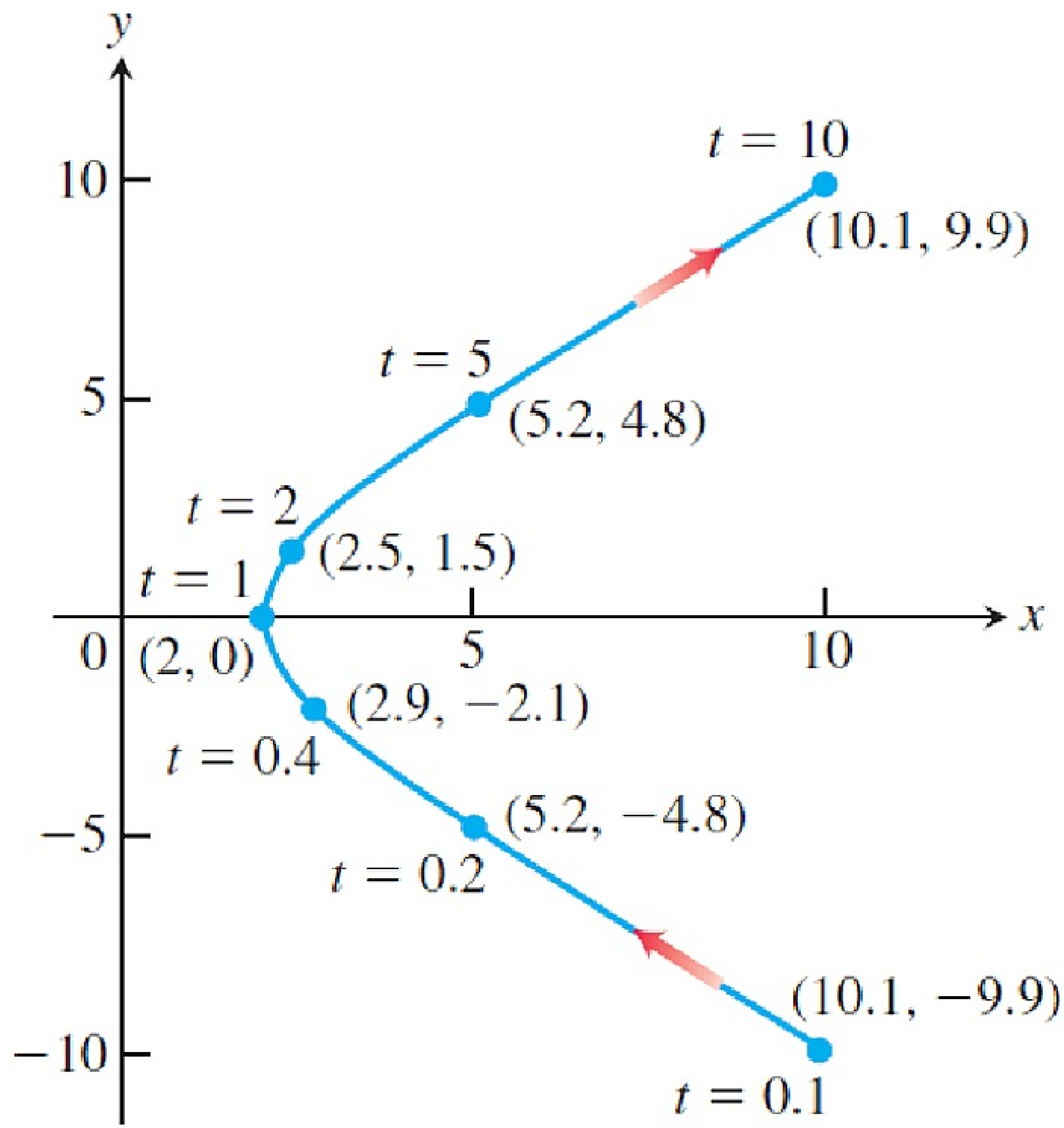

FIGURE 10.7 The curve for $x = t + (1/t)$, $y = t - (1/t)$, $t > 0$ in Example 7. (The part shown is for $0.1 \leq t \leq 10$.)

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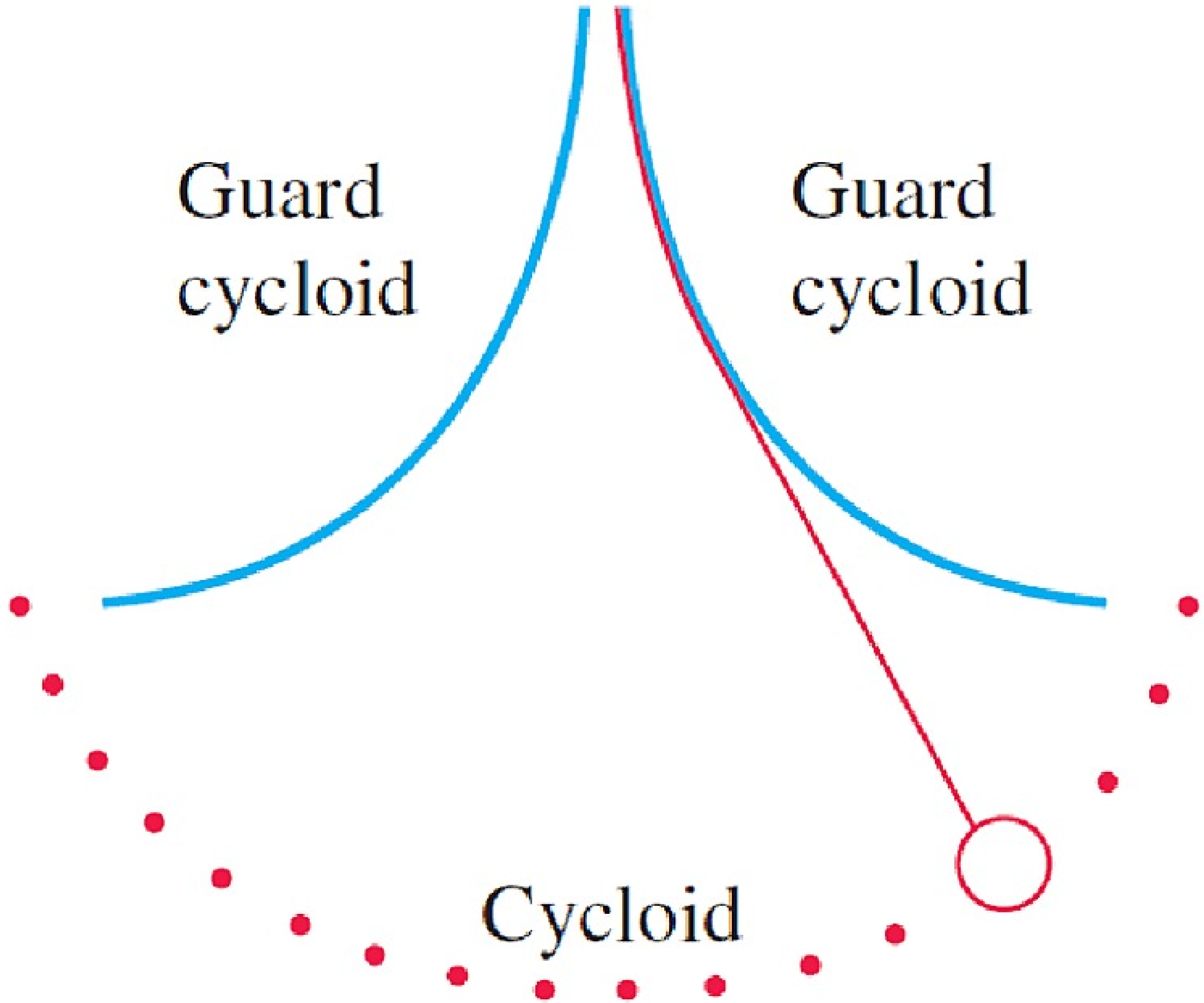

FIGURE 10.8 In Huygens' pendulum clock, the bob swings in a cycloid, so the frequency is independent of the amplitude.

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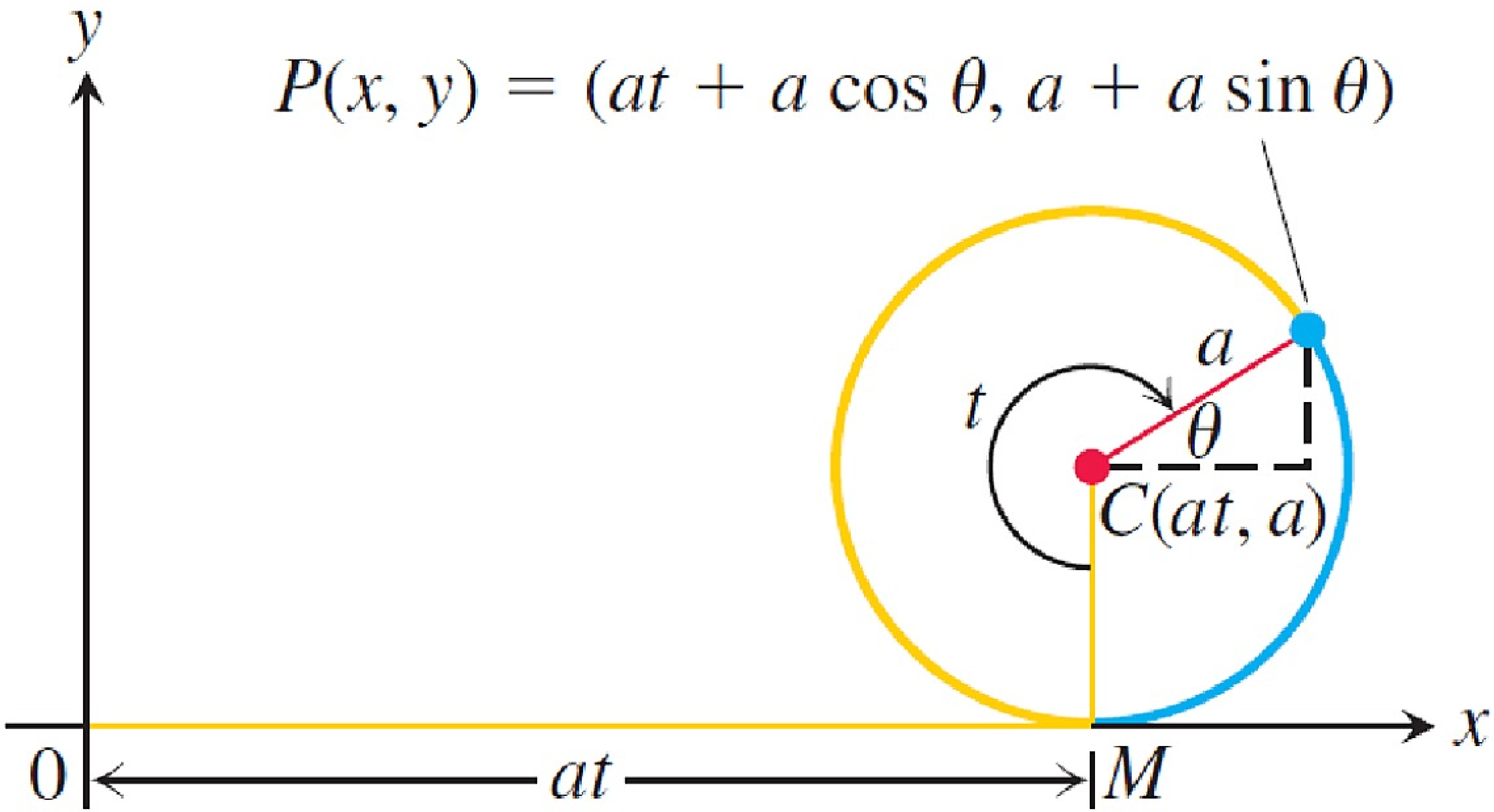

FIGURE 10.9 The position of $P(x, y)$ on the rolling wheel at angle t (Example 8).

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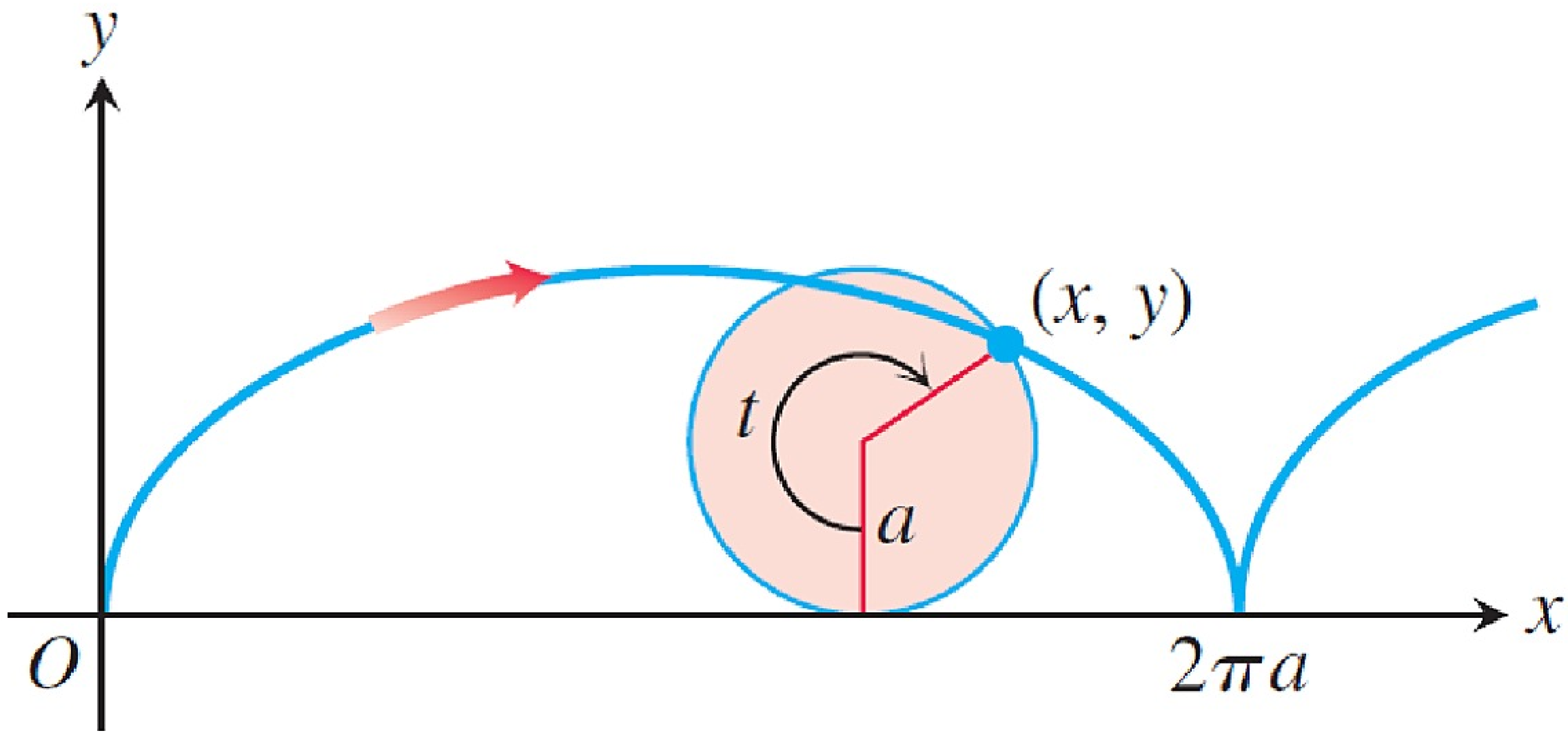

FIGURE 10.10 The cycloid curve $x = a(t - \sin t)$, $y = a(1 - \cos t)$, for $t \geq 0$.

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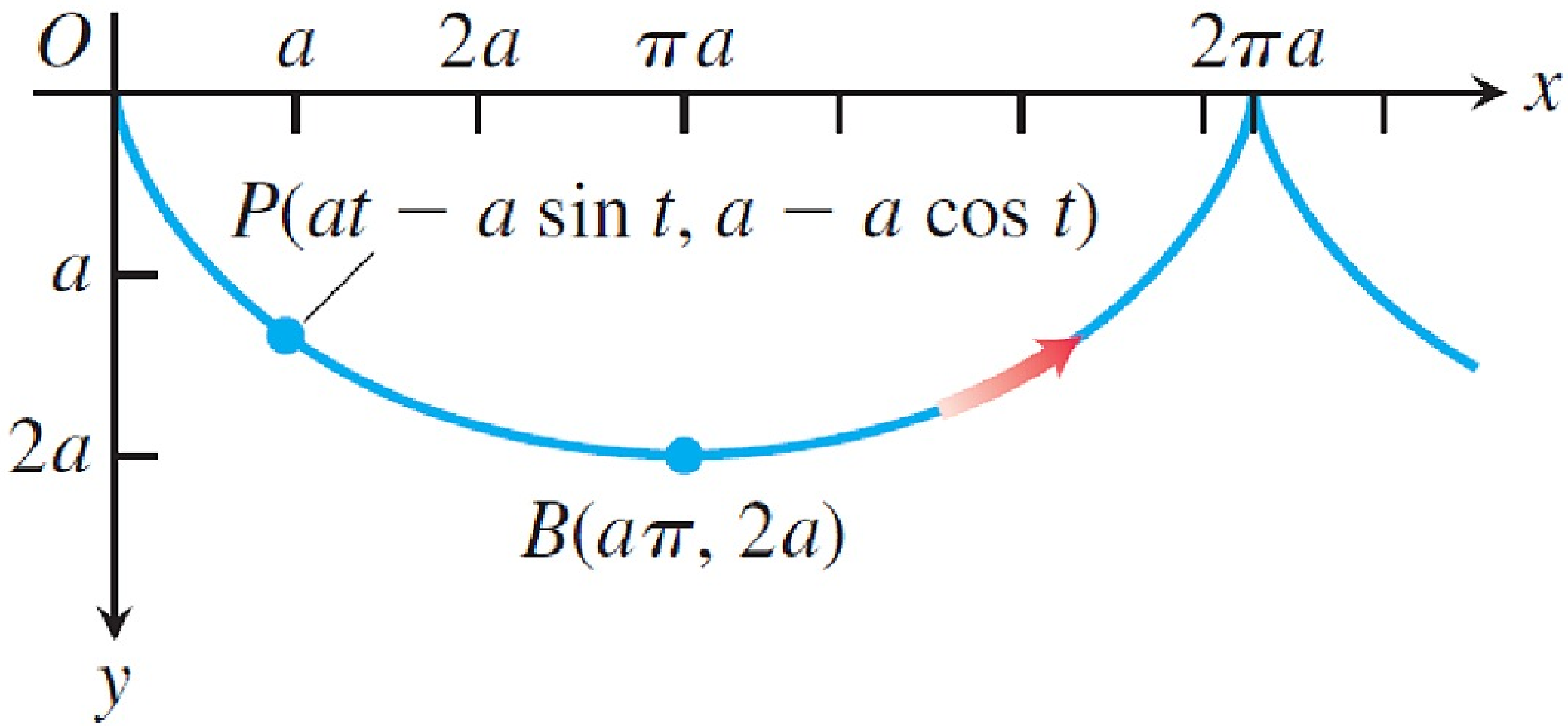

FIGURE 10.11 Turning Figure 10.10 upside down, the y-axis points downward, indicating the direction of the gravitational force. Equations (2) still describe the curve parametrically.

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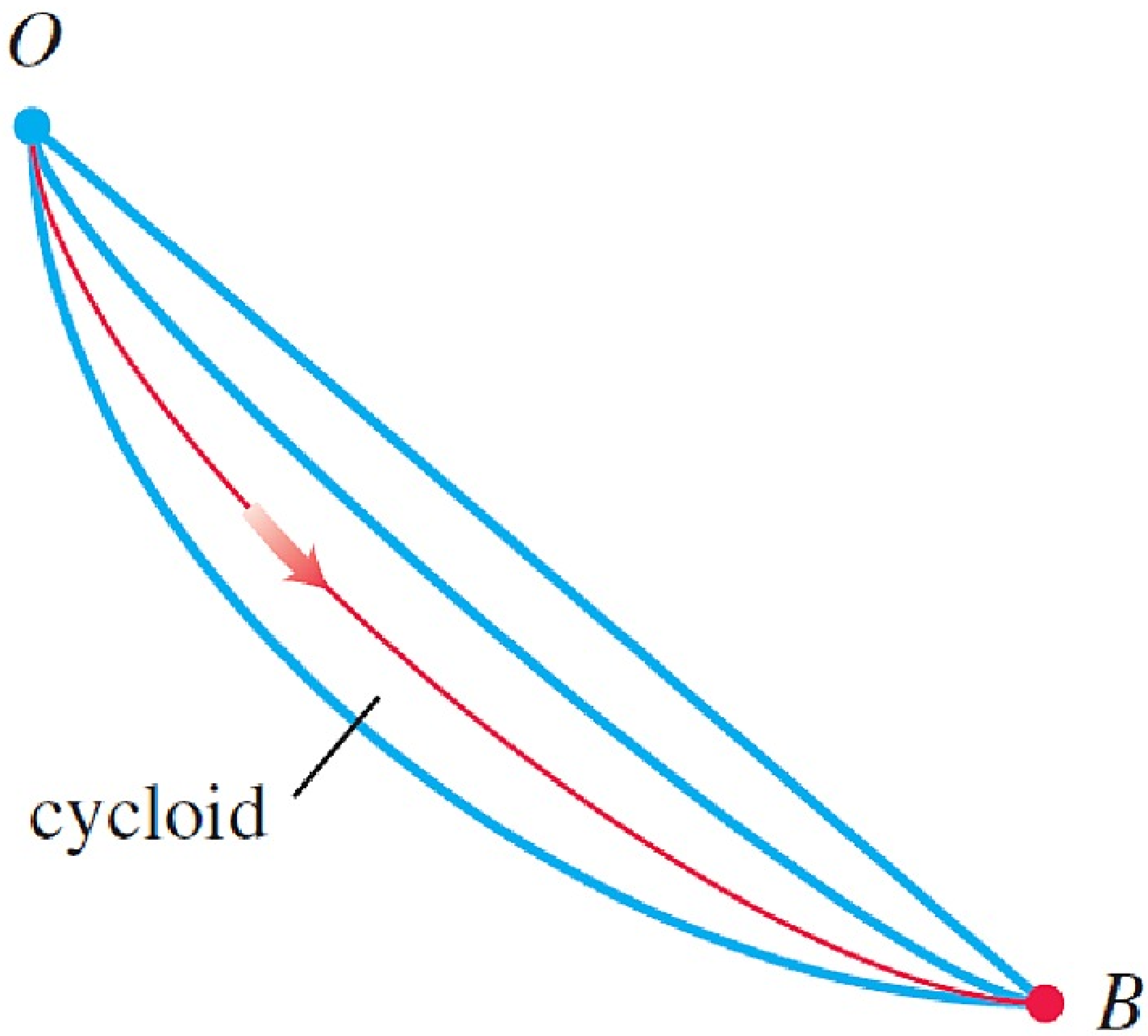

FIGURE 10.12 The cycloid is the unique curve which minimizes the time it takes for a frictionless bead to slide from point O to B.

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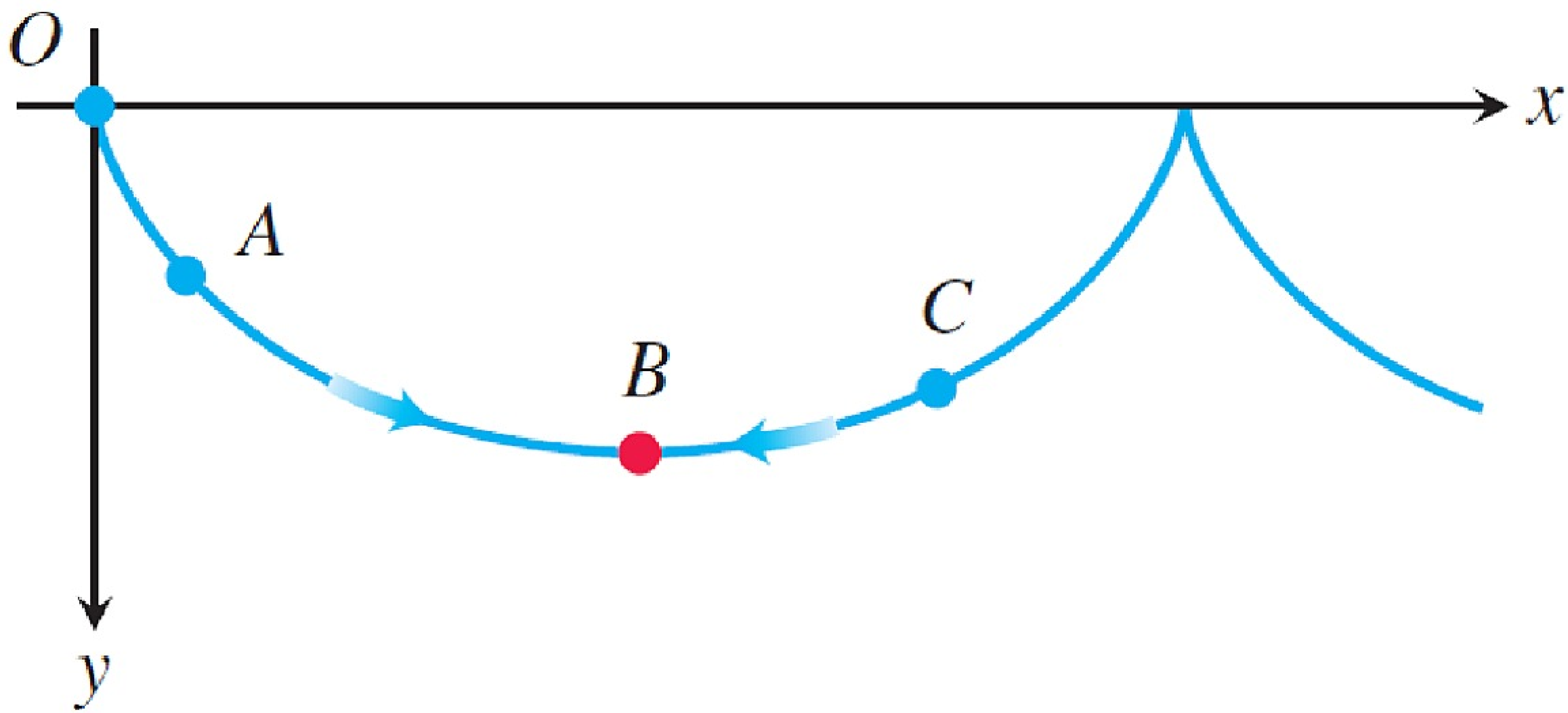

FIGURE 10.13 Beads released simultaneously on the upside-down cycloid at O, A, and C will reach B at the same time.

