# Chapter 11

# Vectors and the Geometry of Space

# Section 11.1

## Three-Dimensional Coordinate Systems

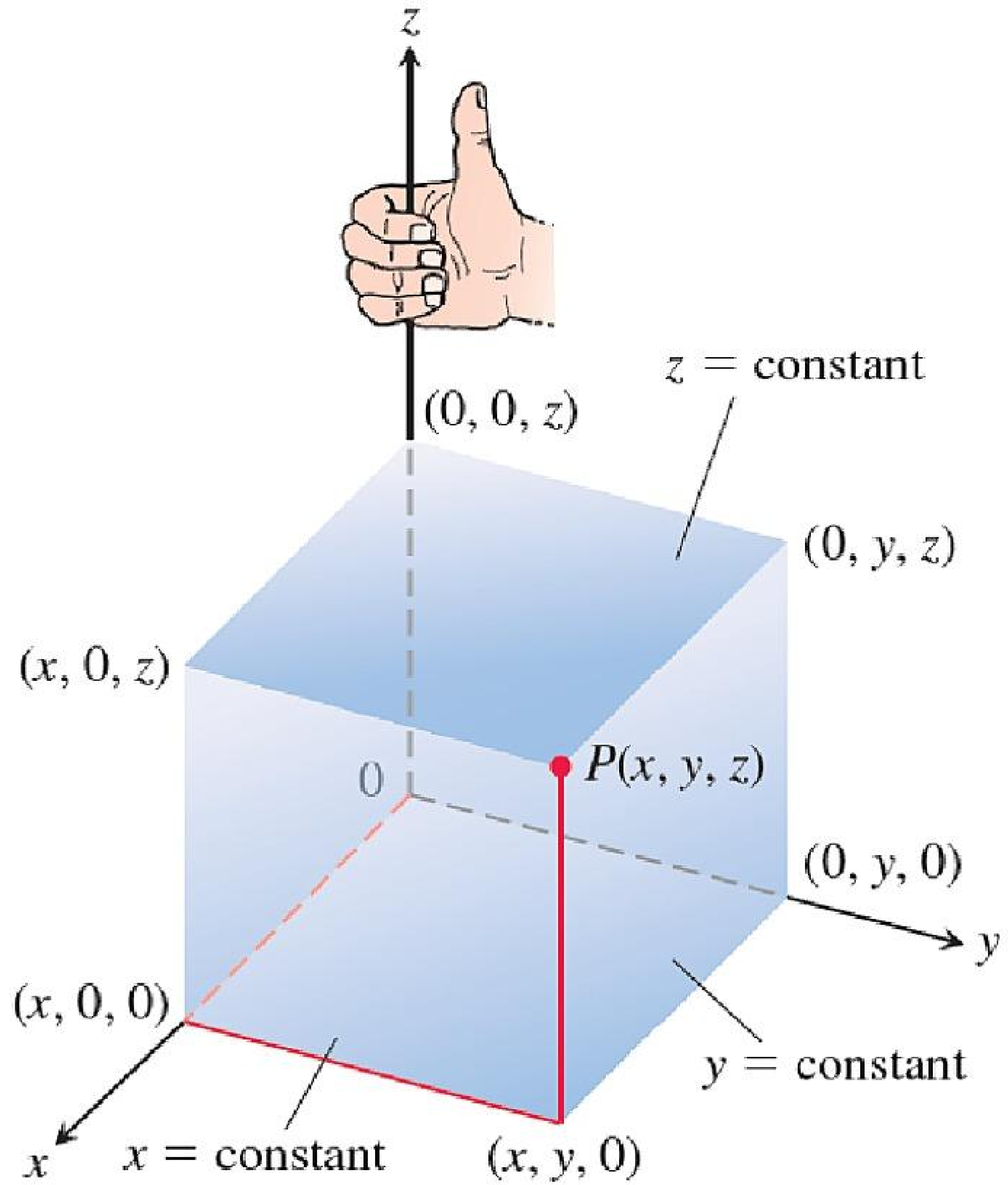


**FIGURE 11.1** The Cartesian coordinate system is right-handed.

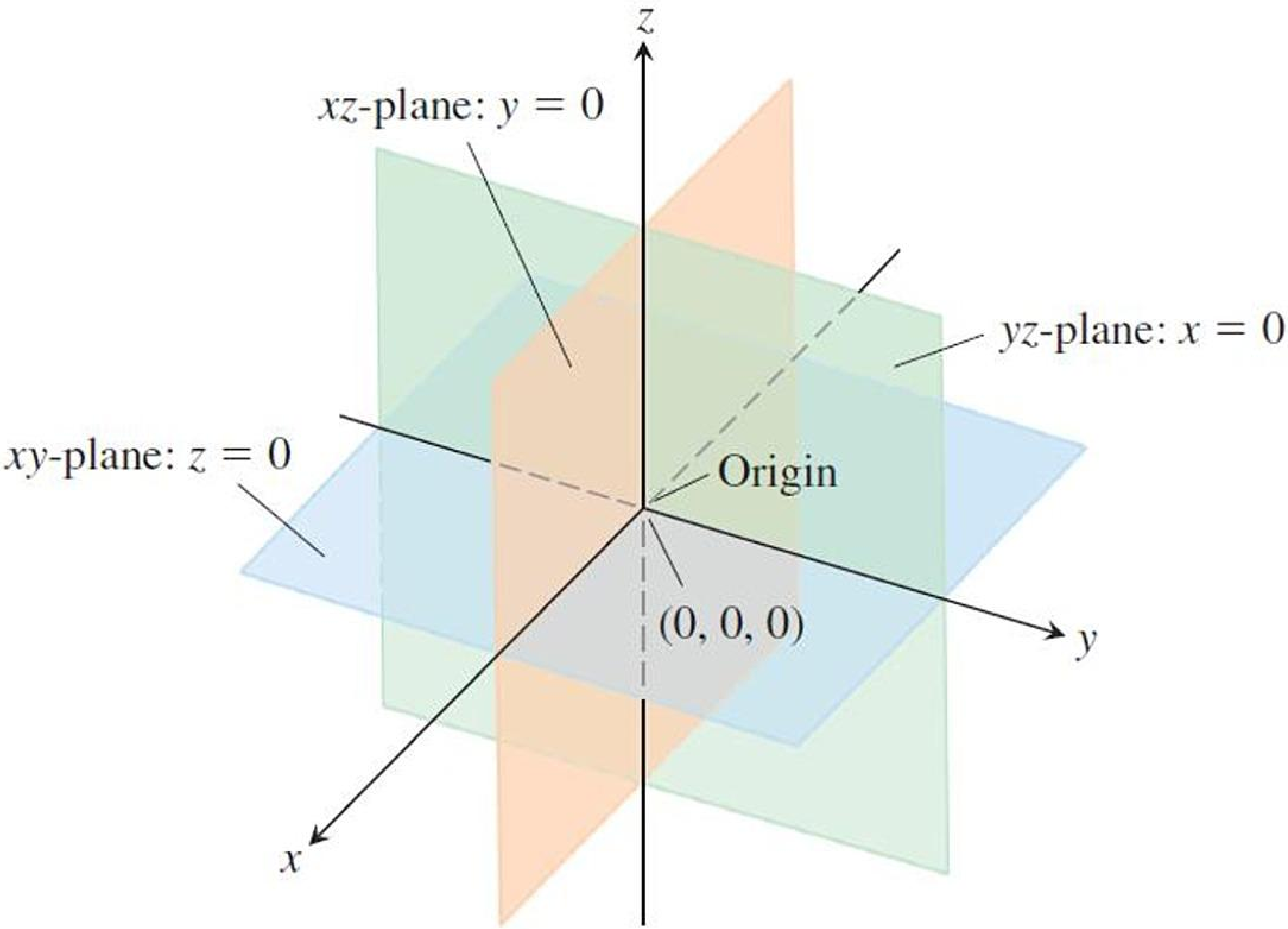


**FIGURE 11.2** The planes $x = 0$, $y = 0$, and $z = 0$ divide space into eight octants.

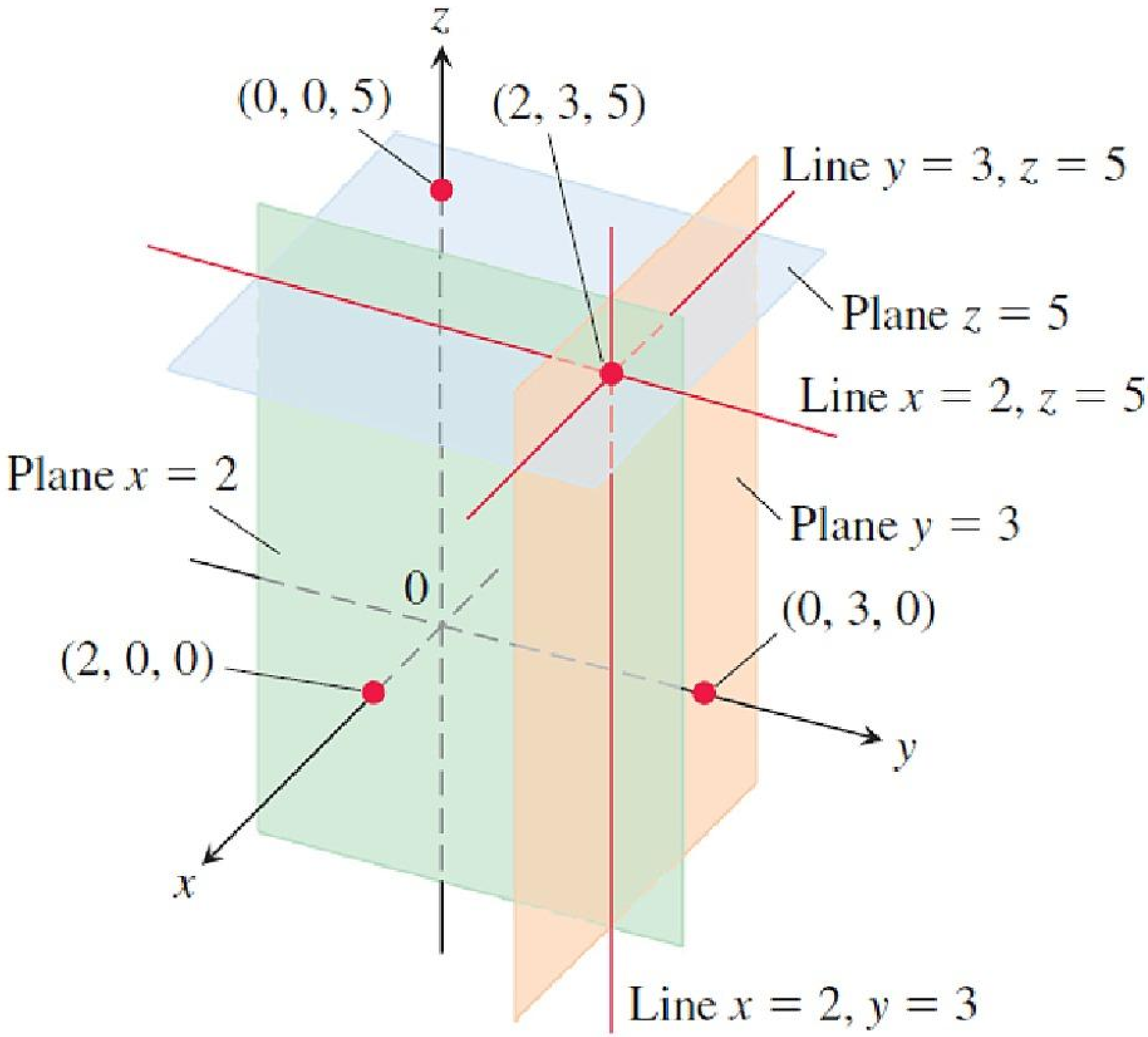


**FIGURE 11.3** The planes $x = 2$, $y = 3$, and $z = 5$ determine three lines through the point $(2, 3, 5)$.

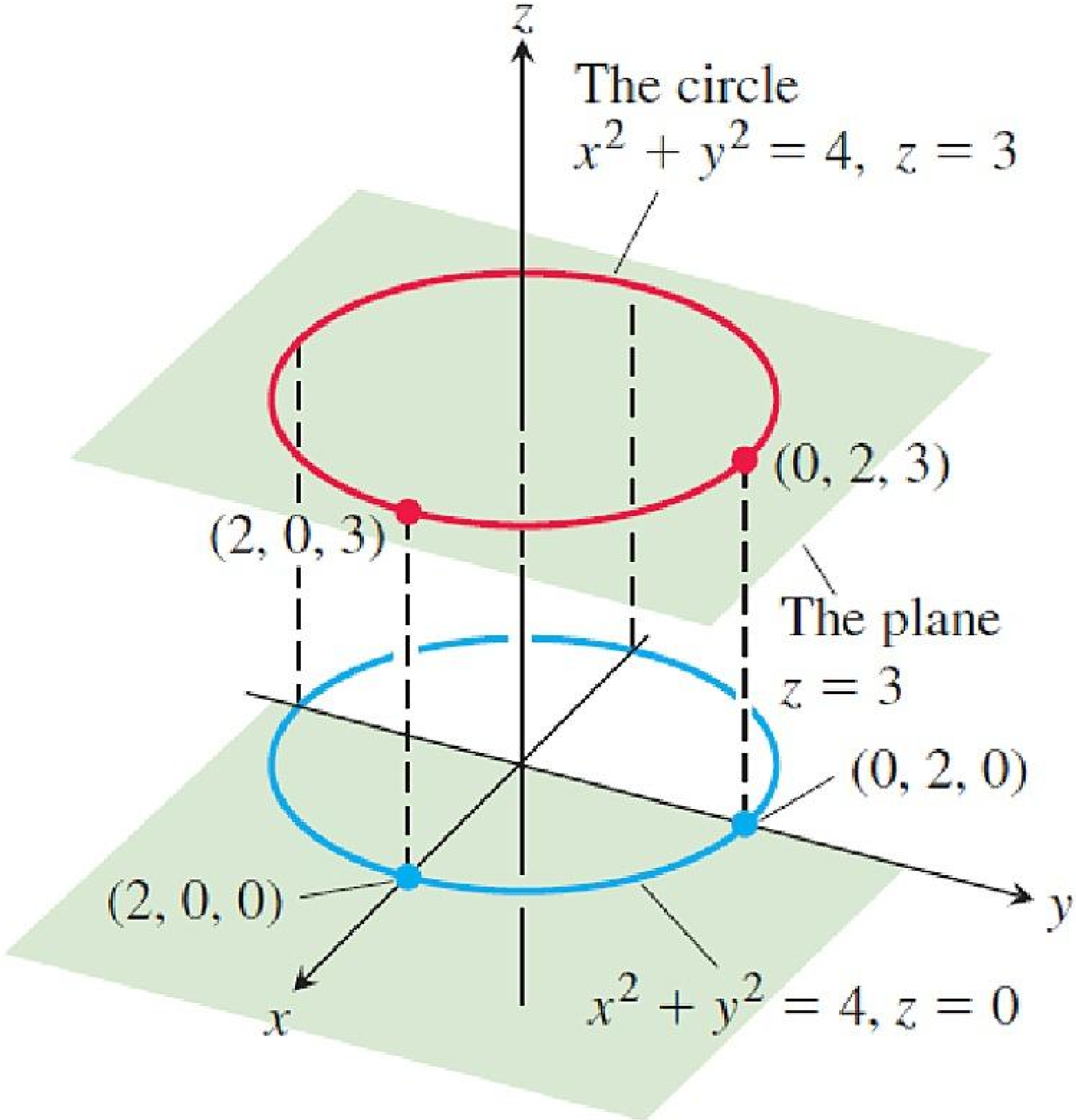


**FIGURE 11.4** The circle $x^2 + y^2 = 4$ in the plane $z = 3$ (Example 2).

**The Distance Between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is**

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$



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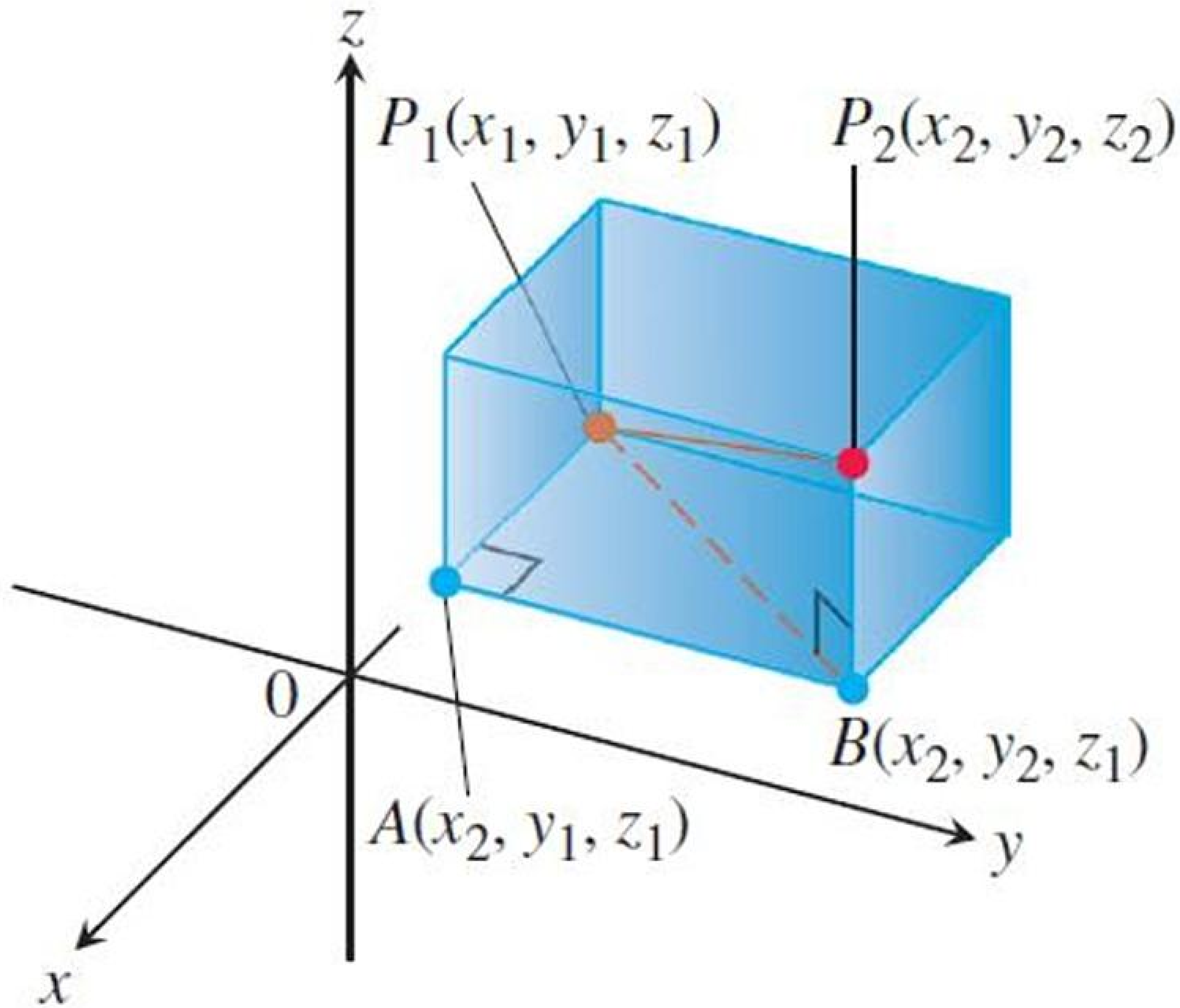



**FIGURE 11.5** We find the distance between $P_1$ and $P_2$ by applying the Pythagorean theorem to the right triangles $P_1AB$ and $P_1BP_2$.

**EXAMPLE 3** The distance between $P_1(2, 1, 5)$ and $P_2(-2, 3, 0)$ is

$$
\begin{aligned}
|P_1P_2| &= \sqrt{(-2-2)^2 + (3-1)^2 + (0-5)^2} \\
&= \sqrt{16 + 4 + 25} \\
&= \sqrt{45} \approx 6.708 .
\end{aligned}
$$

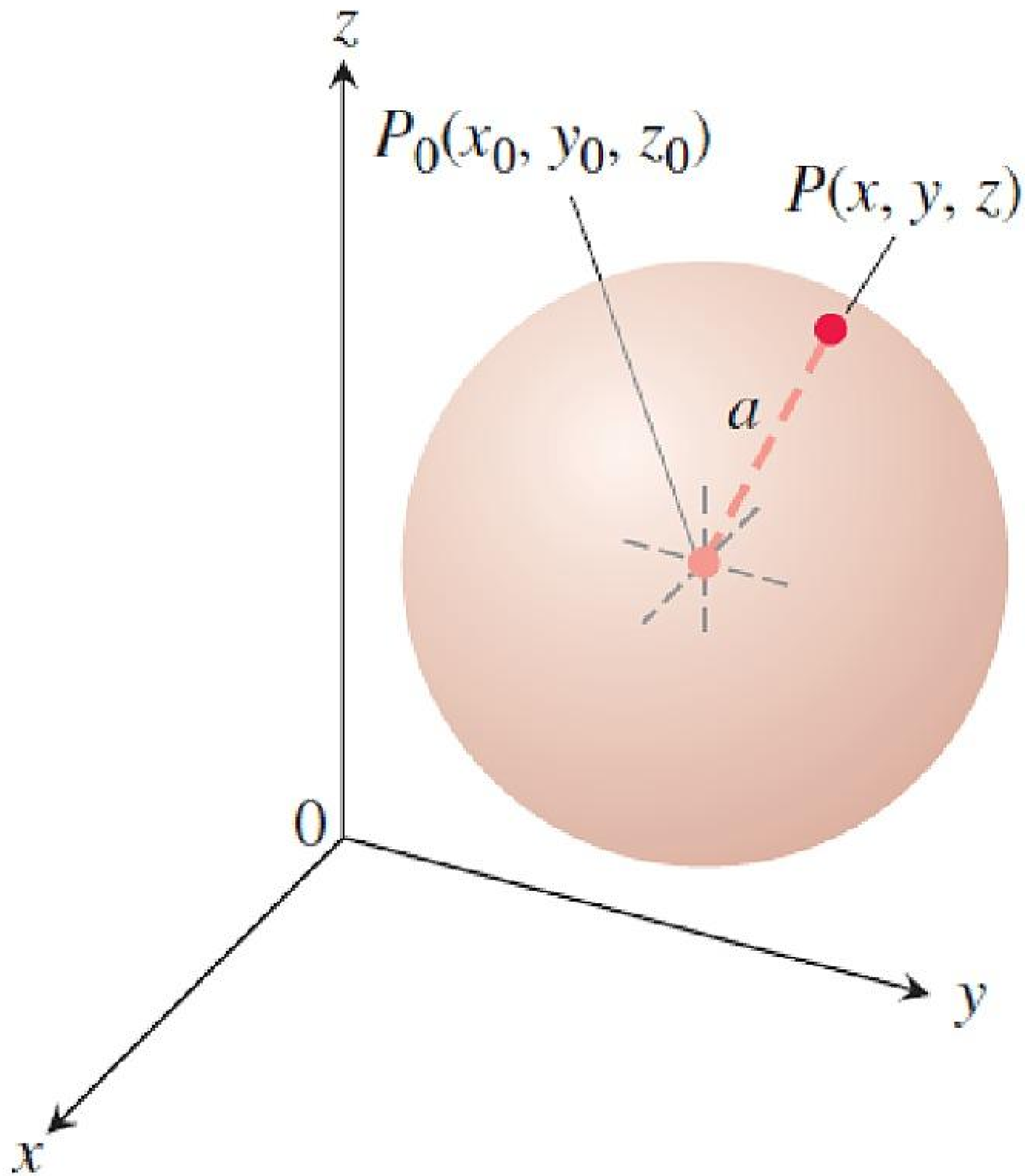



**FIGURE 11.6** The sphere of radius $a$ centered at the point $(x_0, y_0, z_0)$.

**The Standard Equation for the Sphere of Radius $a$ and Center $(x_0, y_0, z_0)$**

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

**EXAMPLE 4** Find the center and radius of the sphere

$$x^2 + y^2 + z^2 + 3x - 4z + 1 = 0.$$

**Solution** We find the center and radius of a sphere the way we find the center and radius of a circle: Complete the squares on the $x$-, $y$-, and $z$-terms as necessary and write each quadratic as a squared linear expression. Then, from the equation in standard form, read off the center and radius. For the sphere here, we have

$$\begin{aligned} x^2 + y^2 + z^2 + 3x - 4z + 1 &= 0 \\ (x^2 + 3x) + y^2 + (z^2 - 4z) &= -1 \\ \left(x^2 + 3x + \left(\frac{3}{2}\right)^2\right) + y^2 + \left(z^2 - 4z + \left(\frac{-4}{2}\right)^2\right) &= -1 + \left(\frac{3}{2}\right)^2 + \left(\frac{-4}{2}\right)^2 \\ \left(x + \frac{3}{2}\right)^2 + y^2 + (z - 2)^2 &= -1 + \frac{9}{4} + 4 = \frac{21}{4}. \end{aligned}$$

From this standard form, we read that $x_0 = -3/2$, $y_0 = 0$, $z_0 = 2$, and $a = \sqrt{21}/2$. The center is $(-3/2, 0, 2)$. The radius is $\sqrt{21}/2$.

