Concurrent Execution

- ***Key idea:*** **Any interleaving of instructions is possible, and some give an unexpected result!**
  - I_i denotes that thread i executes instruction I
  - $\%rdx_i$ is the content of %rdx in thread i's context

i (thread)	$instr_i$	$\%rdx_1$	$\%rdx_2$	cnt
1	H_1	-	-	0
1	L_1	0	-	0
1	U_1	1	-	0
1	S_1	1	-	1
2	H_2	-	-	1
2	L_2	-	1	1
2	U_2	-	2	1
2	S_2	-	2	2
2	T_2	-	2	2
1	T_1	1	-	2

Thread 1
critical section

Thread 2
critical section

OK

Concurrent Execution (cont)

- Incorrect ordering: two threads increment the counter, but the result is 1 instead of 2

i (thread)	$instr_i$	$\%rdx_1$	$\%rdx_2$	cnt
1	H_1	-	-	0
1	L_1	0	-	0
1	U_1	1	-	0
2	H_2	-	-	0
2	L_2	-	0	0
1	S_1	1	-	1
1	T_1	1	-	1
2	U_2	-	1	1
2	S_2	-	1	1
2	T_2	-	1	1

Oops!

Concurrent Execution (cont)

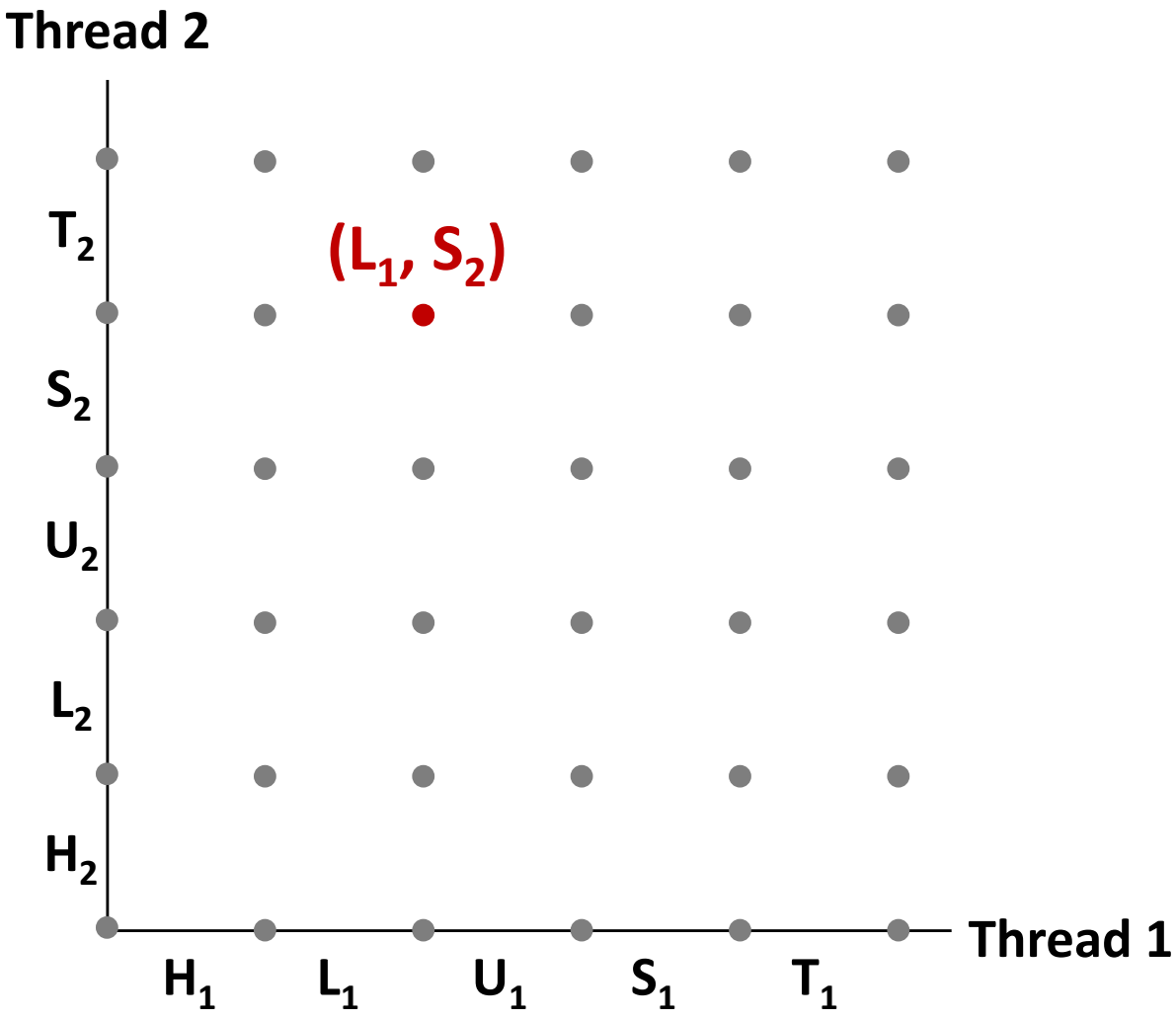
- How about this ordering?