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Section 10.2

Calculus with Parametric Curves

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Parametric Formula for dy/dx

If all three derivatives exist and $dx/dt \neq 0$,

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}. \tag{1}$$

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Parametric Formula for d^2y/dx^2

If the equations $x = f(t)$, $y = g(t)$ define y as a twice-differentiable function of x, then at any point where $dx/dt \neq 0$ and $y' = dy/dx$,

$$\frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}. \tag{2}$$

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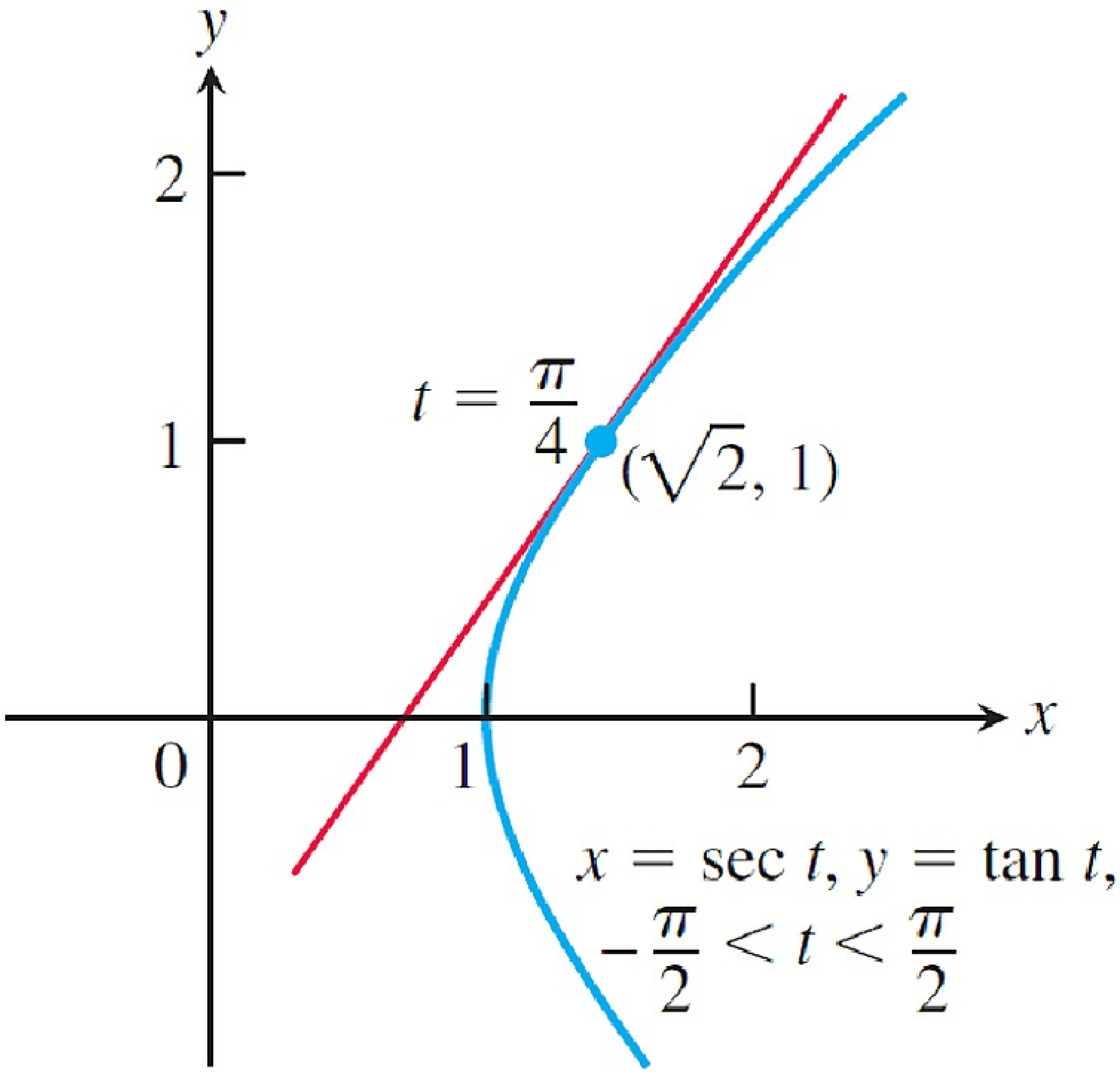

FIGURE 10.14 The curve in Example 1 is the right-hand branch of the hyperbola $x^2 - y^2 = 1$.

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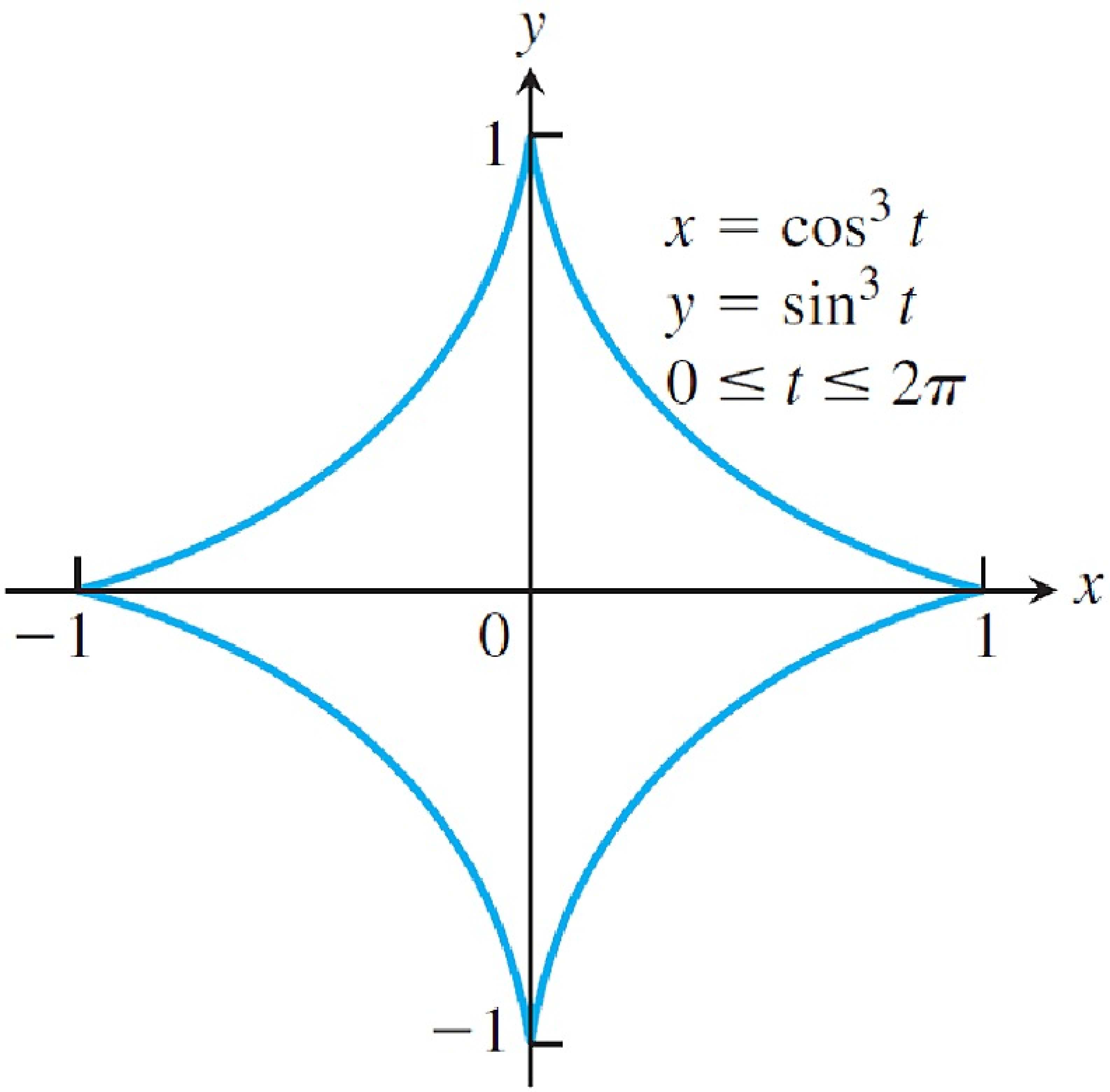

FIGURE 10.15 The astroid in Example 3.

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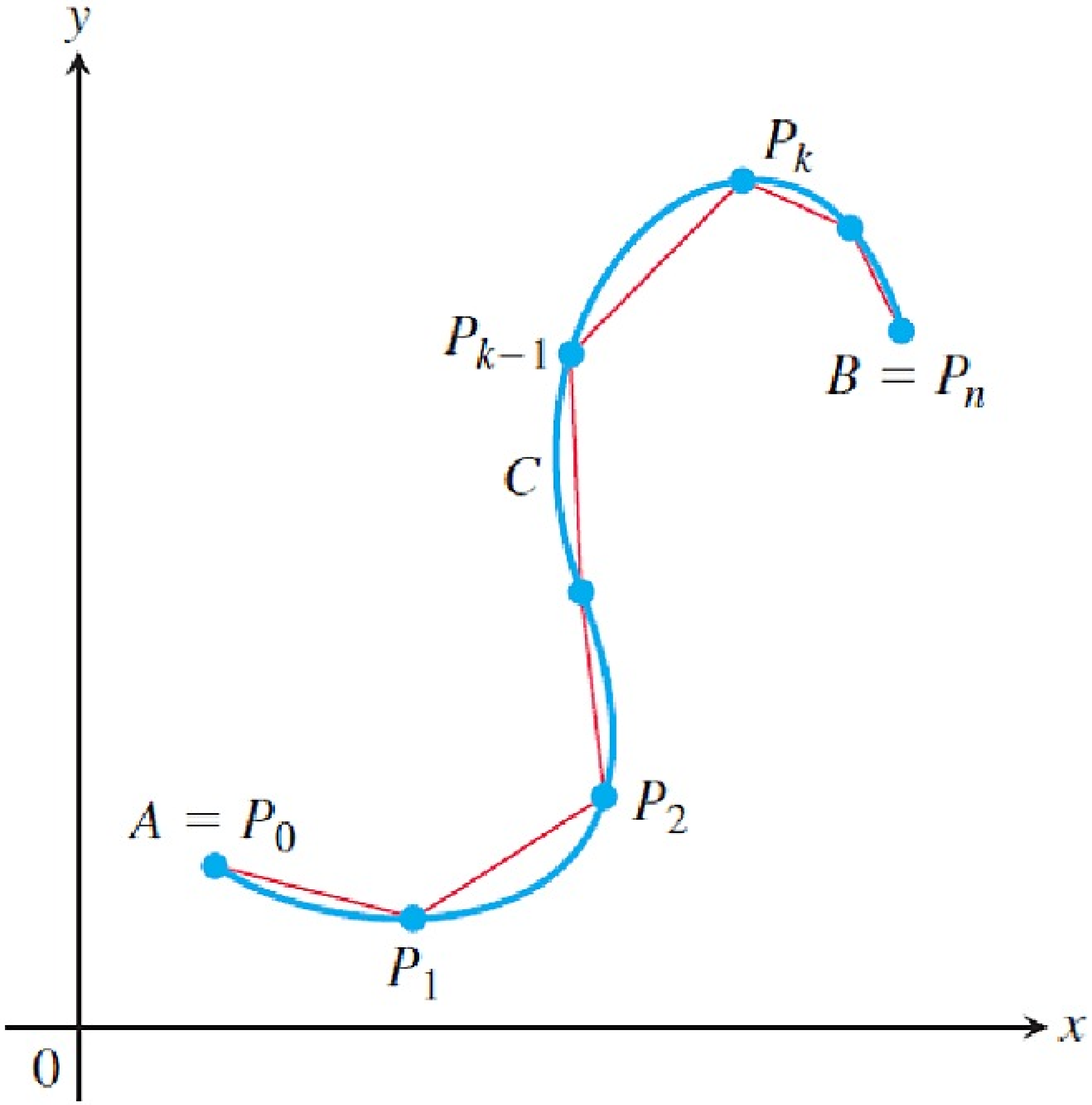

FIGURE 10.16 The length of the smooth curve C from A to B is approximated by the sum of the lengths of the polygonal path (straight-line segments) starting at $A = P_0$, then to P_1, and so on, ending at $B = P_n$.

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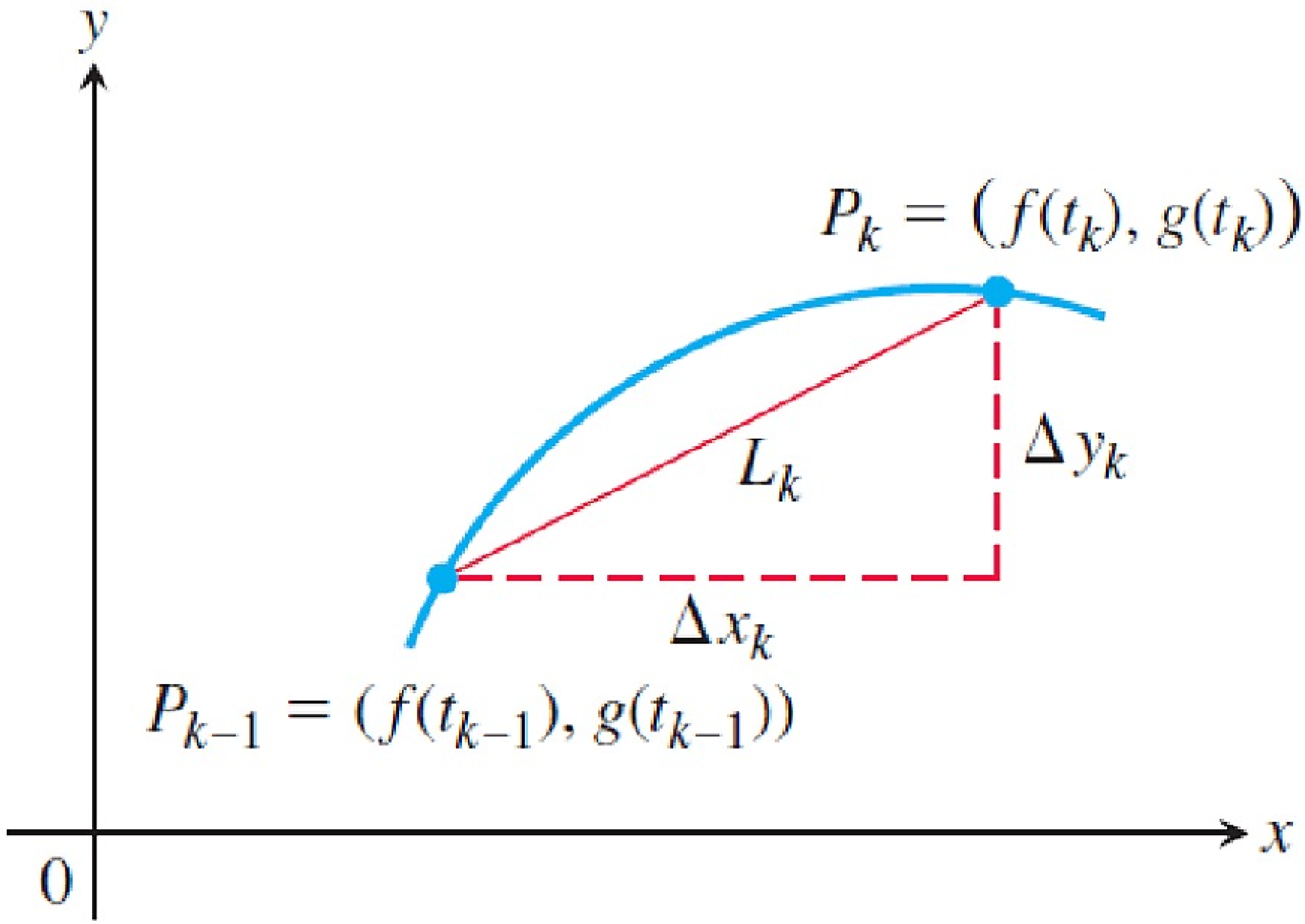

FIGURE 10.17 The arc $P_{k-1}P_k$ is approximated by the straight-line segment shown here, which has length $L_k = \sqrt{(\Delta x_k)^2 + (\Delta y_k)^2}$.

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DEFINITION If a curve C is defined parametrically by $x = f(t)$ and $y = g(t)$, $a \le t \le b$, where f' and g' are continuous and not simultaneously zero on $[a, b]$, and C is traversed exactly once as t increases from $t = a$ to $t = b$, then **the length of C** is the definite integral

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2}\, dt.$$

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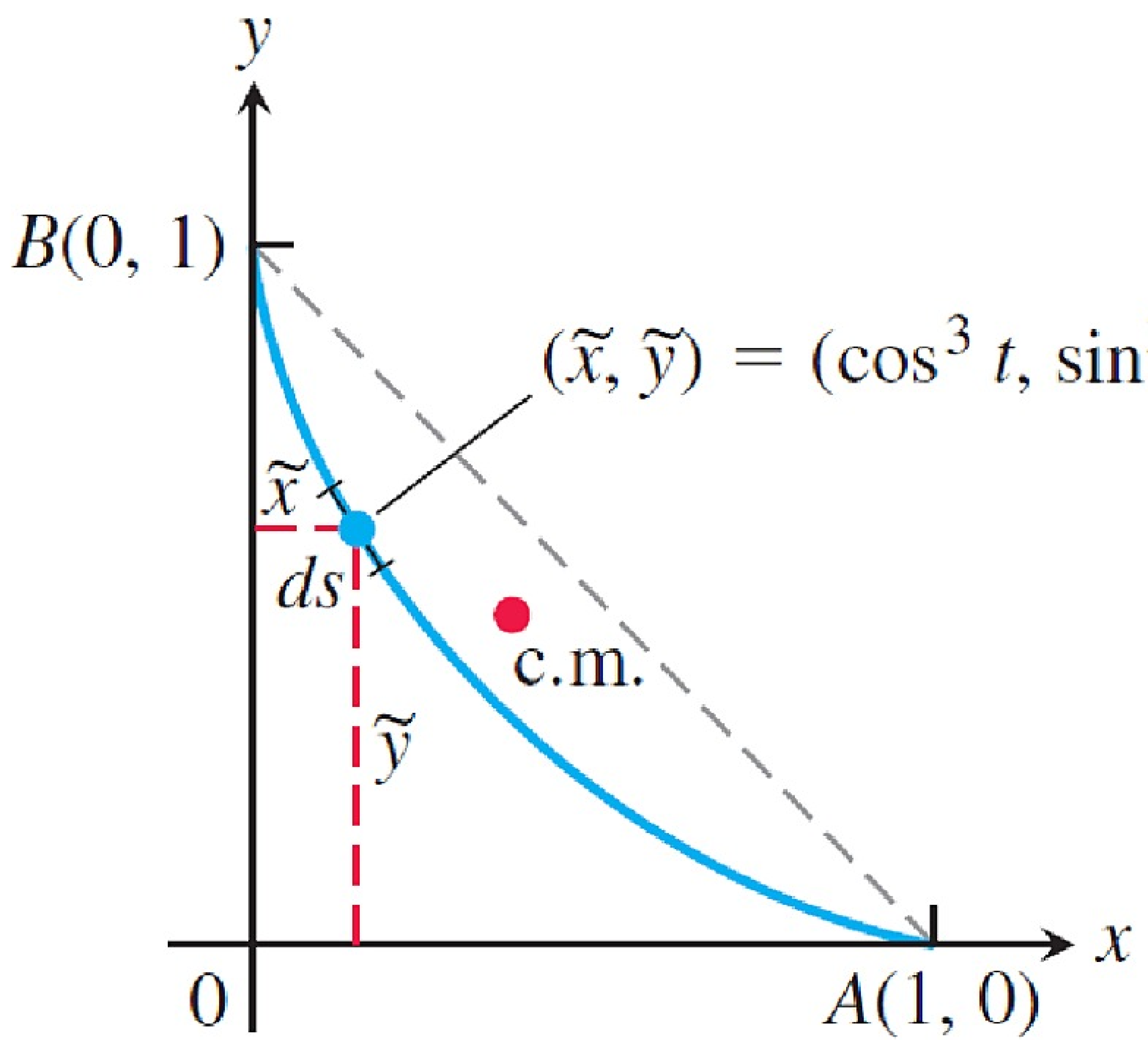

FIGURE 10.18 The centroid (c.m.) of the astroid arc in Example 7.

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Area of Surface of Revolution for Parametrized Curves

If a smooth curve $x = f(t), y = g(t), a \leq t \leq b$, is traversed exactly once as t increases from a to b, then the areas of the surfaces generated by revolving the curve about the coordinate axes are as follows.

1. Revolution about the x-axis ($y \geq 0$):

$$S = \int_a^b 2\pi y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \tag{5}$$

2. Revolution about the y-axis ($x \geq 0$):

$$S = \int_a^b 2\pi x \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt \tag{6}$$

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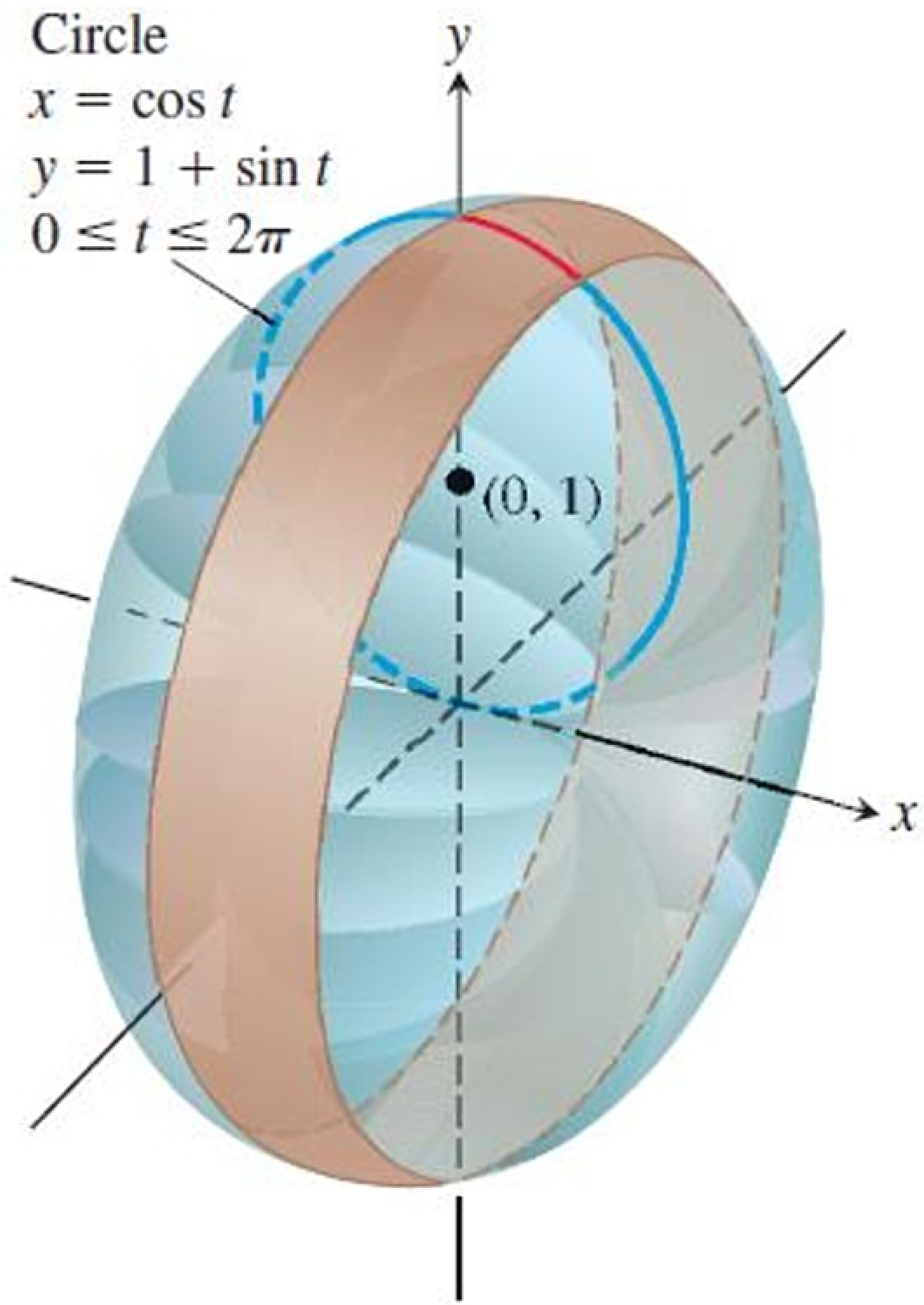

FIGURE 10.19 In Example 9 we calculate the area of the surface of revolution swept out by this parametrized curve.