# Section 11.2

## Vectors

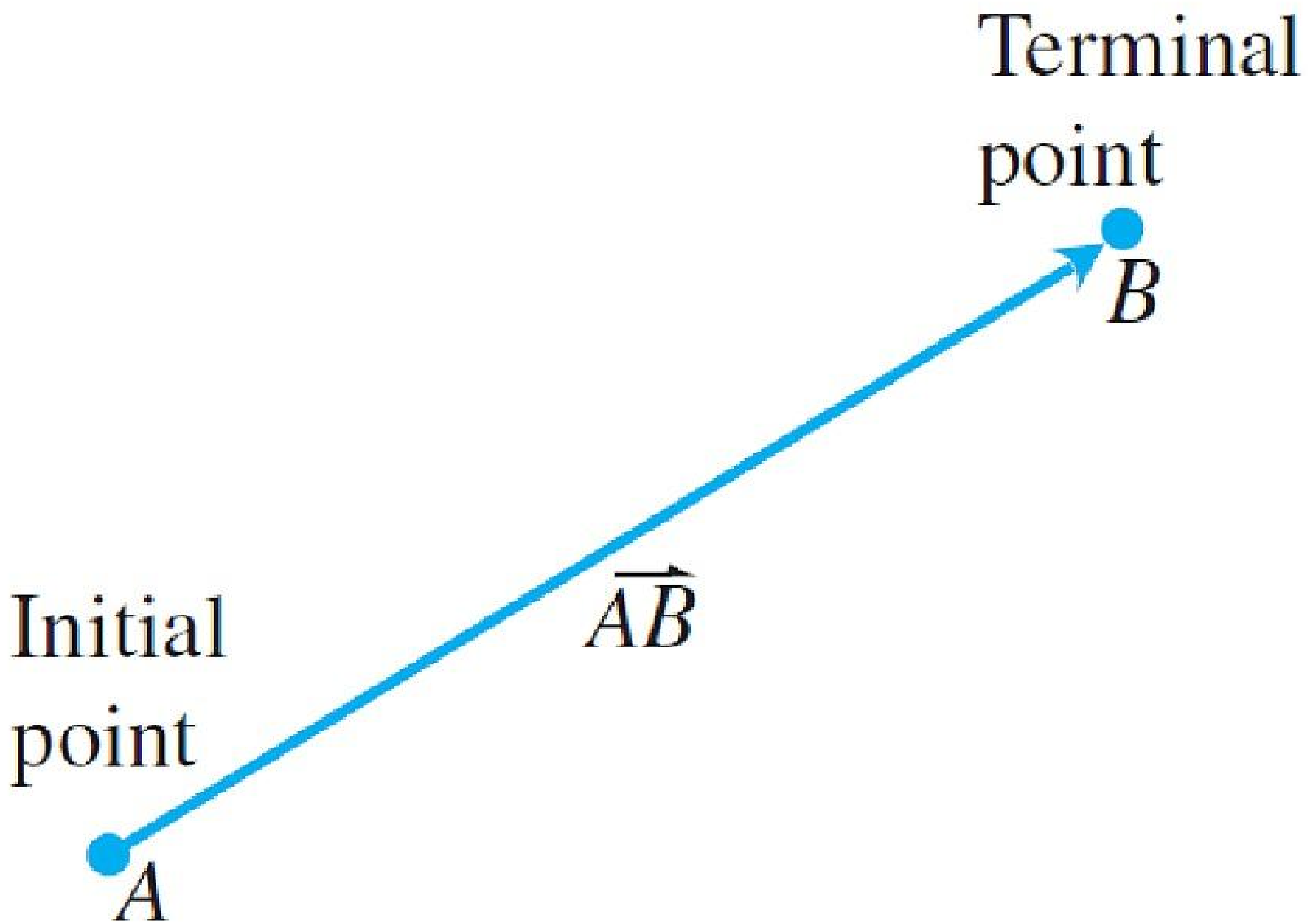



**FIGURE 11.7** The directed line segment $\overrightarrow{AB}$ is called a vector.

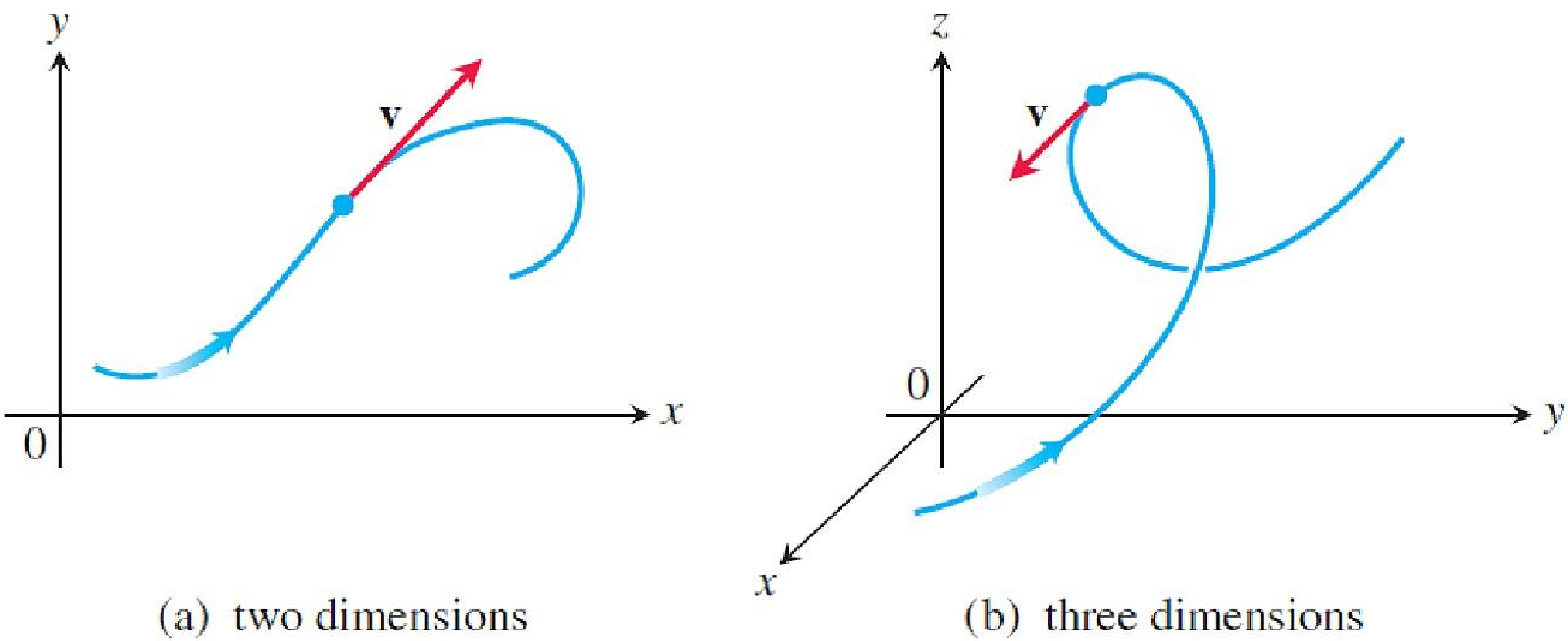



**FIGURE 11.8** The velocity vector of a particle moving along a path (a) in the plane (b) in space. The arrowhead on the path indicates the direction of motion of the particle.

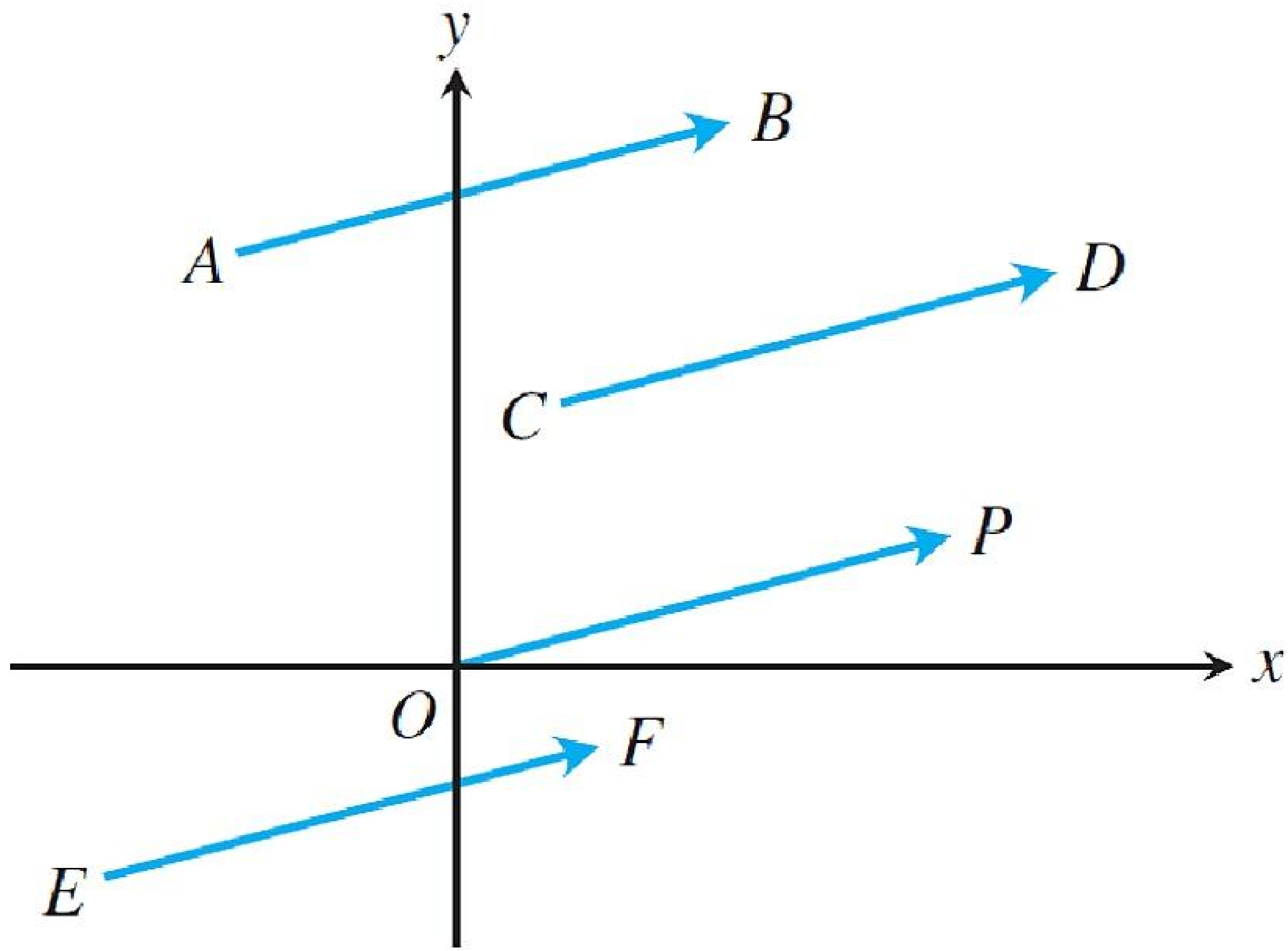



**FIGURE 11.9** The four arrows in the plane (directed line segments) shown here have the same length and direction. They therefore represent the same vector, and we write $\overrightarrow{AB} = \overrightarrow{CD} = \overrightarrow{OP} = \overrightarrow{EF}$.

**DEFINITIONS** The vector represented by the directed line segment $\overrightarrow{AB}$ has **initial point** $A$ and **terminal point** $B$ and its **length** is denoted by $|\overrightarrow{AB}|$. Two vectors are **equal** if they have the same length and direction.

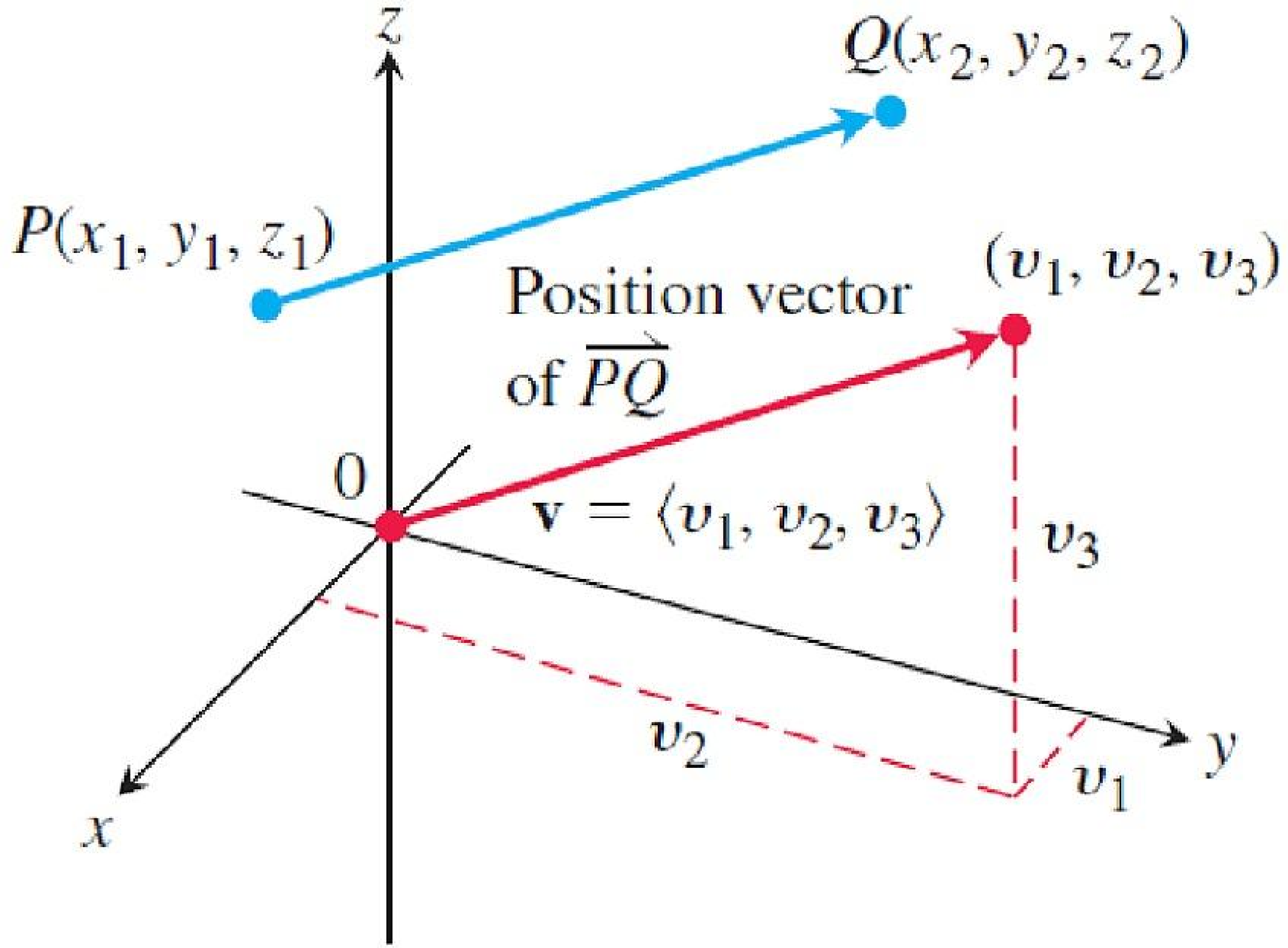



**FIGURE 11.10** A vector $\overrightarrow{PQ}$ in standard position has its initial point at the origin. The directed line segments $\overrightarrow{PQ}$ and $\mathbf{v}$ are parallel and have the same length.

## DEFINITION

If $\mathbf{v}$ is a **two-dimensional** vector in the plane equal to the vector with initial point at the origin and terminal point $(v_1, v_2)$, then the **component form** of $\mathbf{v}$ is

$$\mathbf{v} = \langle v_1, v_2 \rangle .$$

If $\mathbf{v}$ is a **three-dimensional** vector equal to the vector with initial point at the origin and terminal point $(v_1, v_2, v_3)$, then the **component form** of $\mathbf{v}$ is

$$\mathbf{v} = \langle v_1, v_2, v_3 \rangle .$$

The **magnitude** or **length** of the vector $\mathbf{v} = \overrightarrow{PQ}$ is the nonnegative number

$$|\mathbf{v}| = \sqrt{v_1^2 + v_2^2 + v_3^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

(See Figure 12.10.)

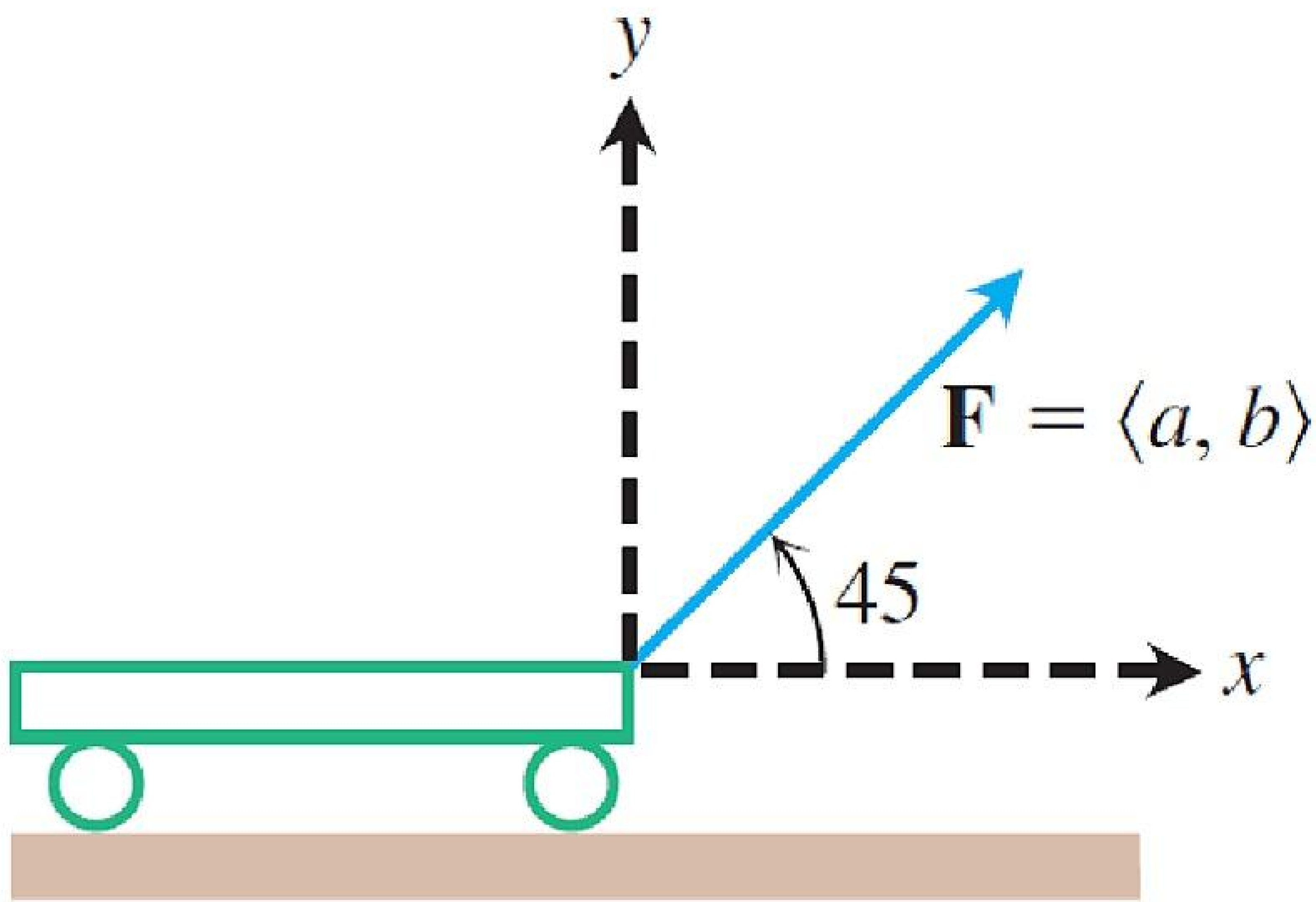



**FIGURE 11.11** The force pulling the cart forward is represented by the vector **F** whose horizontal component is the effective force (Example 2).

**DEFINITIONS** Let $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ be vectors with $k$ a scalar.

**Addition:** $\mathbf{u} + \mathbf{v} = \langle u_1 + v_1, u_2 + v_2, u_3 + v_3 \rangle$

**Scalar multiplication:** $k\mathbf{u} = \langle ku_1, ku_2, ku_3 \rangle$

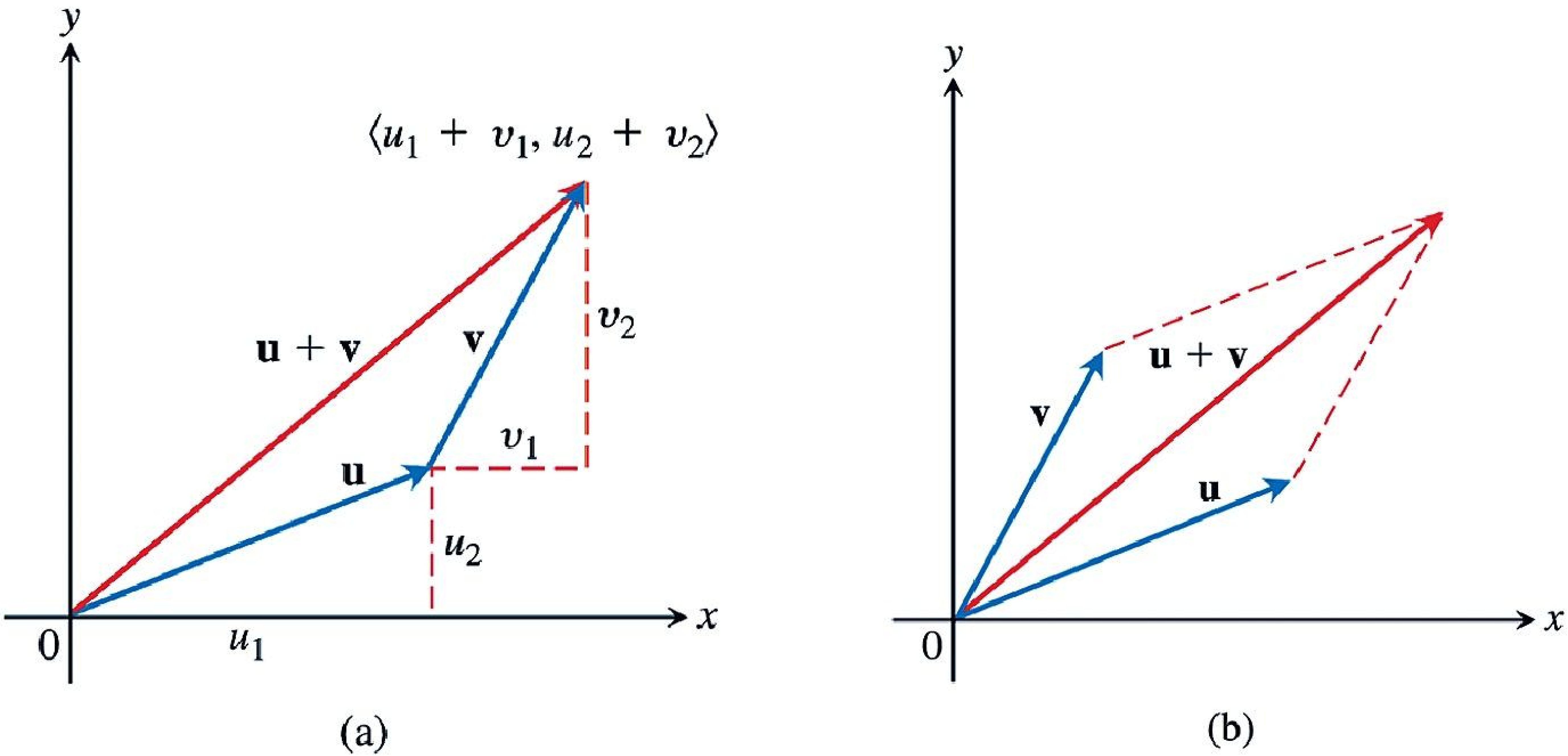



**FIGURE 11.12** (a) Geometric interpretation of the vector sum. (b) The parallelogram law of vector addition in which both vectors are in standard position.

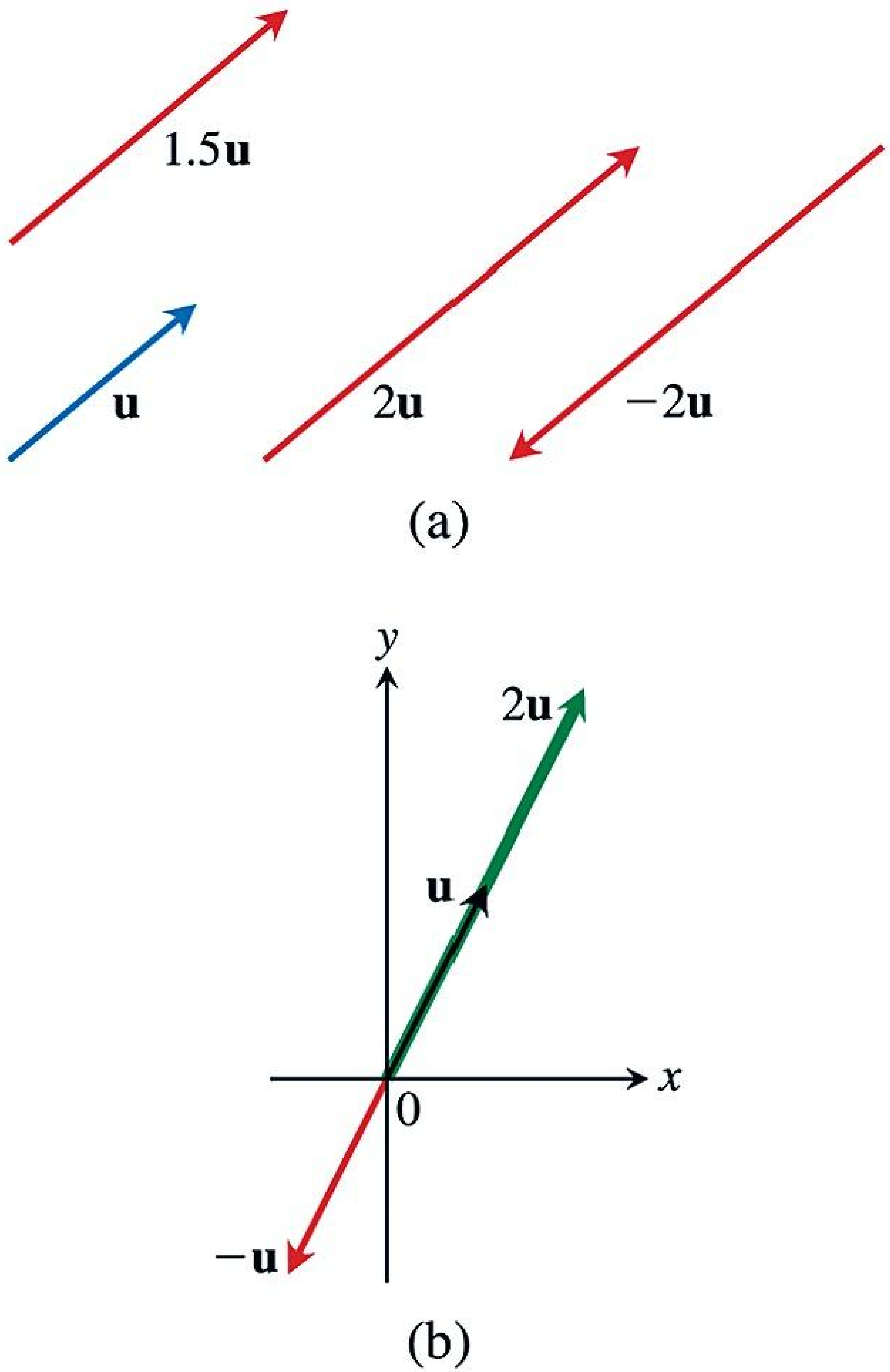



**FIGURE 11.13** (a) Scalar multiples of **u**. (b) Scalar multiples of a vector **u** in standard position.

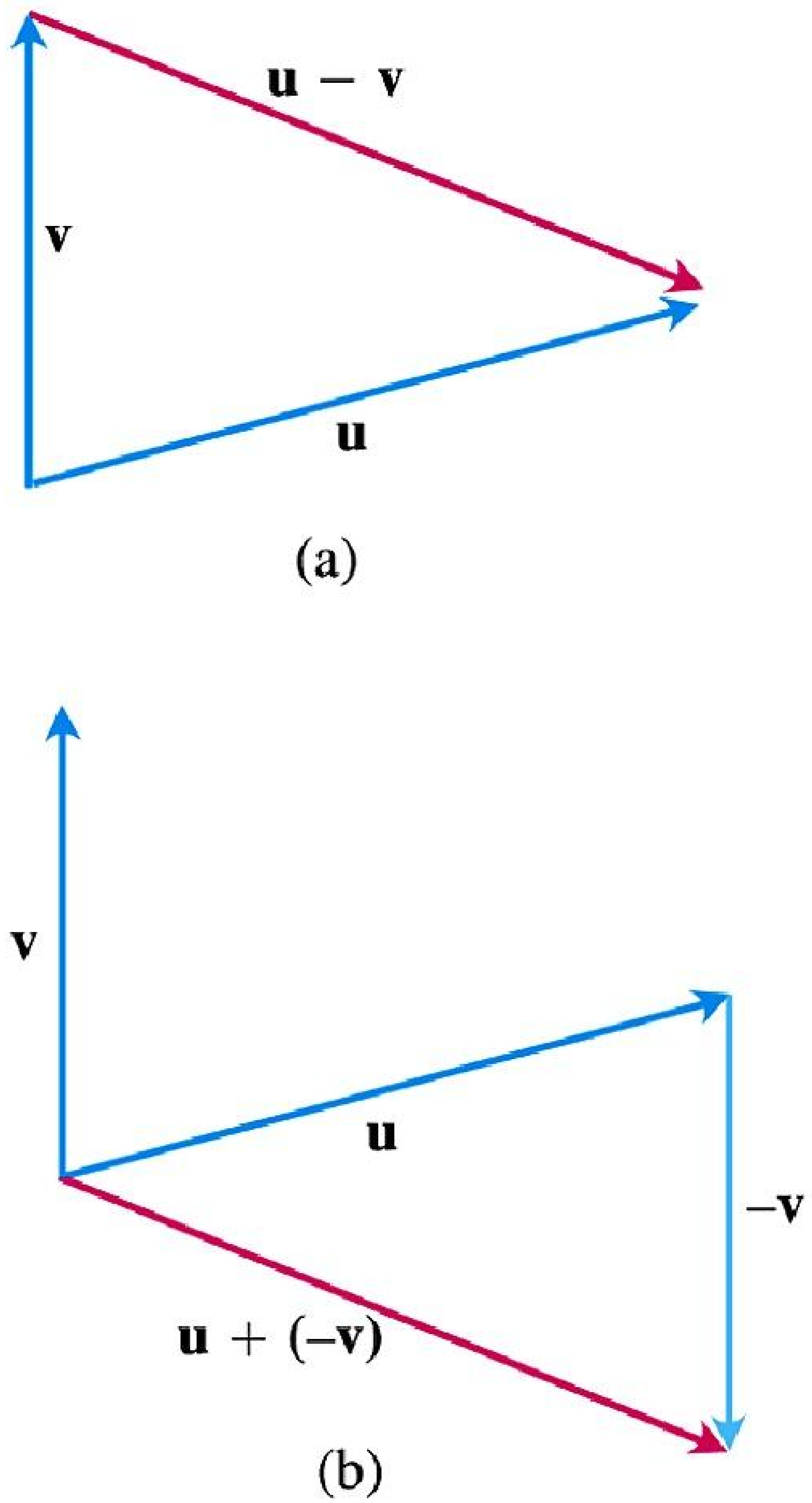



**FIGURE 11.14** (a) The vector $\mathbf{u} - \mathbf{v}$, when added to $\mathbf{v}$, gives $\mathbf{u}$. (b) $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$.