i (thread)	$instr_i$	$\%rdx_1$	$\%rdx_2$	cnt
1	H_1			0
1	L_1	0		
2	H_2			
2	L_2		0	
2	U_2		1	
2	S_2		1	1
1	U_1	1		
1	S_1	1		1
1	T_1			1
2	T_2			1

Oops!

- We can analyze the behavior using a *progress graph*

Progress Graphs



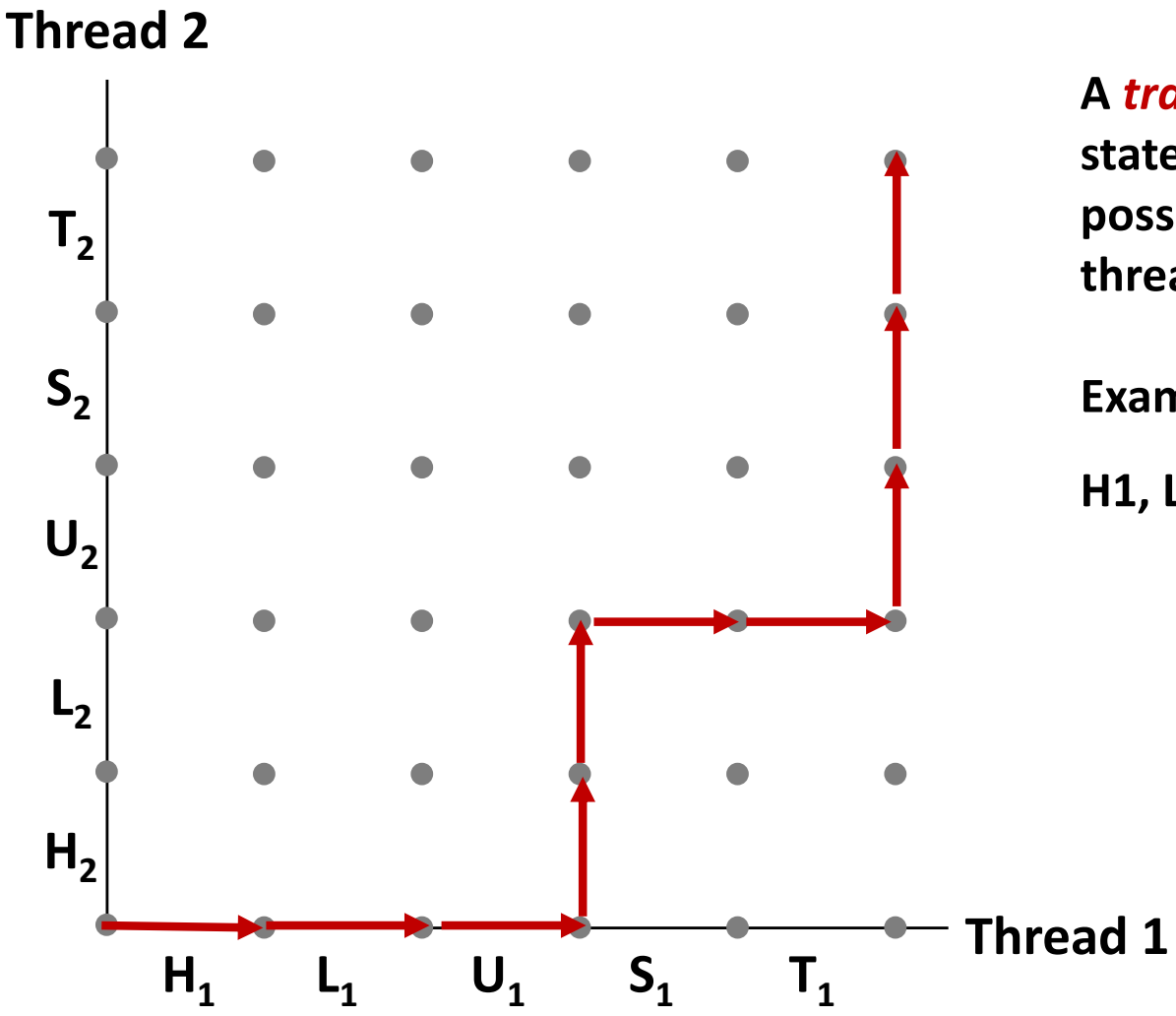
A *progress graph* depicts the discrete *execution state space* of concurrent threads.