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10.3

Polar Coordinates

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Polar Coordinates

$$P(r, \theta)$$

Directed distance from O to P

Directed angle from initial ray to OP

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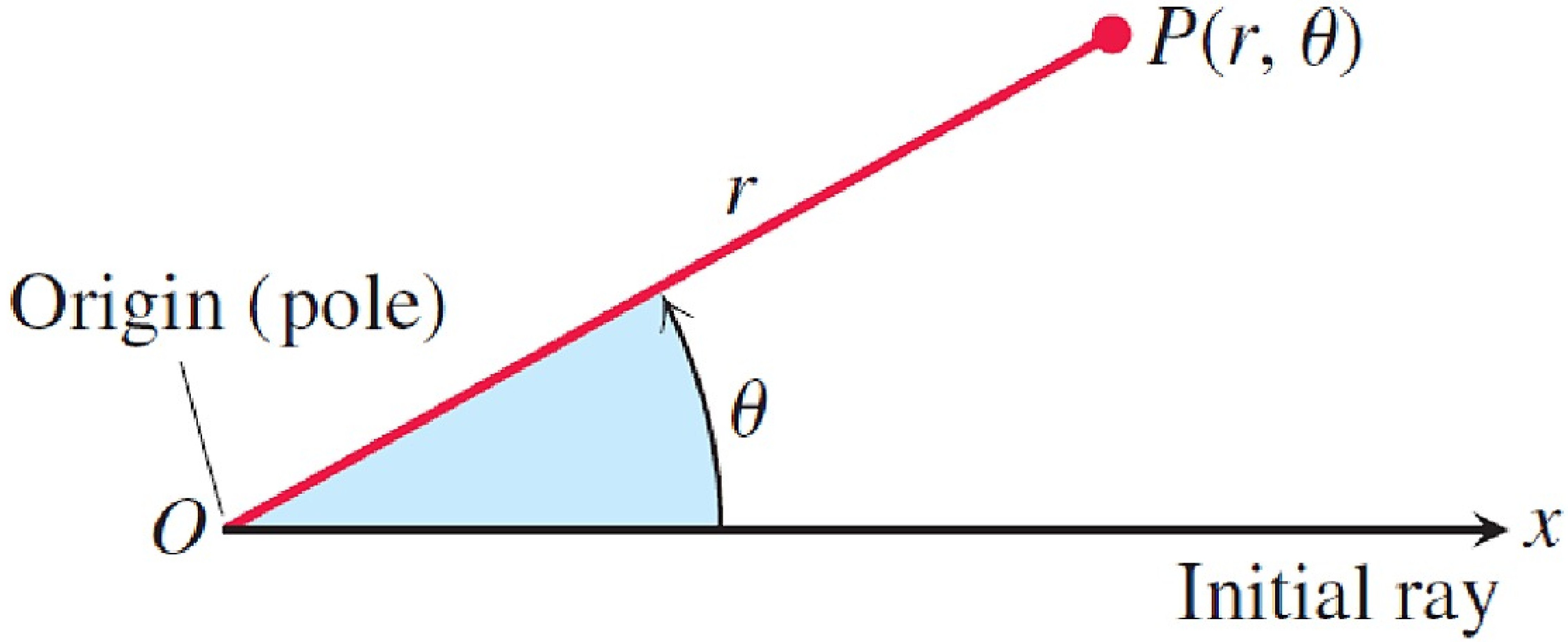

FIGURE 10.20 To define polar coordinates for the plane, we start with an origin, called the pole, and an initial ray.

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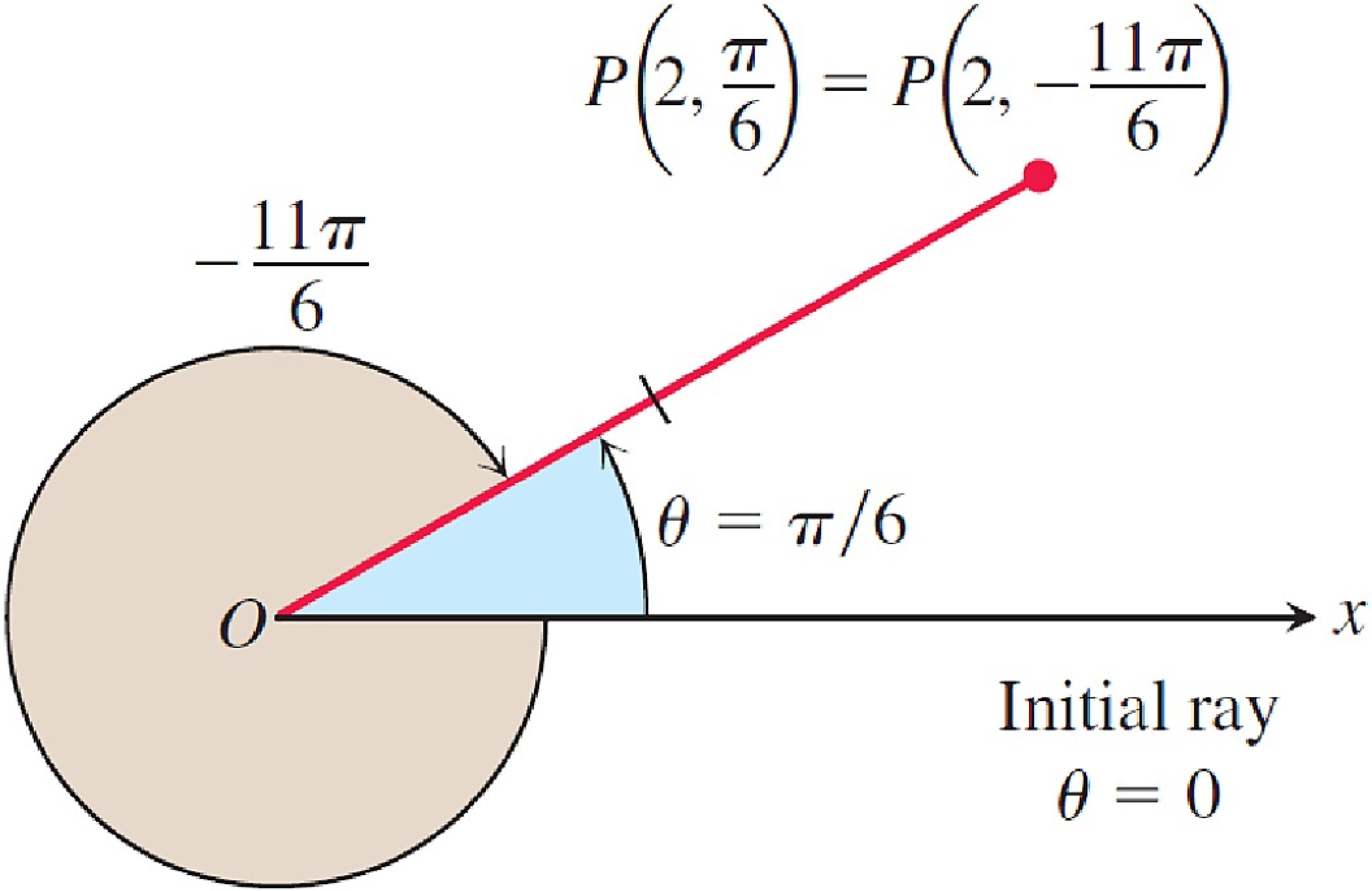

FIGURE 10.21 Polar coordinates are not unique.

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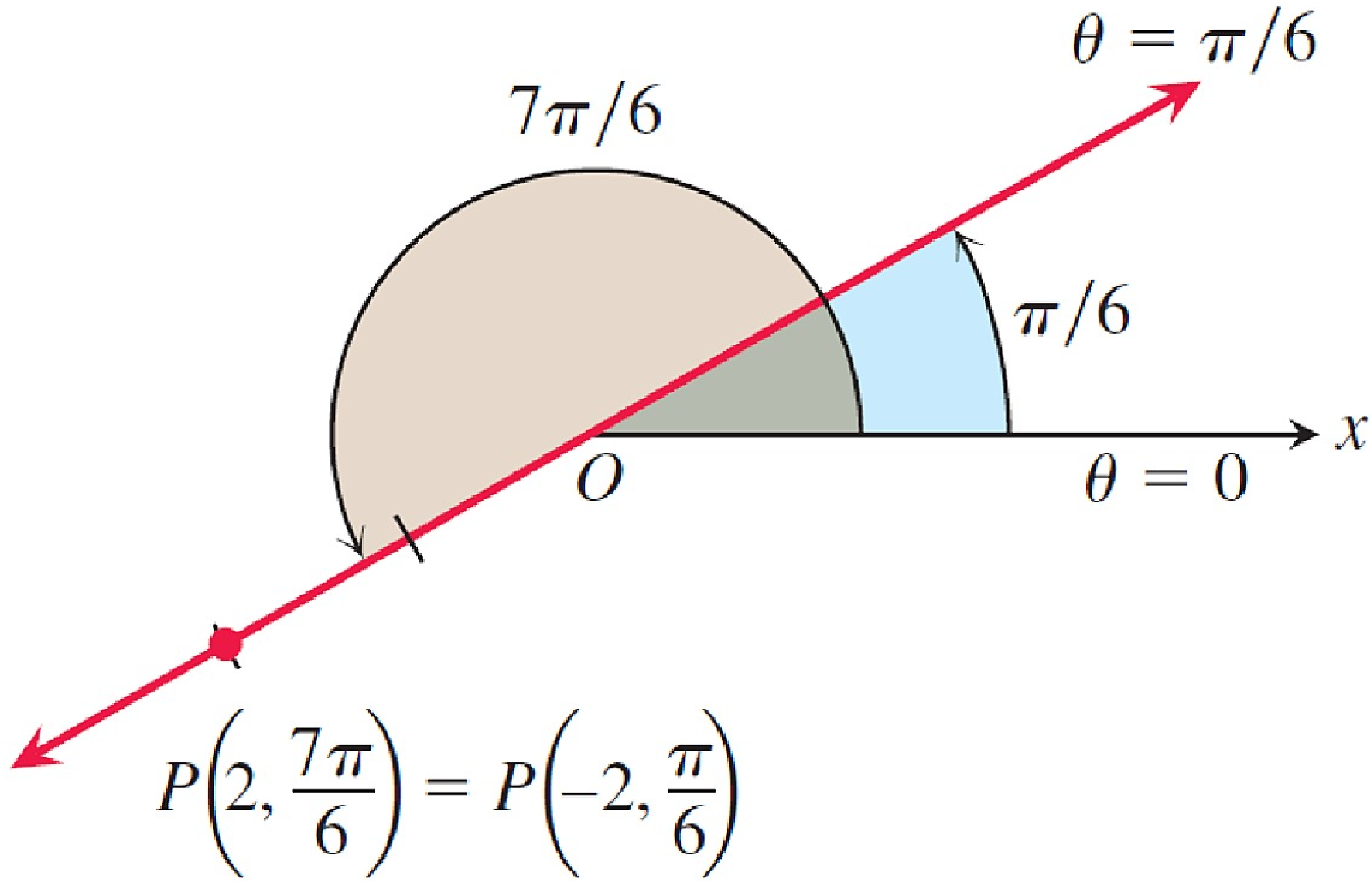

FIGURE 10.22 Polar coordinates can have negative r-values.

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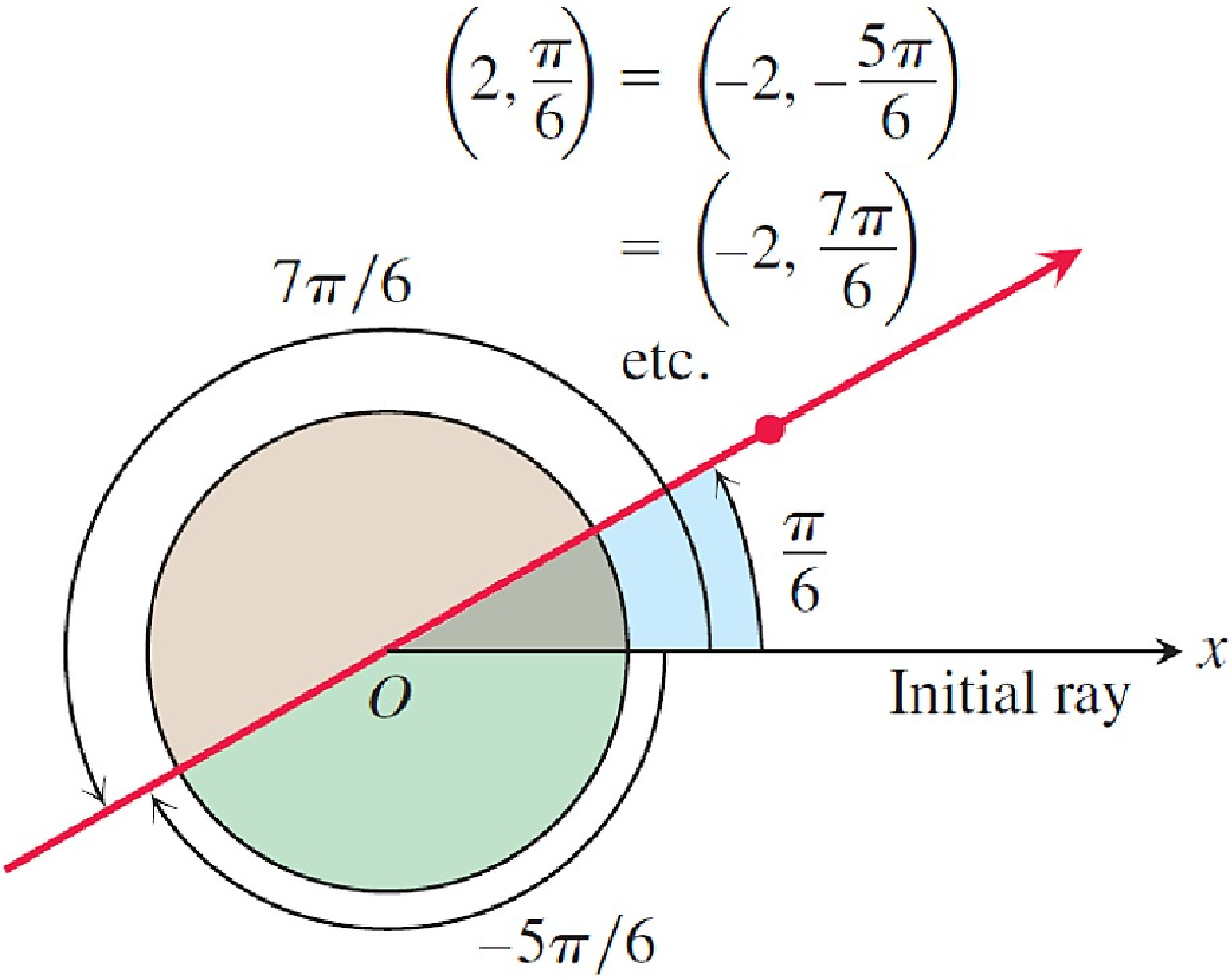

FIGURE 10.23 The point $P(2, \pi/6)$ has infinitely many polar coordinate pairs (Example 1).

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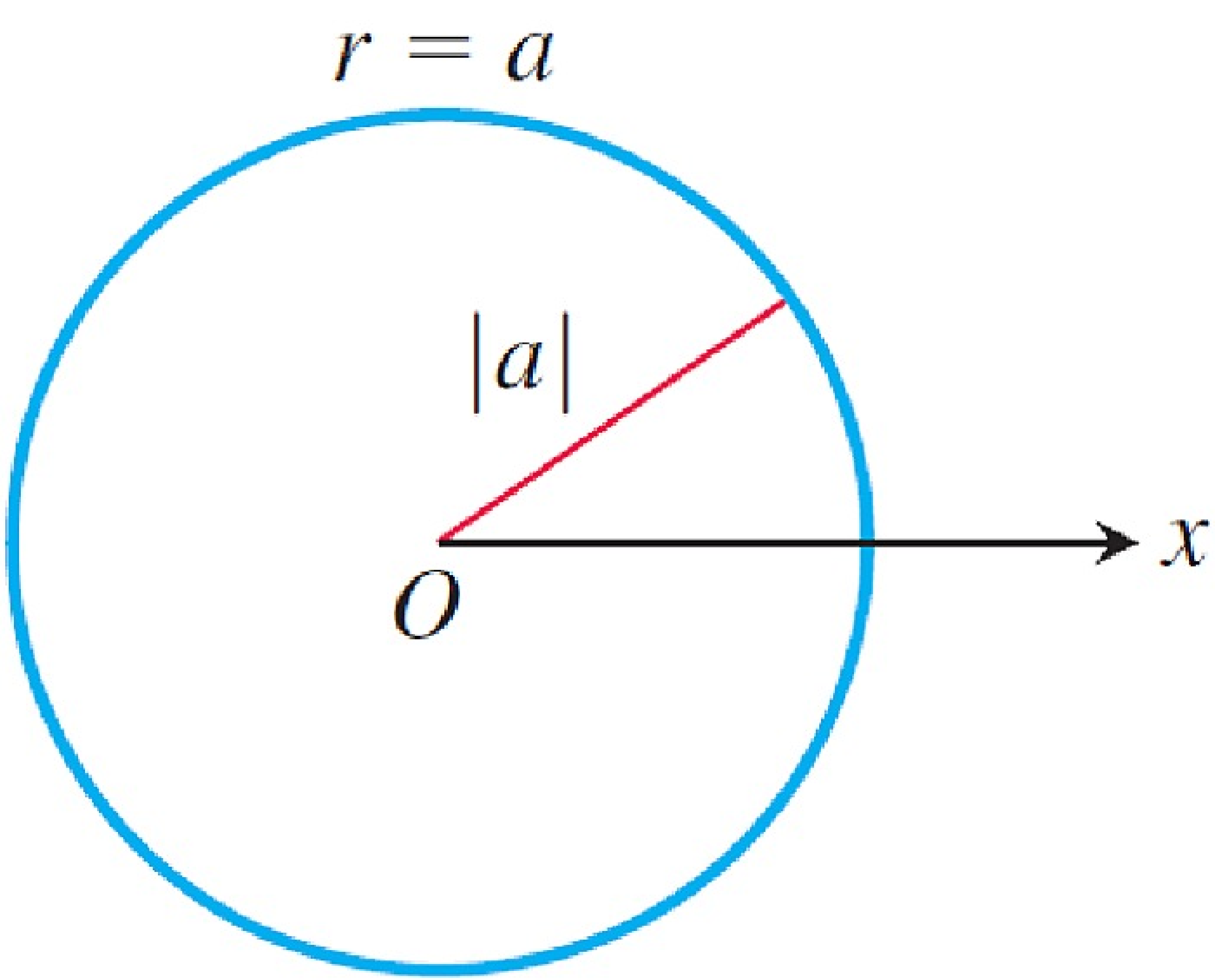

FIGURE 10.24 The polar equation for a circle is $r = a$.

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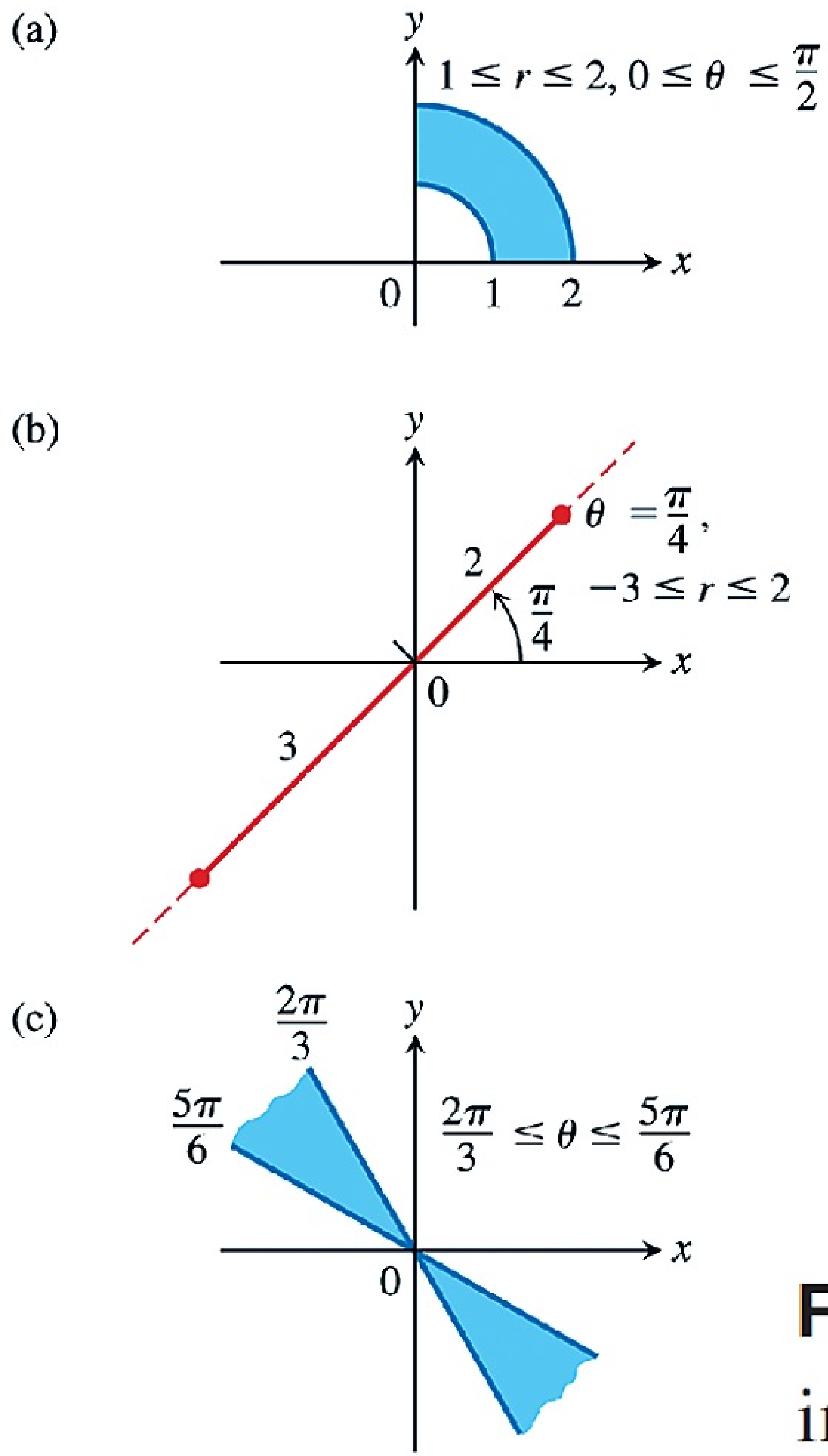

FIGURE 10.25 The graphs of typical inequalities in r and θ (Example 3).

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Equations Relating Polar and Cartesian Coordinates

$$x = r\cos\theta, \qquad y = r\sin\theta, \qquad r^2 = x^2 + y^2, \qquad \tan\theta = \frac{y}{x}$$

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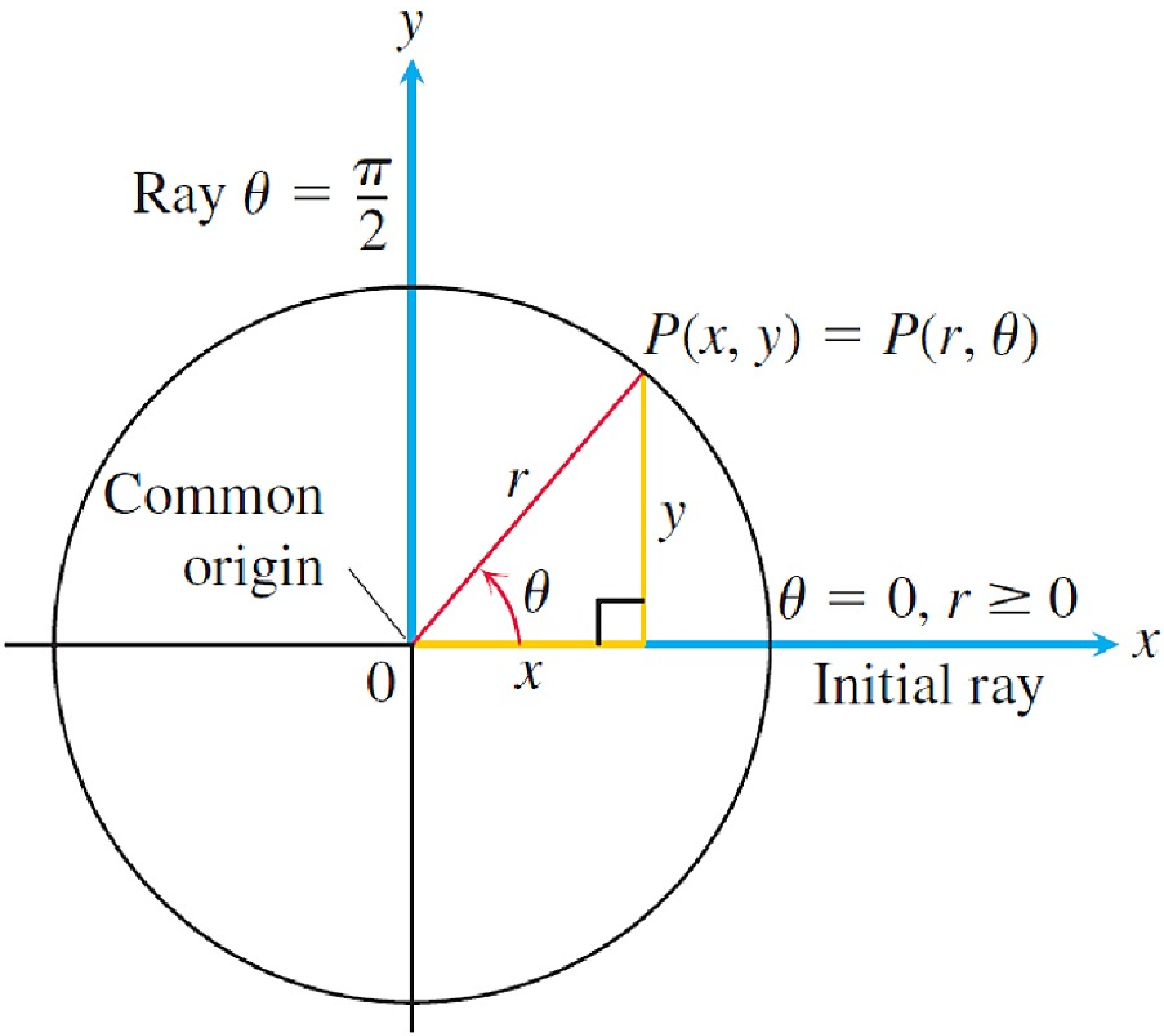

FIGURE 10.26 The usual way to relate polar and Cartesian coordinates.