**EXAMPLE 3** Let $\mathbf{u} = \langle -1, 3, 1 \rangle$ and $\mathbf{v} = \langle 4, 7, 0 \rangle$. Find the components of

**(a)** $2\mathbf{u} + 3\mathbf{v}$ **(b)** $\mathbf{u} - \mathbf{v}$ **(c)** $\left| \frac{1}{2}\mathbf{u} \right|$.

**Solution**

**(a)** $2\mathbf{u} + 3\mathbf{v} = 2\langle -1, 3, 1 \rangle + 3\langle 4, 7, 0 \rangle = \langle -2, 6, 2 \rangle + \langle 12, 21, 0 \rangle = \langle 10, 27, 2 \rangle$

**(b)** $\mathbf{u} - \mathbf{v} = \langle -1, 3, 1 \rangle - \langle 4, 7, 0 \rangle = \langle -1 - 4, 3 - 7, 1 - 0 \rangle = \langle -5, -4, 1 \rangle$

**(c)** $\left| \frac{1}{2}\mathbf{u} \right| = \left| \left\langle -\frac{1}{2}, \frac{3}{2}, \frac{1}{2} \right\rangle \right| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{3}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{2}\sqrt{11}.$


Pearson

## Properties of Vector Operations

Let $\mathbf{u}$, $\mathbf{v}$, $\mathbf{w}$ be vectors and $a$, $b$ be scalars.

1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
2. $(\mathbf{u} + \mathbf{v}) + \mathbf{w} = \mathbf{u} + (\mathbf{v} + \mathbf{w})$
3. $\mathbf{u} + \mathbf{0} = \mathbf{u}$
4. $\mathbf{u} + (-\mathbf{u}) = \mathbf{0}$
5. $0\mathbf{u} = \mathbf{0}$
6. $1\mathbf{u} = \mathbf{u}$
7. $a(b\mathbf{u}) = (ab)\mathbf{u}$
8. $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
9. $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$

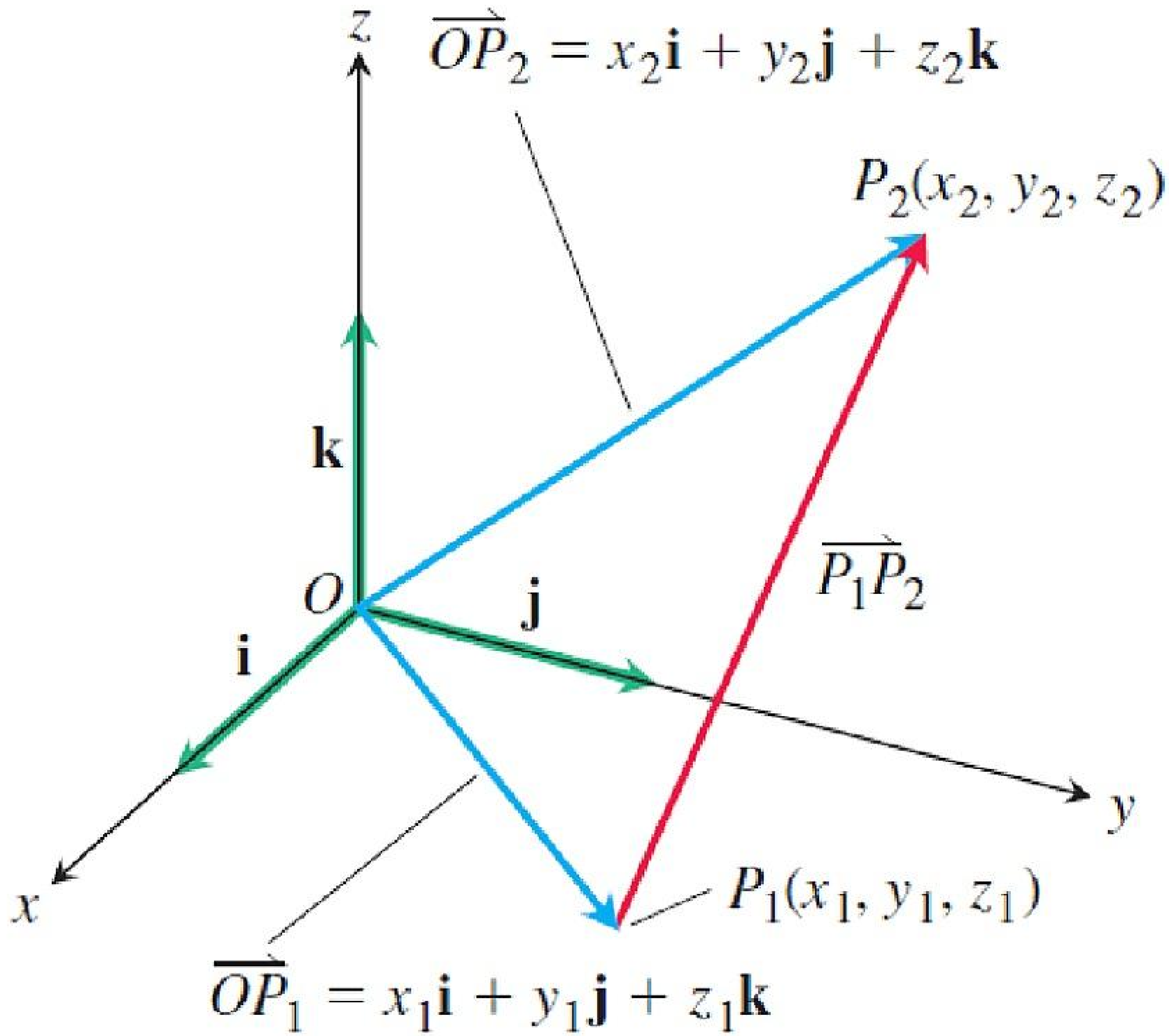



**FIGURE 11.15** The vector from $P_1$ to $P_2$ is $\overrightarrow{P_1P_2} = (x_2 - x_1)\mathbf{i} + (y_2 - y_1)\mathbf{j} + (z_2 - z_1)\mathbf{k}$.

**EXAMPLE 5** If $\mathbf{v} = 3\mathbf{i} - 4\mathbf{j}$ is a velocity vector, express $\mathbf{v}$ as a product of its speed times its direction of motion.

**Solution** Speed is the magnitude (length) of $\mathbf{v}$:

$$|\mathbf{v}| = \sqrt{(3)^2 + (-4)^2} = \sqrt{9 + 16} = 5.$$

The unit vector $\mathbf{v}/|\mathbf{v}|$ is the direction of $\mathbf{v}$:

$$\frac{\mathbf{v}}{|\mathbf{v}|} = \frac{3\mathbf{i} - 4\mathbf{j}}{5} = \frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}.$$

So

$$\mathbf{v} = 3\mathbf{i} - 4\mathbf{j} = \underbrace{5}_{\substack{\text{Length} \\ \text{(speed)}}}\underbrace{\left(\frac{3}{5}\mathbf{i} - \frac{4}{5}\mathbf{j}\right)}_{\text{Direction of motion}}.$$

If $\mathbf{v} \neq \mathbf{0}$, then

**1.** $\dfrac{\mathbf{v}}{|\mathbf{v}|}$ is a unit vector in the direction of $\mathbf{v}$;

**2.** the equation $\mathbf{v} = |\mathbf{v}|\dfrac{\mathbf{v}}{|\mathbf{v}|}$ expresses $\mathbf{v}$ as its length times its direction.

The **midpoint** $M$ of the line segment joining points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is the point

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right).$$

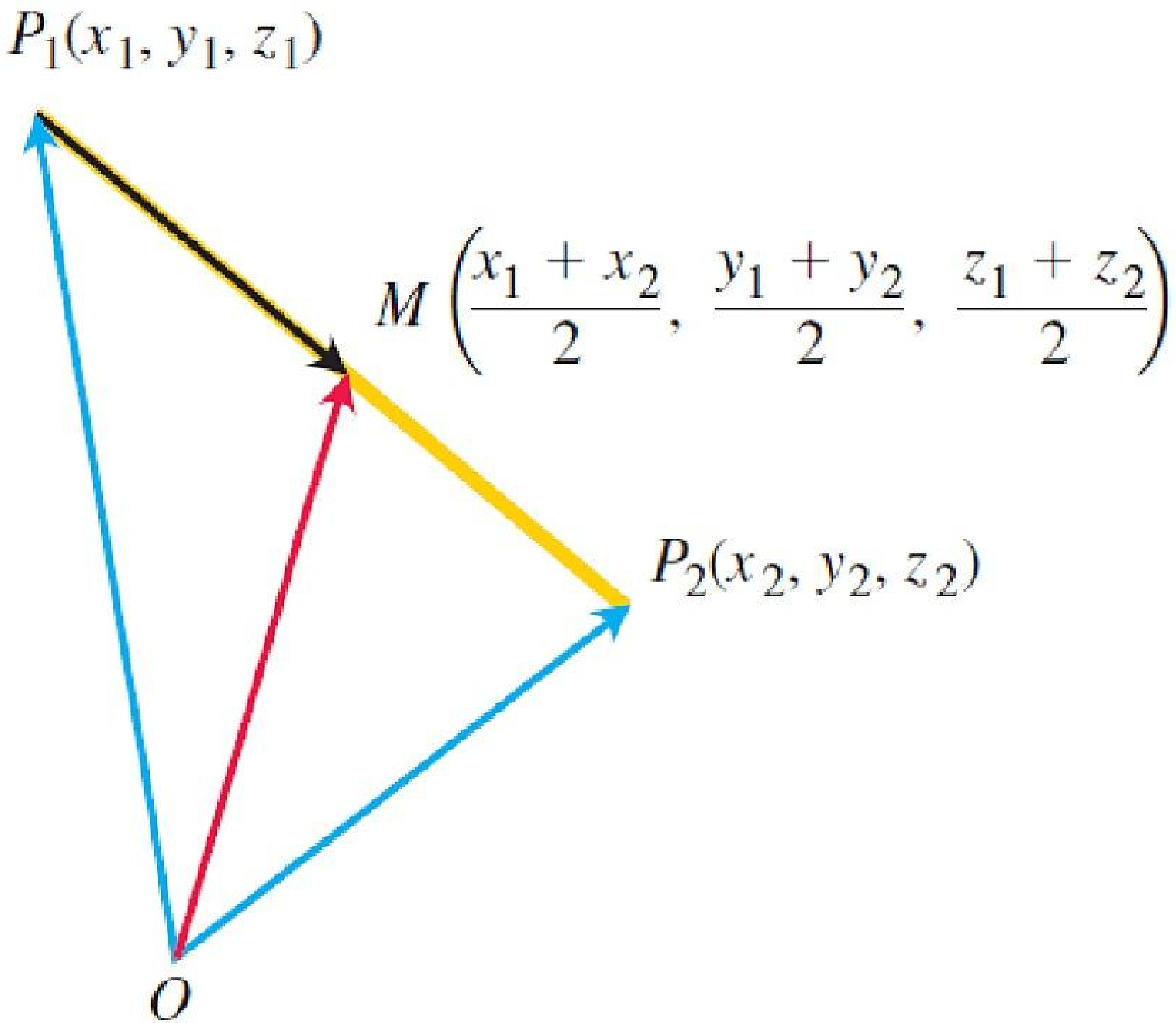



**FIGURE 11.16** The coordinates of the midpoint are the averages of the coordinates of $P_1$ and $P_2$.

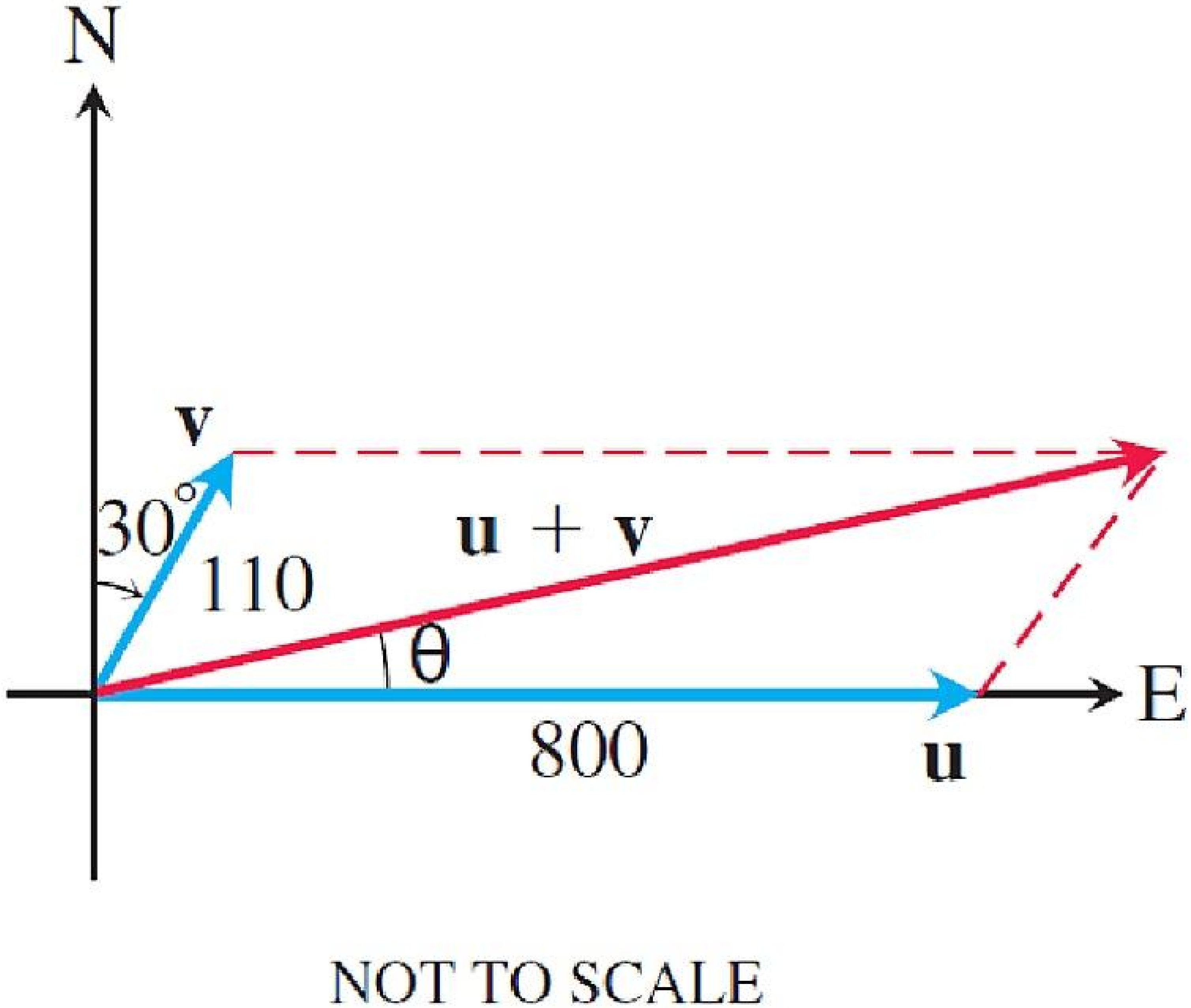



**FIGURE 11.17** Vectors representing the velocities of the airplane **u** and tailwind **v** in Example 8.

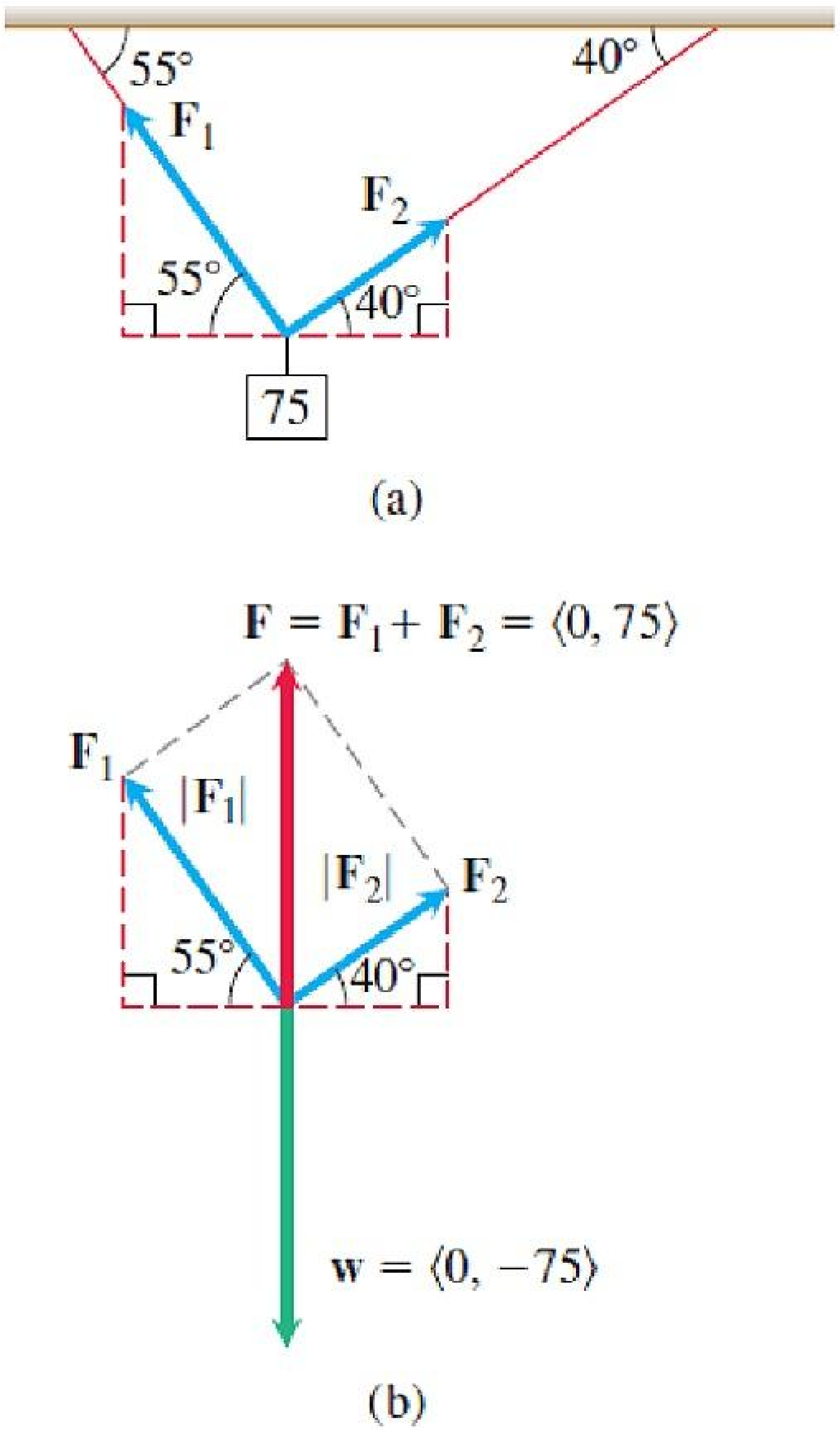



**FIGURE 11.18** The suspended weight in Example 9.

# Section 11.3

## The Dot Product

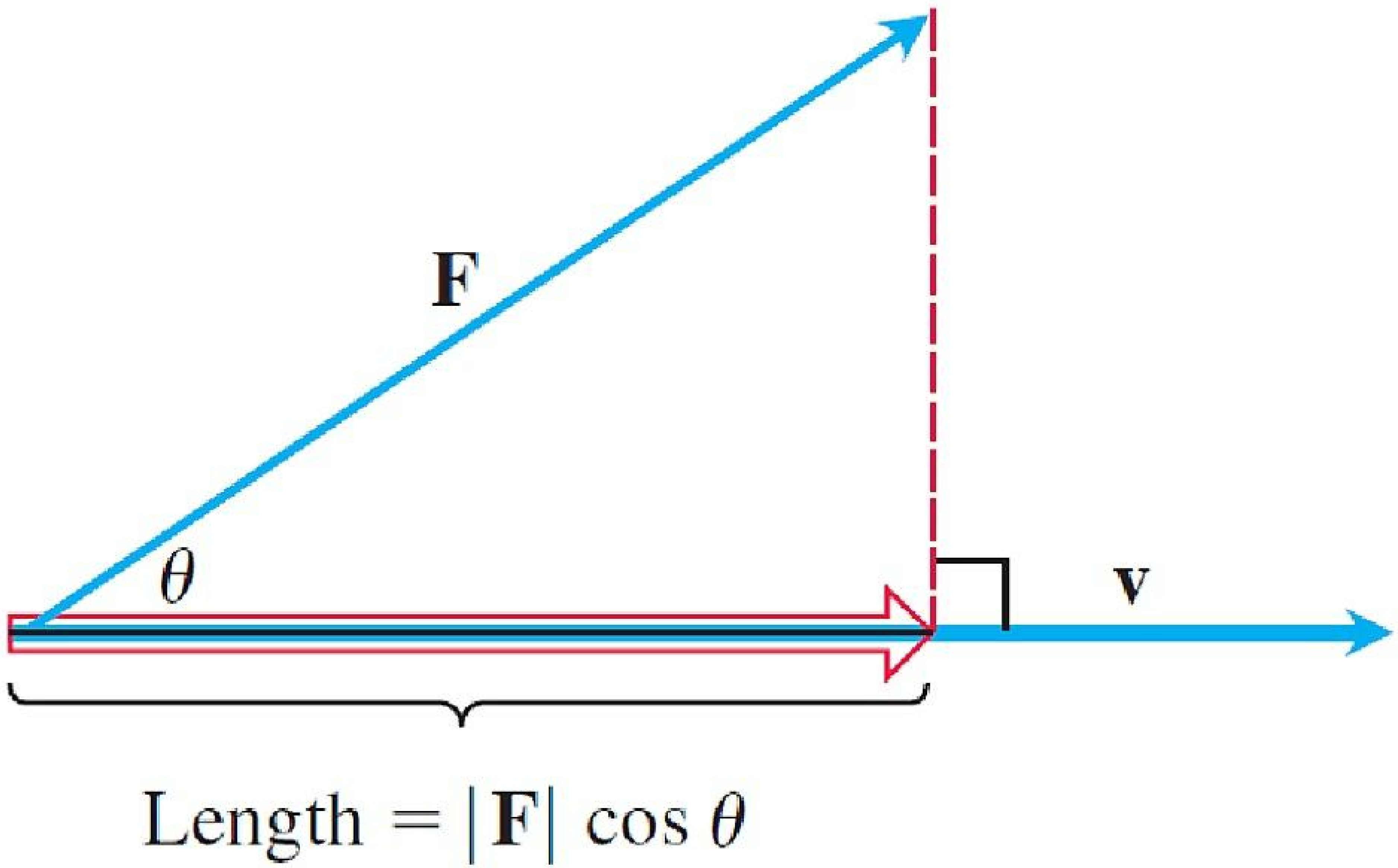



**FIGURE 11.19** The magnitude of the force $\mathbf{F}$ in the direction of vector $\mathbf{v}$ is the length $|\mathbf{F}| \cos \theta$ of the projection of $\mathbf{F}$ onto $\mathbf{v}$.

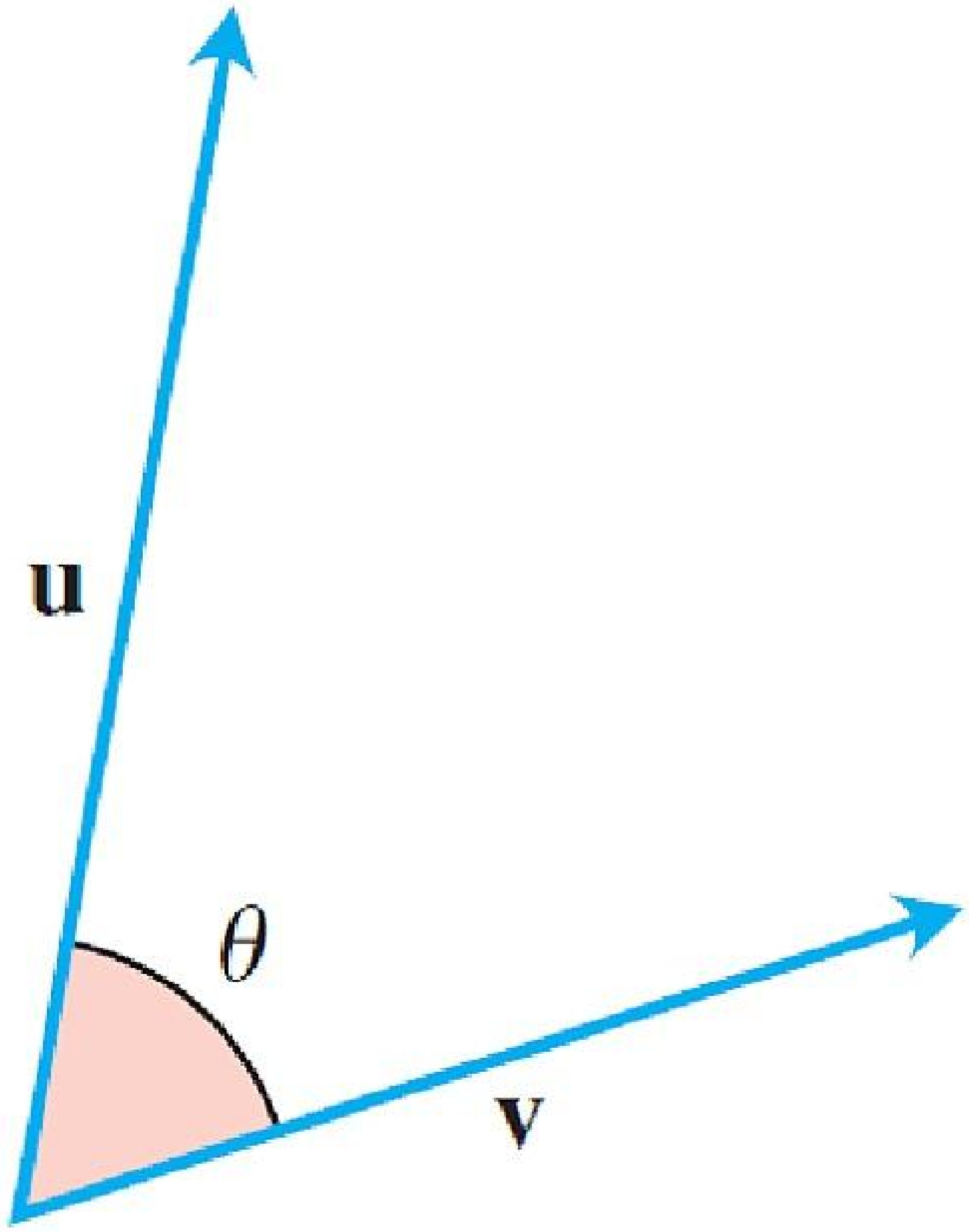



**FIGURE 11.20** The angle between **u** and **v** given by Theorem 1 lies in the interval $[0, \pi]$.

**THEOREM 1—Angle Between Two Vectors** The angle $\theta$ between two nonzero vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is given by

$$\theta = \cos^{-1}\left(\frac{u_1 v_1 + u_2 v_2 + u_3 v_3}{|\mathbf{u}|\,|\mathbf{v}|}\right).$$

**DEFINITION** The **dot product** $\mathbf{u} \cdot \mathbf{v}$ (**"u dot v"**) of vectors $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ and $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$ is the scalar

$$\mathbf{u} \cdot \mathbf{v} = u_1 v_1 + u_2 v_2 + u_3 v_3 .$$

## EXAMPLE 1

**(a)** $$\langle 1, -2, -1\rangle \cdot \langle -6, 2, -3\rangle = (1)(-6) + (-2)(2) + (-1)(-3)$$
$$= -6 - 4 + 3 = -7$$

**(b)** $$\left(\frac{1}{2}\mathbf{i} + 3\mathbf{j} + \mathbf{k}\right) \cdot (4\mathbf{i} - \mathbf{j} + 2\mathbf{k}) = \left(\frac{1}{2}\right)(4) + (3)(-1) + (1)(2) = 1$$

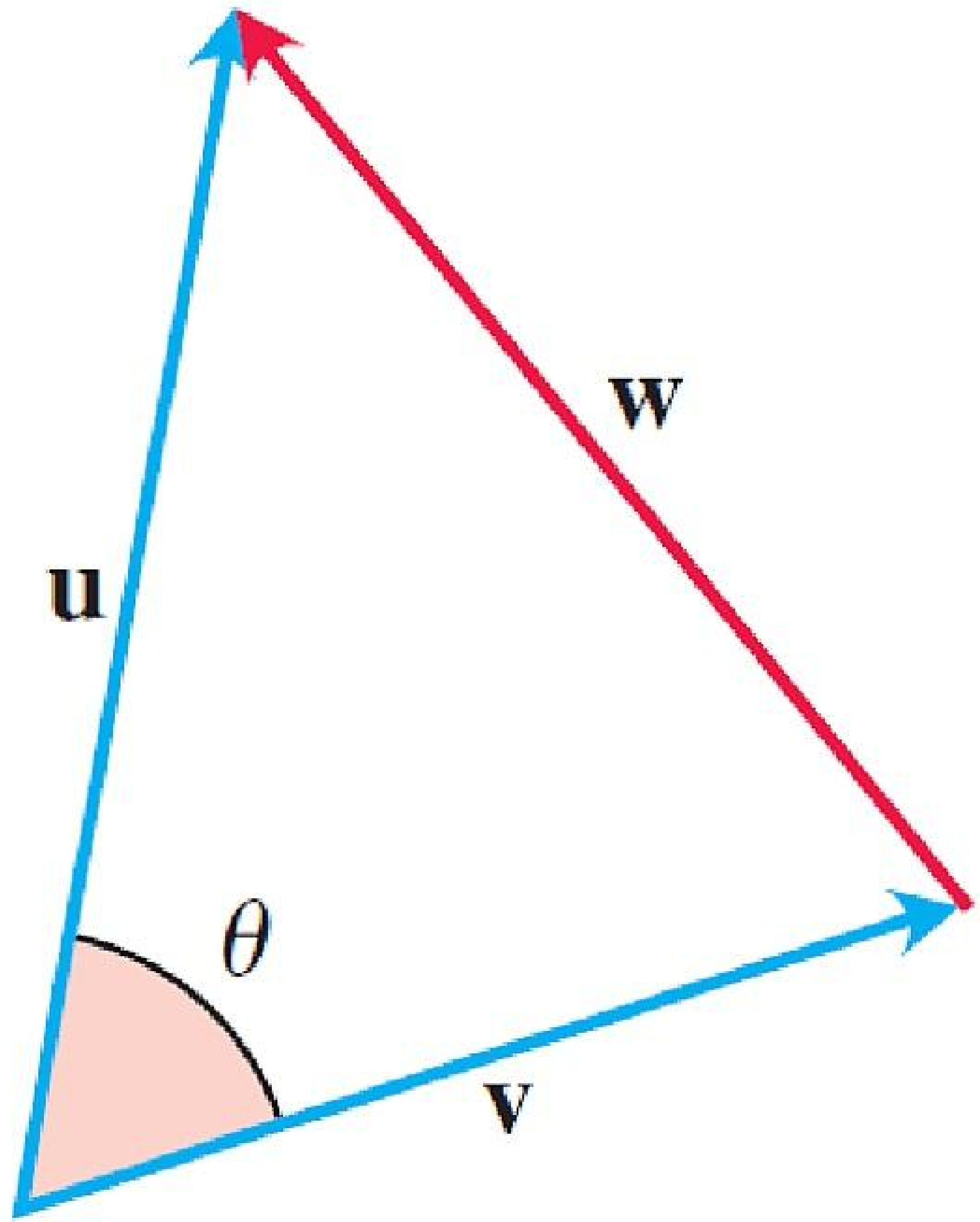



**FIGURE 11.21** The parallelogram law of addition of vectors gives $\mathbf{w} = \mathbf{u} - \mathbf{v}$.

## Dot Product and Angles

The angle between two nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ is $\theta = \cos^{-1}\left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{u}||\mathbf{v}|}\right)$.