Each axis corresponds to the sequential order of instructions in a thread.

Each point corresponds to a possible *execution state* ($Inst_1$, $Inst_2$).

E.g., (L_1, S_2) denotes state where thread 1 has completed L_1 and thread 2 has completed S_2.

Trajectories in Progress Graphs

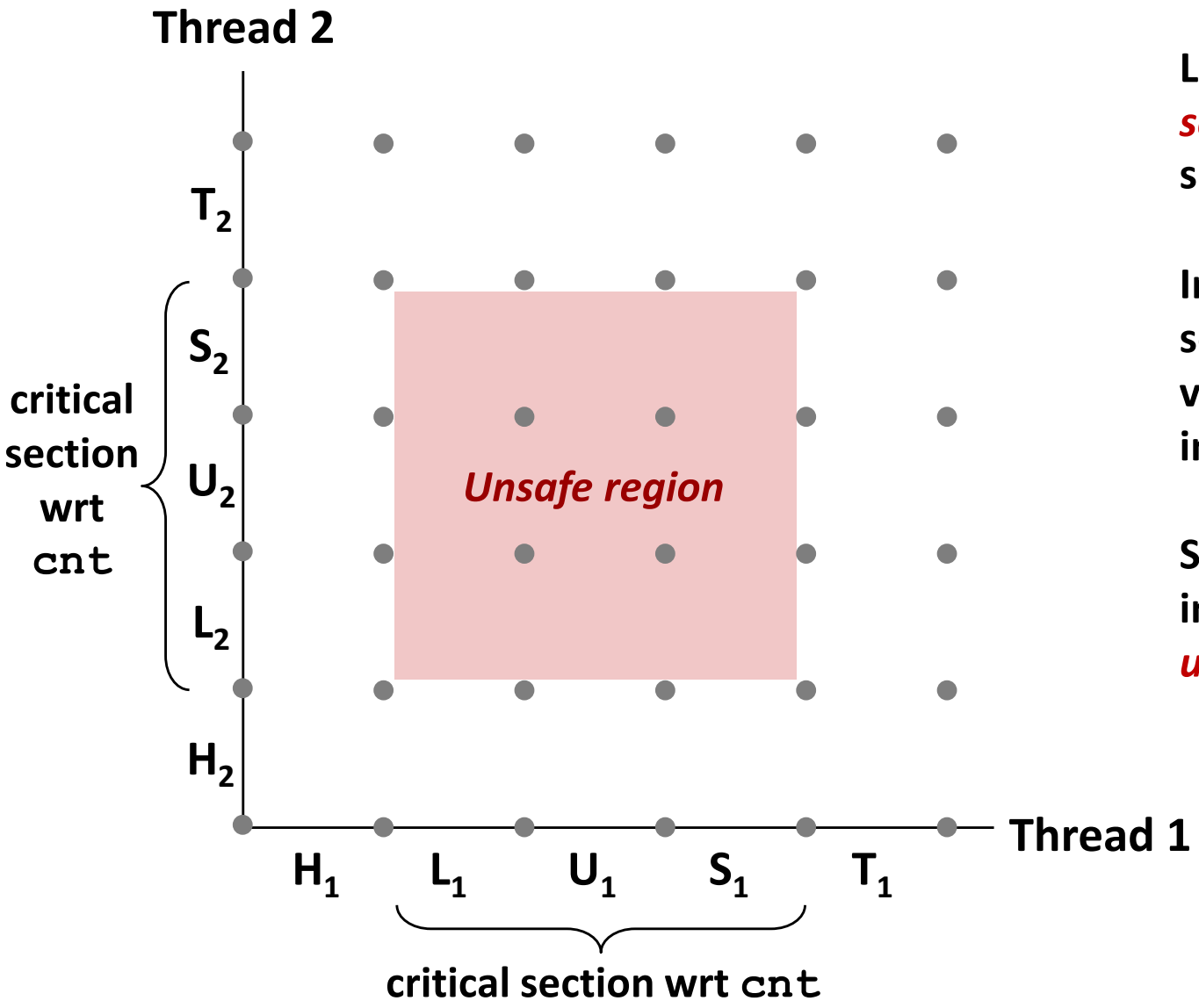


A *trajectory* is a sequence of legal state transitions that describes one possible concurrent execution of the threads.