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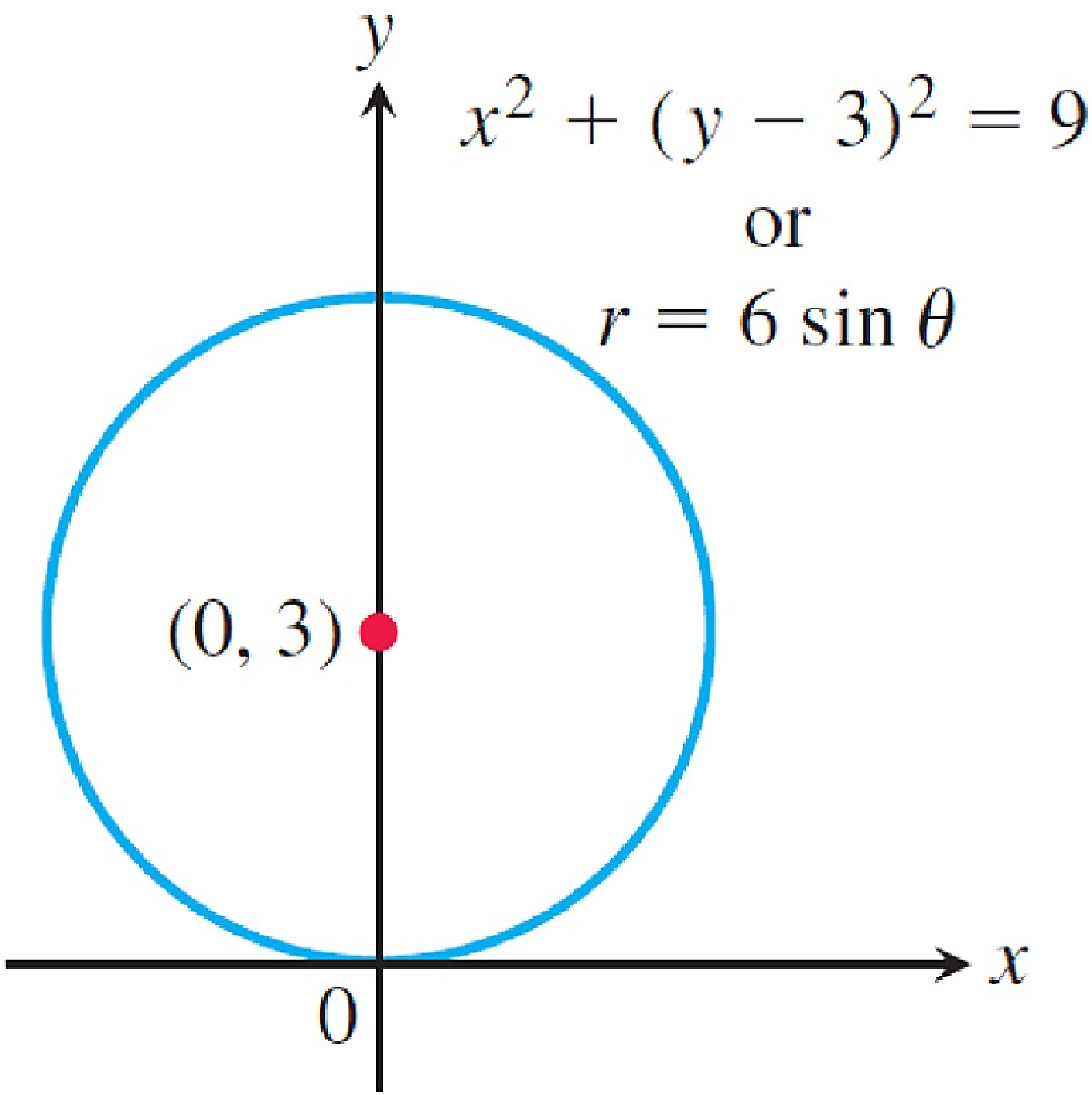

FIGURE 10.27 The circle in Example 5.

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Section 10.4

Graphing Polar Coordinate Equations

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Symmetry Tests for Polar Graphs in the Cartesian xy-Plane

1. *Symmetry about the x-axis:* If the point (r, θ) lies on the graph, then the point $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph (Figure 10.28a).
2. *Symmetry about the y-axis:* If the point (r, θ) lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph (Figure 10.28b).
3. *Symmetry about the origin:* If the point (r, θ) lies on the graph, then the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph (Figure 10.28c).

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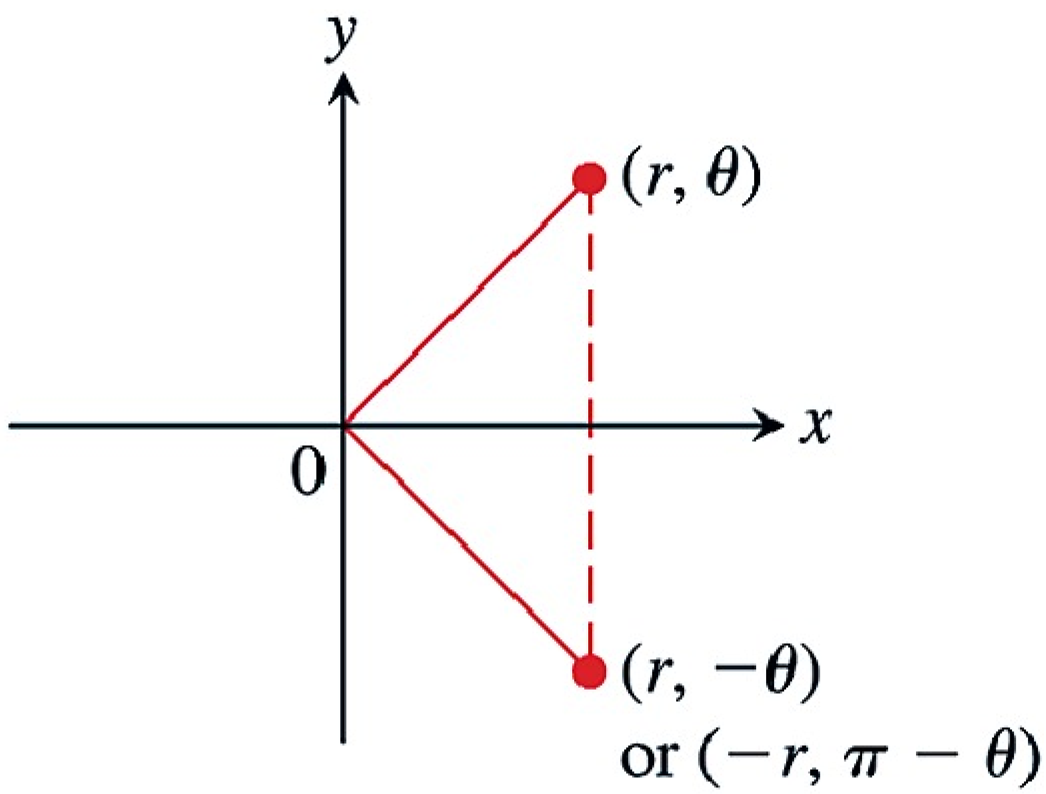
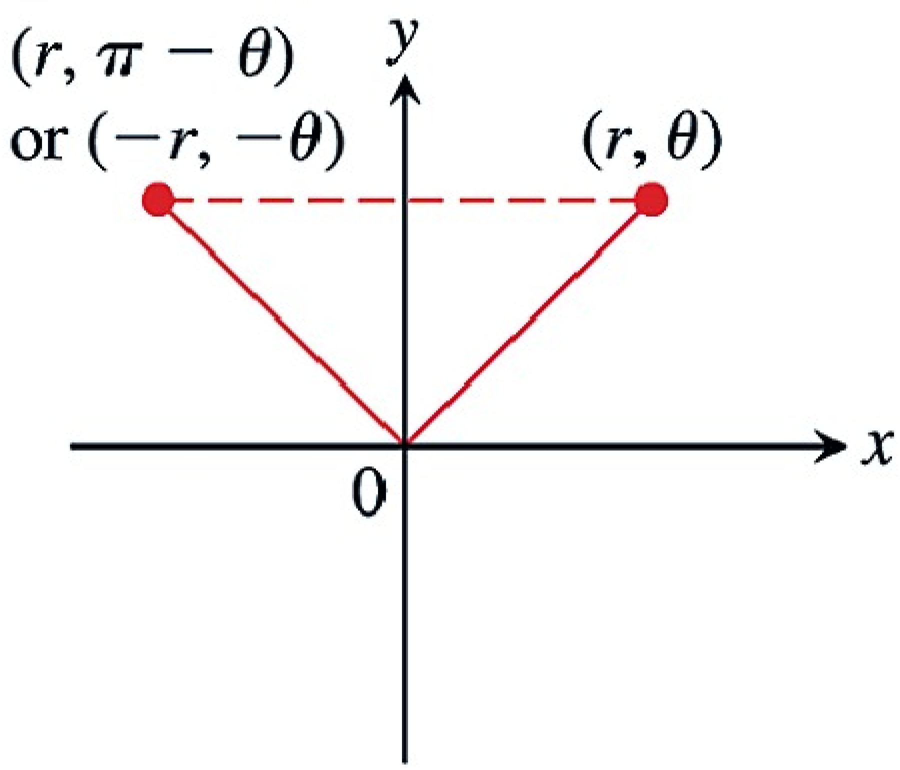
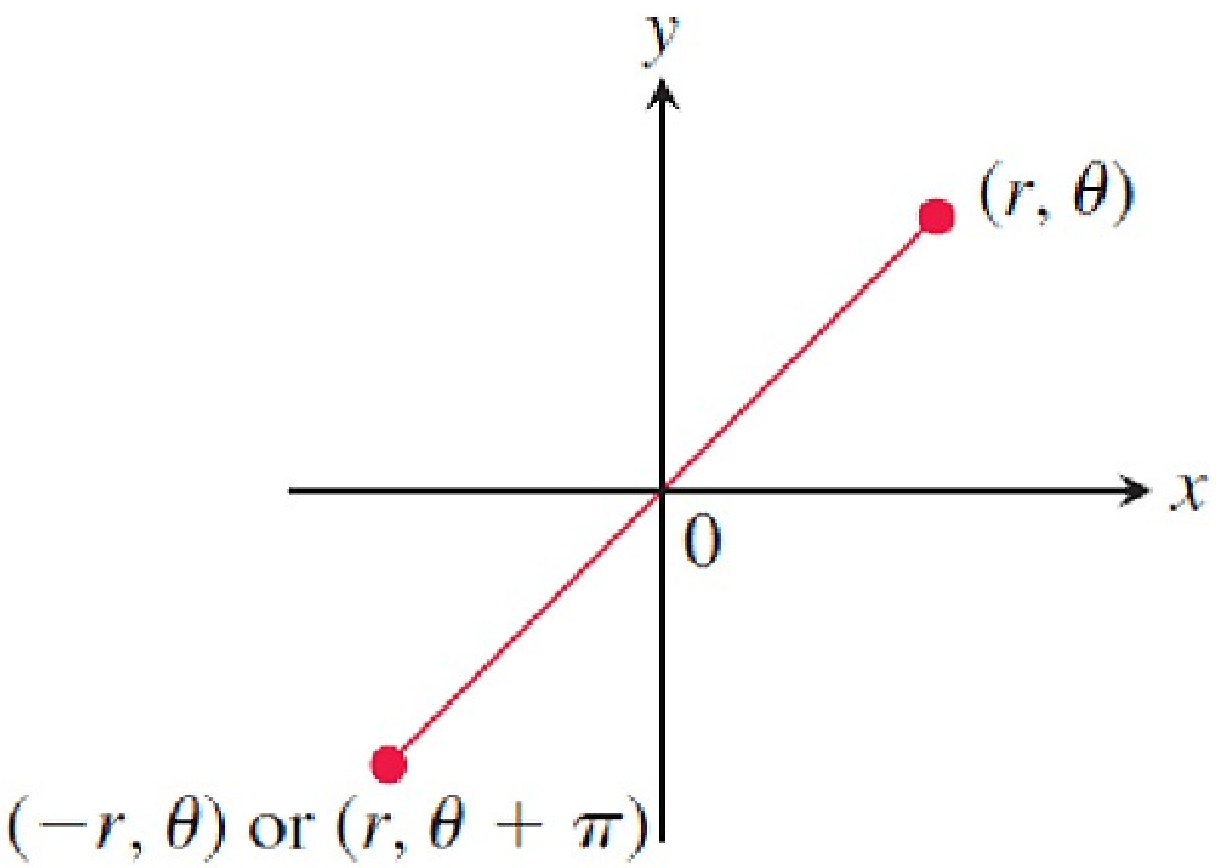


(a) About the x-axis



(b) About the y-axis



(c) About the origin

FIGURE 10.28 Three tests for symmetry in polar coordinates.

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Slope of the Curve $r = f(\theta)$ in the Cartesian xy-Plane

$$\left.\frac{dy}{dx}\right|_{(r,\,\theta)} = \frac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta} \tag{1}$$

provided $dx/d\theta \neq 0$ at (r, θ).

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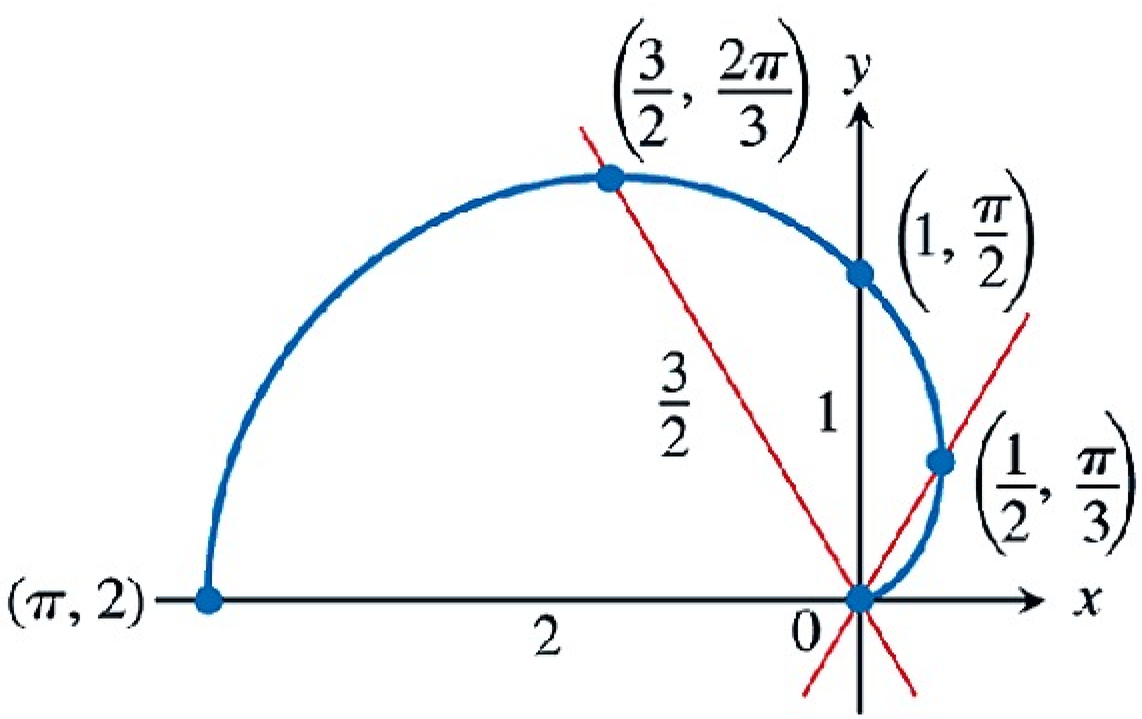
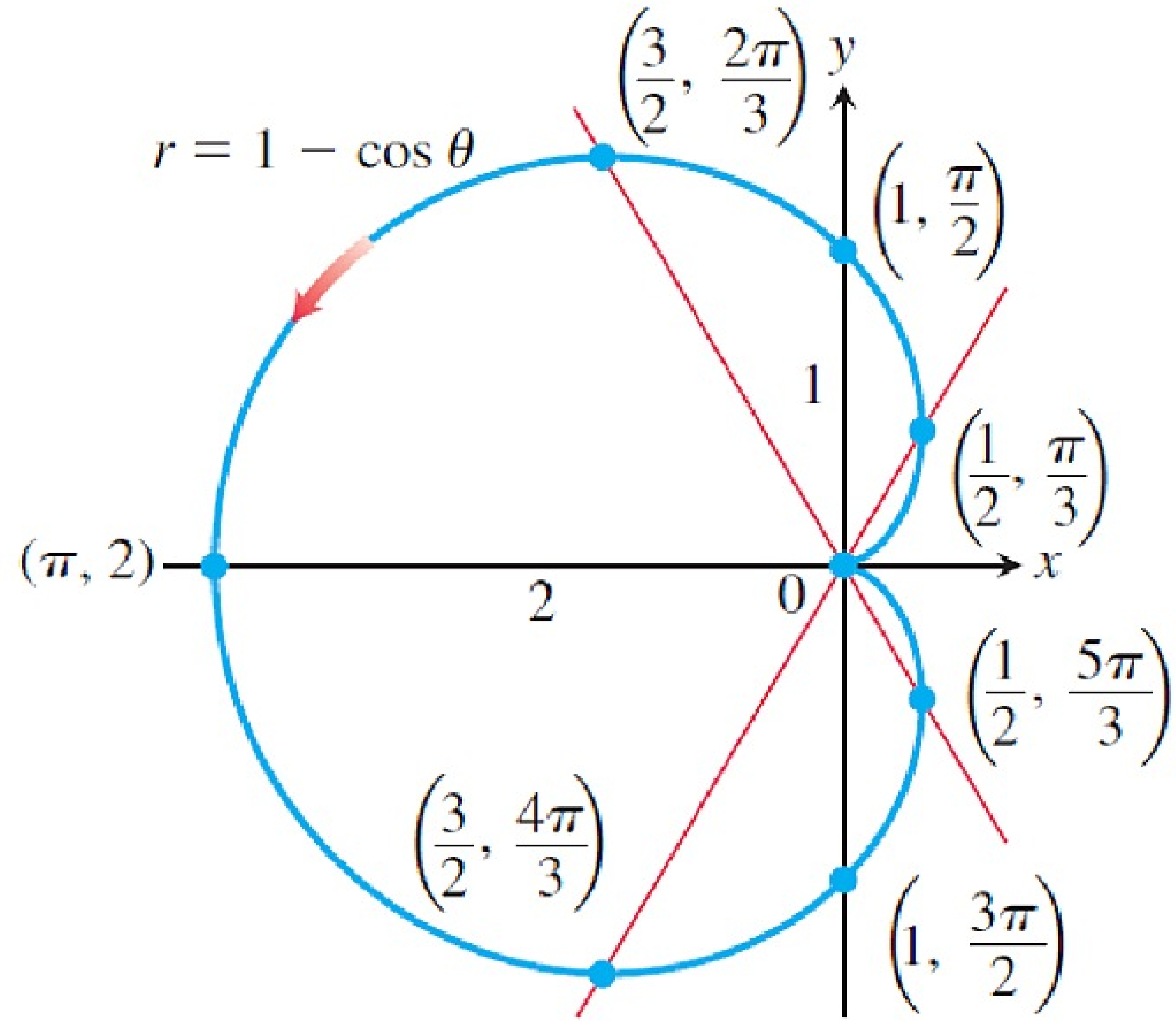
θ	$r = 1 - \cos\theta$
0	0
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{2}$
π	2

(a)



(b)



(c)

FIGURE 10.29 The steps in graphing the cardioid $r = 1 - \cos\theta$ (Example 1). The arrow shows the direction of increasing θ.

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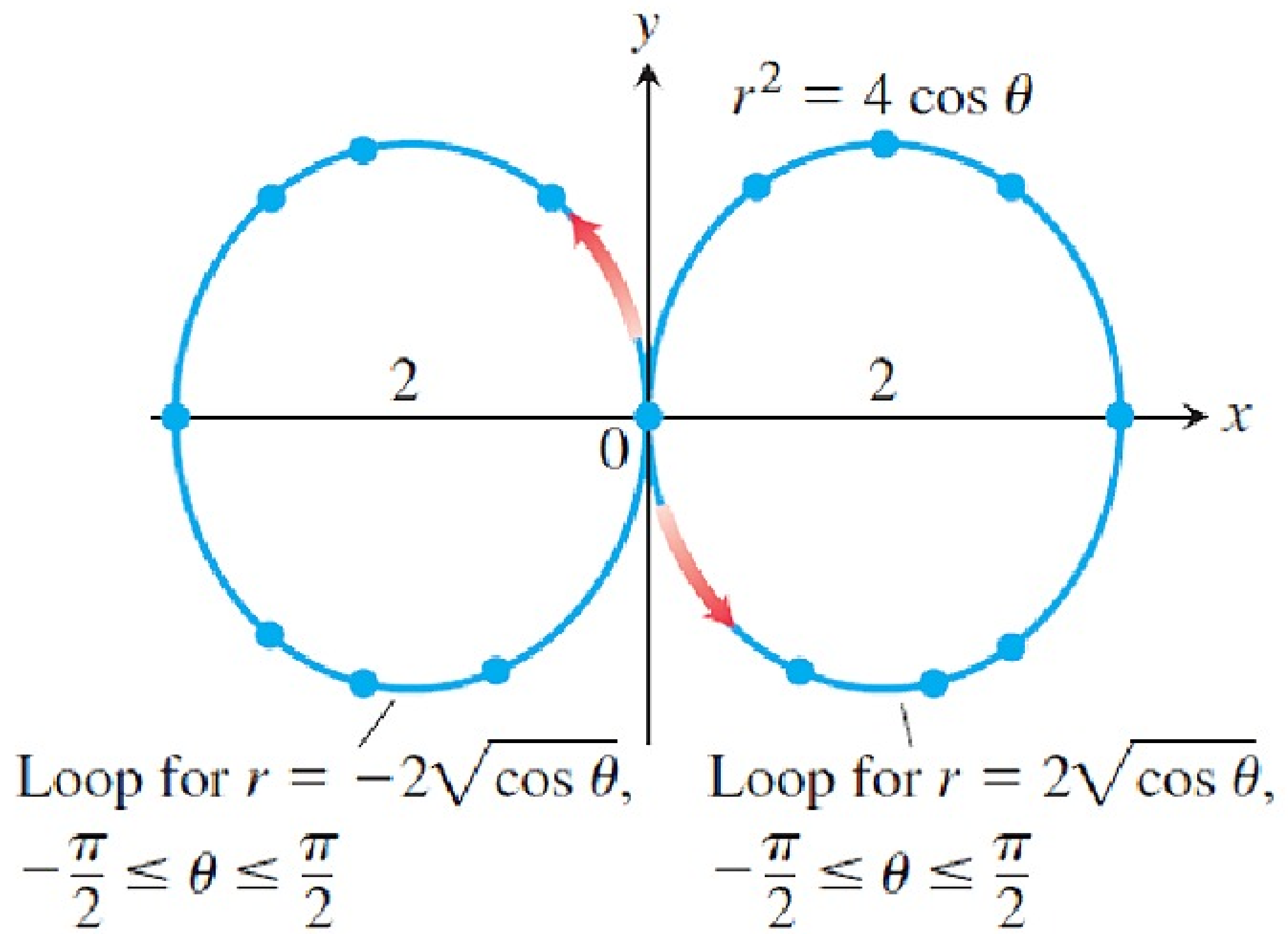
θ	$\cos\theta$	$r = \pm 2\sqrt{\cos\theta}$
0	1	± 2
$\pm\frac{\pi}{6}$	$\frac{\sqrt{3}}{2}$	$\approx \pm 1.9$
$\pm\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\approx \pm 1.7$
$\pm\frac{\pi}{3}$	$\frac{1}{2}$	$\approx \pm 1.4$
$\pm\frac{\pi}{2}$	0	0

(a)



(b)

FIGURE 10.30 The graph of $r^2 = 4\cos\theta$. The arrows show the direction of increasing θ. The values of r in the table are rounded (Example 2).

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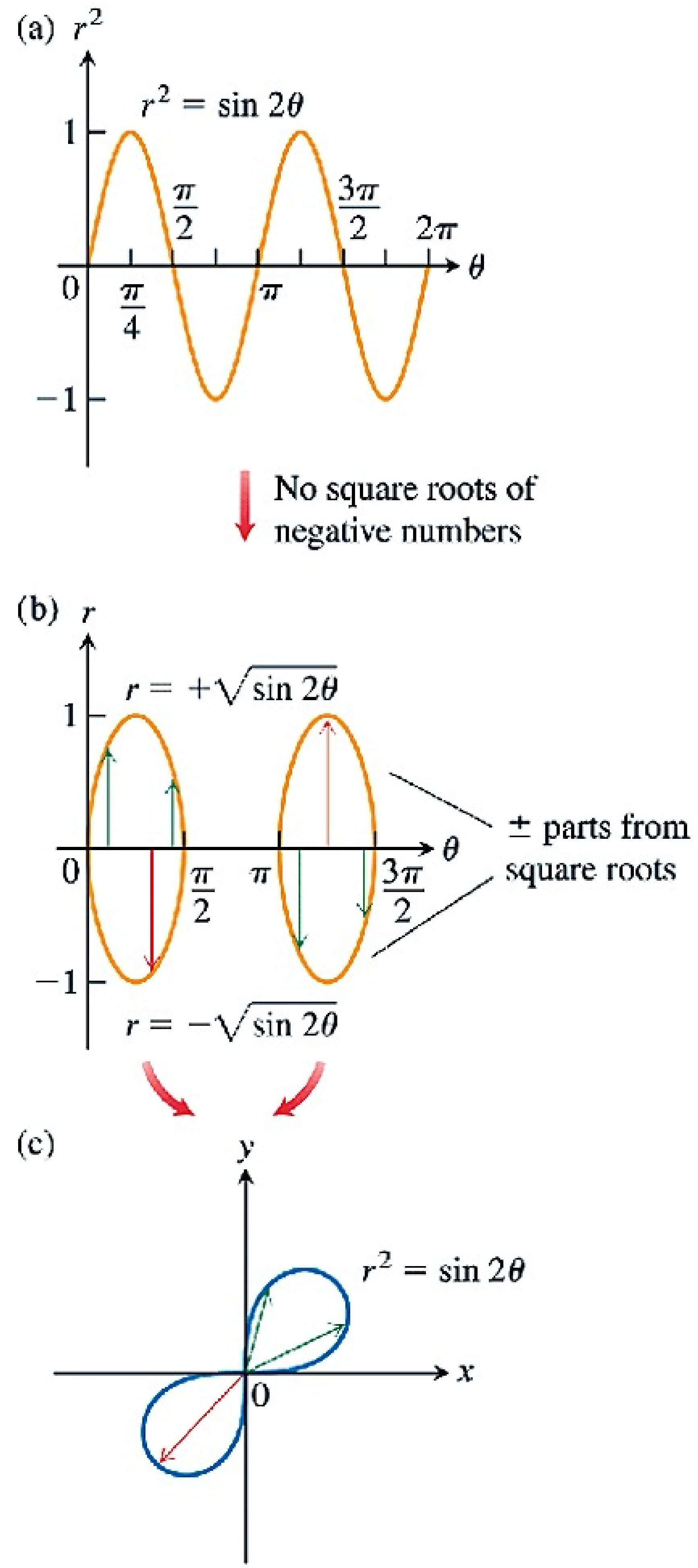

FIGURE 10.31 To plot $r = f(\theta)$ in the Cartesian $r\theta$-plane in (b), we first plot $r^2 = \sin 2\theta$ in the $r^2\theta$-plane in (a) and then ignore the values of θ for which $\sin 2\theta$ is negative. The radii from the sketch in (b) cover the polar graph of the lemniscate in (c) twice (Example 3).