The dot product of two vectors $\mathbf{u}$ and $\mathbf{v}$ is given by $\mathbf{u} \cdot \mathbf{v} = |\mathbf{u}||\mathbf{v}|\cos\theta$.

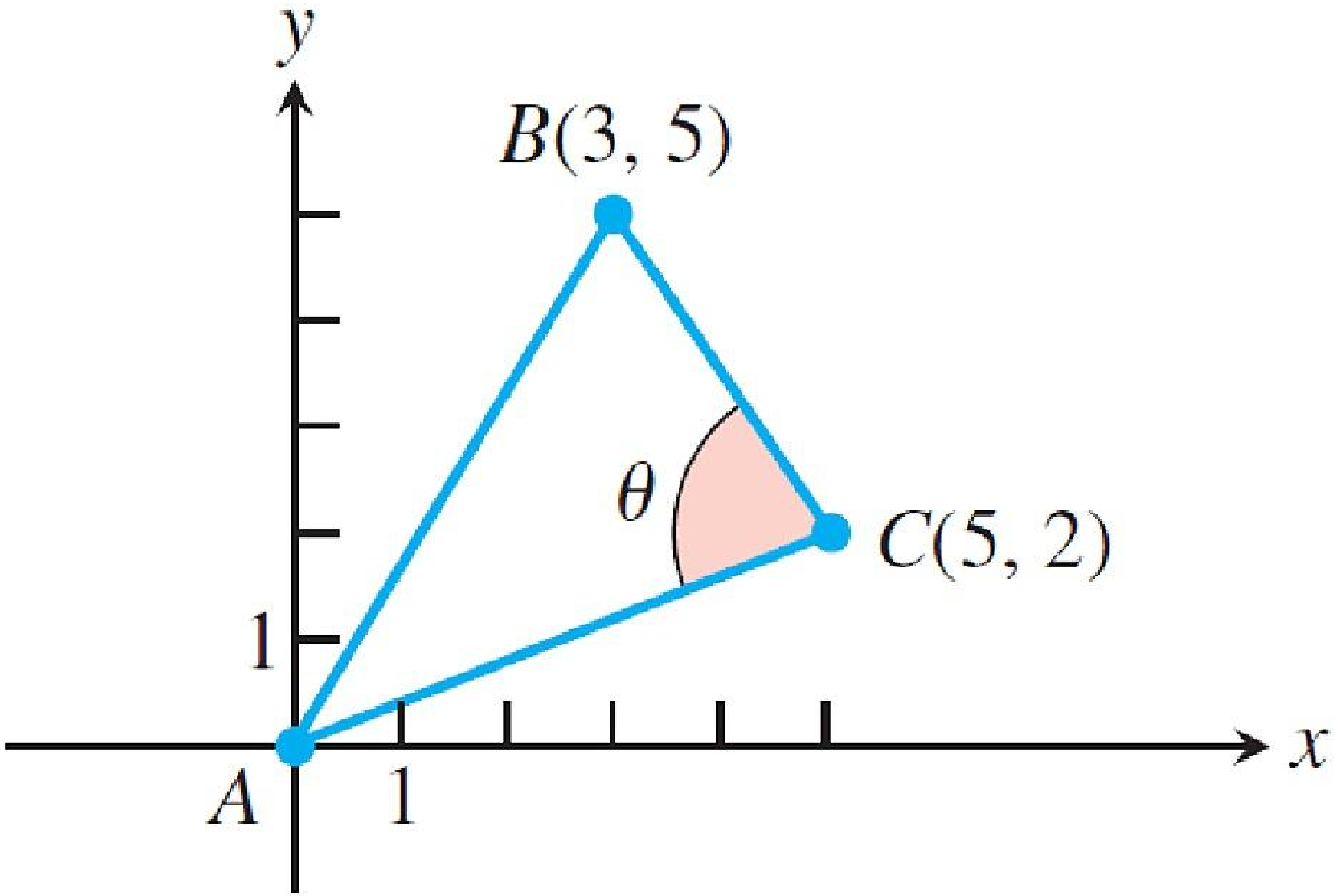



**FIGURE 11.22** The triangle in Example 3.

**DEFINITION** Vectors $\mathbf{u}$ and $\mathbf{v}$ are orthogonal if $\mathbf{u} \cdot \mathbf{v} = 0$.

**Properties of the Dot Product**

If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are any vectors and $c$ is a scalar, then

1. $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$
2. $(c\mathbf{u}) \cdot \mathbf{v} = \mathbf{u} \cdot (c\mathbf{v}) = c(\mathbf{u} \cdot \mathbf{v})$
3. $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$
4. $\mathbf{u} \cdot \mathbf{u} = |\mathbf{u}|^2$
5. $\mathbf{0} \cdot \mathbf{u} = 0$.

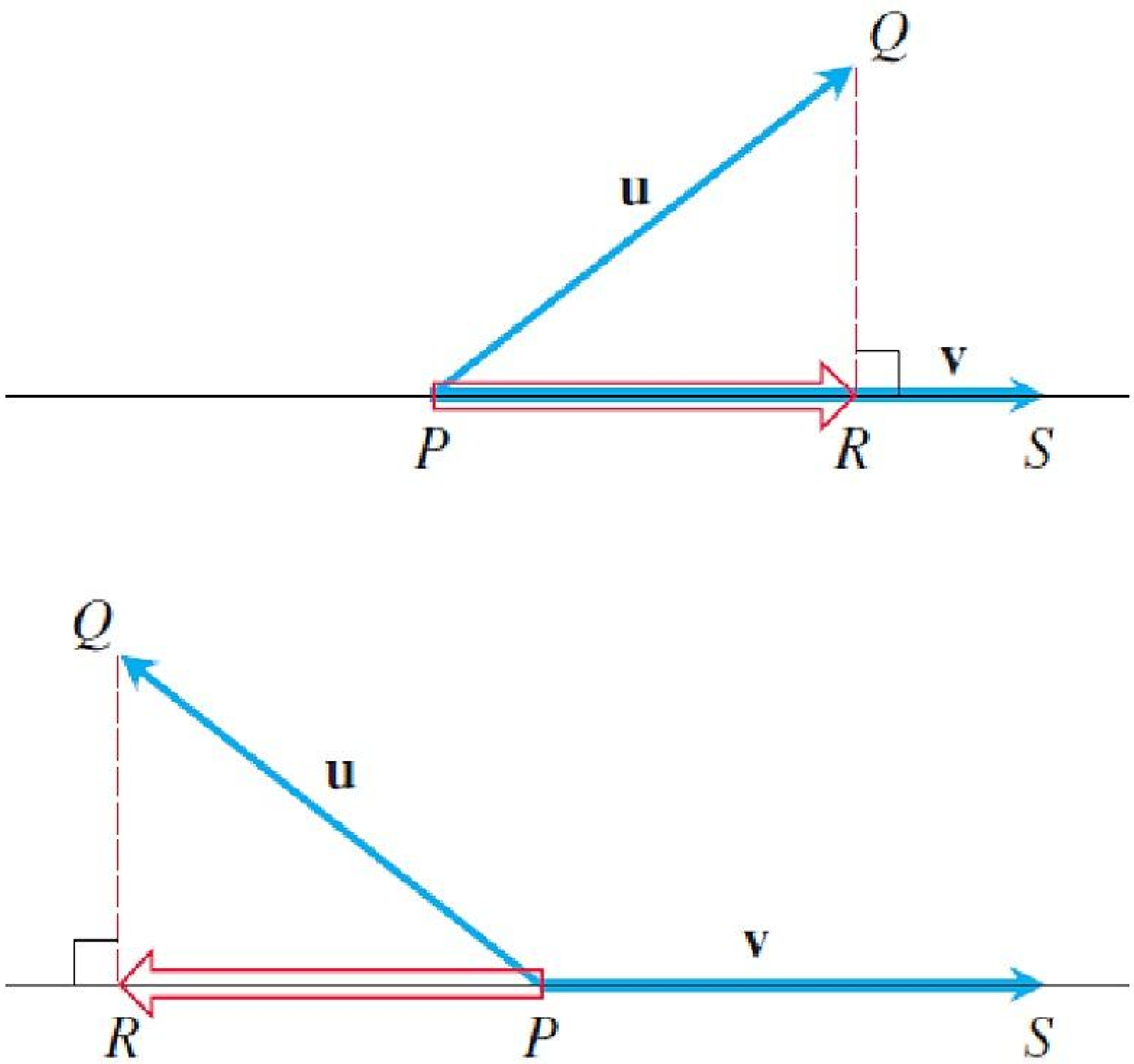



**FIGURE 11.23** The vector projection of **u** onto **v**.

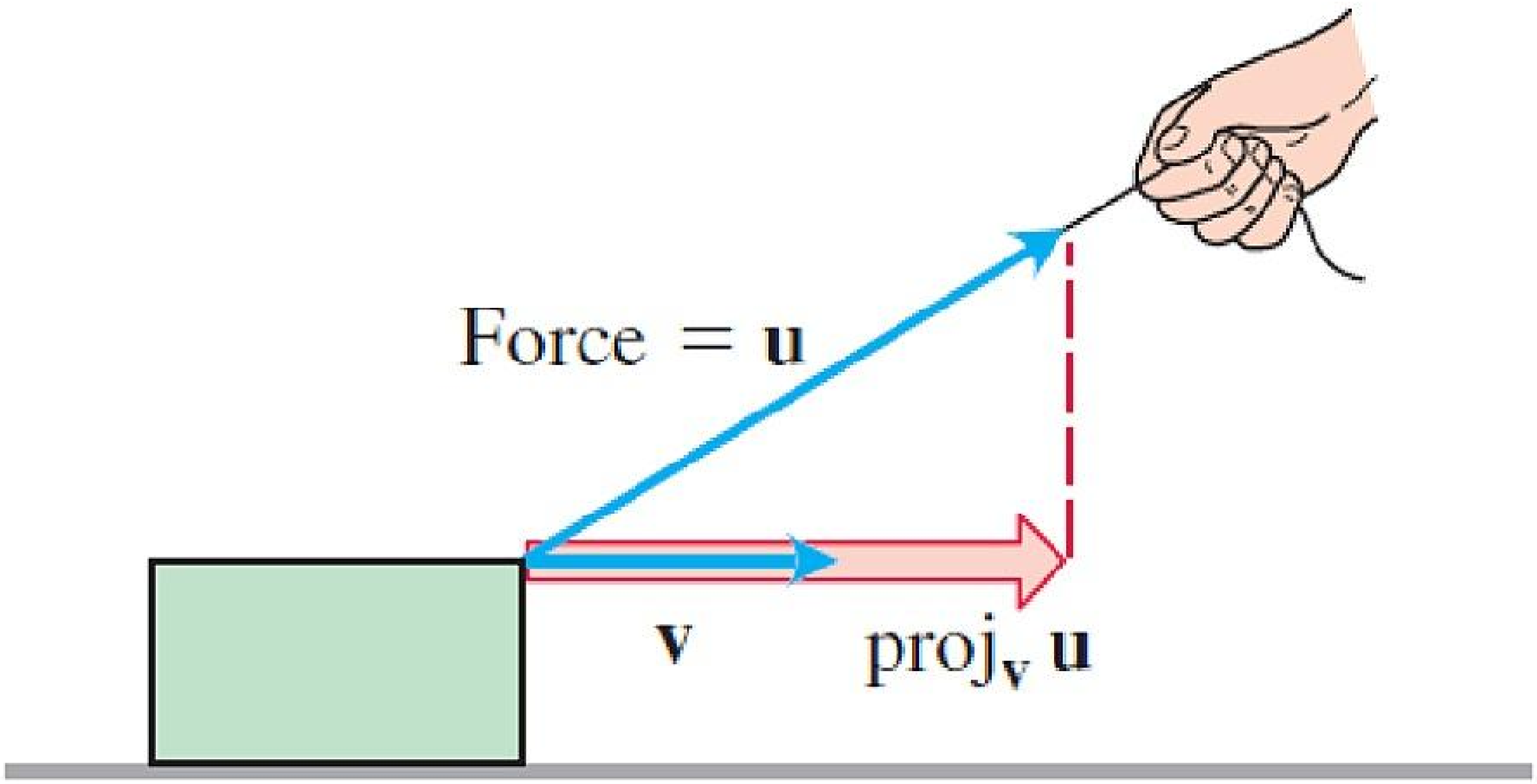



**FIGURE 11.24** If we pull on the box with force $\mathbf{u}$, the effective force moving the box forward in the direction $\mathbf{v}$ is the projection of $\mathbf{u}$ onto $\mathbf{v}$.

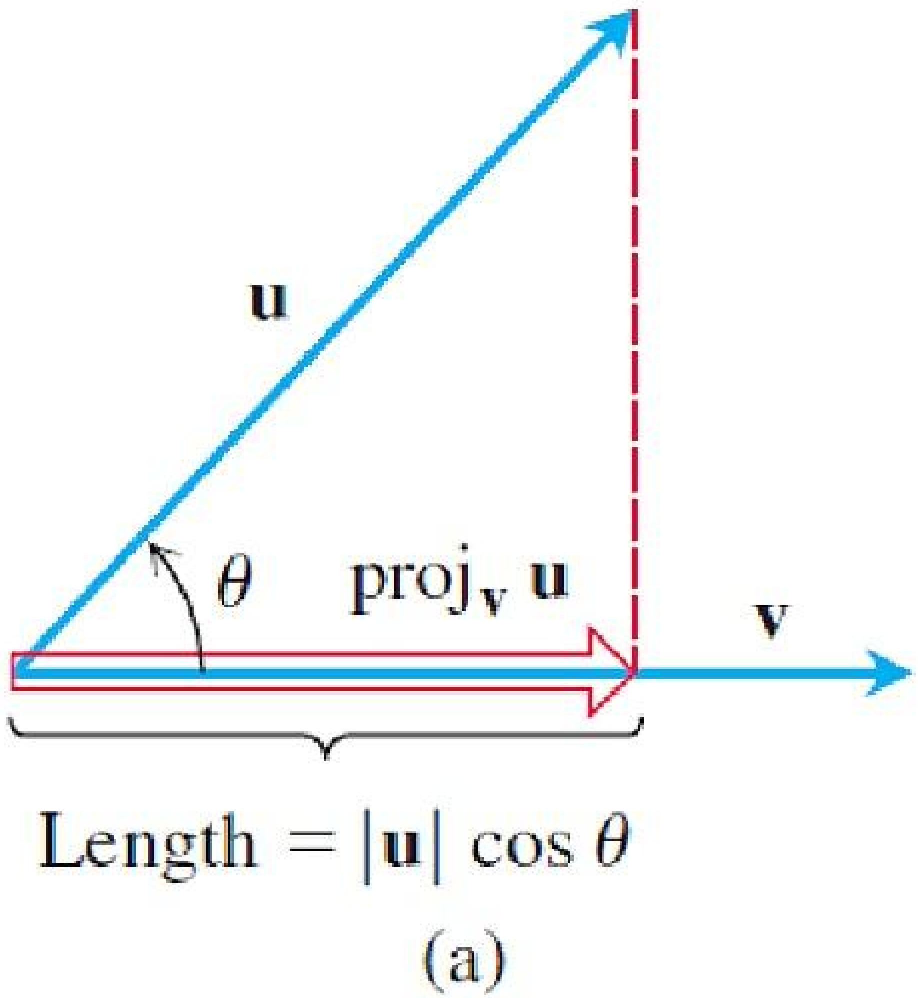



**FIGURE 11.25** The length of $\text{proj}_{\mathbf{v}}\,\mathbf{u}$ is (a) $|\mathbf{u}|\cos\theta$ if $\cos\theta \geq 0$ and (b) $-|\mathbf{u}|\cos\theta$ if $\cos\theta < 0$.

The vector projection of **u** onto **v** is the vector

$$\operatorname{proj}_{\mathbf{v}} \mathbf{u} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|^2}\right)\mathbf{v} = \left(\frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|}\right)\frac{\mathbf{v}}{|\mathbf{v}|}. \tag{1}$$

The scalar component of **u** in the direction of **v** is the scalar

$$|\mathbf{u}| \cos\theta = \frac{\mathbf{u} \cdot \mathbf{v}}{|\mathbf{v}|} = \mathbf{u} \cdot \frac{\mathbf{v}}{|\mathbf{v}|}. \tag{2}$$

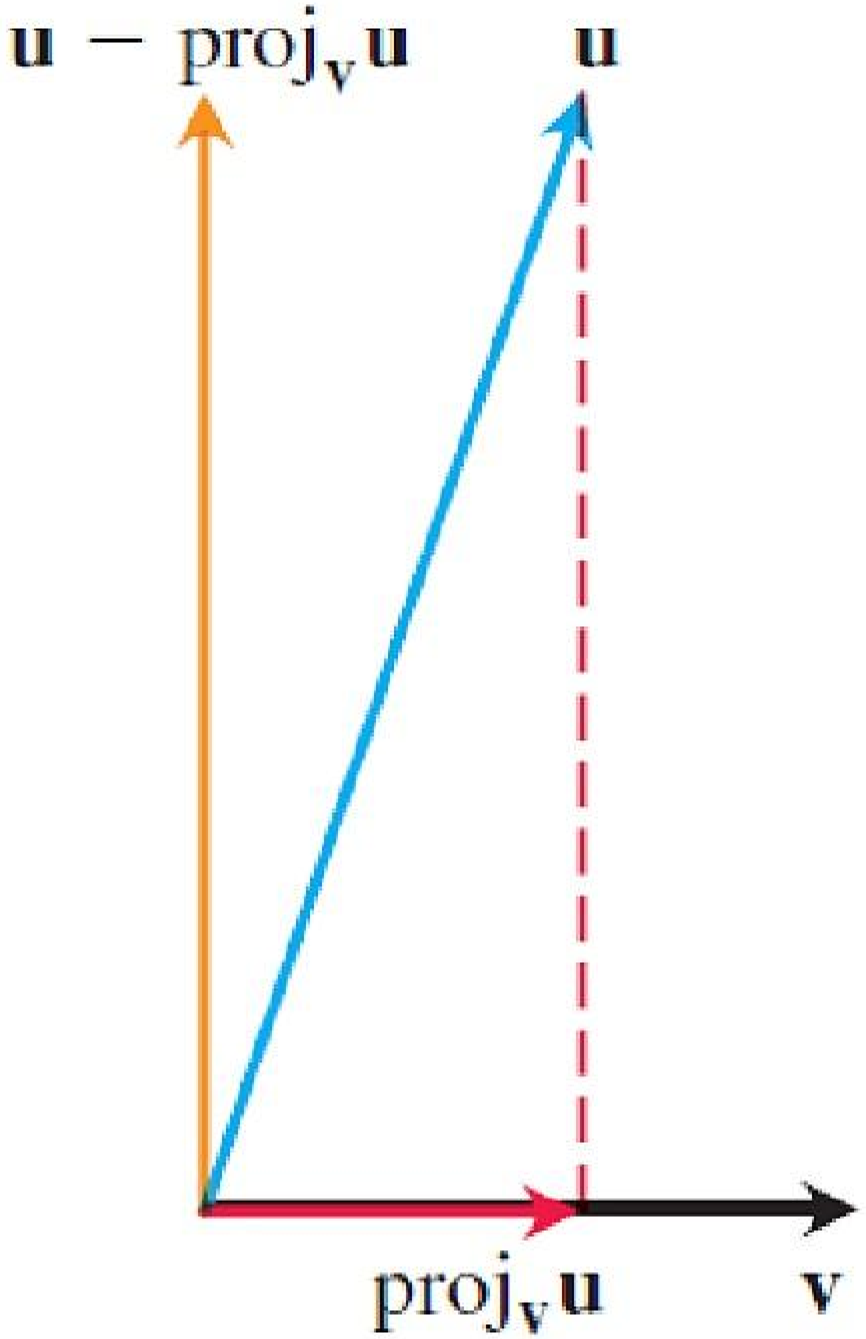



**FIGURE 11.26** The vector $\mathbf{u}$ is the sum of two perpendicular vectors: a vector $\text{proj}_{\mathbf{v}}\,\mathbf{u}$, parallel to $\mathbf{v}$, and a vector $\mathbf{u} - \text{proj}_{\mathbf{v}}\,\mathbf{u}$, perpendicular to $\mathbf{v}$.

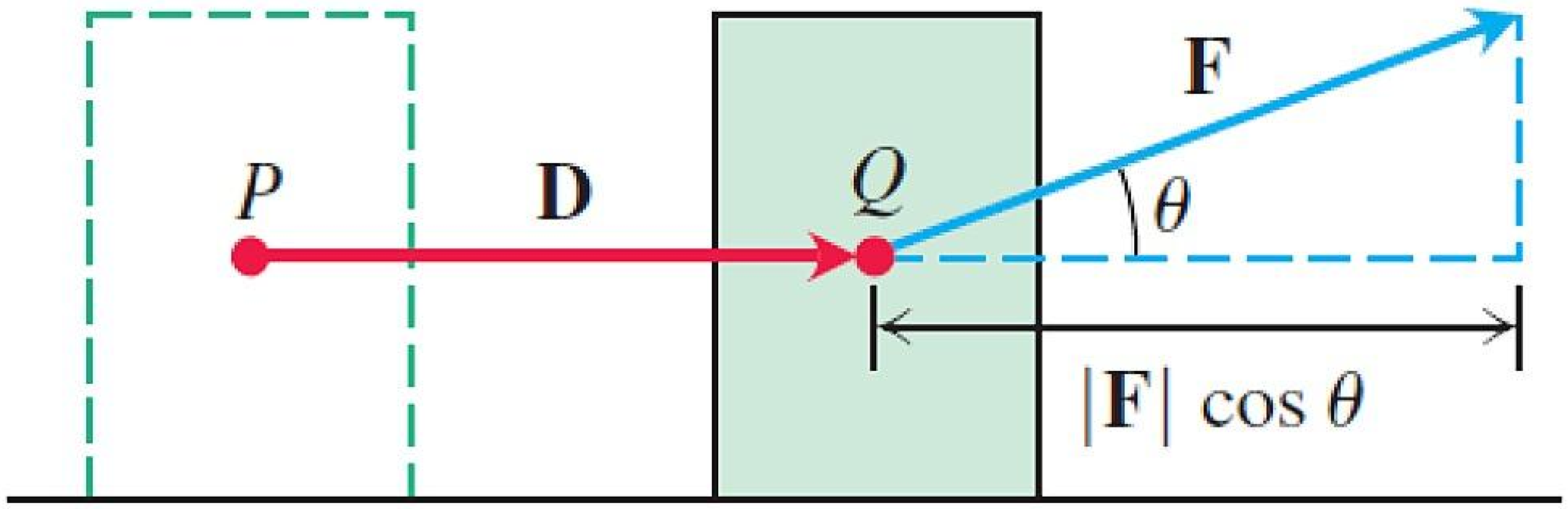



**FIGURE 11.27** The work done by a constant force $\mathbf{F}$ during a displacement $\mathbf{D}$ is $(|\mathbf{F}| \cos \theta)|\mathbf{D}|$, which is the dot product $\mathbf{F} \cdot \mathbf{D}$.