Example:

H1, L1, U1, H2, L2, S1, T1, U2, S2, T2

Critical Sections and Unsafe Regions

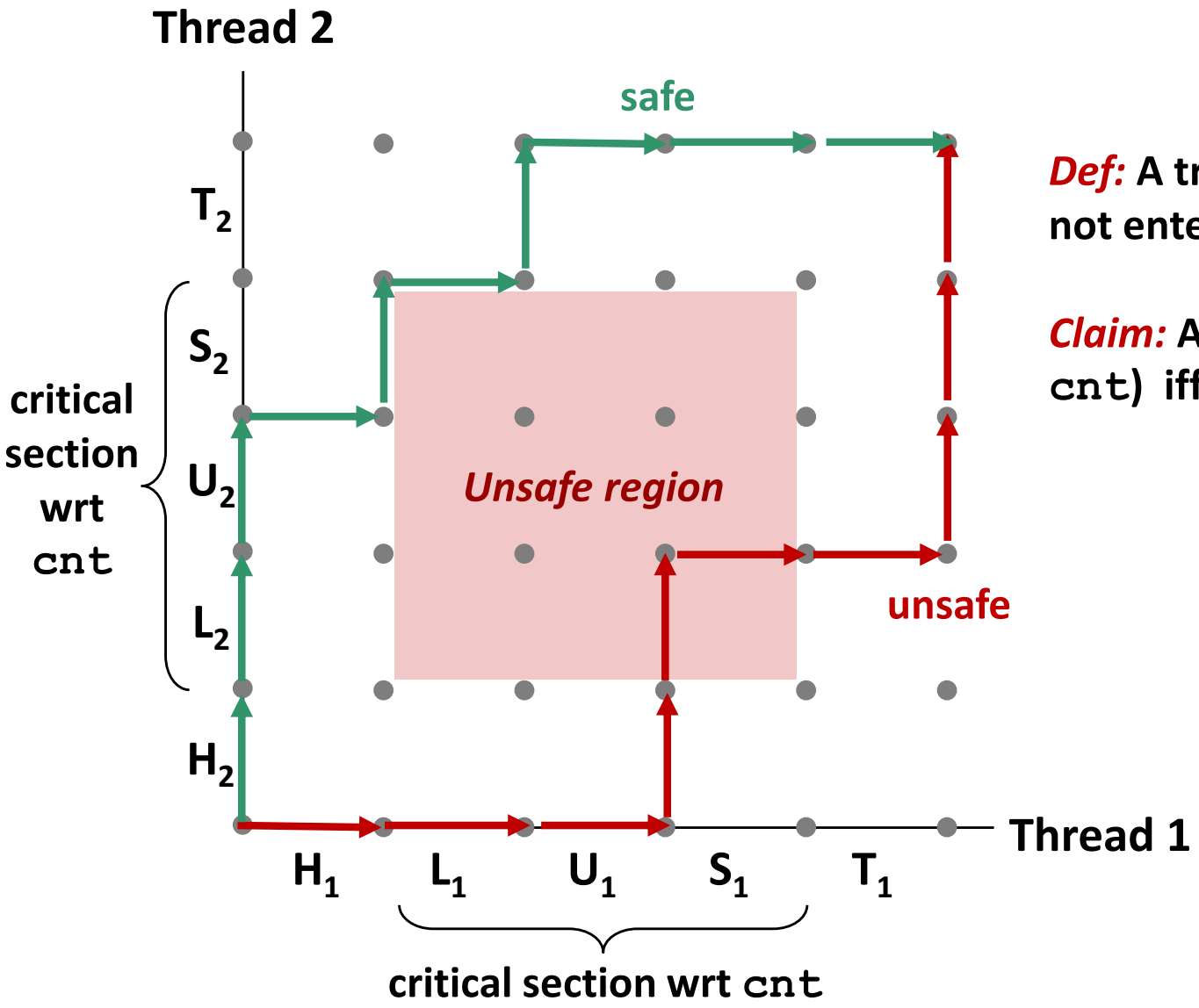


L, U, and S form a *critical section* with respect to the shared variable `cnt`

Instructions in critical sections (wrt some shared variable) should not be interleaved

Sets of states where such interleaving occurs form *unsafe regions*

Critical Sections and Unsafe Regions



Def: **A trajectory is *safe* iff it does not enter any unsafe region**

Claim: **A trajectory is correct (wrt `cnt`) iff it is safe**