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Section 10.7

Areas and Lengths in Polar Coordinates

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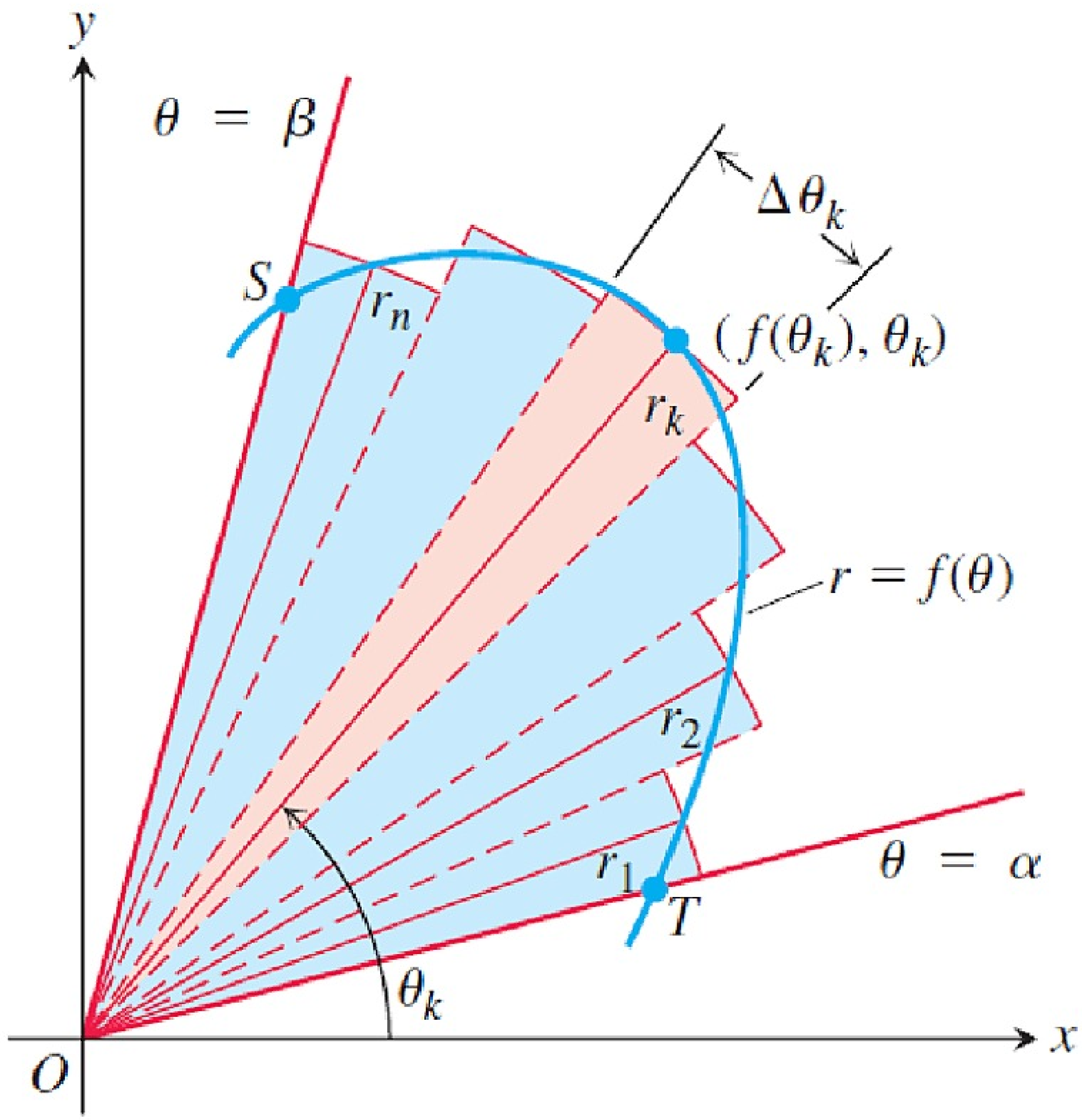

FIGURE 10.32 To derive a formula for the area of region *OTS*, we approximate the region with fan-shaped circular sectors.

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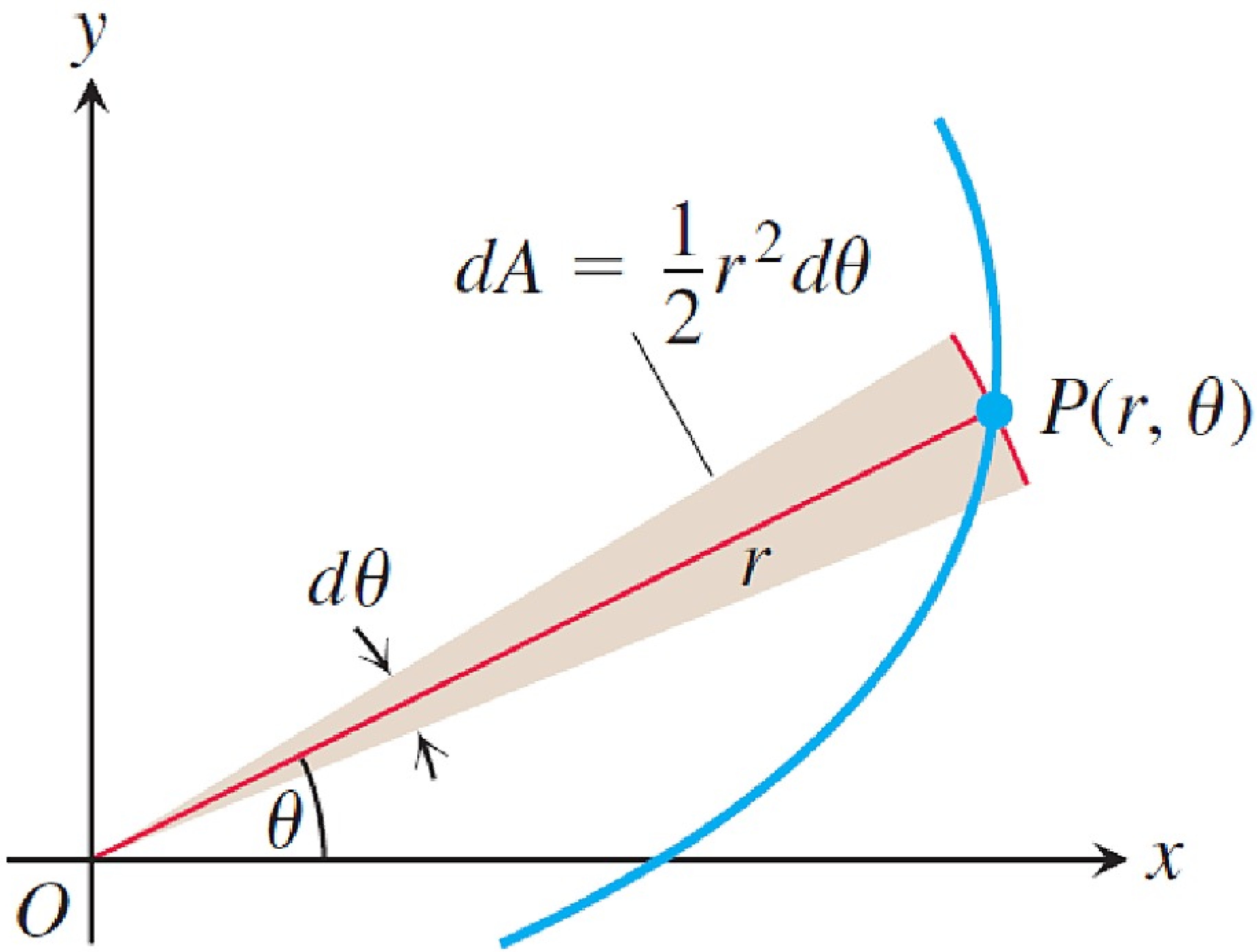

FIGURE 10.33 The area differential dA for the curve $r = f(\theta)$.

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Area of the Fan-Shaped Region Between the Origin and the Curve $r = f(\theta)$ when $\alpha \le \theta \le \beta$, $r \ge 0$, and $\beta - \alpha \le 2\pi$.

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r^2 \, d\theta$$

This is the integral of the **area differential** (Figure 10.33)

$$dA = \frac{1}{2} r^2 \, d\theta = \frac{1}{2} \big(f(\theta) \big)^2 d\theta.$$

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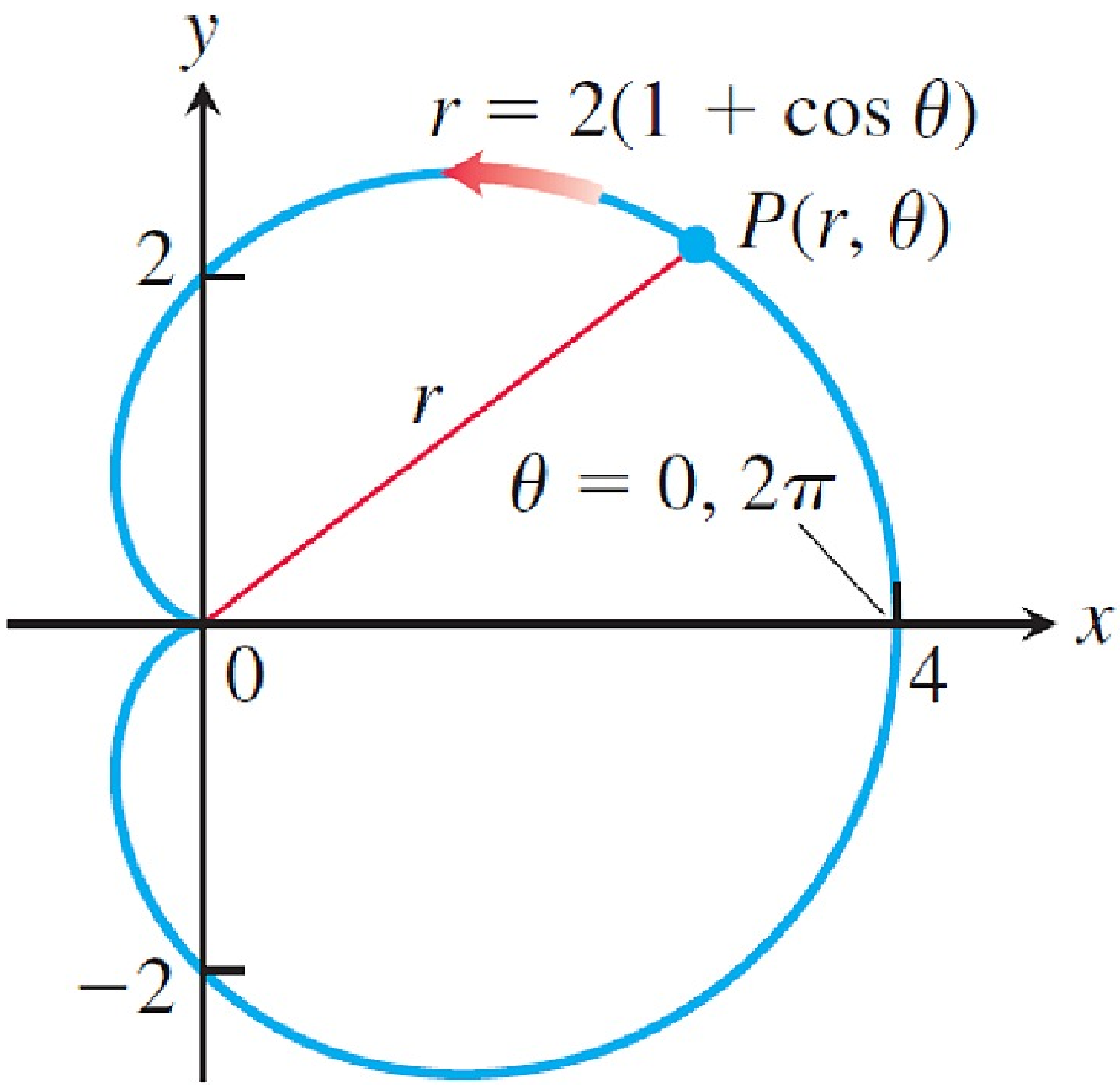

FIGURE 10.34 The cardioid in Example 1.

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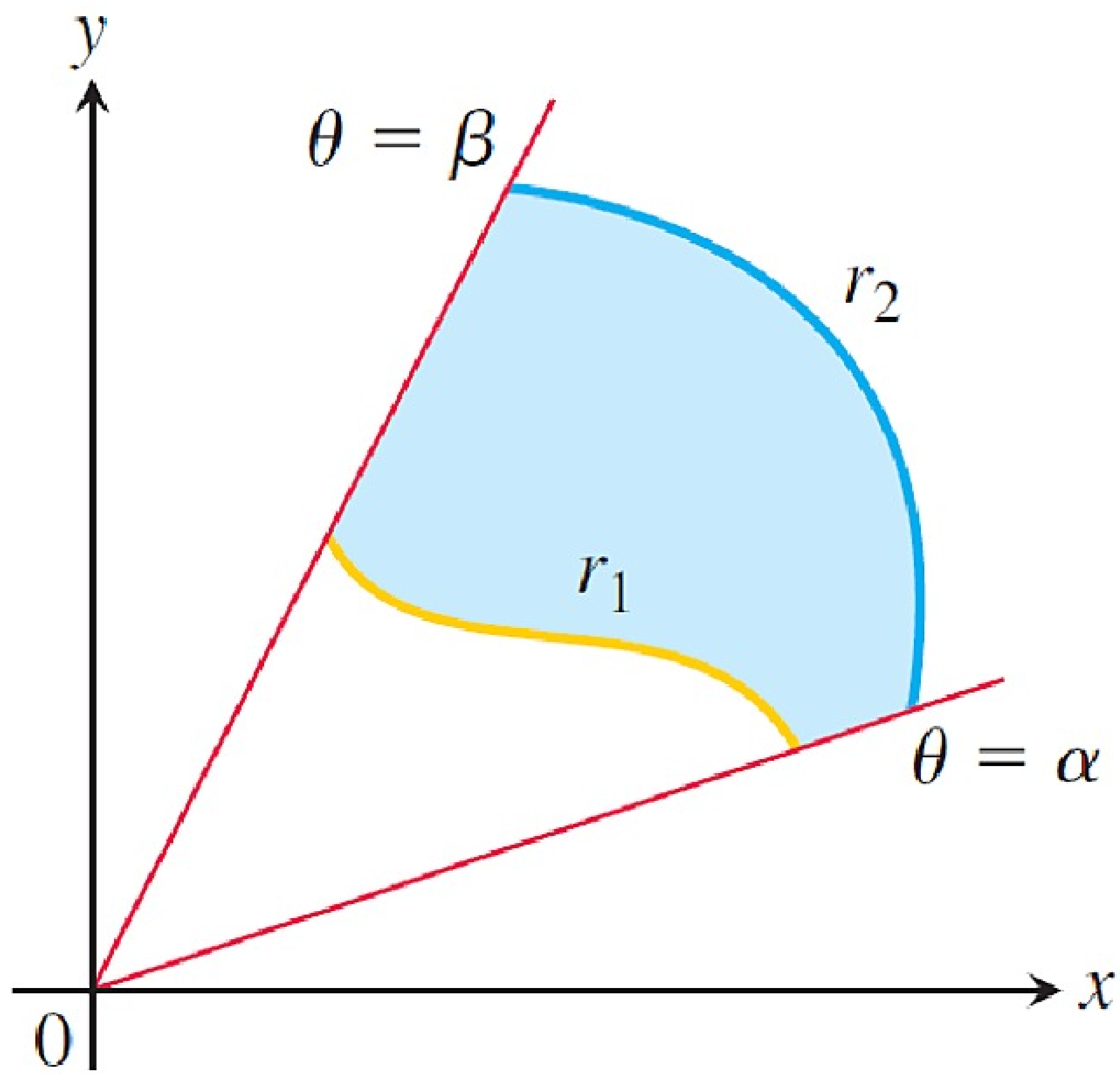

FIGURE 10.35 The area of the shaded region is calculated by subtracting the area of the region between r_1 and the origin from the area of the region between r_2 and the origin.

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Area of the Region $0 \le r_1(\theta) \le r \le r_2(\theta), \alpha \le \theta \le \beta$, and $\beta - \alpha \le 2\pi$.

$$A = \int_{\alpha}^{\beta} \frac{1}{2} r_2^{\,2}\, d\theta - \int_{\alpha}^{\beta} \frac{1}{2} r_1^{\,2}\, d\theta = \int_{\alpha}^{\beta} \frac{1}{2}\left(r_2^{\,2} - r_1^{\,2}\right) d\theta \qquad (1)$$

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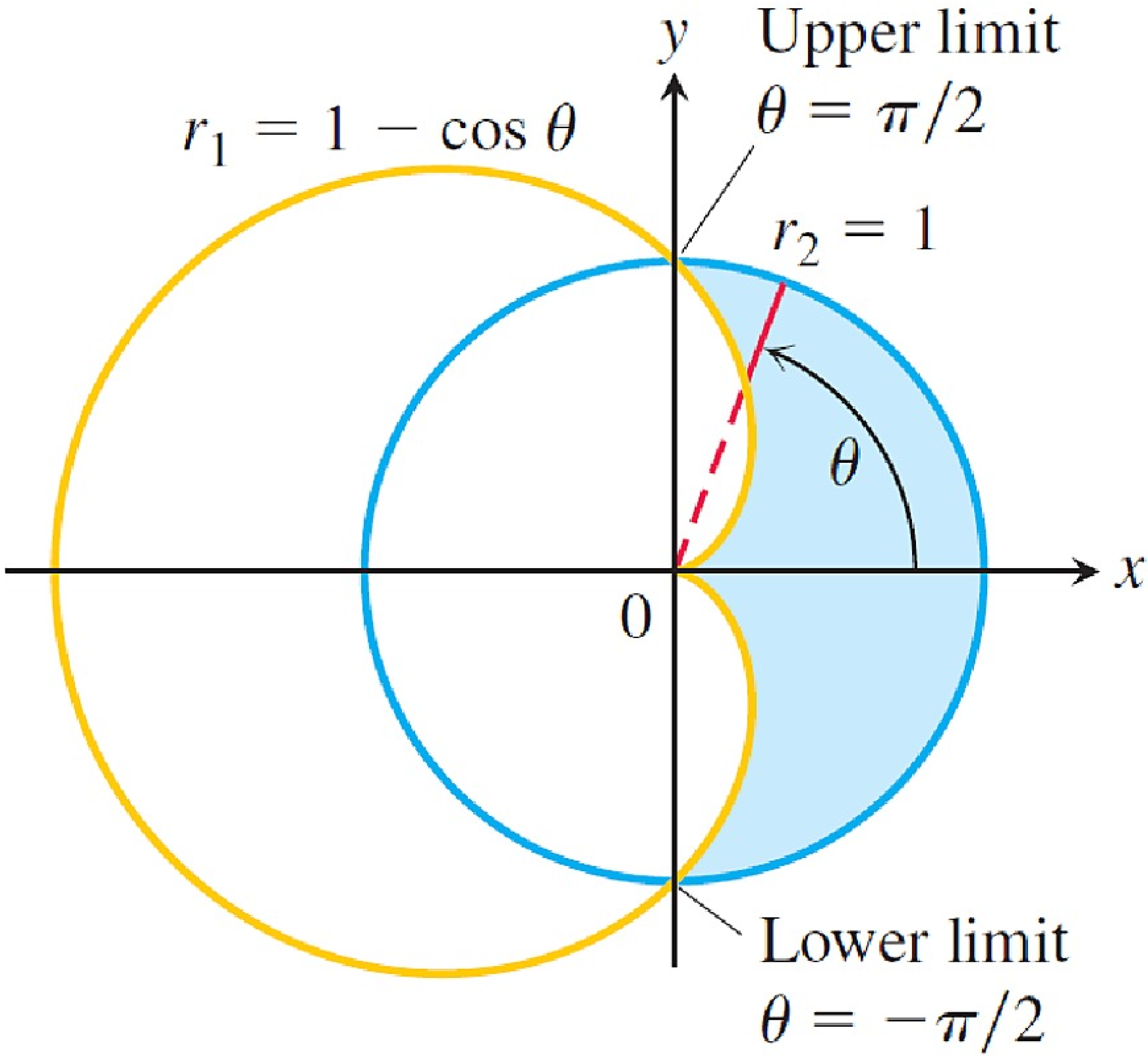

FIGURE 10.36 The region and limits of integration in Example 2.

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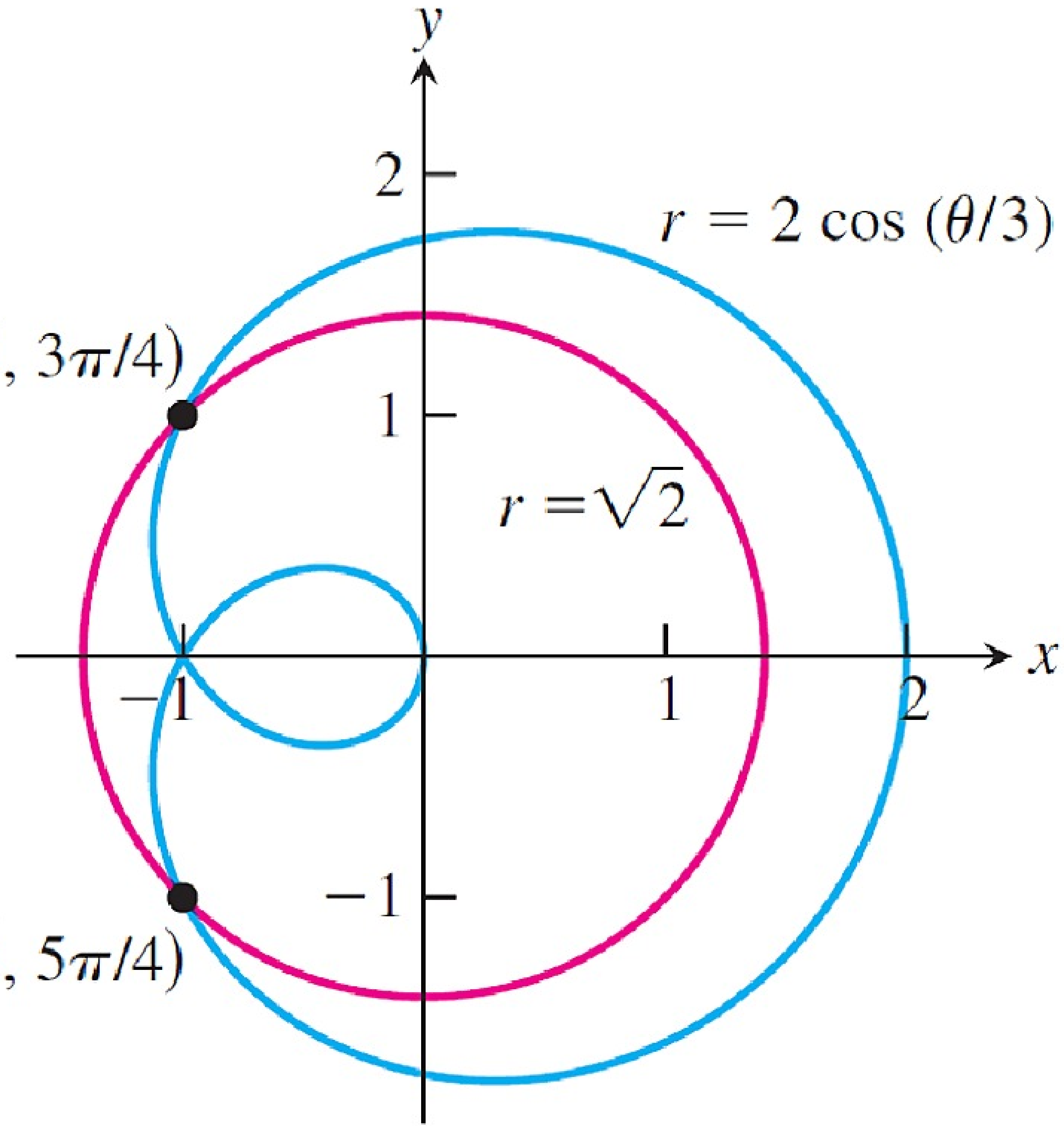

FIGURE 10.37 The curves $r = 2\cos(\theta/3)$ and $r = \sqrt{2}$ intersect at two points (Example 3).