**DEFINITION** The **work** done by a constant force $\mathbf{F}$ acting through a displacement $\mathbf{D} = \overrightarrow{PQ}$ is

$$W = \mathbf{F} \cdot \mathbf{D}.$$

# Section 11.4

## The Cross Product

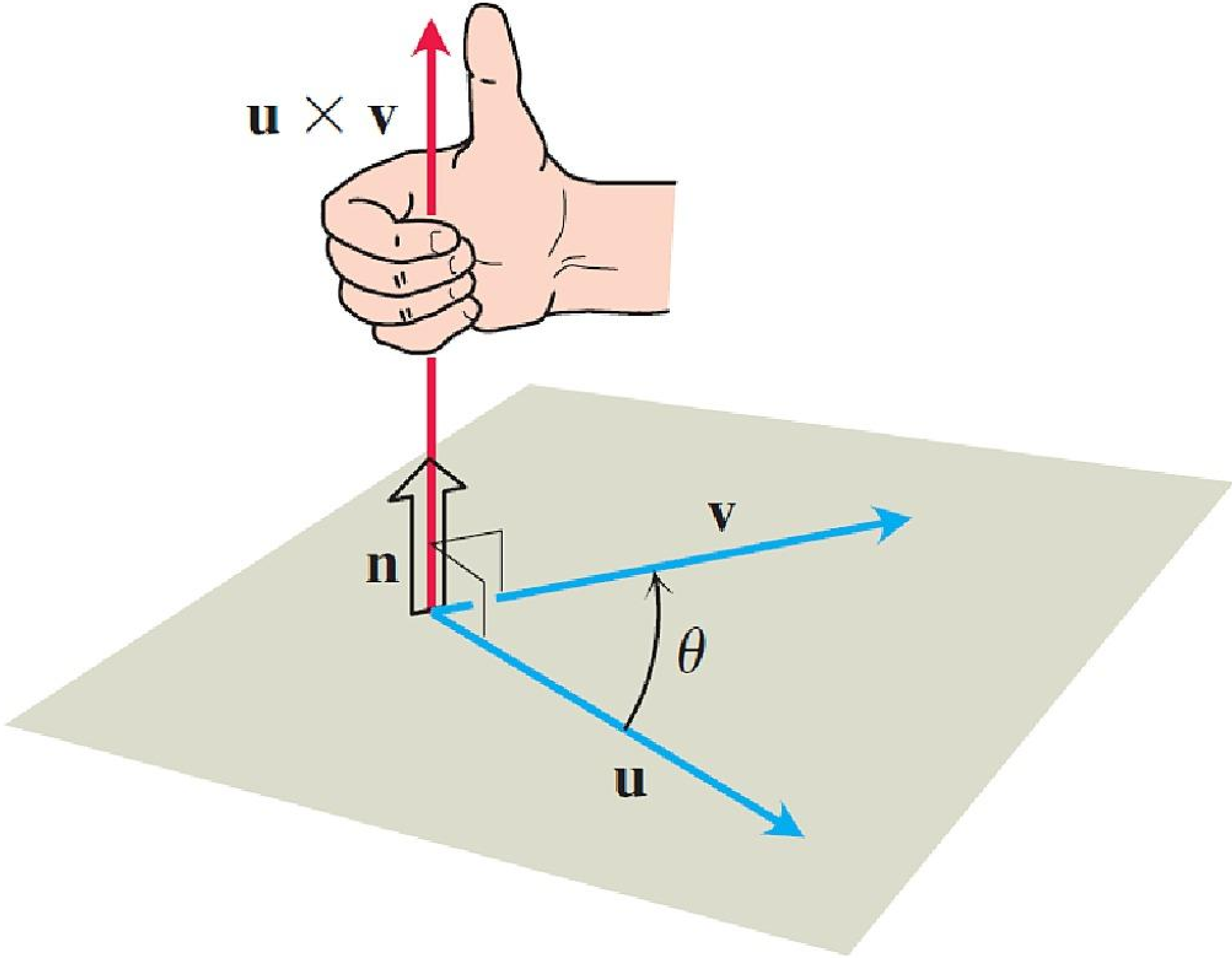



**FIGURE 11.28** The construction of $\mathbf{u} \times \mathbf{v}$.

**DEFINITION** The **cross product u $\times$ v** (**“u cross v”**) is the vector

$$\mathbf{u} \times \mathbf{v} = (|\mathbf{u}||\mathbf{v}| \sin \theta)\, \mathbf{n}.$$

### Parallel Vectors

Nonzero vectors $\mathbf{u}$ and $\mathbf{v}$ are parallel if and only if $\mathbf{u} \times \mathbf{v} = \mathbf{0}$.

### Properties of the Cross Product

If $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are any vectors and $r$, $s$ are scalars, then

1. $(r\mathbf{u}) \times (s\mathbf{v}) = (rs)(\mathbf{u} \times \mathbf{v})$
2. $\mathbf{u} \times (\mathbf{v} + \mathbf{w}) = \mathbf{u} \times \mathbf{v} + \mathbf{u} \times \mathbf{w}$
3. $\mathbf{v} \times \mathbf{u} = -(\mathbf{u} \times \mathbf{v})$
4. $(\mathbf{v} + \mathbf{w}) \times \mathbf{u} = \mathbf{v} \times \mathbf{u} + \mathbf{w} \times \mathbf{u}$
5. $\mathbf{0} \times \mathbf{u} = \mathbf{0}$
6. $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$

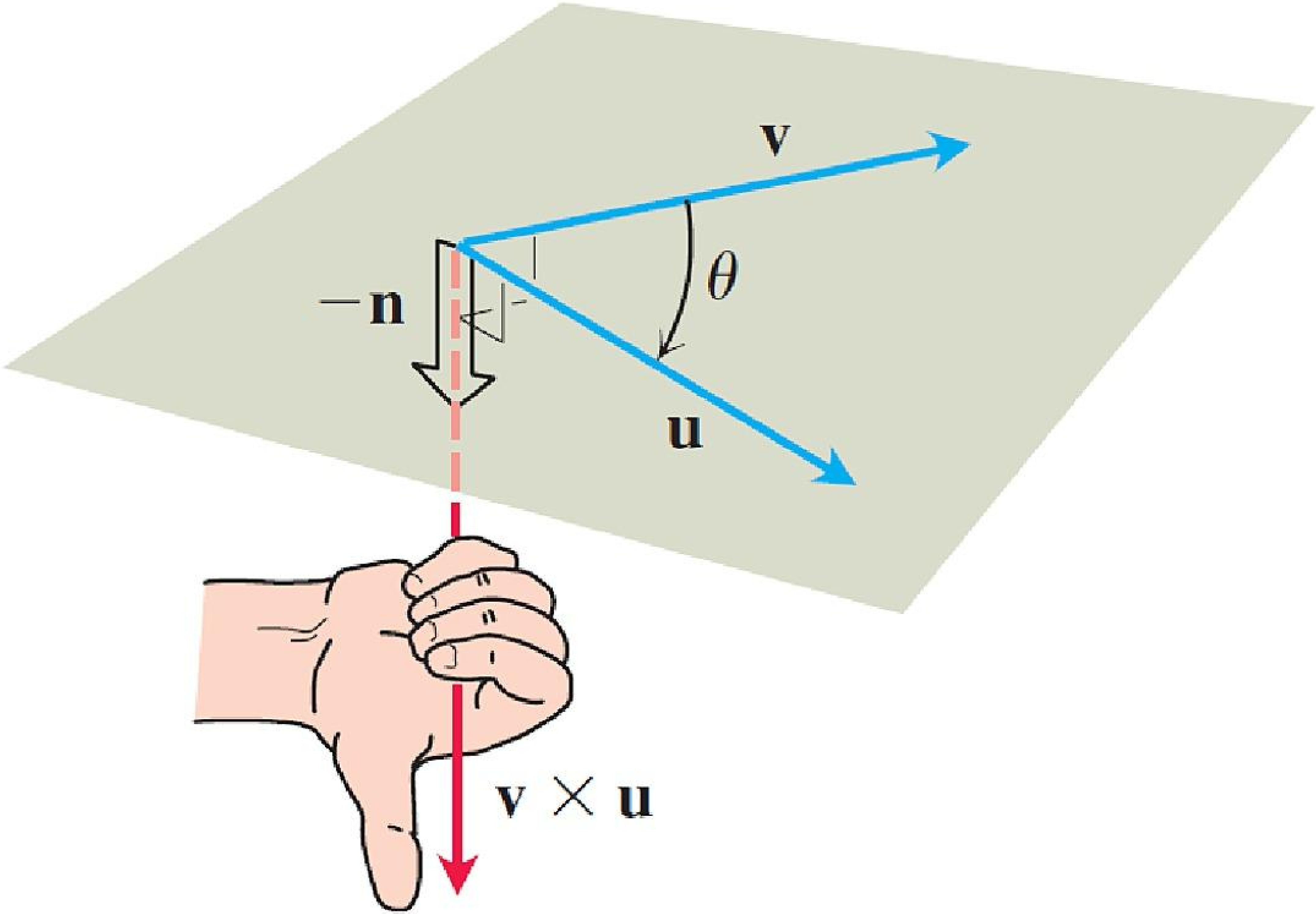



**FIGURE 11.29** The construction of $\mathbf{v} \times \mathbf{u}$.

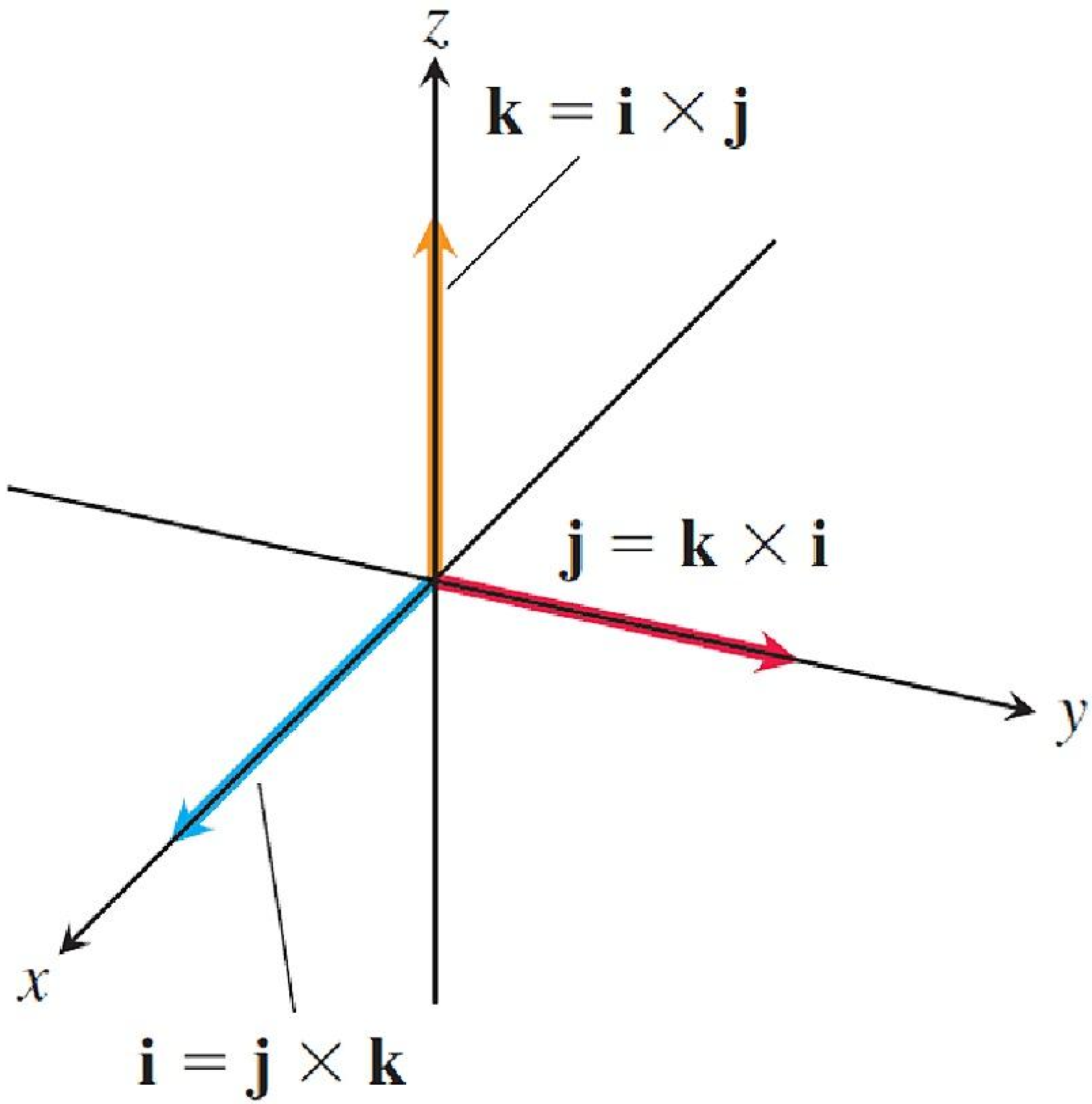



**FIGURE 11.30** The pairwise cross products of $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$.

$$
|\mathbf{u} \times \mathbf{v}| = |\mathbf{u}||\mathbf{v}|\,|\sin\theta|\,|\mathbf{n}| = |\mathbf{u}||\mathbf{v}|\sin\theta .
$$

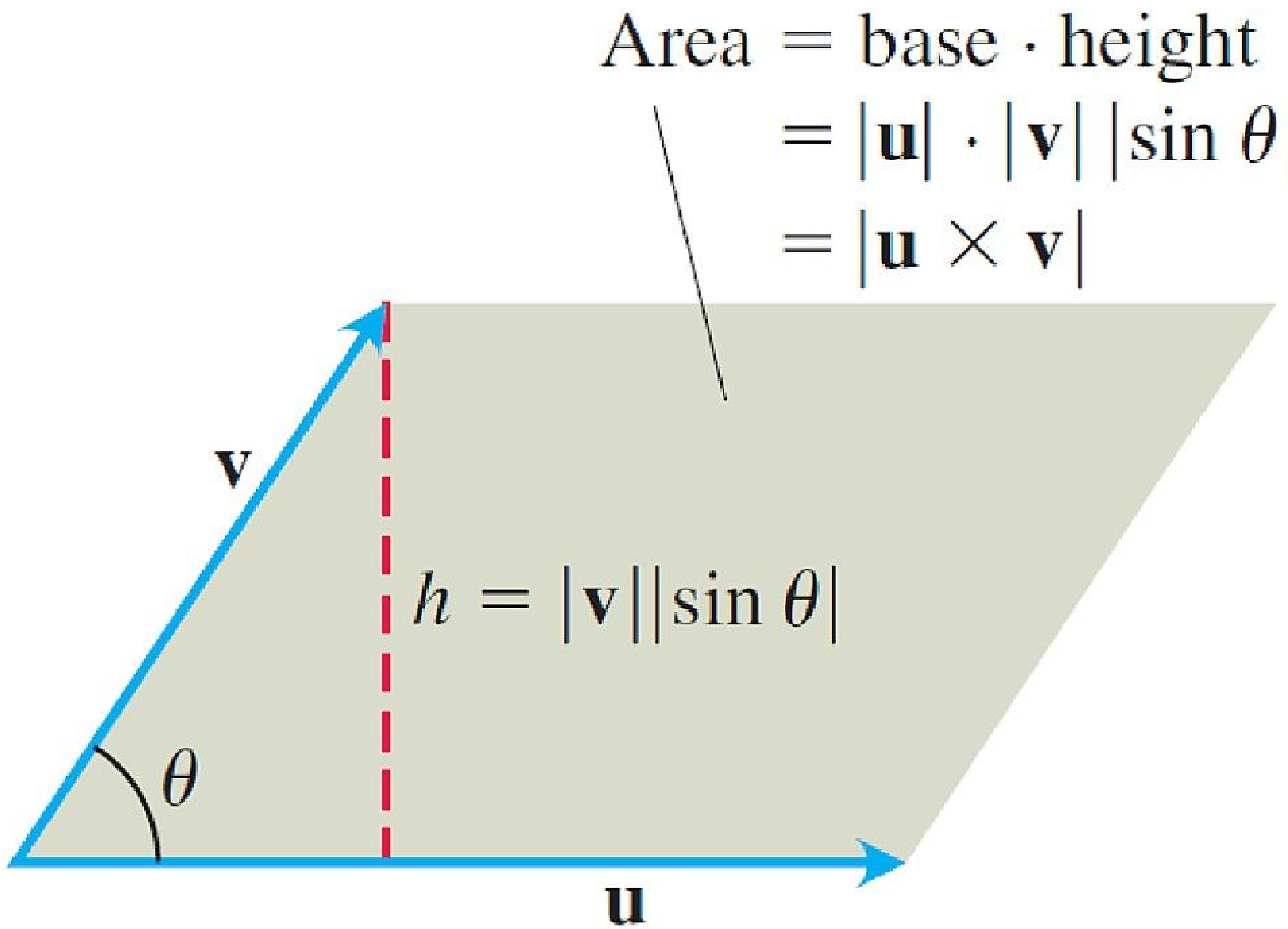



**FIGURE 11.31** The parallelogram determined by **u** and **v**.

**Calculating the Cross Product as a Determinant**

If $\mathbf{u} = u_1\mathbf{i} + u_2\mathbf{j} + u_3\mathbf{k}$ and $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$, then

$$\mathbf{u} \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}.$$

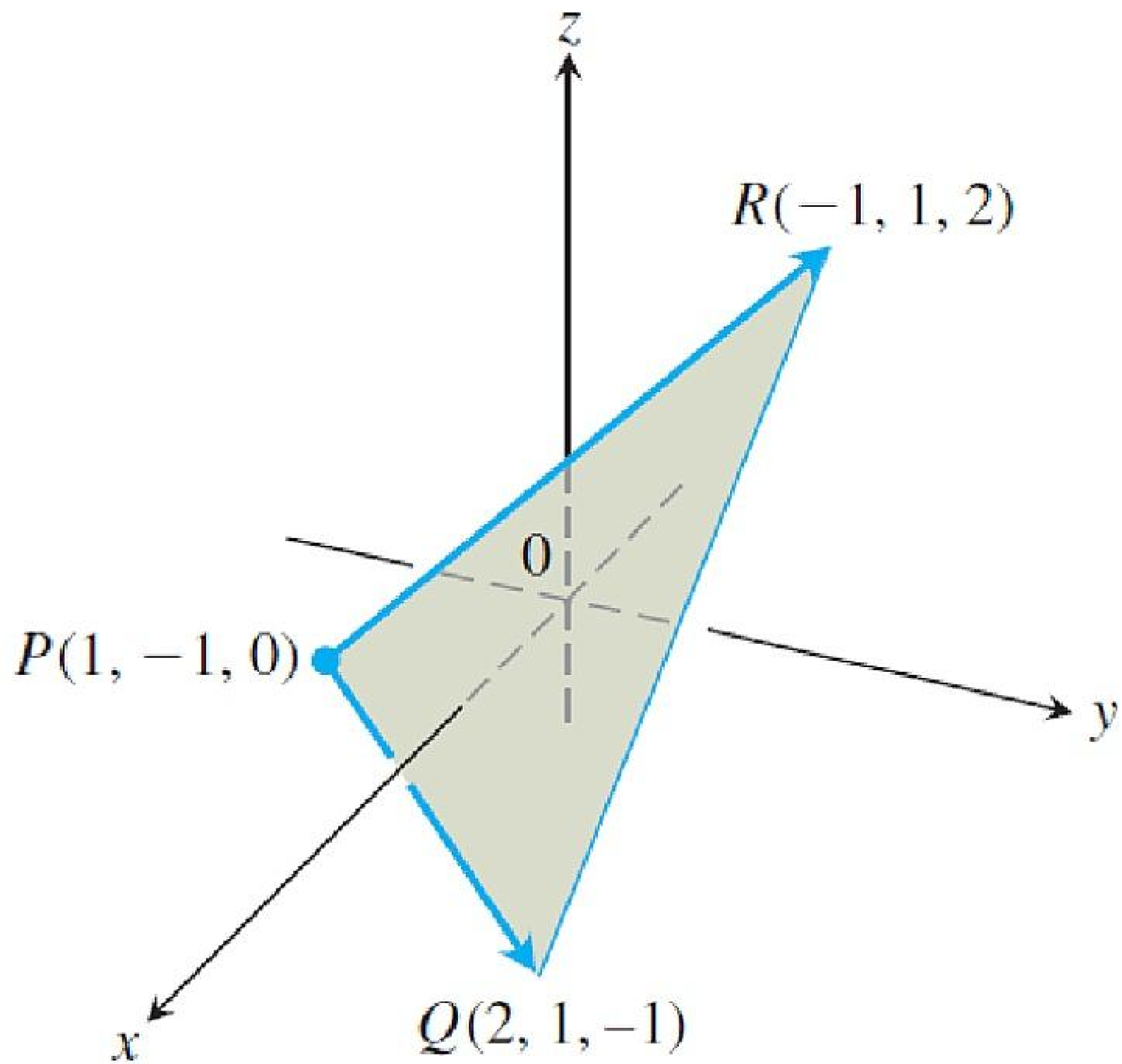
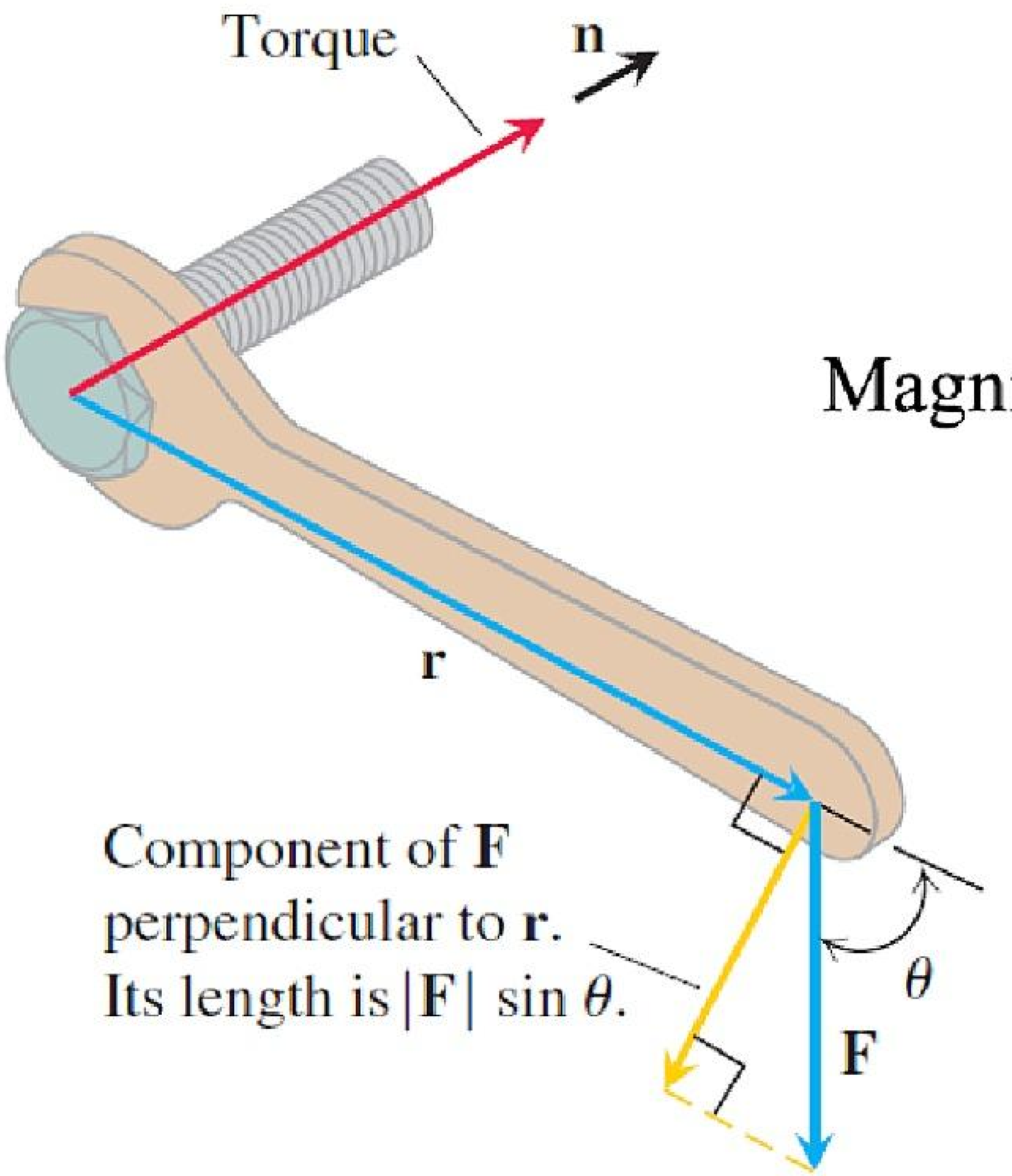



**FIGURE 11.32** The vector $\overrightarrow{PQ} \times \overrightarrow{PR}$ is perpendicular to the plane of triangle $PQR$ (Example 2). The area of triangle $PQR$ is half of $|\overrightarrow{PQ} \times \overrightarrow{PR}|$ (Example 3).




$$\text{Magnitude of torque vector} = |\mathbf{r}||\mathbf{F}| \sin\theta,$$

**FIGURE 11.33** The torque vector describes the tendency of the force $\mathbf{F}$ to drive the bolt forward.

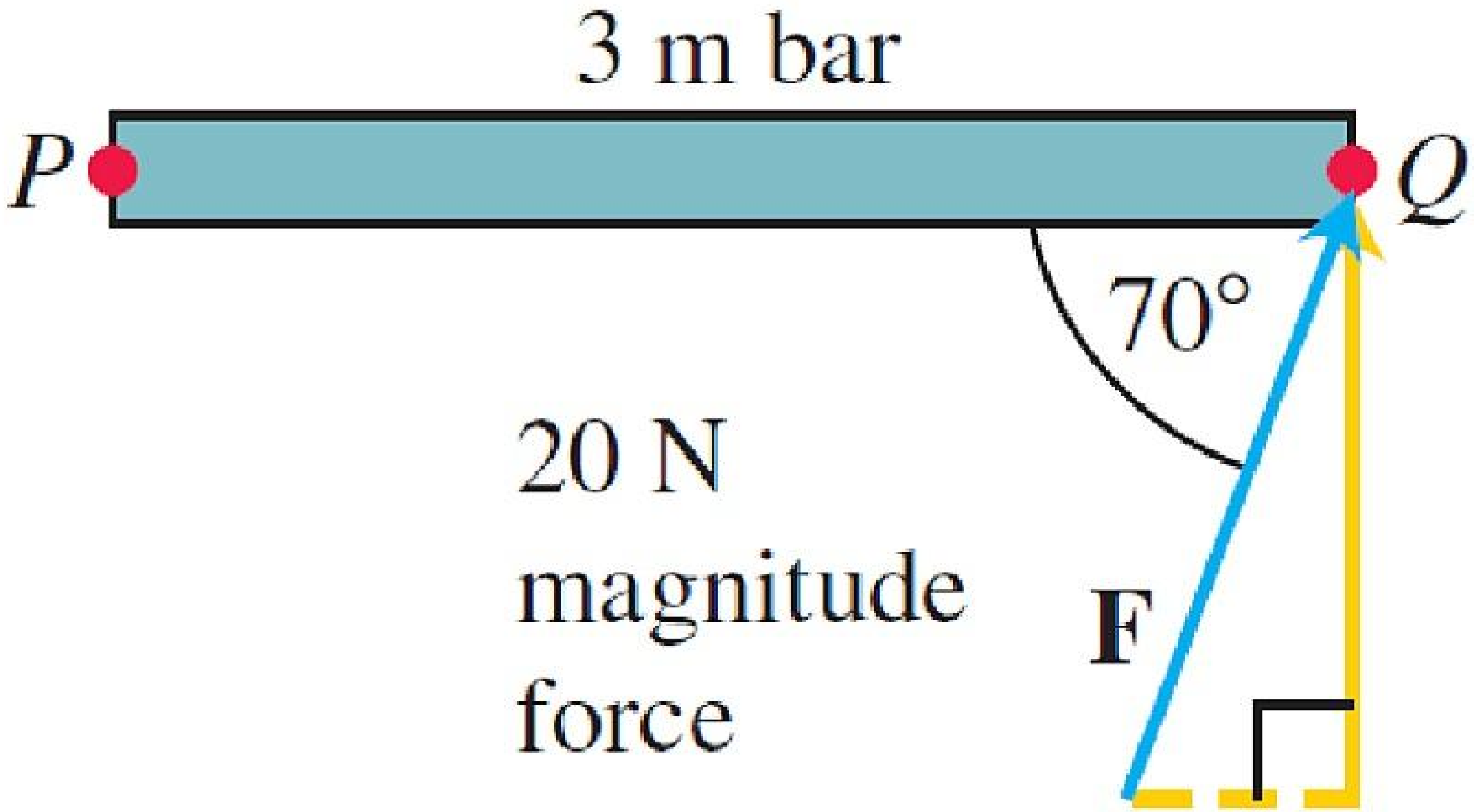



**FIGURE 11.34** The magnitude of the torque exerted by **F** at $P$ is about 56.4 N·m (Example 5). The bar rotates counter-clockwise around $P$.

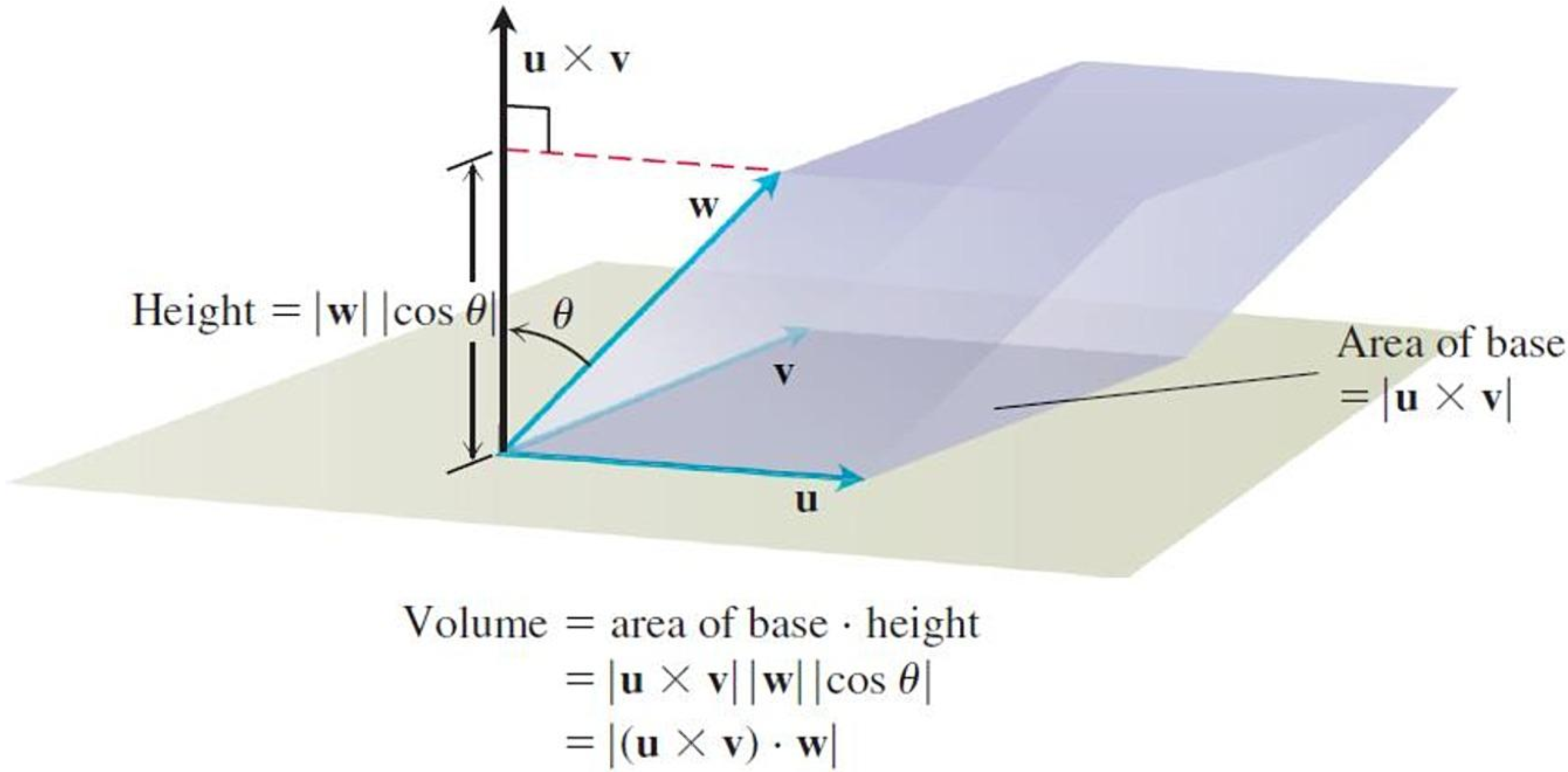



**FIGURE 11.35** The number $|(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}|$ is the volume of a parallelepiped.

## Calculating the Triple Scalar Product as a Determinant

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = \begin{vmatrix} u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \\ w_1 & w_2 & w_3 \end{vmatrix}$$

# Section 11.5

## Lines and Planes in Space

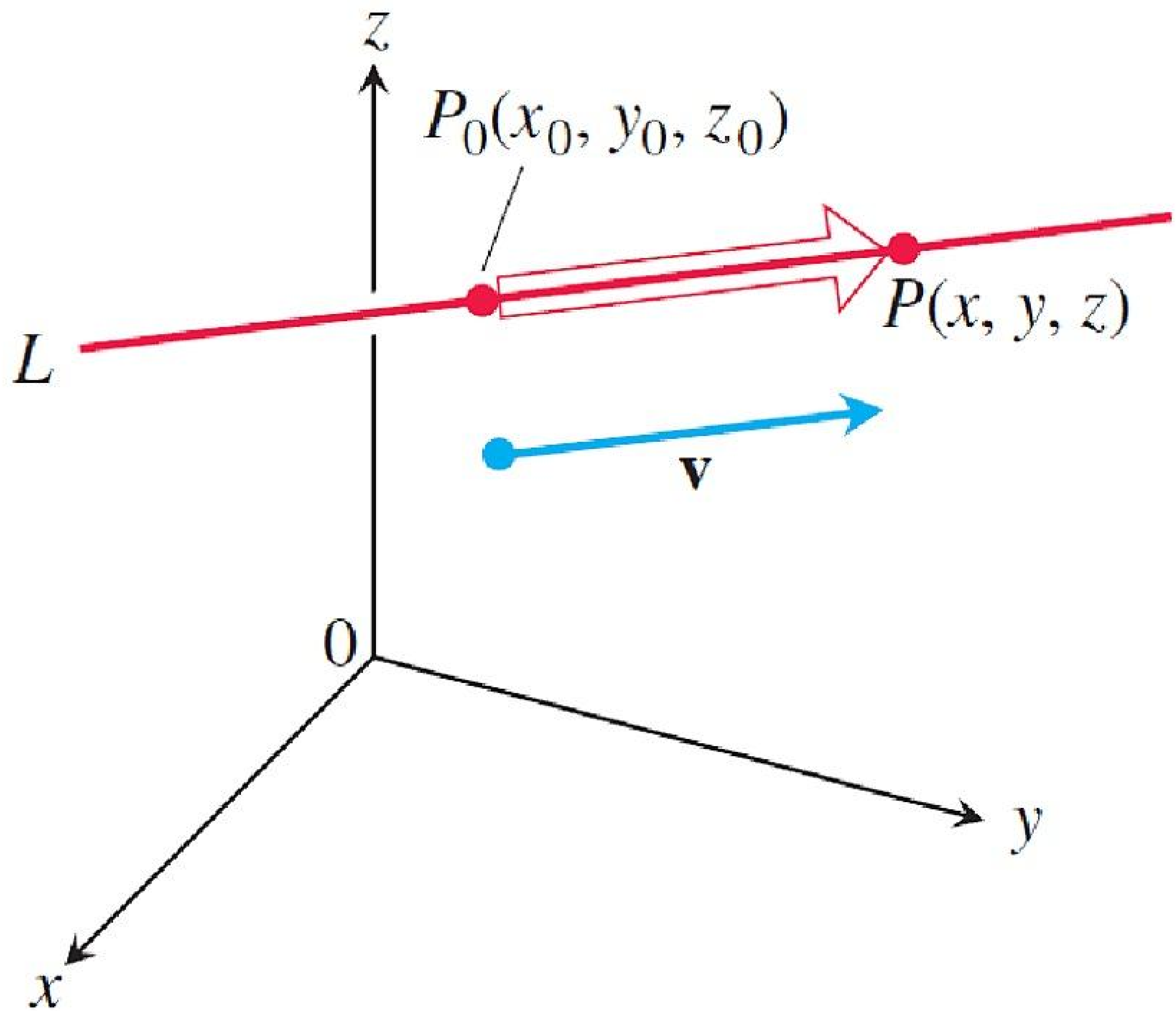



**FIGURE 11.36** A point $P$ lies on $L$ through $P_0$ parallel to $\mathbf{v}$ if and only if $\overrightarrow{P_0P}$ is a scalar multiple of $\mathbf{v}$.

**Vector Equation for a Line**

**A vector equation for the line $L$ through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v}$** is

$$\mathbf{r}(t) = \mathbf{r}_0 + t\mathbf{v}, \qquad -\infty < t < \infty, \tag{2}$$

where $\mathbf{r}$ is the position vector of a point $P(x, y, z)$ on $L$ and $\mathbf{r}_0$ is the position vector of $P_0(x_0, y_0, z_0)$.

## Parametric Equations for a Line

**The standard parametrization of the line through $P_0(x_0, y_0, z_0)$ parallel to $\mathbf{v} = v_1\mathbf{i} + v_2\mathbf{j} + v_3\mathbf{k}$ is**

$$x = x_0 + tv_1, \quad y = y_0 + tv_2, \quad z = z_0 + tv_3, \quad -\infty < t < \infty \tag{3}$$

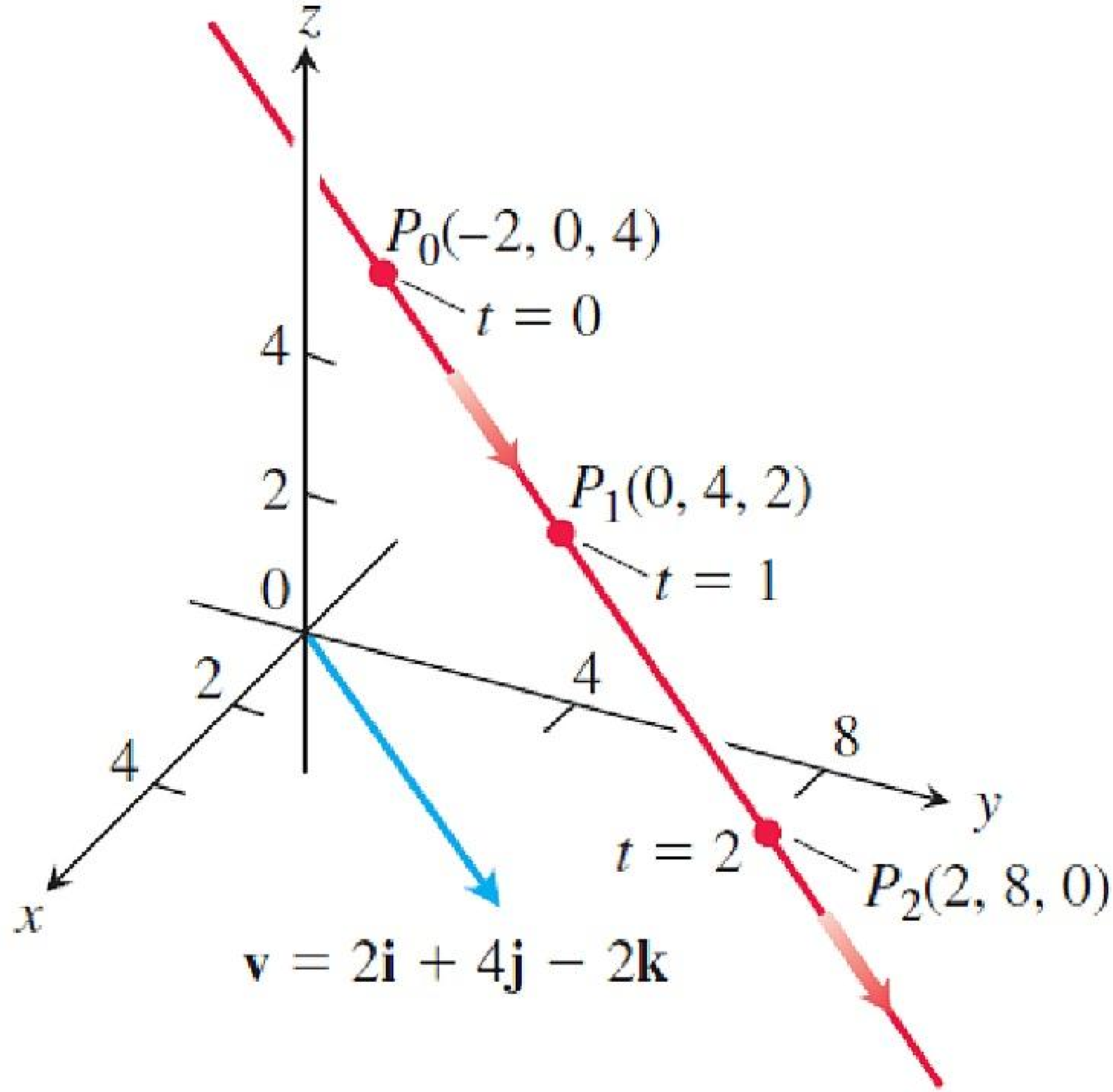



**FIGURE 11.37** Selected points and parameter values on the line in Example 1. The arrows show the direction of increasing $t$.

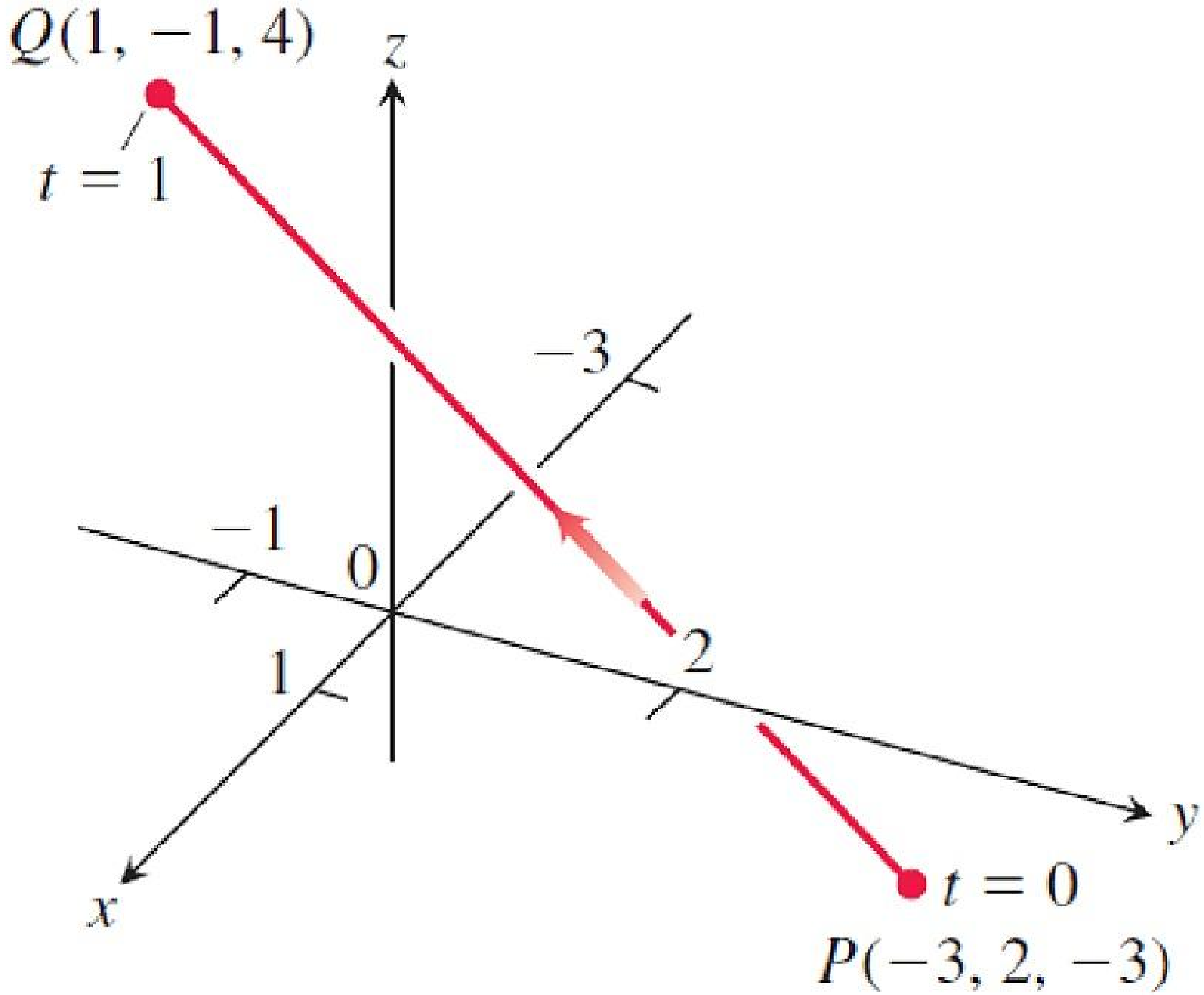



**FIGURE 11.38** Example 3 derives a parametrization of line segment $PQ$. The arrow shows the direction of increasing $t$.

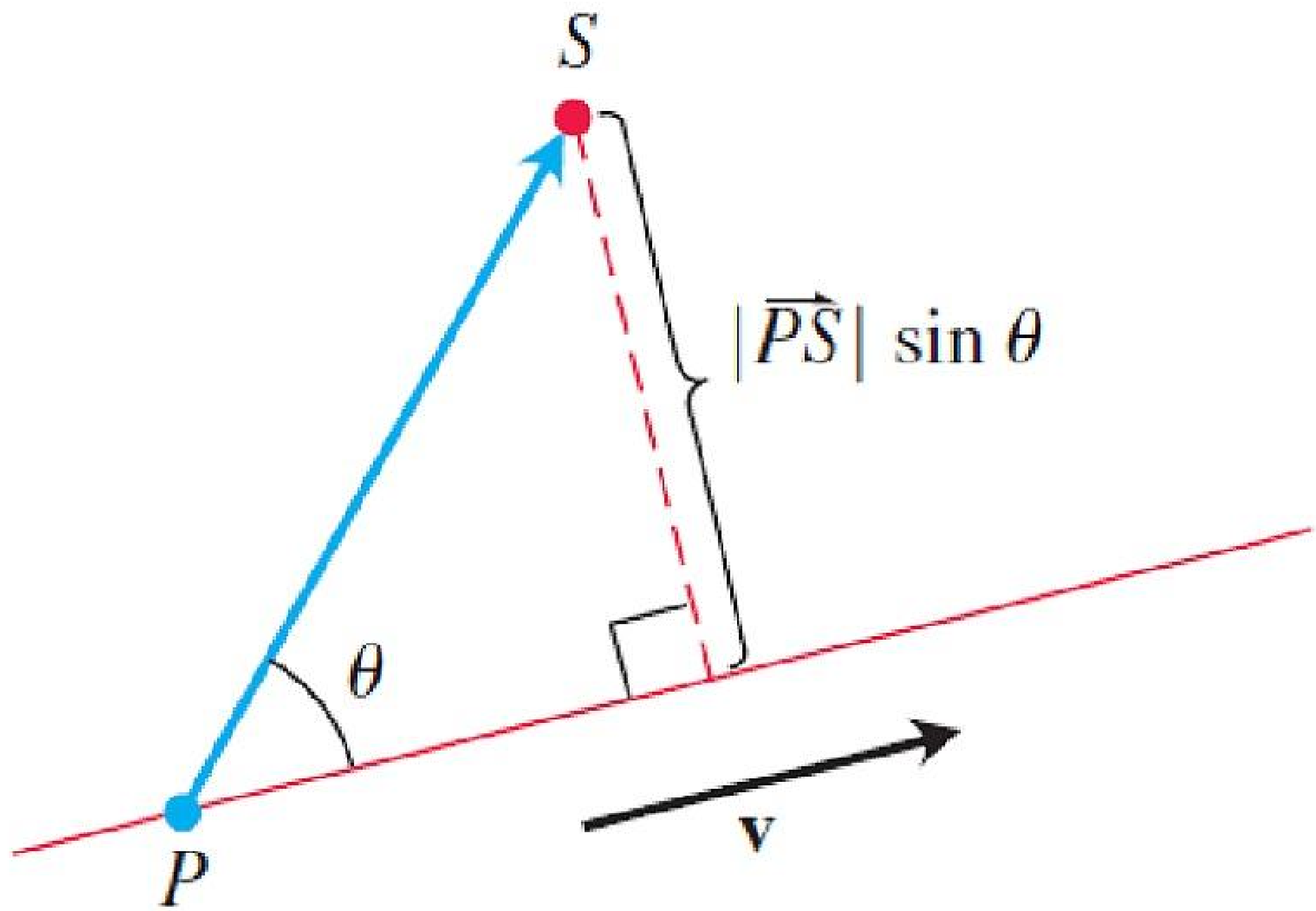



**FIGURE 11.39** The distance from $S$ to the line through $P$ parallel to $\mathbf{v}$ is $|\overrightarrow{PS}|\sin\theta$, where $\theta$ is the angle between $\overrightarrow{PS}$ and $\mathbf{v}$.

**Distance from a Point $S$ to a Line Through $P$ Parallel to $\mathbf{v}$**

$$d = \frac{|\overrightarrow{PS} \times \mathbf{v}|}{|\mathbf{v}|} \tag{5}$$

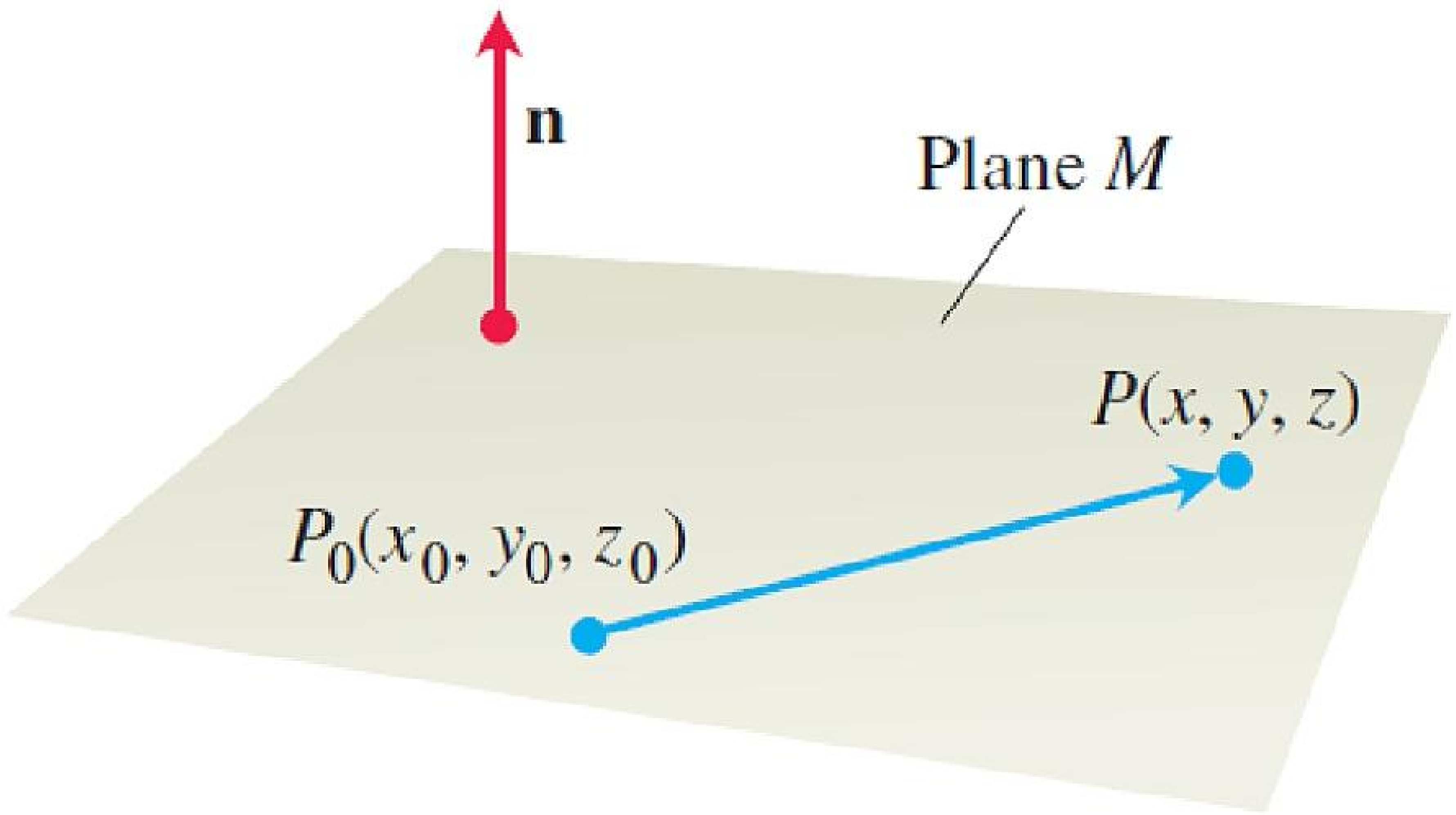



**FIGURE 11.40** The standard equation for a plane in space is defined in terms of a vector normal to the plane: A point $P$ lies in the plane through $P_0$ normal to $\mathbf{n}$ if and only if $\mathbf{n} \cdot \overrightarrow{P_0P} = 0$.

**Equation for a Plane**

The plane through $P_0(x_0, y_0, z_0)$ normal to $\mathbf{n} = A\mathbf{i} + B\mathbf{j} + C\mathbf{k}$ has

**Vector equation:** $\mathbf{n} \cdot \overrightarrow{P_0P} = 0$

**Component equation:** $A(x - x_0) + B(y - y_0) + C(z - z_0) = 0$

**Component equation simplified:** $Ax + By + Cz = D$, where $D = Ax_0 + By_0 + Cz_0$

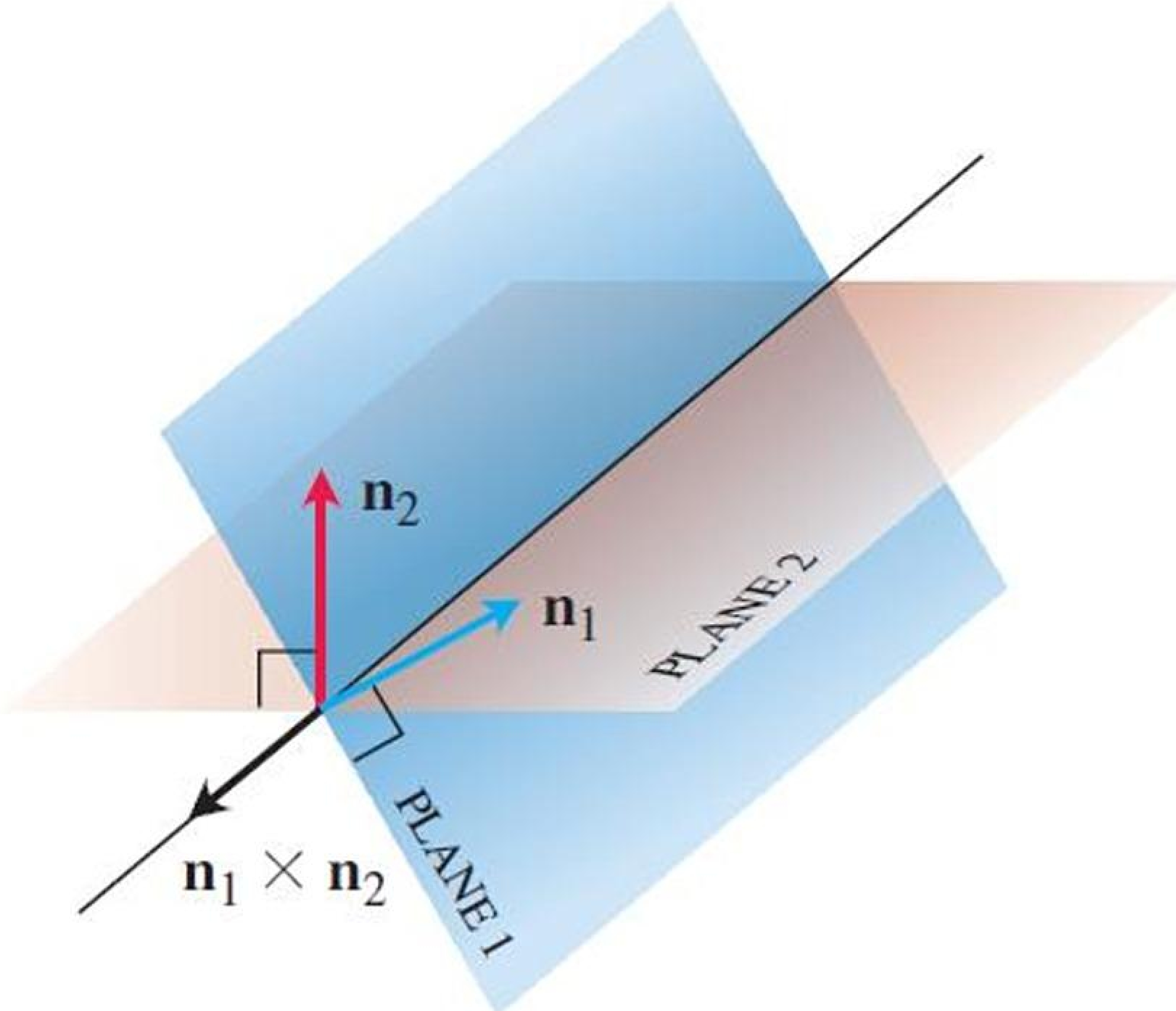



**FIGURE 11.41** How the line of intersection of two planes is related to the planes' normal vectors (Example 8).

## Distance from a Point $S$ to a Plane with Normal $\mathbf{n}$ at Point $P$

$$d = \left| \overrightarrow{PS} \cdot \frac{\mathbf{n}}{|\mathbf{n}|} \right| \tag{6}$$

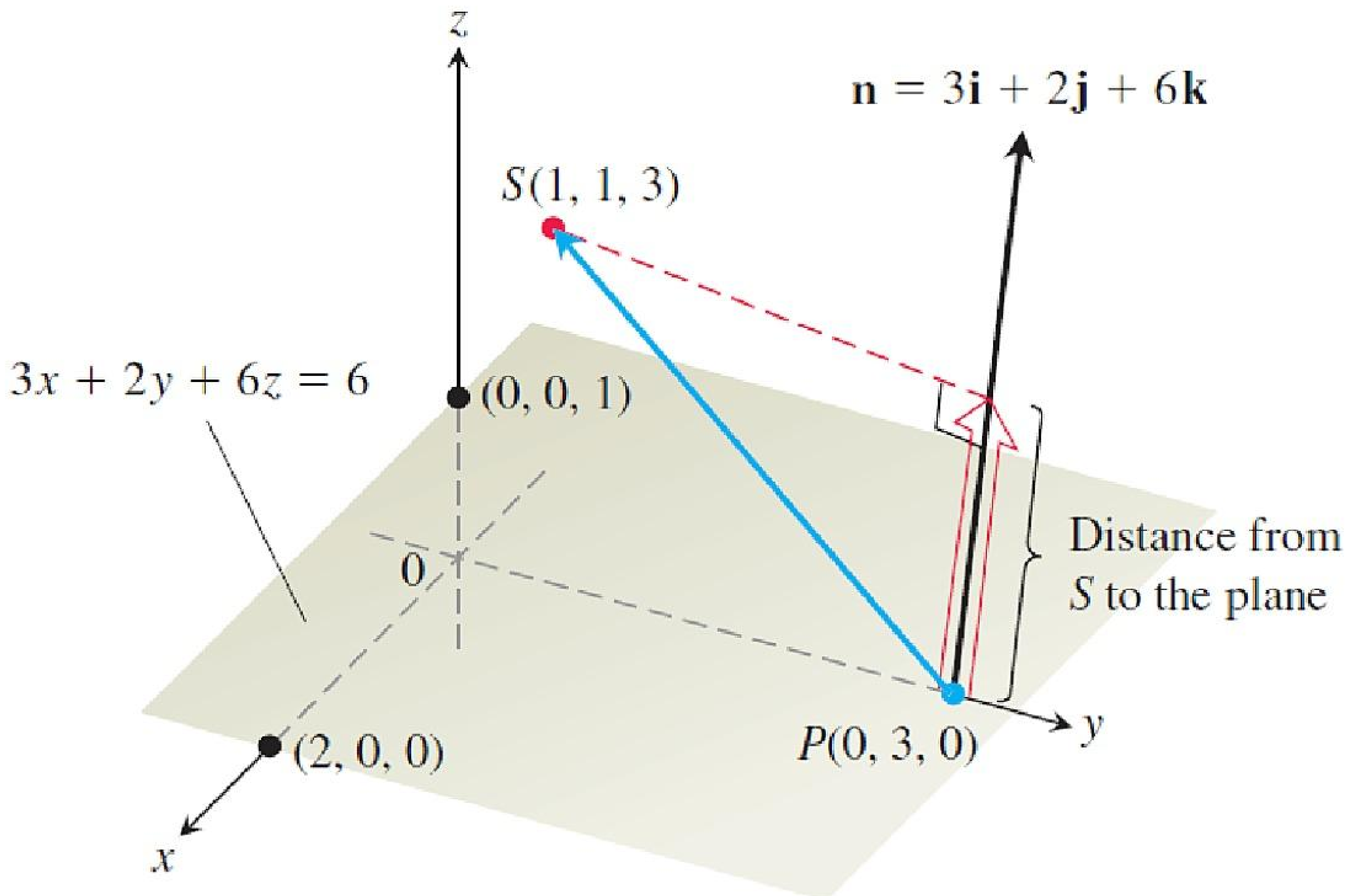



**FIGURE 11.42** The distance from $S$ to the plane is the length of the vector projection of $\overrightarrow{PS}$ onto $\mathbf{n}$ (Example 11).