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Length of a Polar Curve

If $r = f(\theta)$ has a continuous first derivative for $\alpha \le \theta \le \beta$ and if the point $P(r, \theta)$ traces the curve $r = f(\theta)$ exactly once as θ runs from α to β, then the length of the curve is

$$L = \int_{\alpha}^{\beta} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta. \tag{3}$$

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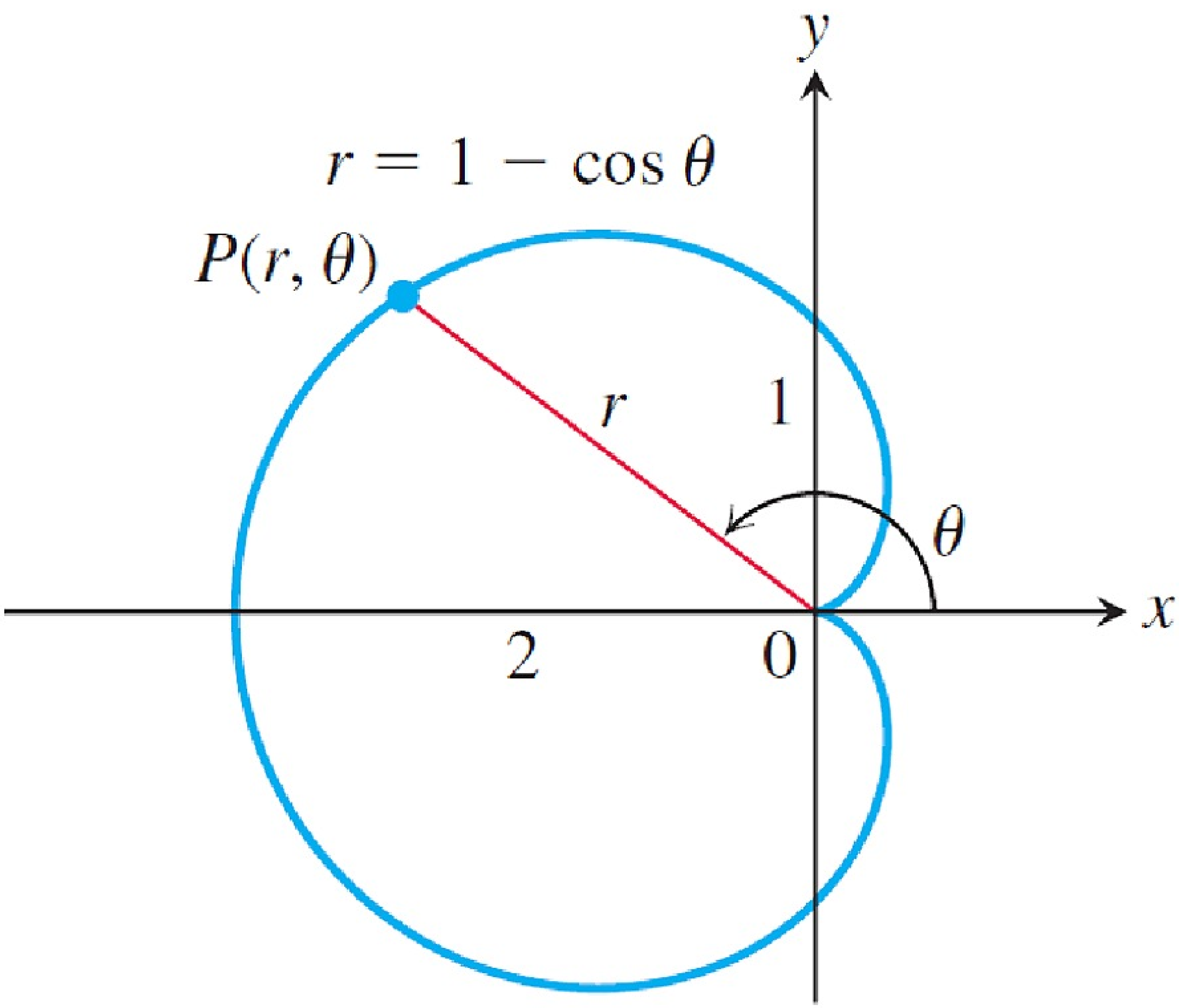

FIGURE 10.38 Calculating the length of a cardioid (Example 4).

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Section 10.6

Conic Sections

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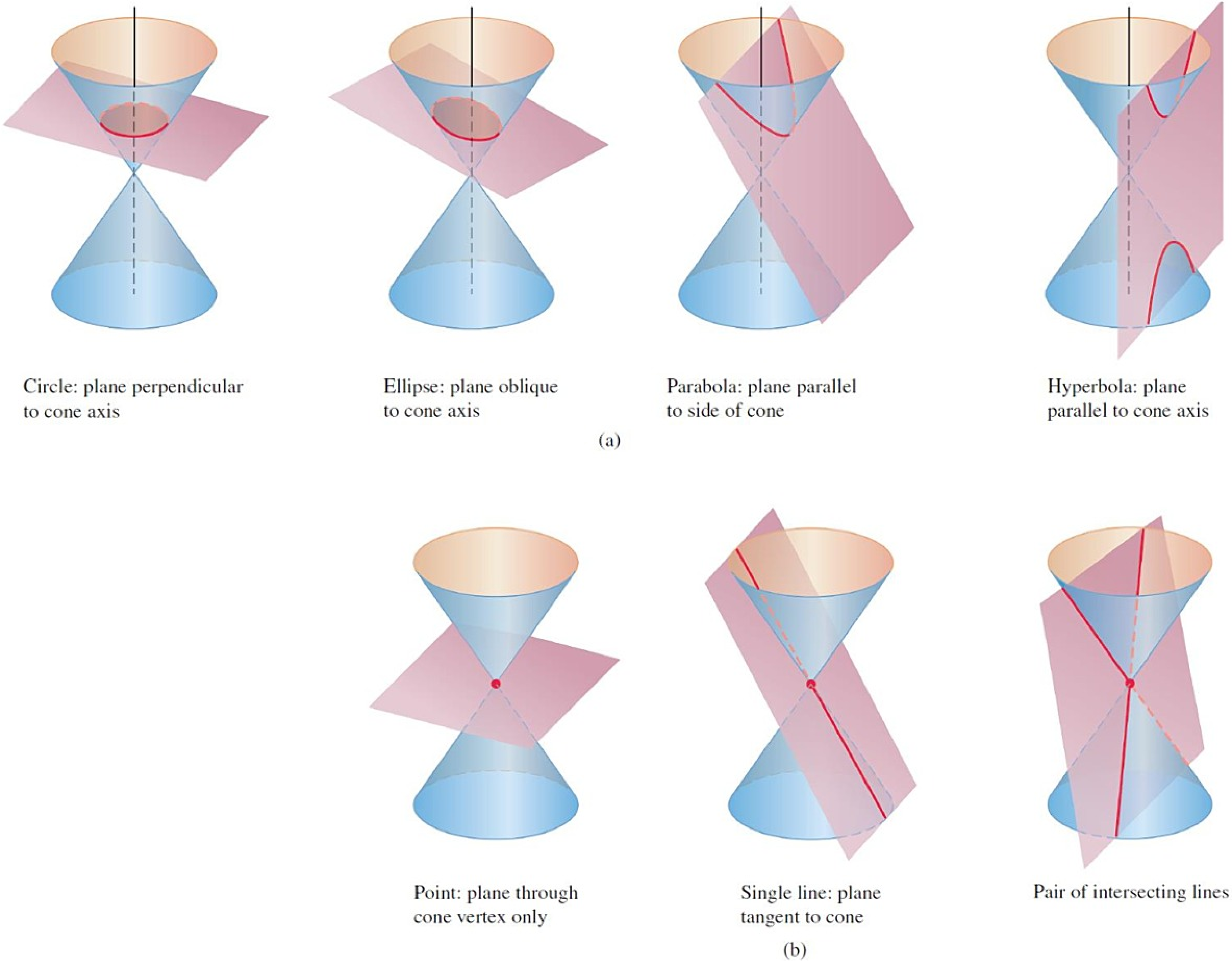

FIGURE 10.39 The standard conic sections (a) are the curves in which a plane cuts a *double* cone. Hyperbolas come in two parts, called *branches*. The point and lines obtained by passing the plane through the cone's vertex (b) are *degenerate* conic sections.

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DEFINITIONS A set that consists of all the points in a plane equidistant from a given fixed point and a given fixed line in the plane is a **parabola**. The fixed point is the **focus** of the parabola. The fixed line is the **directrix**.

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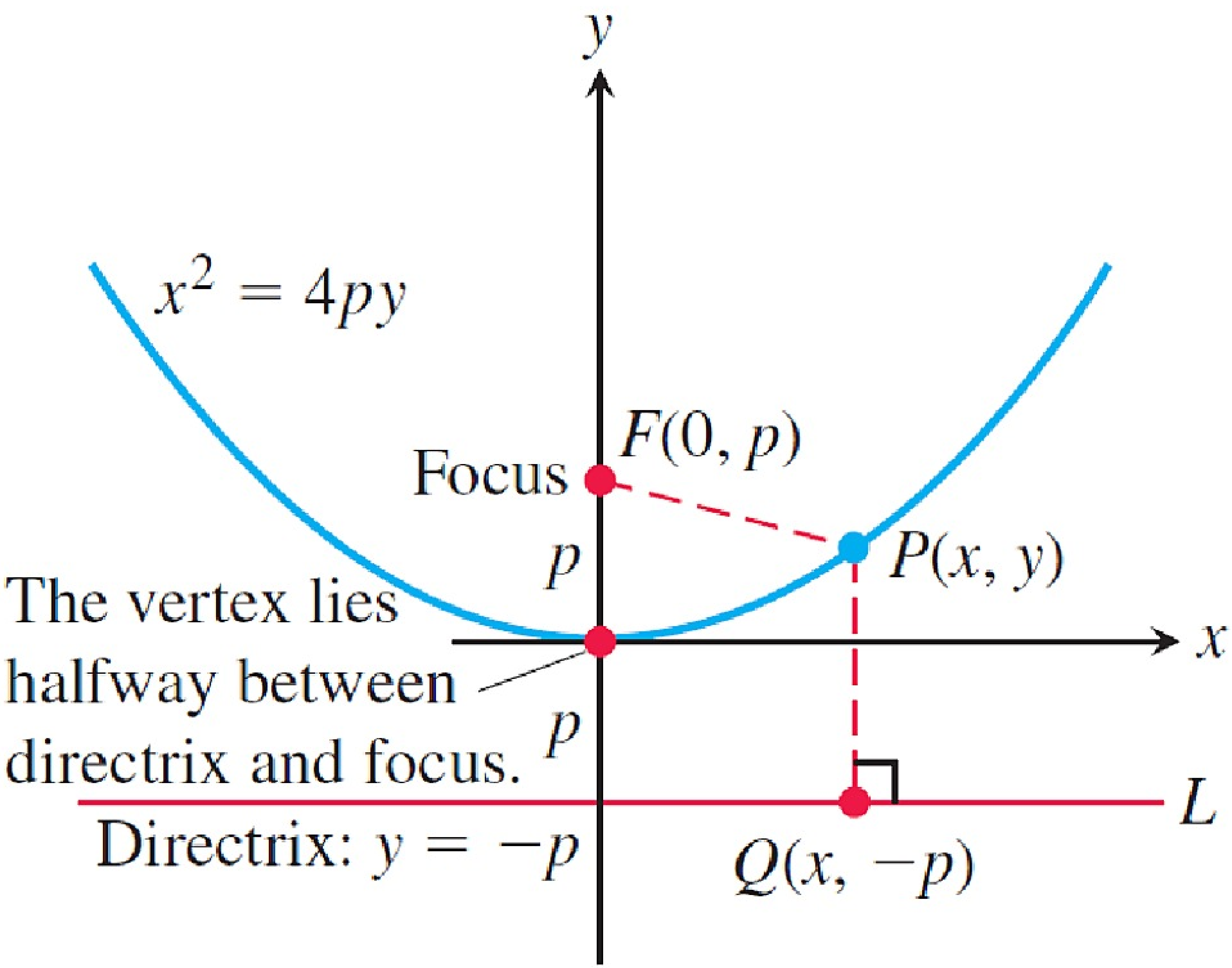

FIGURE 10.40 The standard form of the parabola $x^2 = 4py$, $p > 0$.

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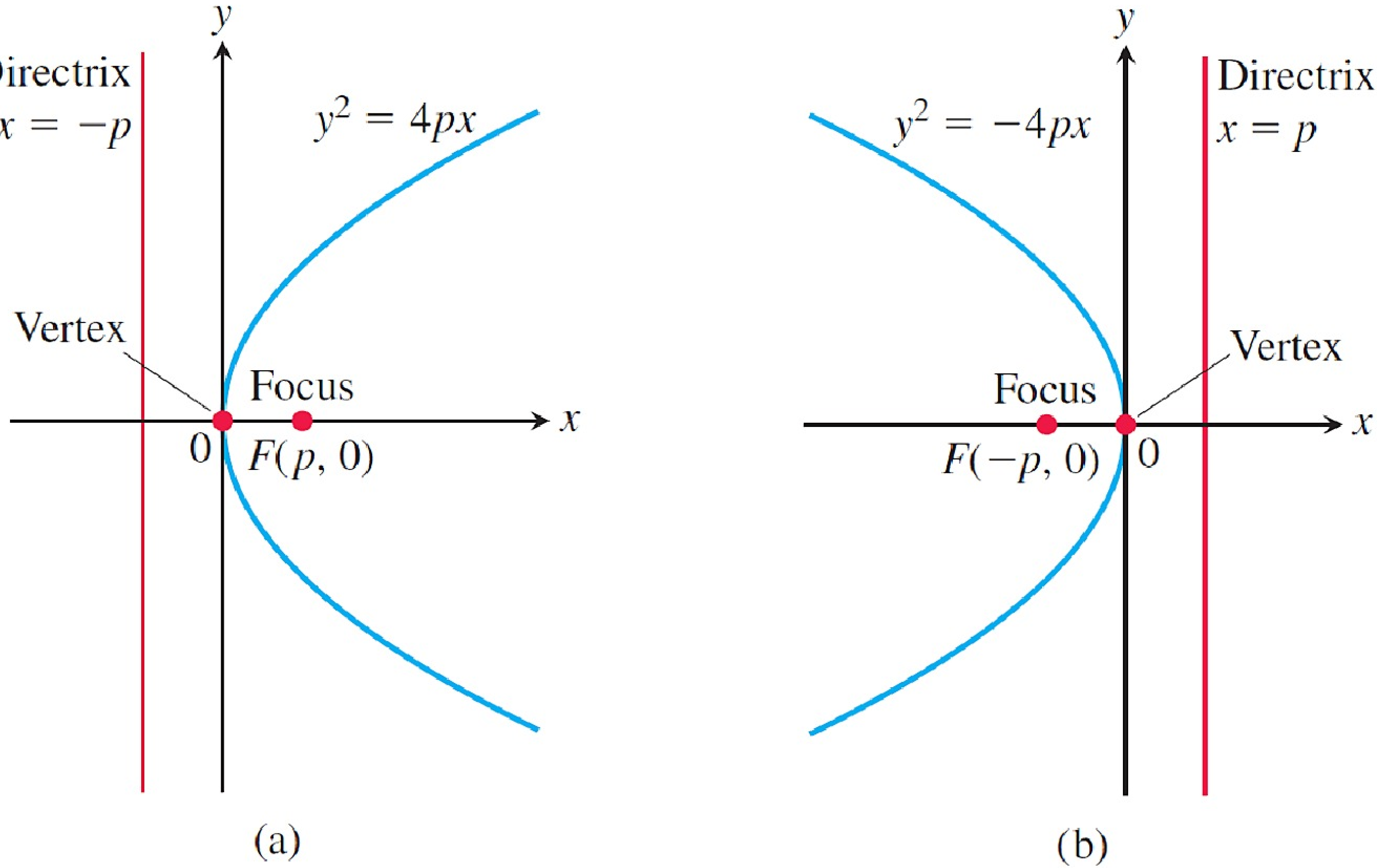

FIGURE 10.41 (a) The parabola $y^2 = 4px$. (b) The parabola $y^2 = -4px$.

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DEFINITIONS An **ellipse** is the set of points in a plane whose distances from two fixed points in the plane have a constant sum. The two fixed points are the foci of the ellipse.

The line through the foci of an ellipse is the ellipse's **focal axis**. The point on the axis halfway be-tween the foci is the **center**. The points where the focal axis and ellipse cross are the ellipse's **vertices** (Figure 10.42).

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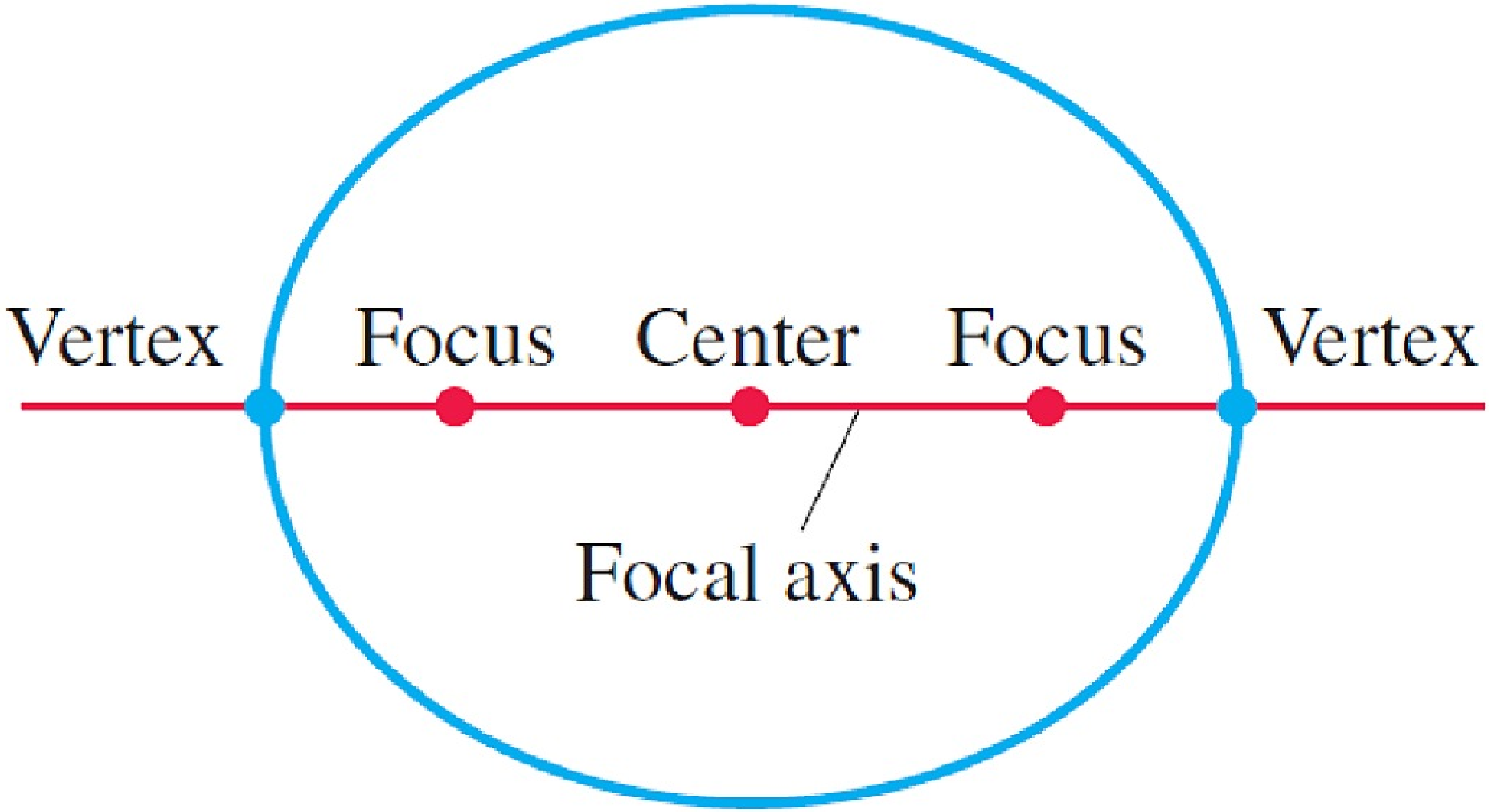

FIGURE 10.42 Points on the focal axis of an ellipse.

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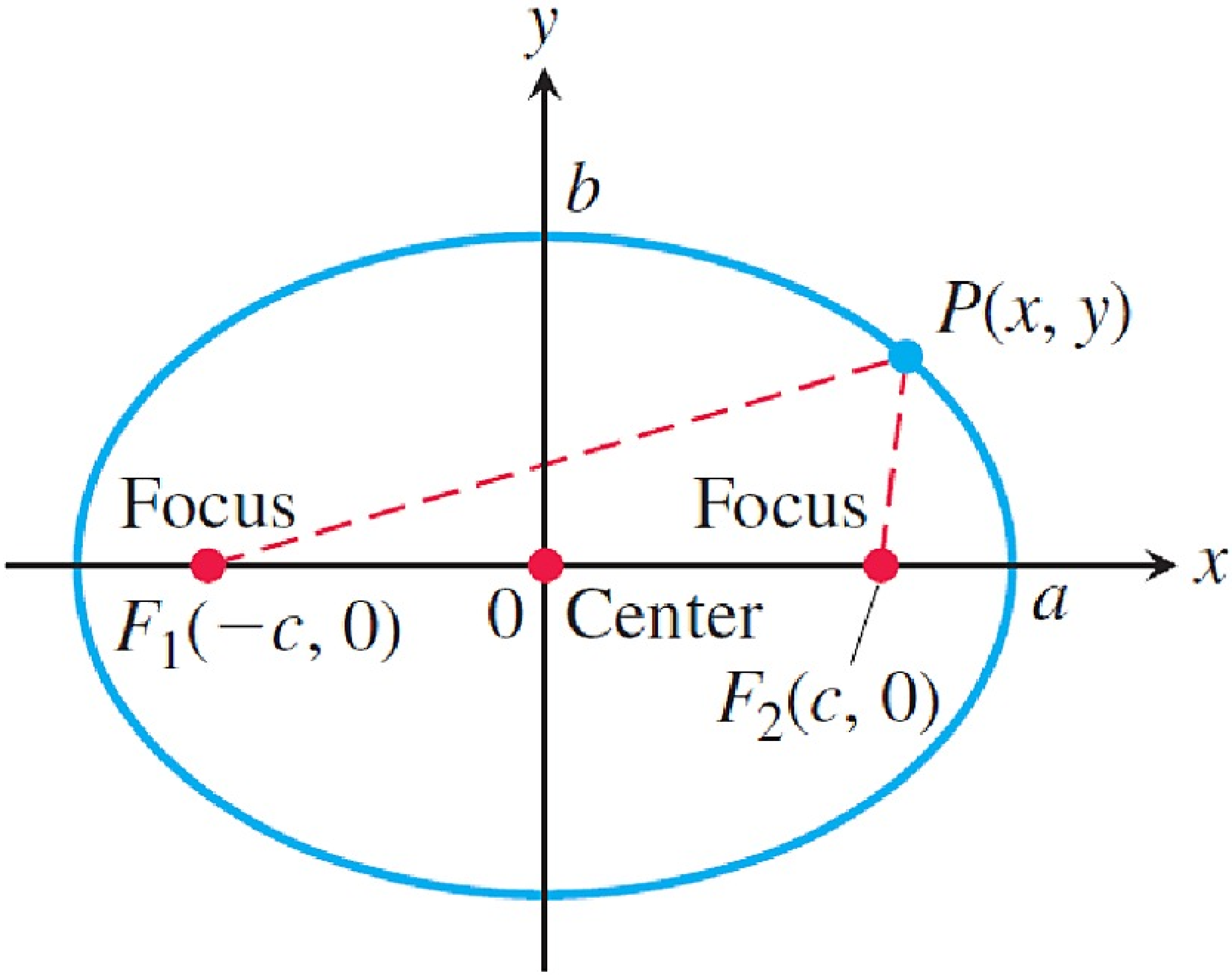

FIGURE 10.43 The ellipse defined by the equation $PF_1 + PF_2 = 2a$ is the graph of the equation $(x^2/a^2) + (y^2/b^2) = 1$, where $b^2 = a^2 - c^2$.

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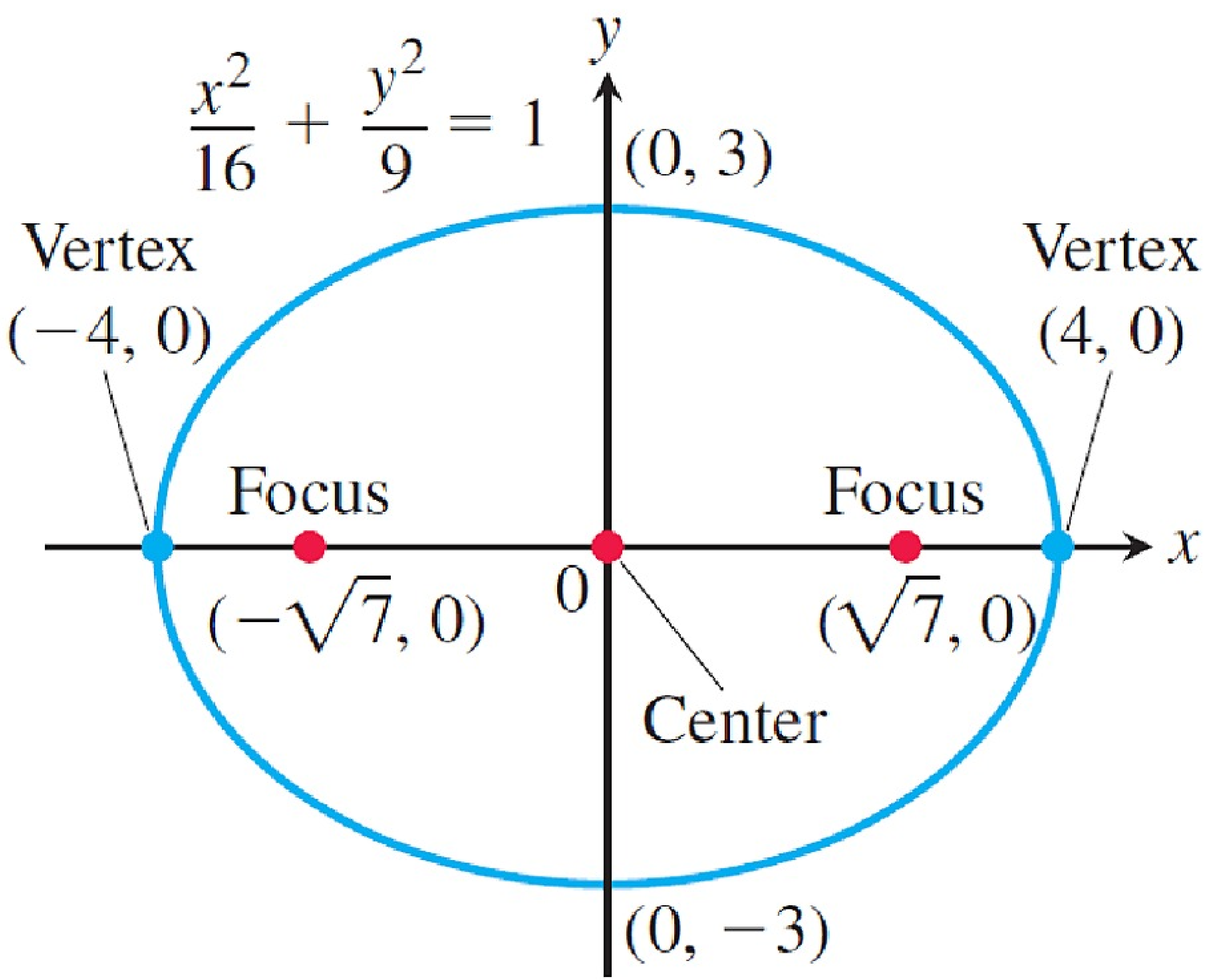

FIGURE 10.44 An ellipse with its major axis horizontal (Example 2).

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Standard-Form Equations for Ellipses Centered at the Origin

Foci on the x-axis: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1 \quad (a > b)$

Center-to-focus distance: $c = \sqrt{a^2 - b^2}$

Foci: $(\pm c, 0)$

Vertices: $(\pm a, 0)$

Foci on the y-axis: $\dfrac{x^2}{b^2} + \dfrac{y^2}{a^2} = 1 \quad (a > b)$

Center-to-focus distance: $c = \sqrt{a^2 - b^2}$

Foci: $(0, \pm c)$

Vertices: $(0, \pm a)$

In each case, a is the semimajor axis and b is the semiminor axis.

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DEFINITIONS A **hyperbola** is the set of points in a plane whose distances from two fixed points in the plane have a constant difference. The two fixed points are the foci of the hyperbola.

The line through the foci of a hyperbola is the **focal axis**. The point on the axis halfway between the foci is the hyperbola's **center**. The points where the focal axis and hyperbola cross are the **vertices** (Figure 10.45).

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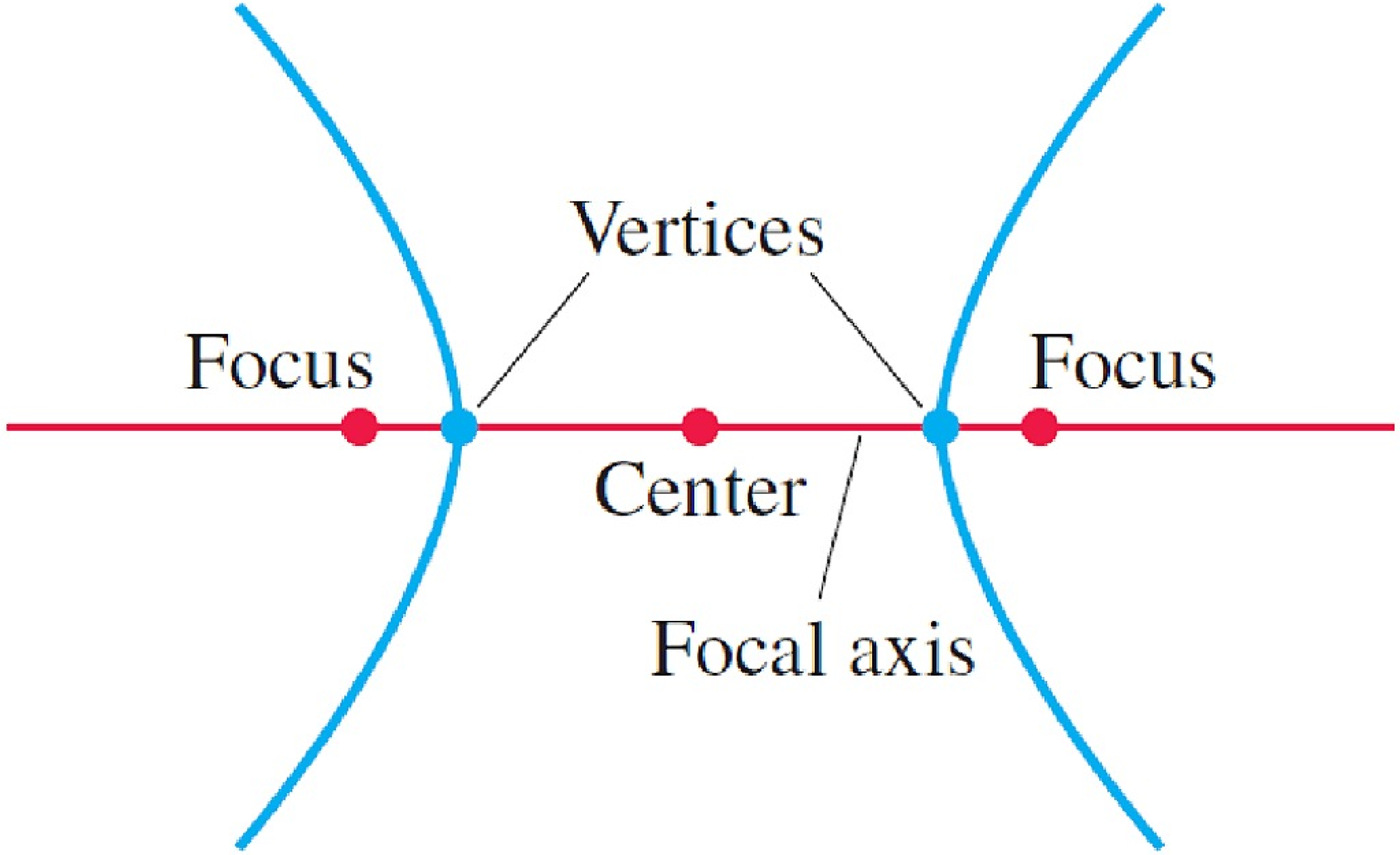

FIGURE 10.45 Points on the focal axis of a hyperbola.

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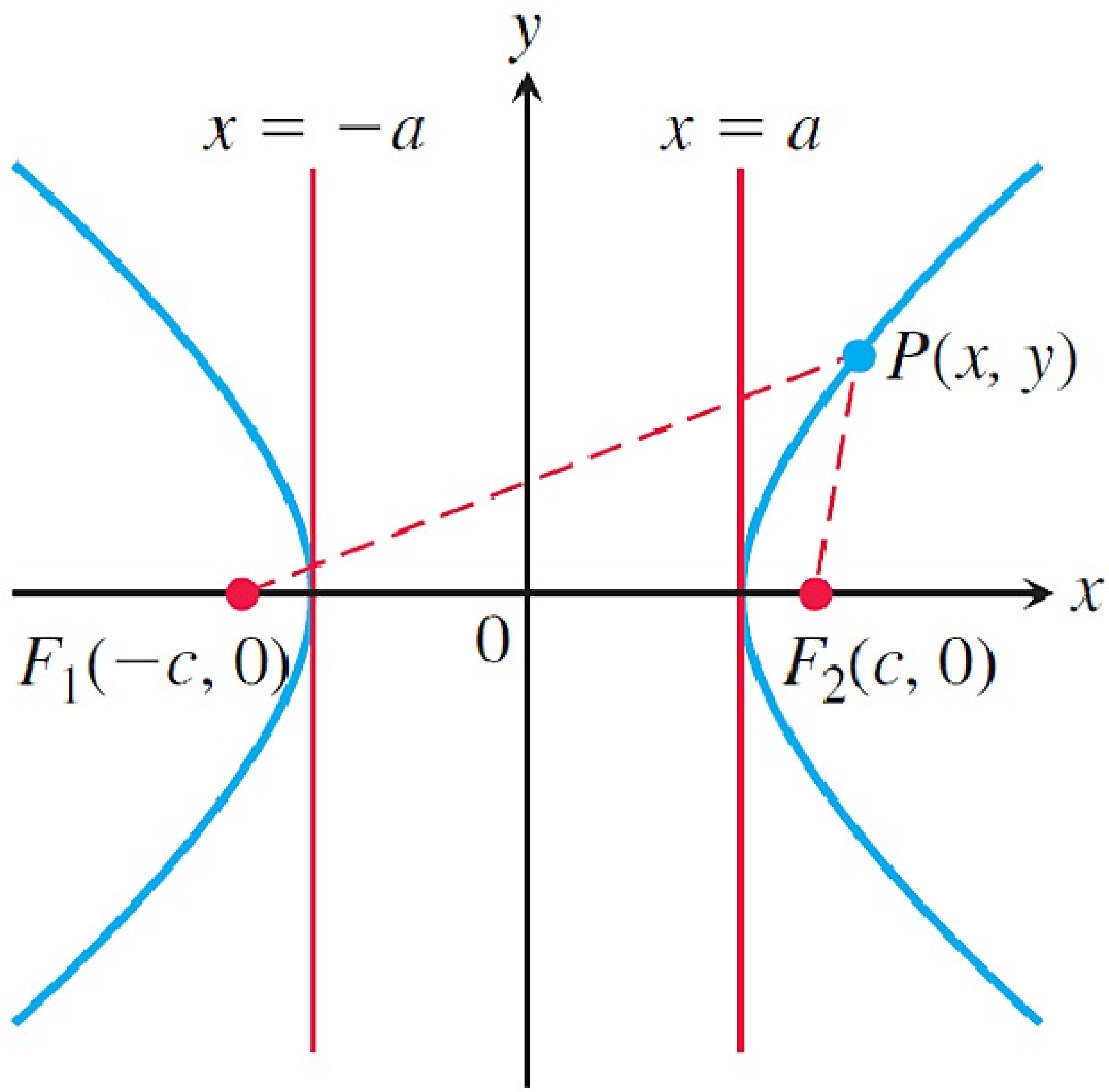

FIGURE 10.46 Hyperbolas have two branches. For points on the right-hand branch of the hyperbola shown here, $PF_1 - PF_2 = 2a$. For points on the left-hand branch, $PF_2 - PF_1 = 2a$. We then let $b = \sqrt{c^2 - a^2}$.

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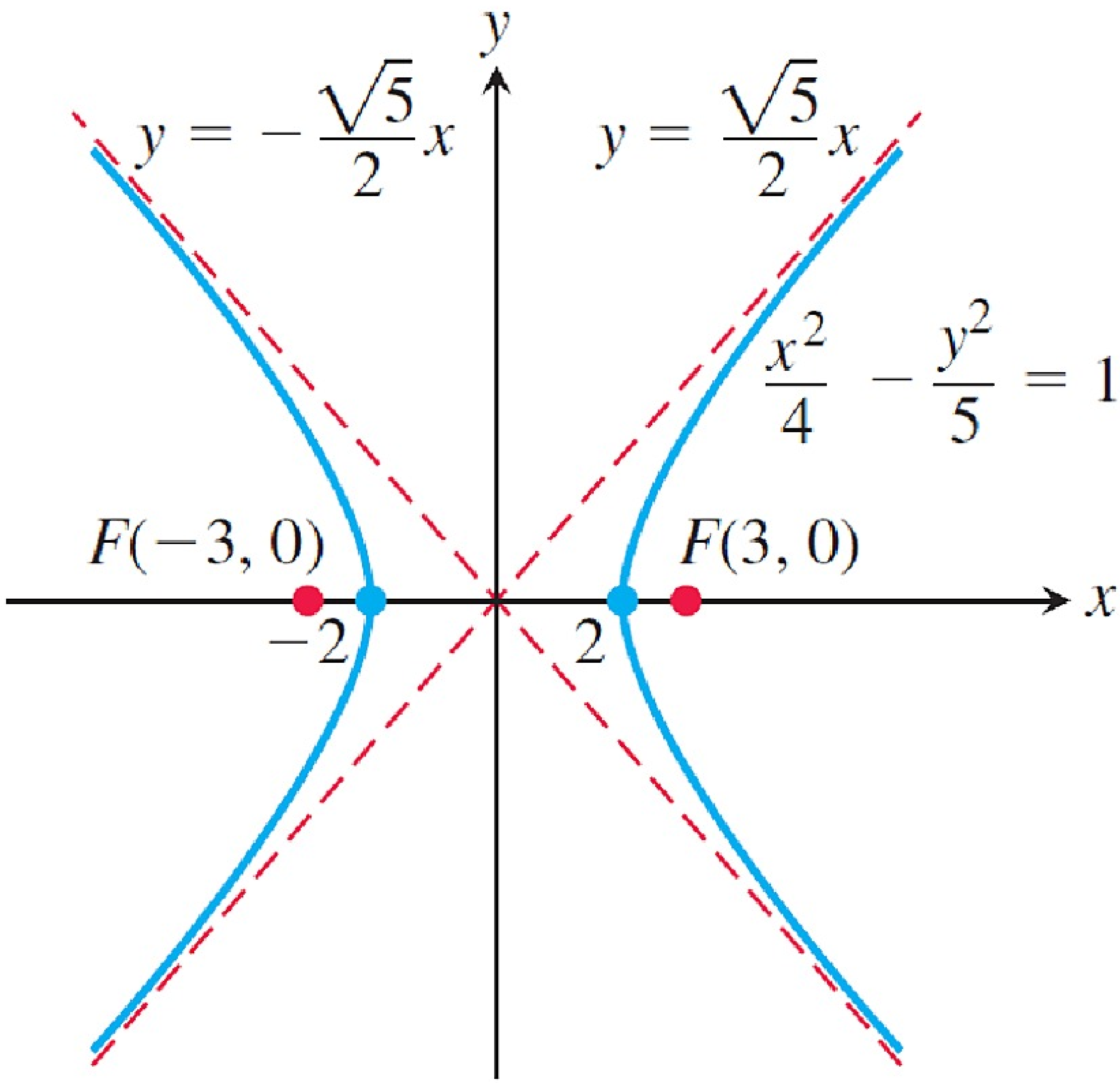

FIGURE 10.47 The hyperbola and its asymptotes in Example 3.

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Standard-Form Equations for Hyperbolas Centered at the Origin

Foci on the x-axis: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Center-to-focus distance: $c = \sqrt{a^2 + b^2}$

Foci: $(\pm c, 0)$

Vertices: $(\pm a, 0)$

Asymptotes: $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 0$ or $y = \pm \dfrac{b}{a} x$

Foci on the y-axis: $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 1$

Center-to-focus distance: $c = \sqrt{a^2 + b^2}$

Foci: $(0, \pm c)$

Vertices: $(0, \pm a)$

Asymptotes: $\dfrac{y^2}{a^2} - \dfrac{x^2}{b^2} = 0$ or $y = \pm \dfrac{a}{b} x$

Notice the difference in the asymptote equations (b/a in the first, a/b in the second).

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Section 10.7

Conics in Polar Coordinates

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DEFINITION

The **eccentricity** of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$ $(a > b)$ is

$$e = \frac{c}{a} = \frac{\sqrt{a^2 - b^2}}{a}.$$

The **eccentricity** of the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$ is

$$e = \frac{c}{a} = \frac{\sqrt{a^2 + b^2}}{a}.$$

The **eccentricity** of a parabola is $e = 1$.

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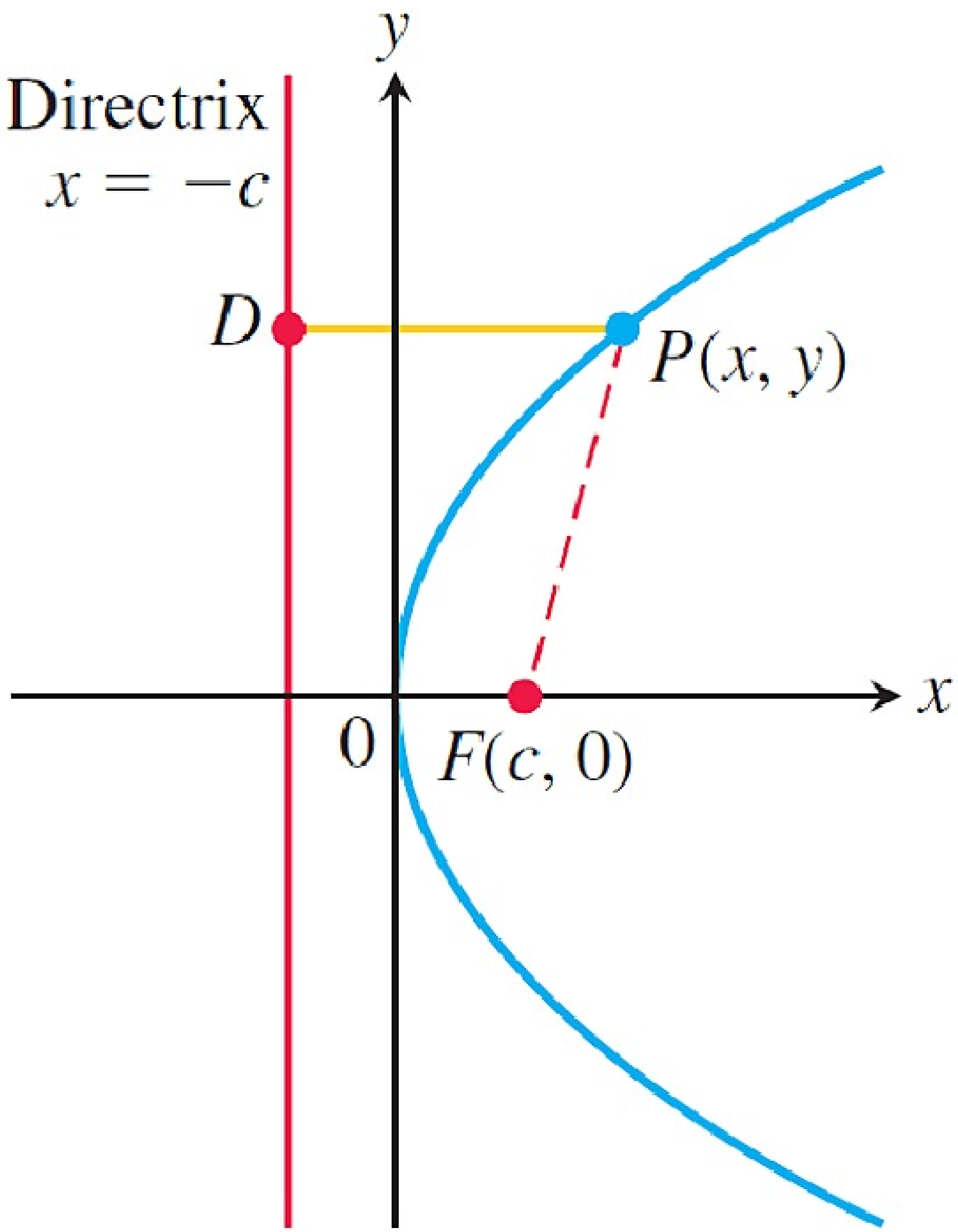

FIGURE 10.48 The distance from the focus F to any point P on a parabola equals the distance from P to the nearest point D on the directrix, so $PF = PD$.

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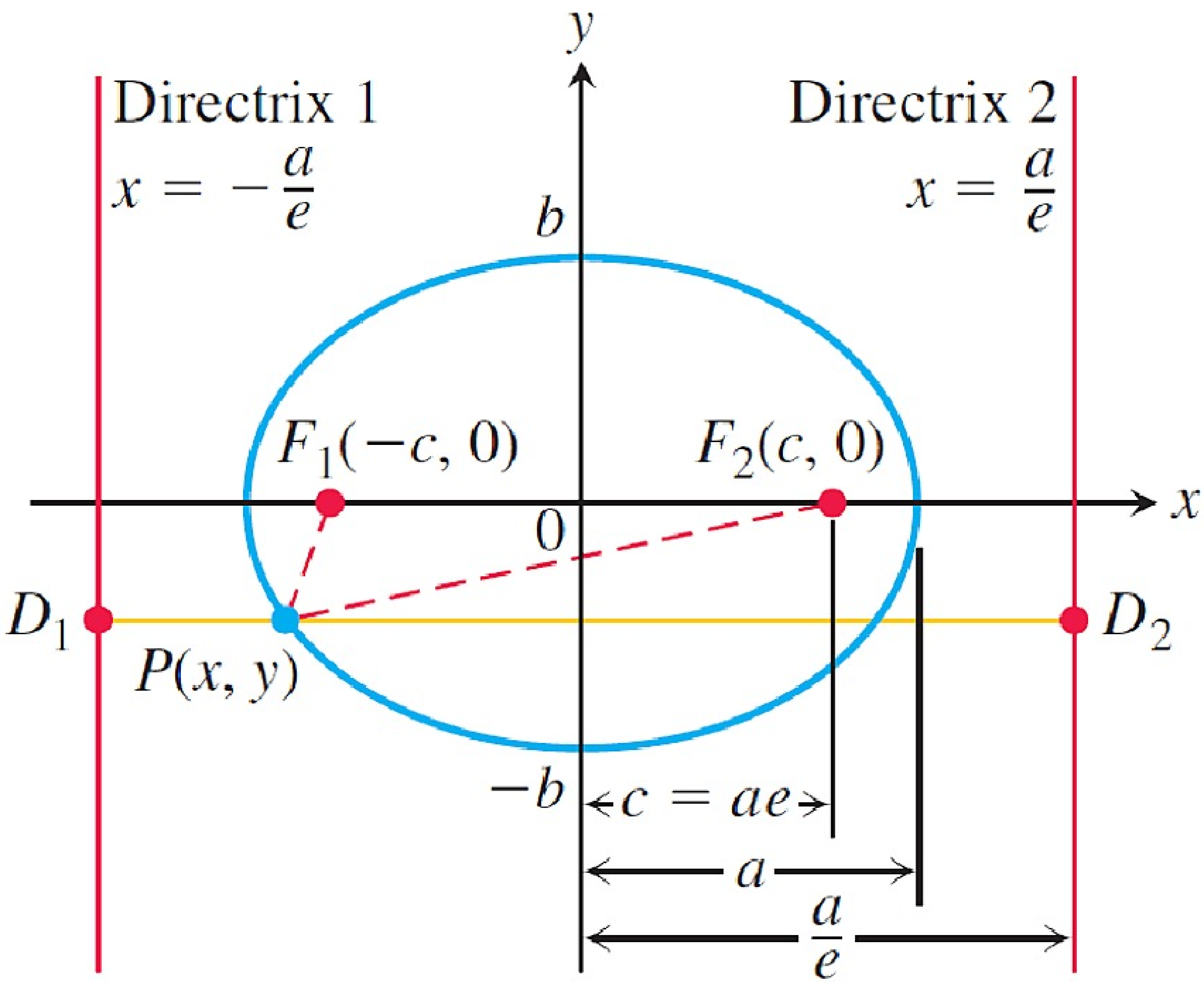

FIGURE 10.49 The foci and directrices of the ellipse $(x^2/a^2) + (y^2/b^2) = 1$. Directrix 1 corresponds to focus F_1 and directrix 2 to focus F_2.

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In both the ellipse and the hyperbola, the eccentricity is the ratio of the distance between the foci to the distance between the vertices (because $c/a = 2c/2a$).

$$\boxed{\text{Eccentricity} = \frac{\text{distance between foci}}{\text{distance between vertices}}}$$

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The "focus–directrix" equation $PF = e \cdot PD$ unites the parabola, ellipse, and hyperbola in the following way. Suppose that the distance PF of a point P from a fixed point F (the focus) is a constant multiple of its distance from a fixed line (the directrix). That is, suppose

$$\boxed{PF = e \cdot PD, \qquad (4)}$$

where e is the constant of proportionality. Then the path traced by P is

(a) a *parabola* if $e = 1$,

(b) an *ellipse* of eccentricity e if $e < 1$, and

(c) a *hyperbola* of eccentricity e if $e > 1$.

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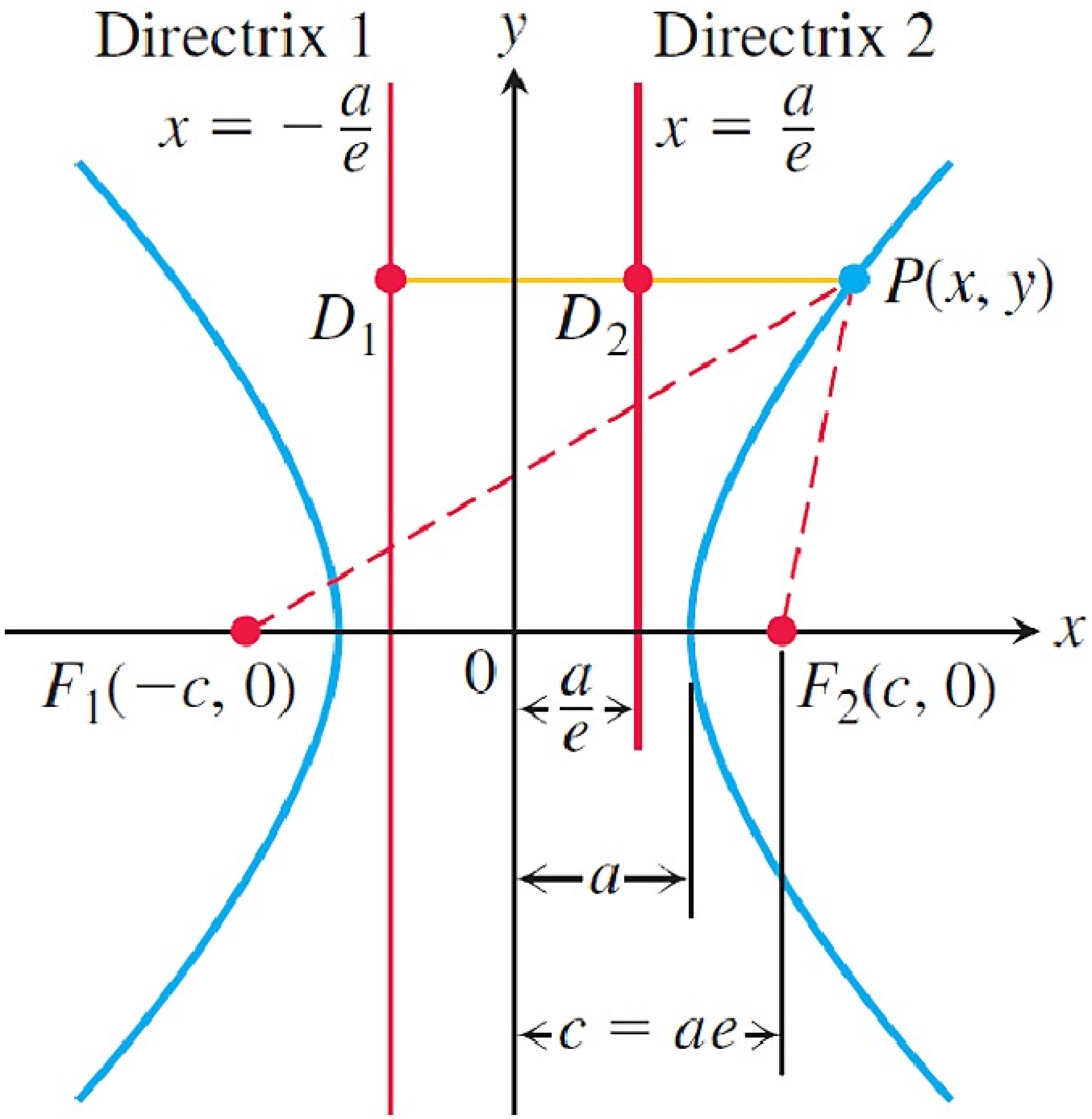

FIGURE 10.50 The foci and directrices of the hyperbola $(x^2/a^2) - (y^2/b^2) = 1$. No matter where P lies on the hyperbola, $PF_1 = e \cdot PD_1$ and $PF_2 = e \cdot PD_2$.