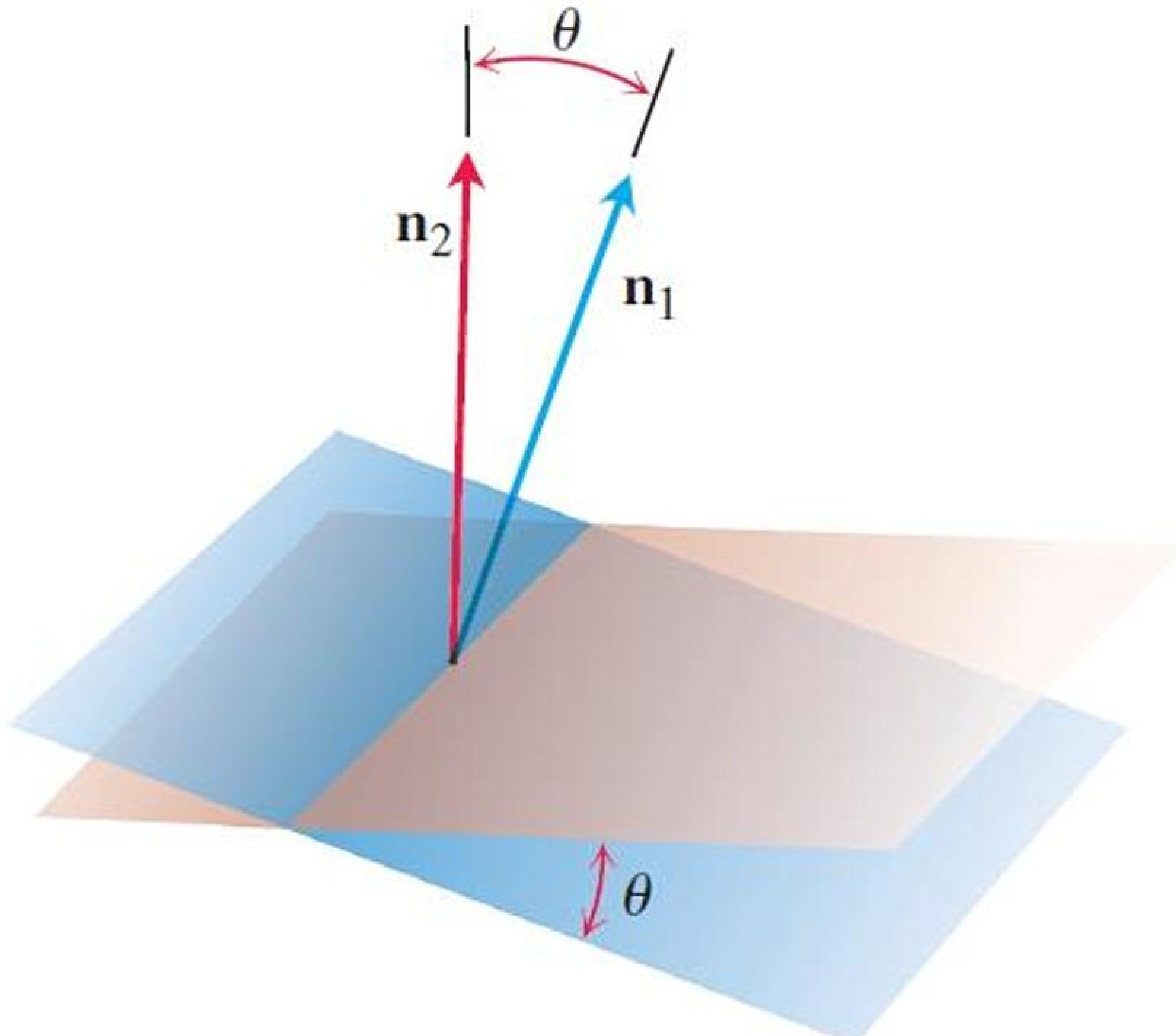



**FIGURE 11.43** The angle between two planes is obtained from the angle between their normals.

# Section 11.6

## Cylinders and Quadric Surfaces

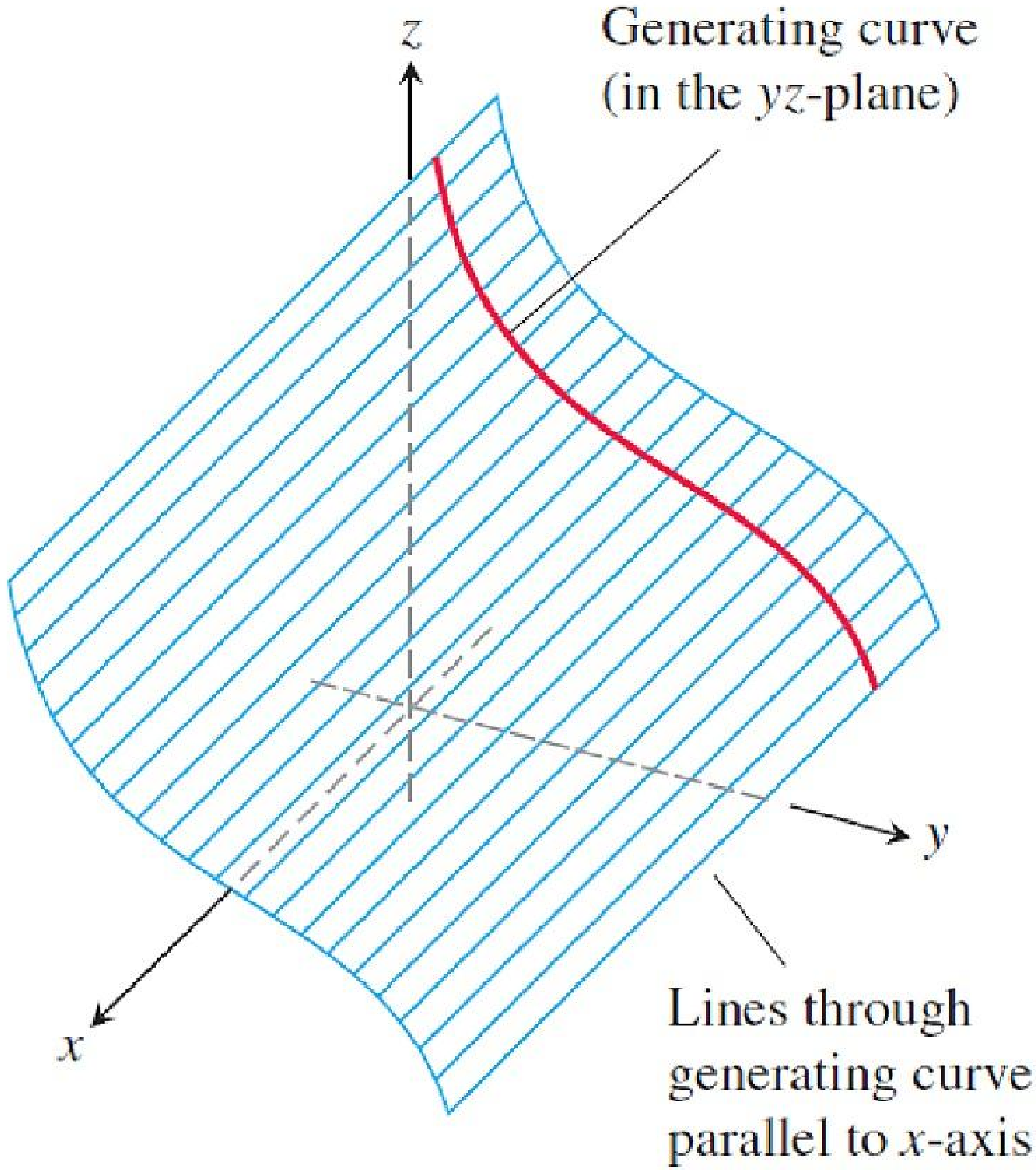



**FIGURE 11.44** A cylinder and generating curve.

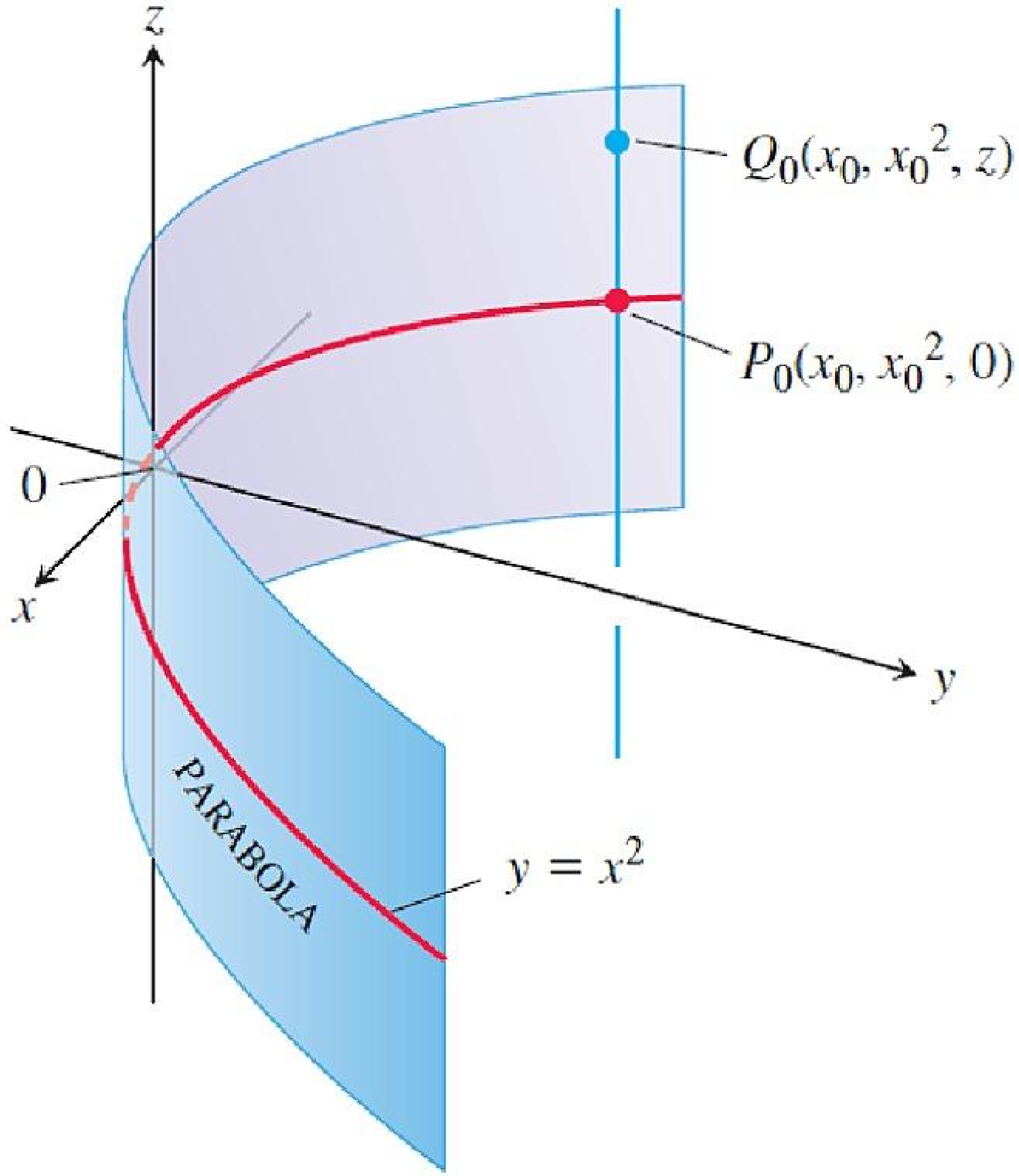



**FIGURE 11.45** Every point of the cylinder in Example 1 has coordinates of the form $(x_0, x_0^2, z)$. We call it "the cylinder $y = x^2$."

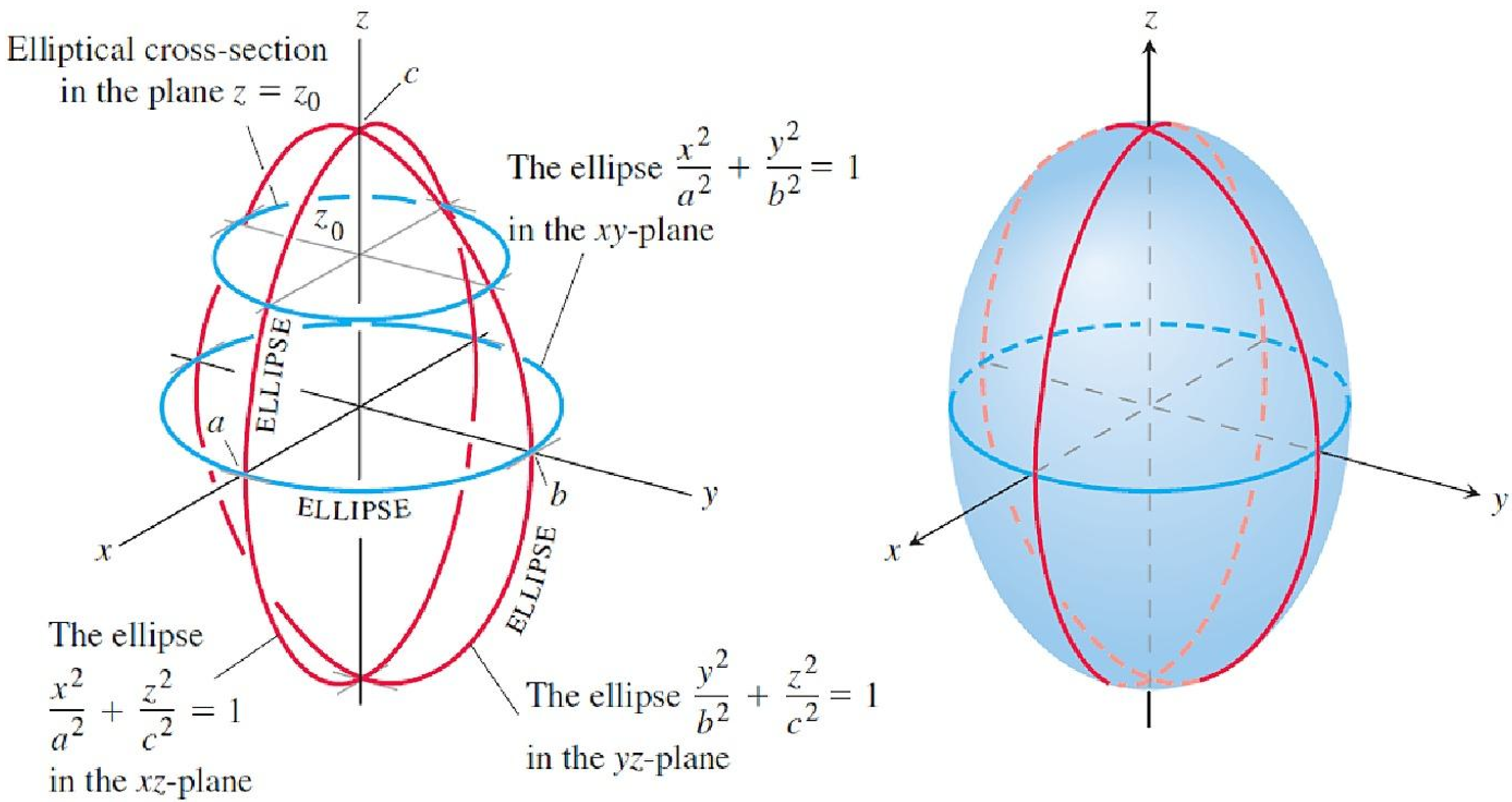



**FIGURE 11.46** The ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

in Example 2 has elliptical cross-sections in each of the three coordinate planes.

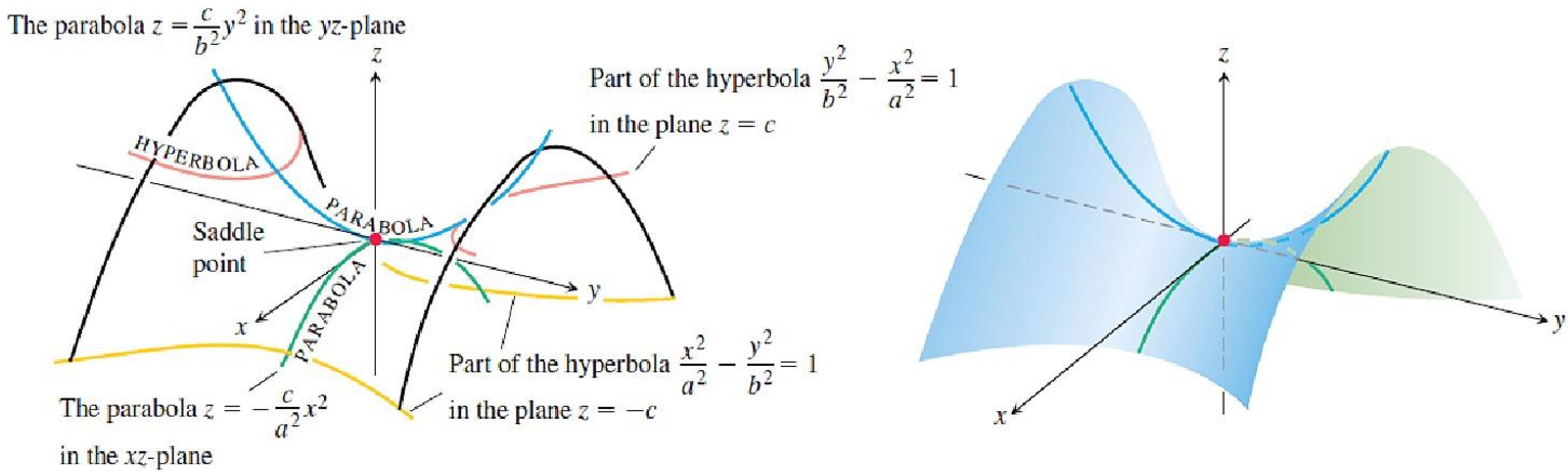
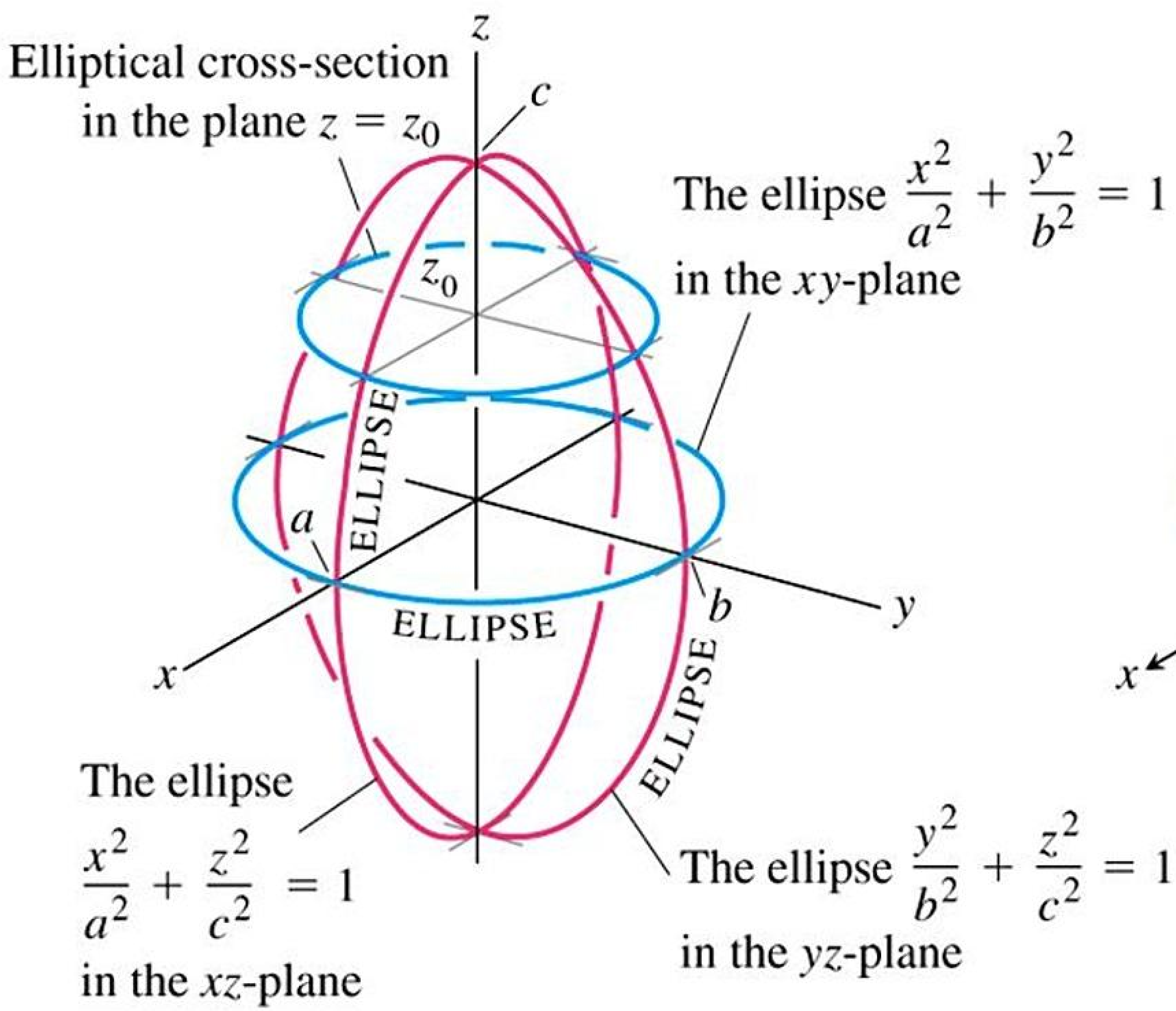
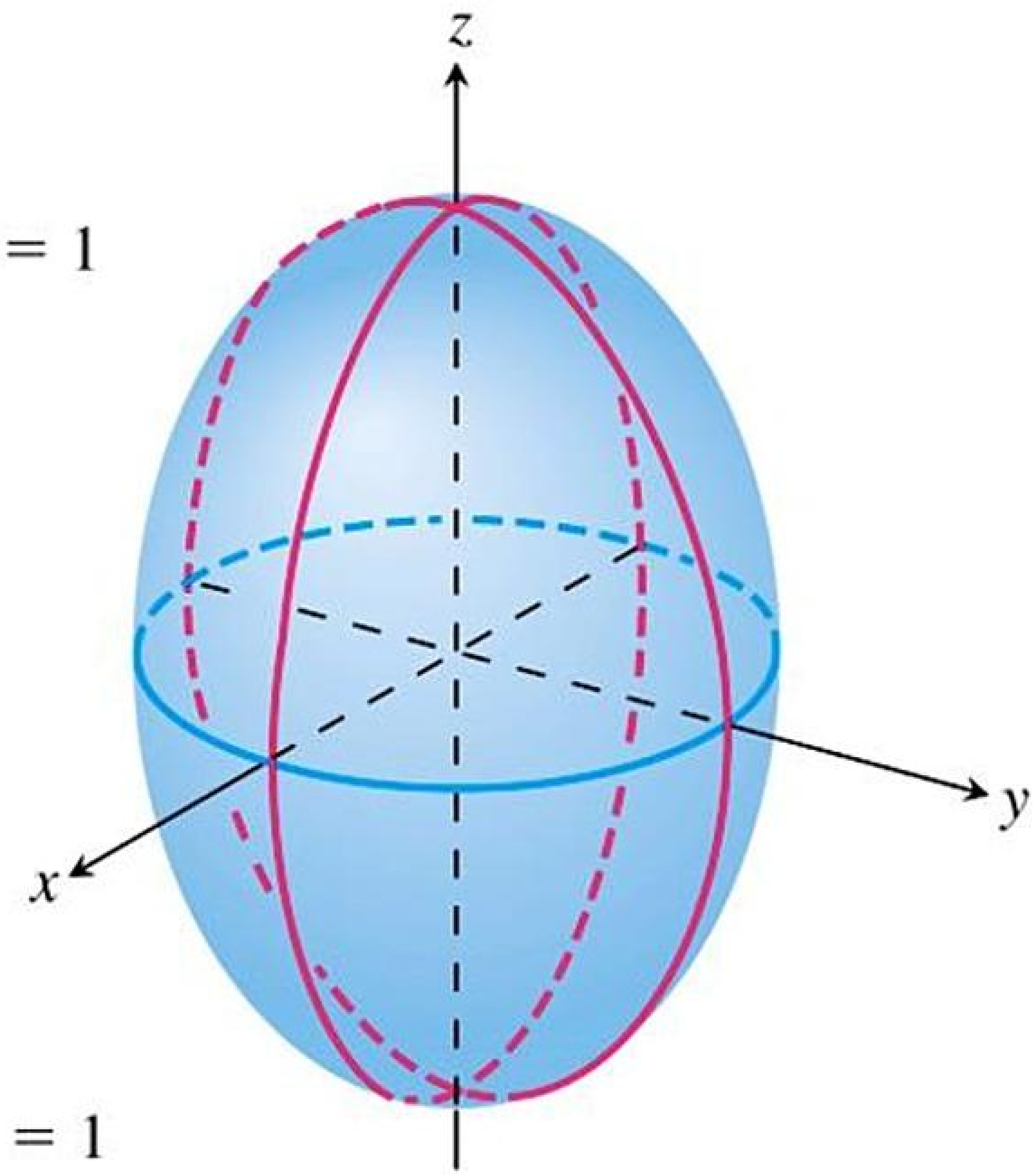



**FIGURE 11.47** The hyperbolic paraboloid $(y^2/b^2) - (x^2/a^2) = z/c$, $c > 0$. The cross-sections in planes perpendicular to the $z$-axis above and below the $xy$-plane are hyperbolas. The cross-sections in planes perpendicular to the other axes are parabolas.