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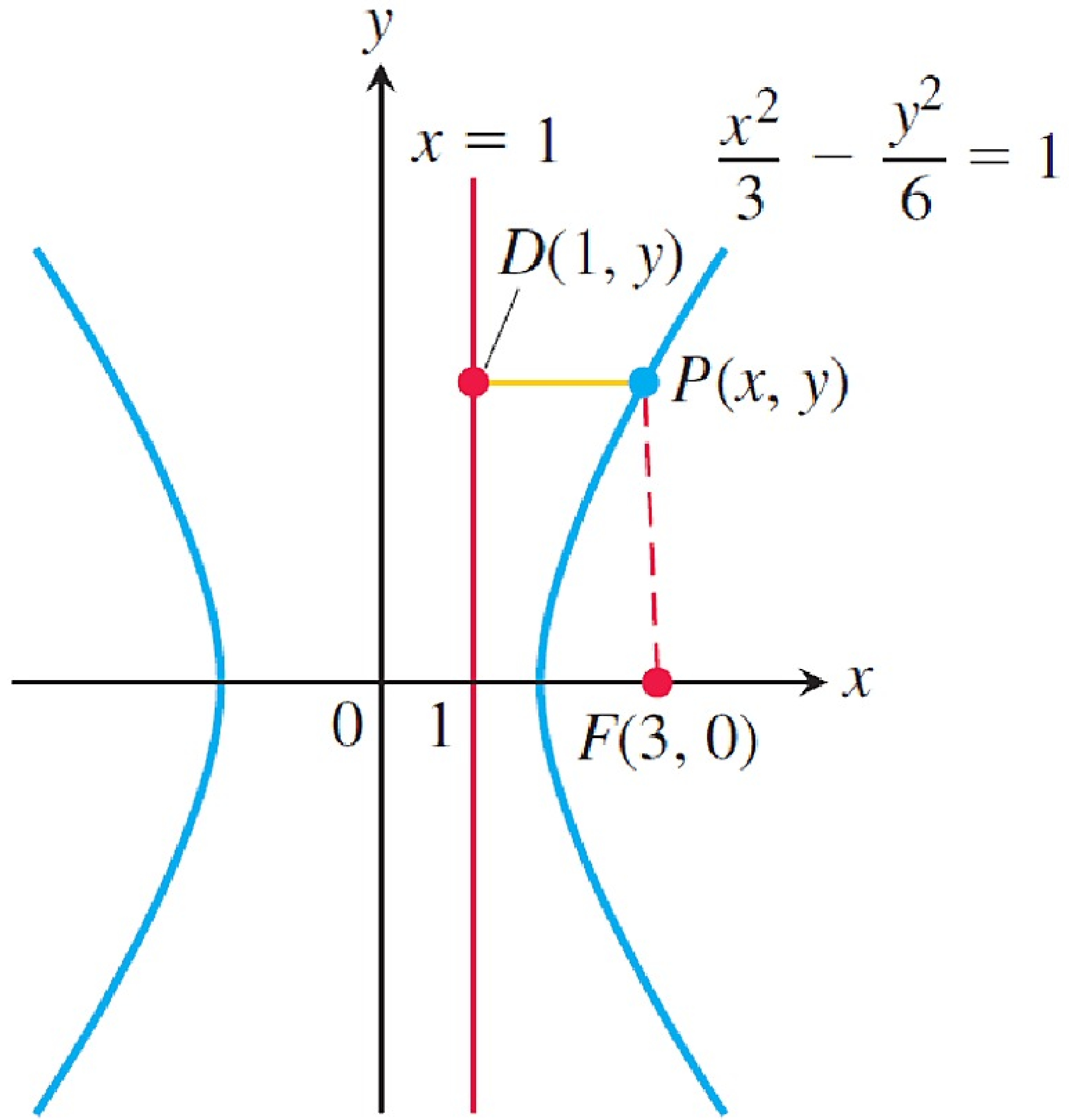

FIGURE 10.51 The hyperbola and directrix in Example 1.

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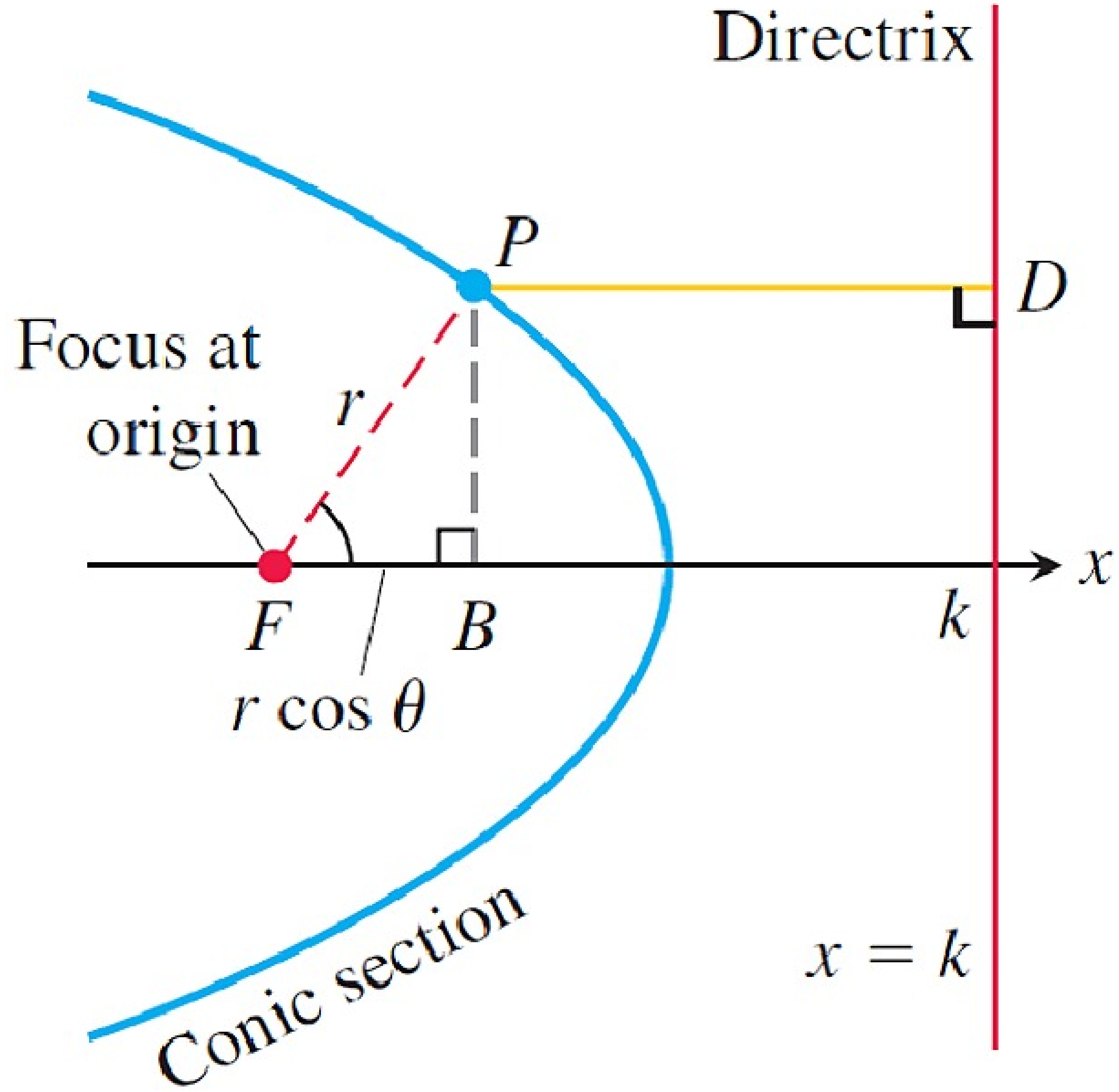

FIGURE 10.52 If a conic section is put in the position with its focus placed at the origin and a directrix perpendicular to the initial ray and right of the origin, we can find its polar equation from the conic's focus–directrix equation.

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Polar Equation for a Conic with Eccentricity e

$$r = \frac{ke}{1 + e\cos\theta}, \tag{5}$$

where $x = k > 0$ is the vertical directrix.

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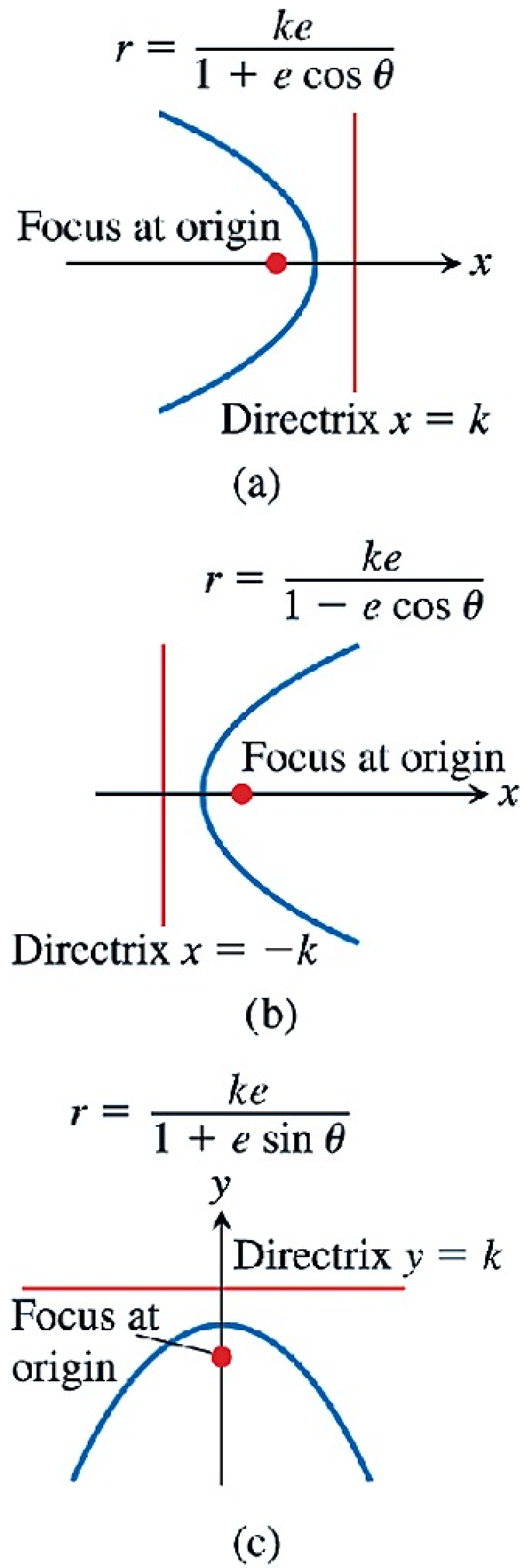

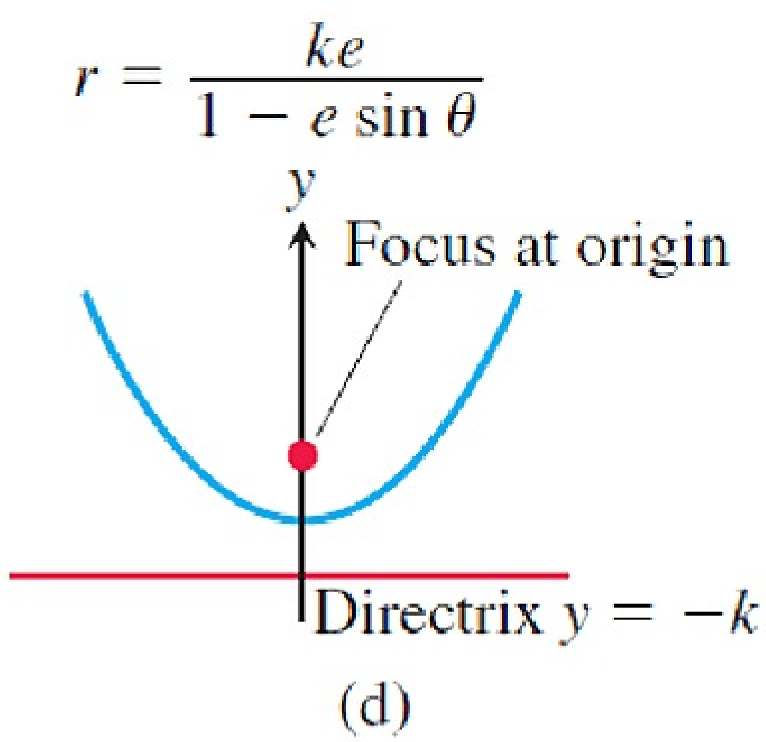

FIGURE 10.53 Equations for conic sections with eccentricity $e > 0$ but different locations of the directrix. The graphs here show a parabola, so $e = 1$.

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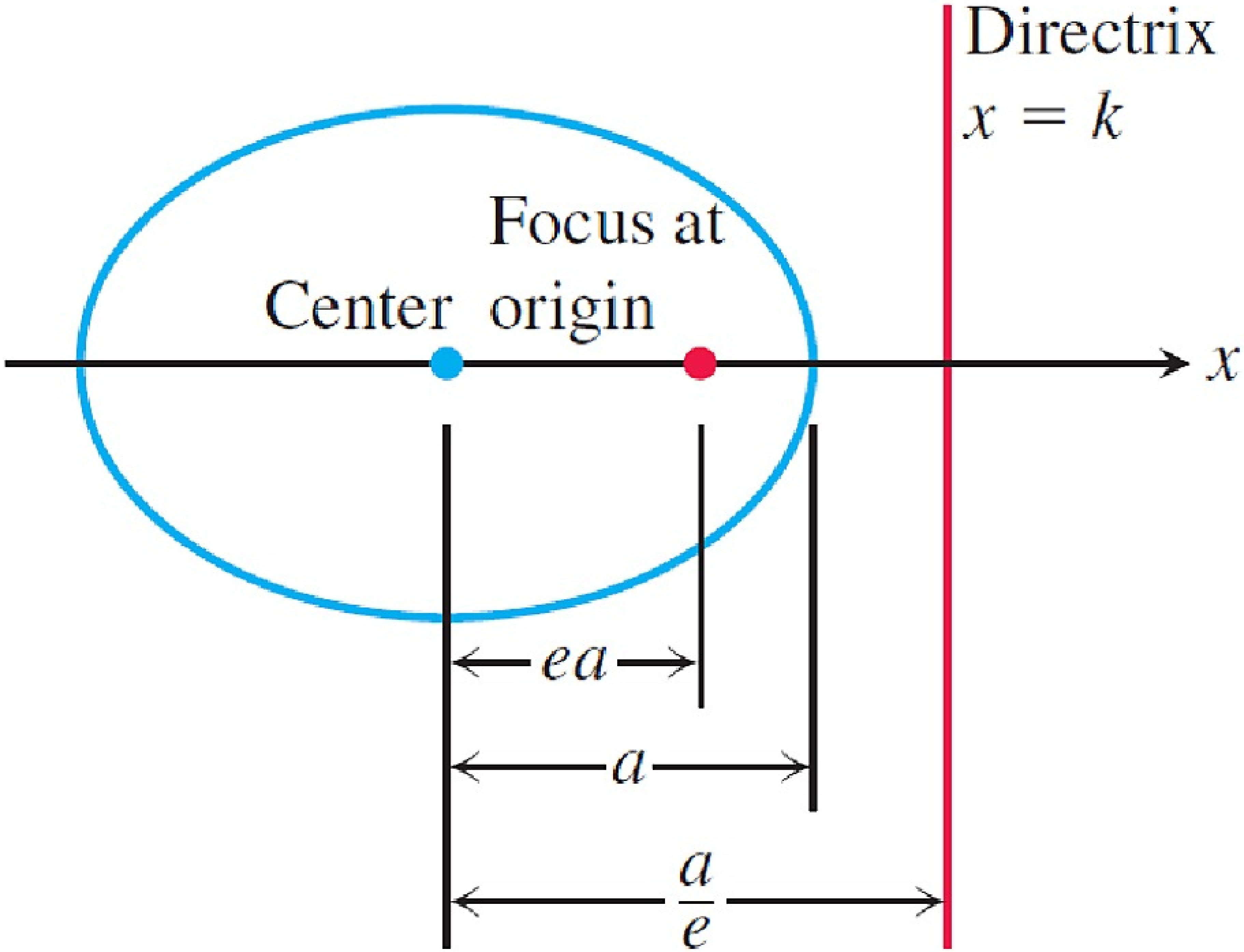

FIGURE 10.54 In an ellipse with semi-major axis a, the focus–directrix distance is $k = (a/e) - ea$, so $ke = a(1 - e^2)$.

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Polar Equation for the Ellipse with Eccentricity e and Semimajor Axis a

$$r = \frac{a(1 - e^2)}{1 + e\cos\theta} \tag{6}$$

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The Standard Polar Equation for Lines

If the point $P_0(r_0, \theta_0)$ is the foot of the perpendicular from the origin to the line L, and $r_0 \geq 0$, then an equation for L is

$$r \cos(\theta - \theta_0) = r_0. \tag{7}$$

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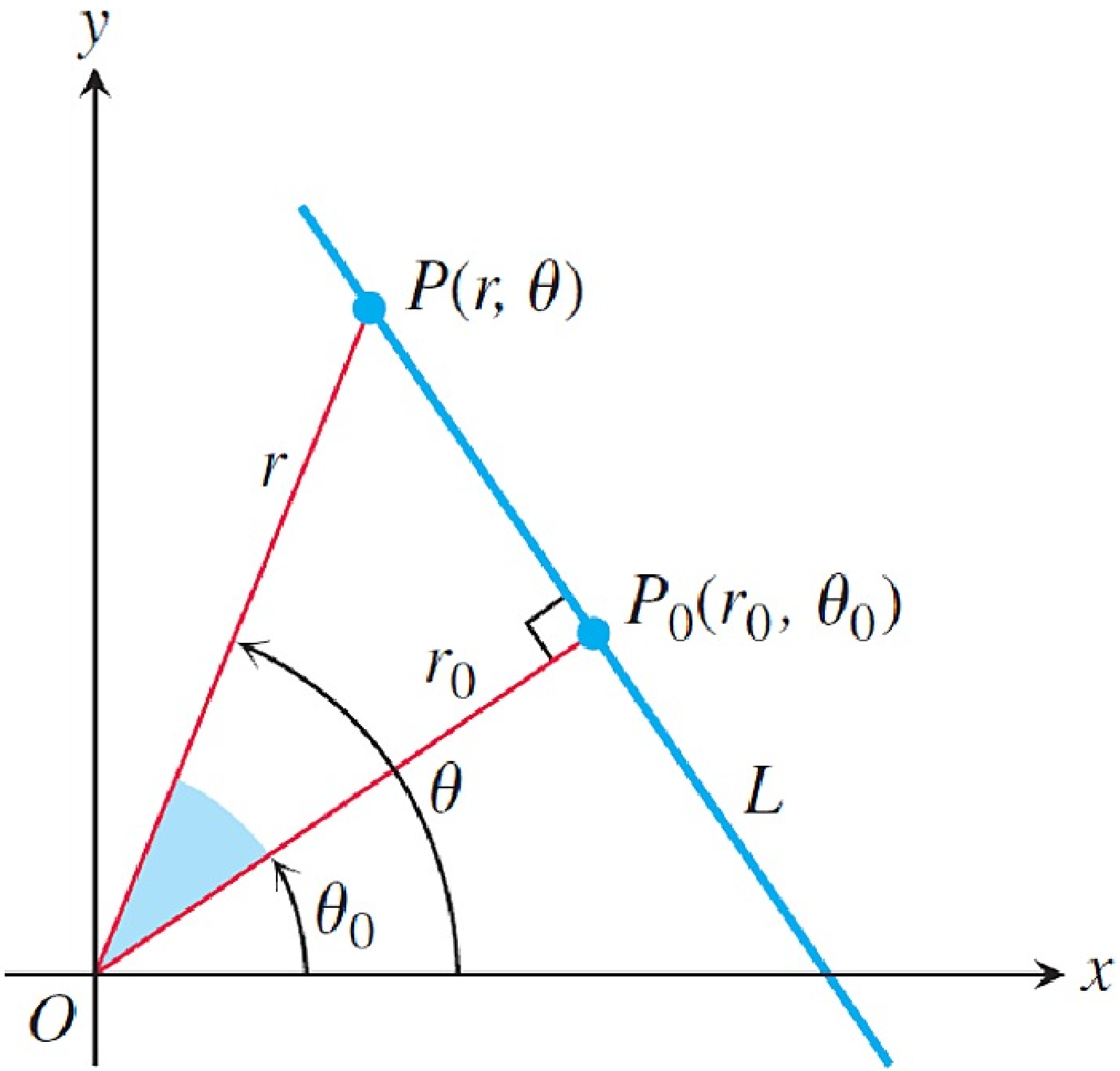

FIGURE 10.55 We can obtain a polar equation for line L by reading the relation $r_0 = r\cos(\theta - \theta_0)$ from the right triangle OP_0P.

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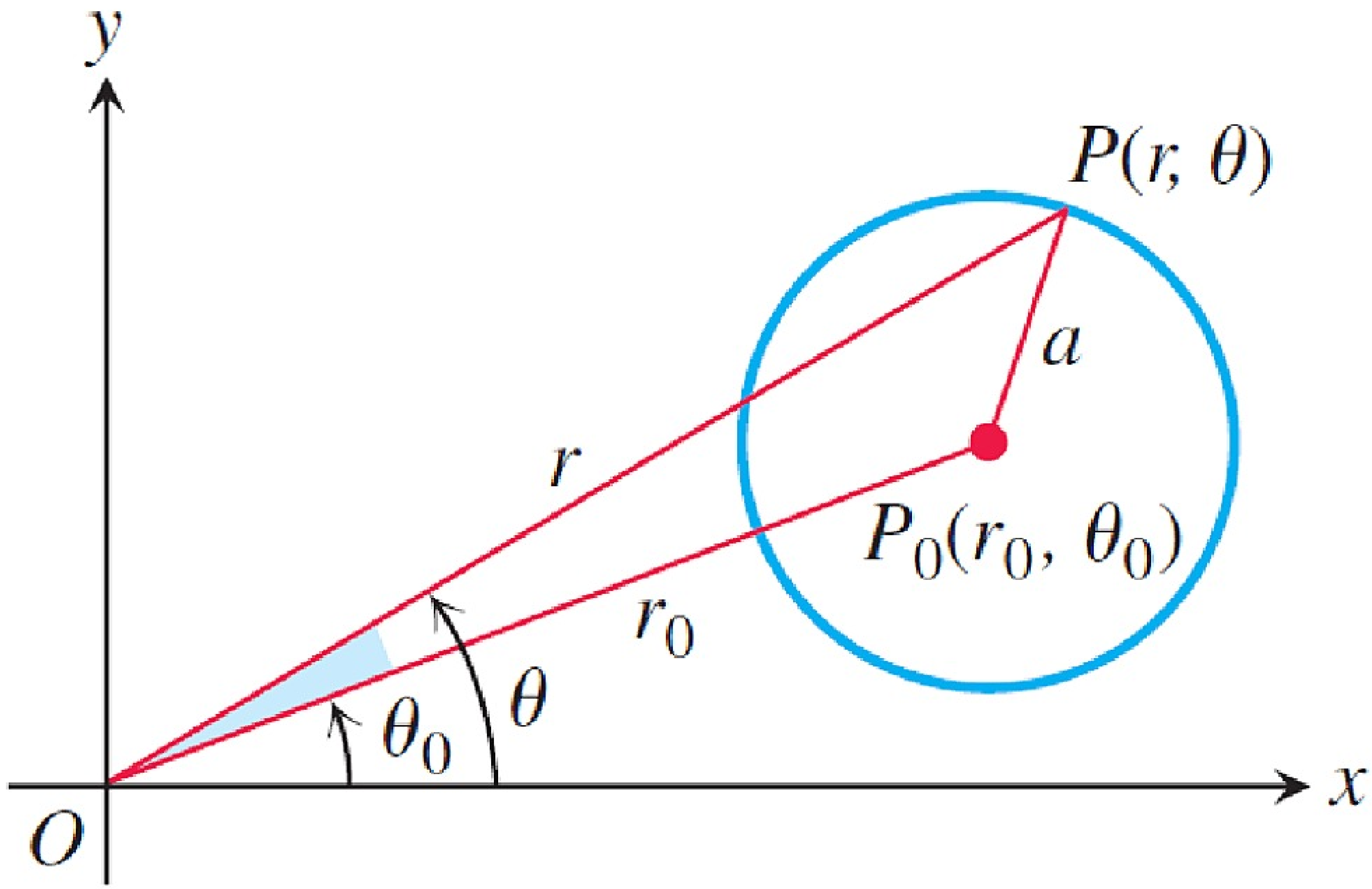

FIGURE 10.56 We can get a polar equation for this circle by applying the Law of Cosines to triangle OP_0P.

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