# **Table 11.1** Graphs of Quadric Surfaces

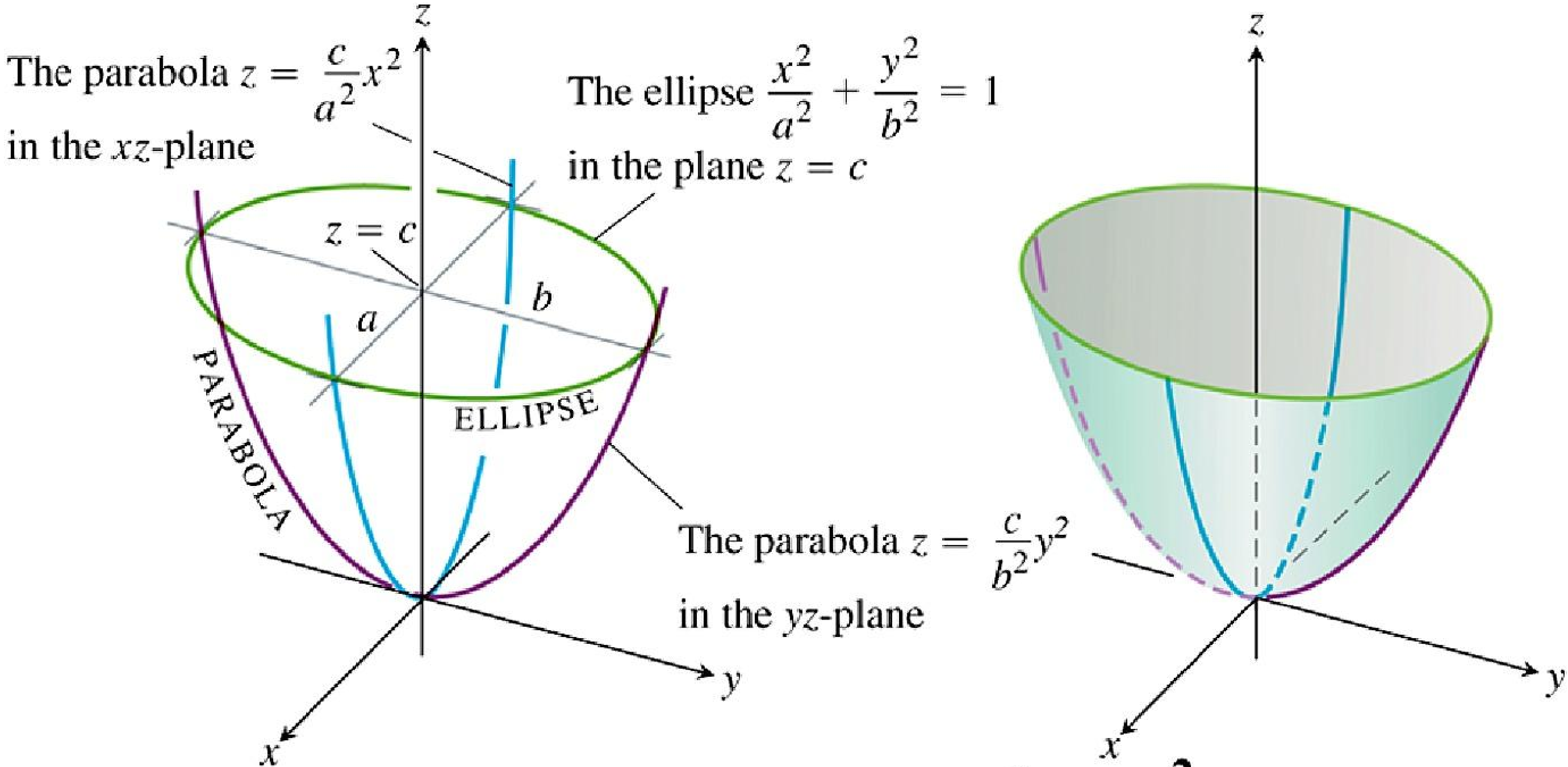



**ELLIPSOID**

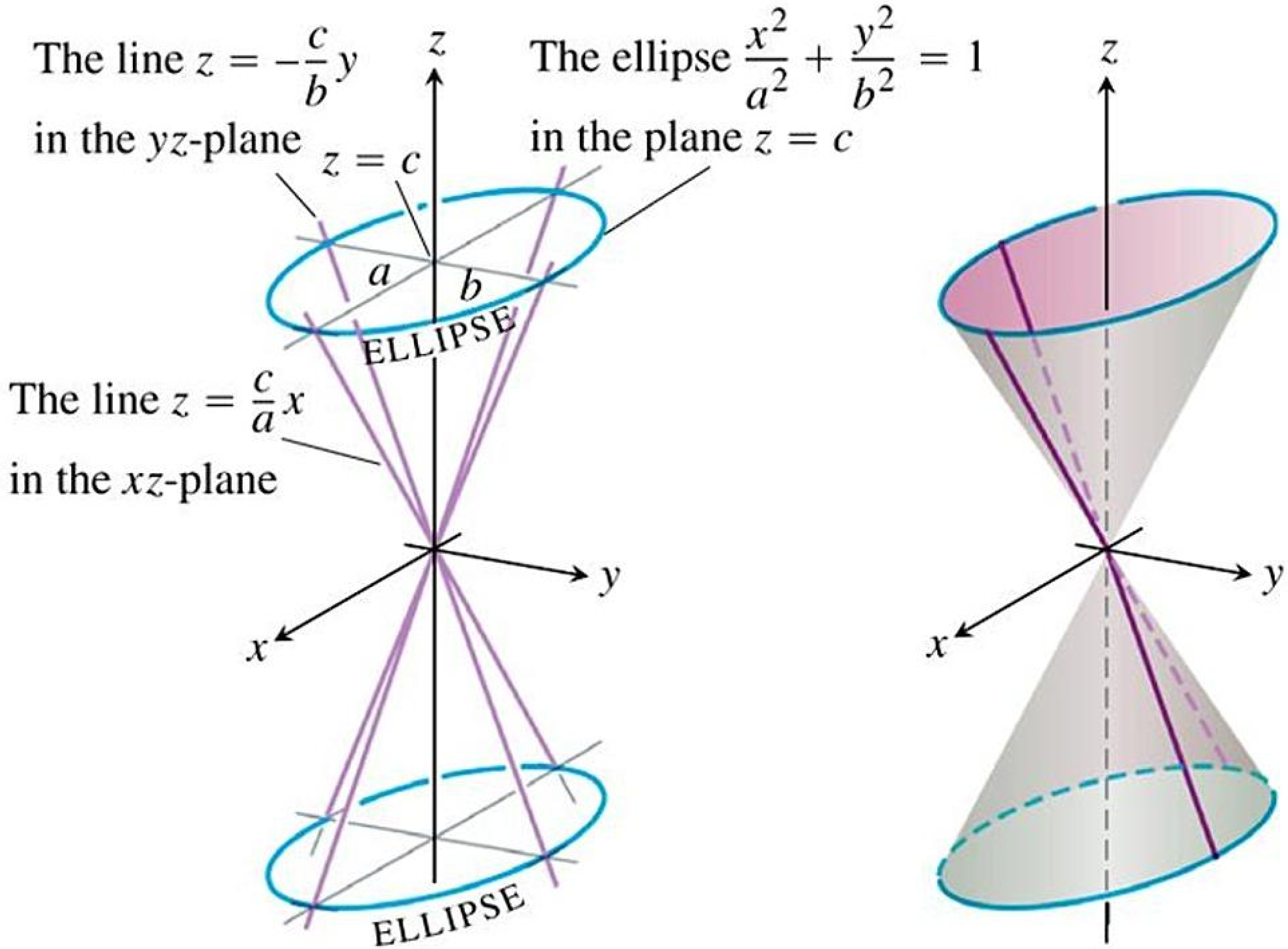
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

# **Table 11.1** Graphs of Quadric Surfaces (cont)




**ELLIPTICAL PARABOLOID** $\quad \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = \dfrac{z}{c}$

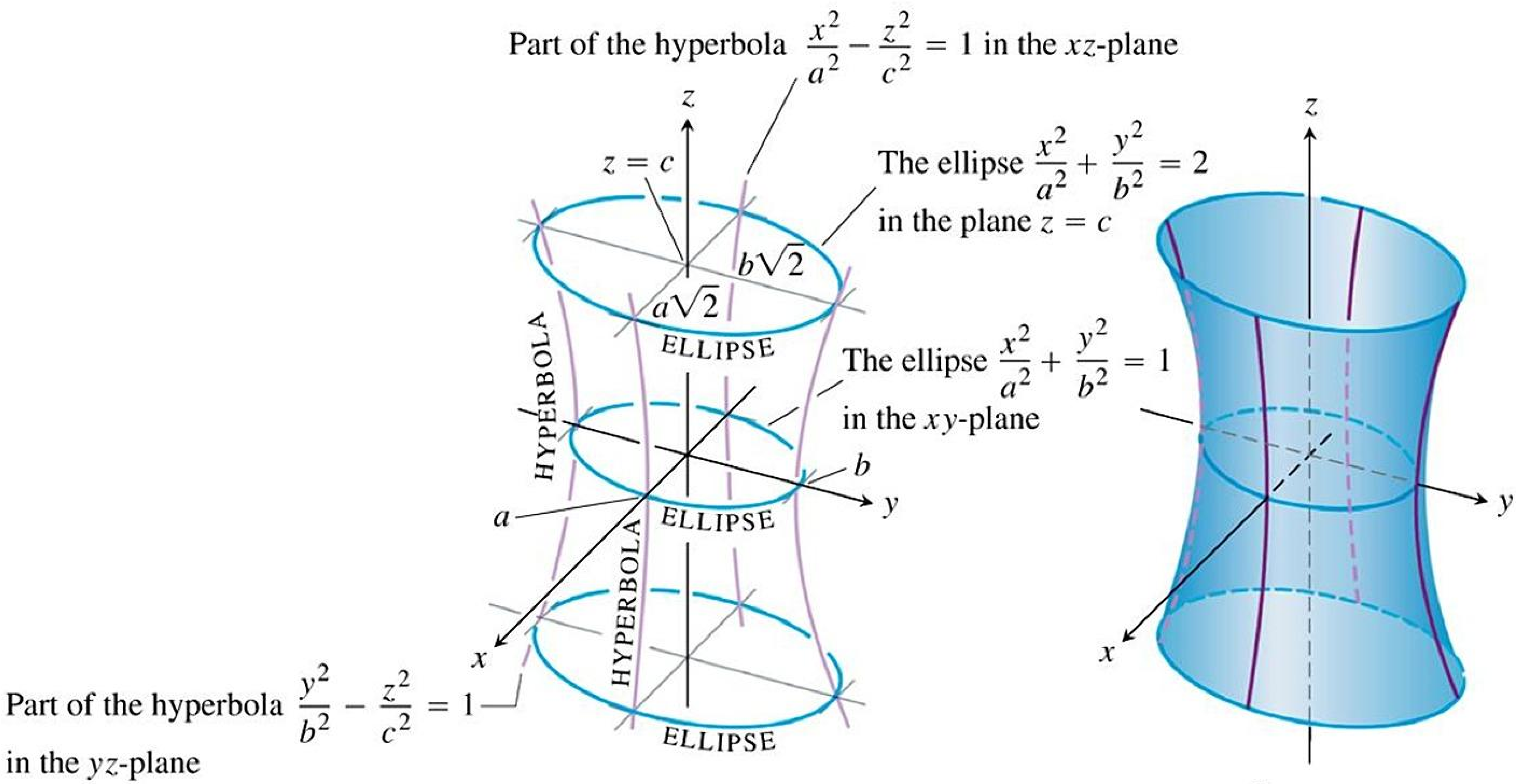
# Table 11.1 Graphs of Quadric Surfaces (cont)




**ELLIPTICAL CONE**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

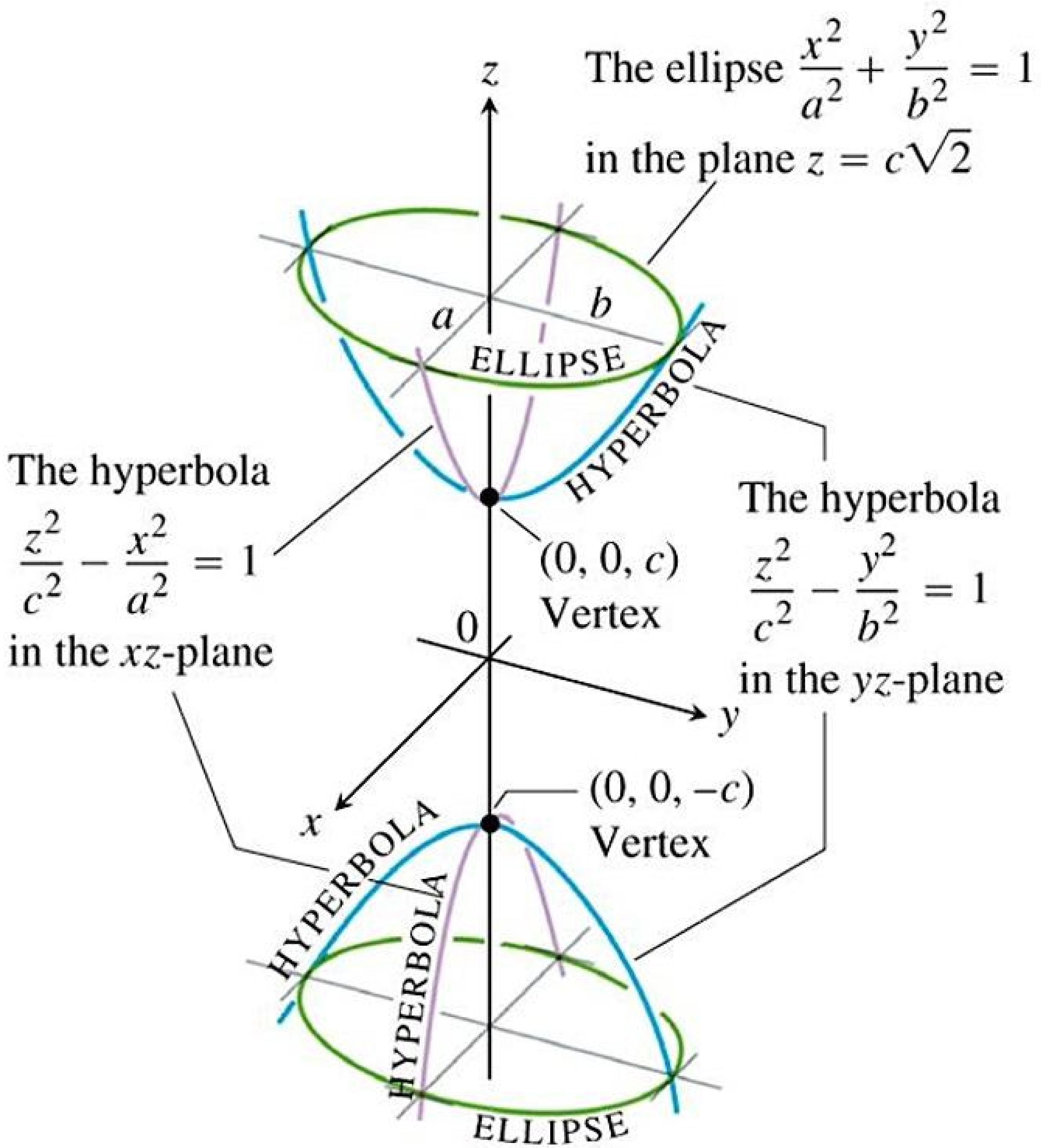
# Table 11.1 Graphs of Quadric Surfaces (cont)

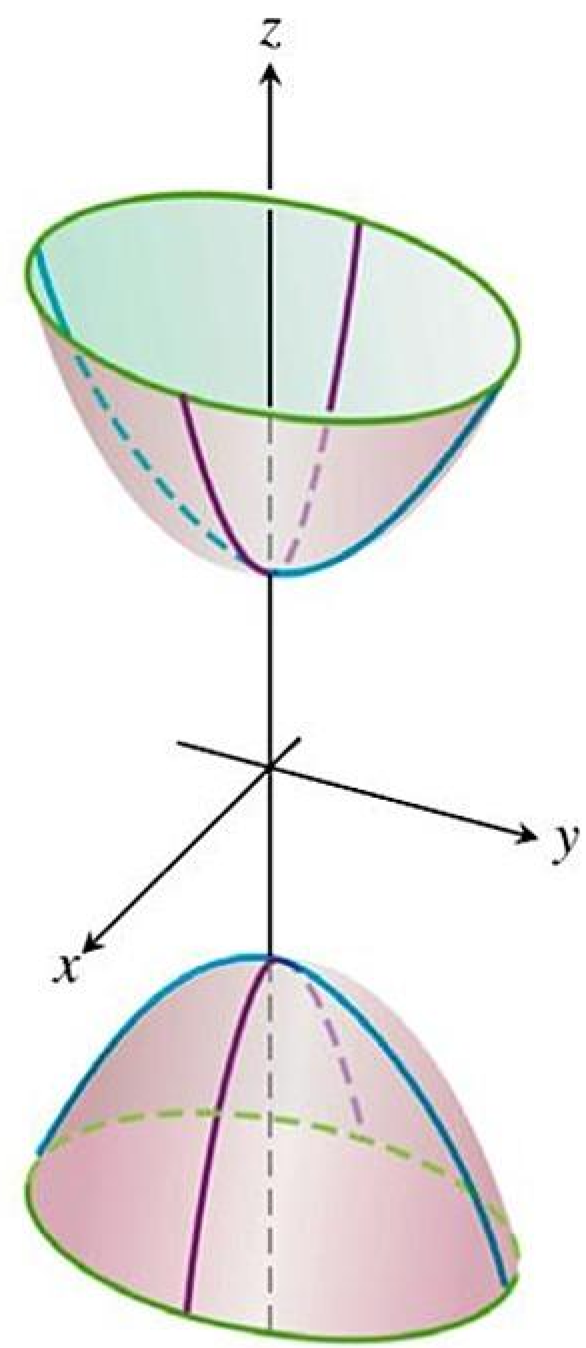



**HYPERBOLOID OF ONE SHEET**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

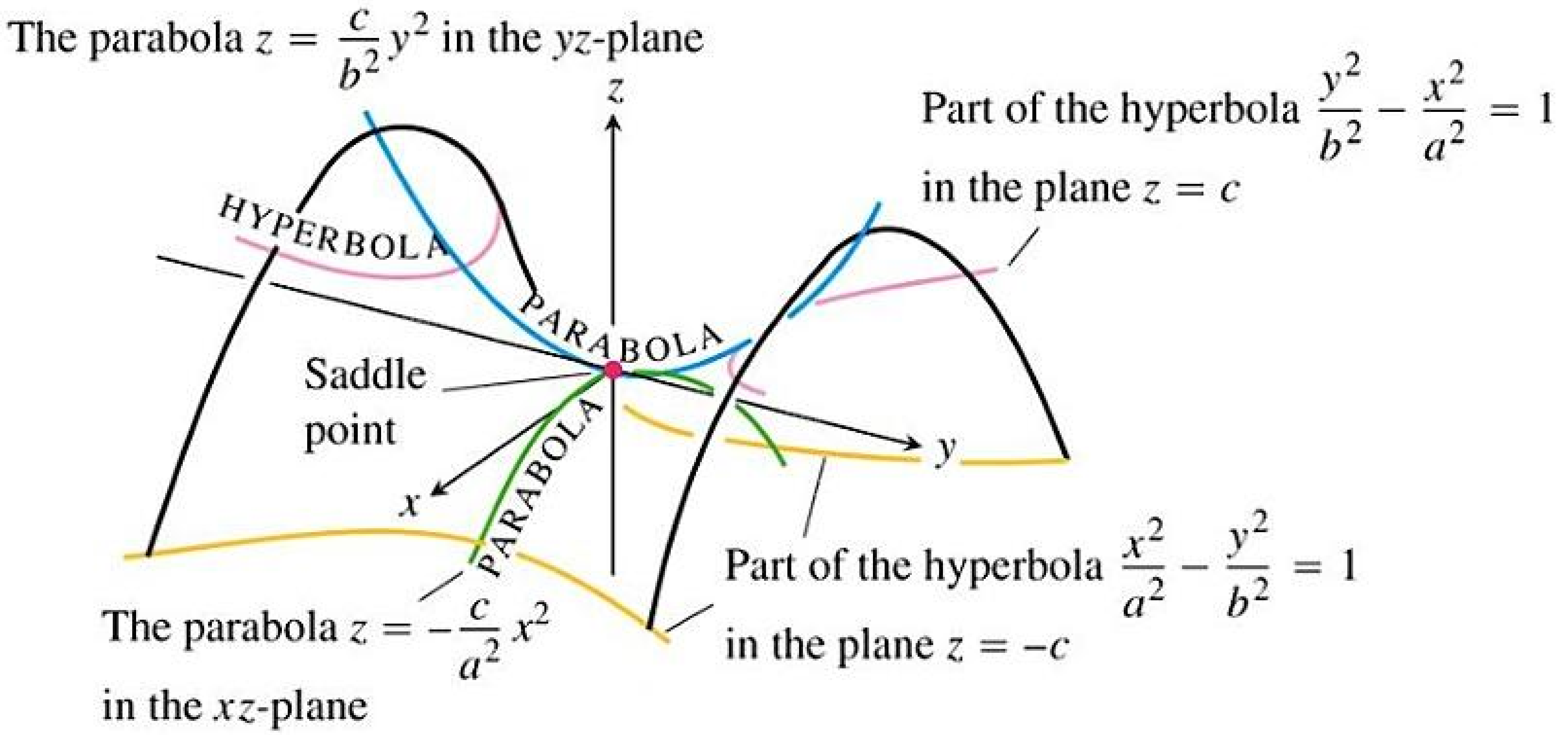
# **Table 11.1** Graphs of Quadric Surfaces (cont)

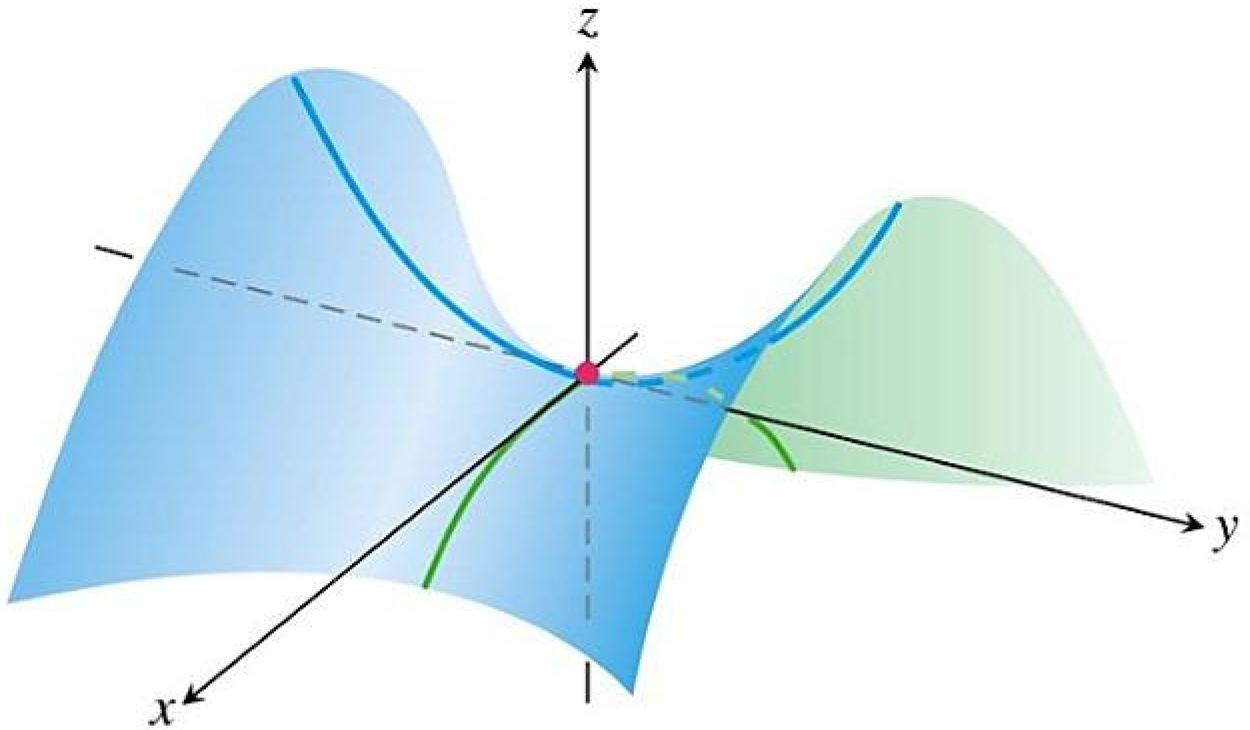



**HYPERBOLOID OF TWO SHEETS** $\frac{z^2}{c^2} - \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

# Table 11.1 Graphs of Quadric Surfaces (cont)




**HYPERBOLIC PARABOLOID** $\frac{y^2}{b^2} - \frac{x^2}{a^2} = \frac{z}{c}, \quad c > 0$

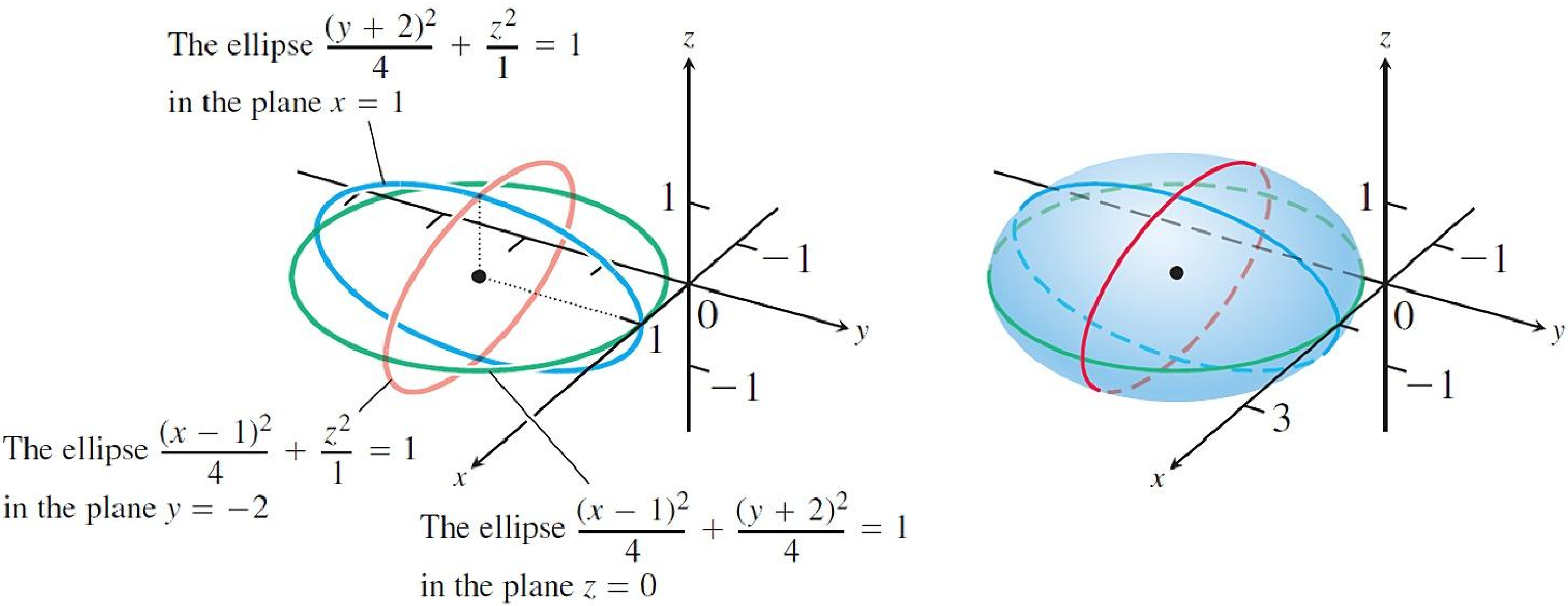



**FIGURE 11.48** An ellipsoid centered at the point $(1, -2, 0)$.