

机器学习第三次作业

1、推导软-SVM 主问题的对偶问题

1. 构造拉格朗日函数

$$L(w, b, \alpha, \epsilon, \beta) = \frac{1}{2} W^T \cdot W + C \sum_{i=1}^n \epsilon_i - \sum_{i=1}^n \alpha_i (y_i (W^T x_i + b) - 1 + \epsilon_i) - \sum_{i=1}^n \beta_i \epsilon_i$$

求导：

$$\begin{aligned} \frac{\partial L}{\partial w} &= W - \sum_{i=1}^n \alpha_i y_i x_i = 0 \\ \frac{\partial L}{\partial b} &= - \sum_{i=1}^n \alpha_i y_i = 0 \\ \frac{\partial L}{\partial \epsilon} &= C - \sum_{i=1}^n \alpha_i - \sum_{i=1}^n \beta_i = 0 \end{aligned} \rightarrow \begin{cases} W = \sum_{i=1}^n \alpha_i y_i x_i \\ \sum_{i=1}^n \alpha_i y_i = 0 \\ C = \sum_{i=1}^n (\alpha_i + \beta_i) \end{cases}$$

将其代入到 $L(w, b, \alpha, \epsilon, \beta)$ 中得

$$\begin{aligned} L &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i y_i x_i^T \alpha_j y_j x_j + \sum_{i=1}^n (\alpha_i + \beta_i) \epsilon_i - \sum_{i=1}^n \alpha_i - \sum_{i=1}^n \alpha_i \epsilon_i - \sum_{i=1}^n \beta_i \epsilon_i \\ &= \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i y_i \alpha_j y_j x_i^T x_j - \sum_{i=1}^n \alpha_i \end{aligned}$$

\therefore 软-SVM 的对偶问题为

$$\begin{cases} \max_{\alpha} \sum_i \alpha_i - \frac{1}{2} \sum_{i,j} \alpha_i \alpha_j y_i y_j x_i^T x_j \\ \sum_i \alpha_i y_i = 0 \\ C \geq \alpha_i \geq 0 \quad \forall i \end{cases}$$

2、上机实验题

(1) 读取数据

```

def load_data():
    """
    读取数据集，X共有1900个特征，训练集有4000个数据，测试集有1000个数据
    :return: train_X, train_y, test_X, test_y
    """
    train_dataFile = '垃圾邮件训练和测试数据/spamTrain.mat'
    test_dataFile = '垃圾邮件训练和测试数据/spamTest.mat'
    # 1899个特征值，4000个数据
    train_data = scio.loadmat(train_dataFile)
    train_X = train_data.get("X")
    train_y = train_data.get("y")
    # 1899个特征值，1000个数据
    test_data = scio.loadmat(test_dataFile)
    test_X = test_data.get("Xtest")
    test_y = test_data.get("ytest")
    return np.array(train_X), np.array(train_y), np.array(test_X), np.array(test_y)

```

(2) 佩加索斯算法

```

def pegasos(X, y, T=1000, lam=1):
    m, n = X.shape # m表示X的样本个数， n表示每个样本的特征个数
    # w = np.random.randn(n, 1)
    # b = np.random.randn(1)
    w = np.zeros((n, 1))
    b = 0
    for t in range(1, T + 1):
        eta = 1.0 / (lam * t)
        i = np.random.randint(m)
        p = predict(w, X[i], b)
        if y[i] * p < 1:
            w = (1.0 - 1 / t) * w + eta * y[i] * (X[i].reshape(-1, 1))
            b += eta * y[i]
        else:
            w = (1.0 - 1 / t) * w
            b = b
    return w, b

```

(3) 计算精度

```

def accuracy(w, b, X, y):
    y_predict = np.array([predict(w, x, b) for x in X])
    y_predict = np.array([1 if i > 0 else -1 for i in y_predict])
    num = 0
    for (i, j) in zip(y_predict, y):
        if i == j:
            num += 1
    acc = num / len(y) * 100
    return acc

```

(4) 保存测试集的标签和预测结果的比较

```
def save(y_test, y_predict):
    fn = "预测值与测试集标签比较.txt"
    y_predict = [[True if i > 0 else False] for i in y_predict]
    y_test = [[True if i > 0 else False] for i in y_test]
    num = 0
    with open(fn, "w") as file_obj:
        string = "(标签, 预测值)"
        file_obj.write(string + string + string + string + "\n")
        for i, j in zip(y_test, y_predict):
            num += 1
            if num % 4 == 0:
                string = str((i, j)) + '    '
                file_obj.write(string + "\n")
            else:
                string = str((i, j)) + '    '
                file_obj.write(string)
```

(5) 试验结果

 SVM ×
在测试集上精度为: 97.2, 时间为: 2.127821922302246

Process finished with exit code 0

 SVM ×
在测试集上精度为: 97.3, 时间为: 1.91546630859375

Process finished with exit code 0

 SVM ×
在测试集上精度为: 97.5, 时间为: 2.176470994949341

Process finished with exit code 0

(标签, 预测值)	(标签, 预测值)	(标签, 预测值)	(标签, 预测值)
[[True], [True]]	[[False], [False]]	[[False], [False]]	[[True], [True]]
[[True], [True]]	[[True], [True]]	[[True], [True]]	[[False], [False]]
[[True], [True]]	[[True], [False]]	[[False], [False]]	[[False], [False]]
[[False], [False]]	[[False], [False]]	[[True], [True]]	[[False], [False]]
[[False], [False]]	[[True], [True]]	[[True], [True]]	[[True], [True]]
[[False], [False]]	[[False], [True]]	[[False], [False]]	[[False], [False]]
[[True], [True]]	[[False], [False]]	[[False], [False]]	[[True], [True]]
[[False], [False]]	[[False], [False]]	[[False], [False]]	[[False], [False]]
[[False], [False]]	[[False], [False]]	[[True], [True]]	[[False], [False]]
[[True], [True]]	[[False], [True]]	[[True], [True]]	[[False], [False]]
[[True], [True]]	[[True], [True]]	[[True], [True]]	[[True], [True]]
[[True], [True]]	[[False], [False]]	[[False], [False]]	[[False], [False]]
[[True], [True]]	[[False], [True]]	[[False], [False]]	[[False], [False]]
[[True], [True]]	[[False], [False]]	[[False], [False]]	[[False], [False]]
[[False], [False]]	[[True], [True]]	[[False], [False]]	[[False], [False]]
[[False], [False]]	[[False], [False]]	[[False], [False]]	[[False], [False]]
[[True], [True]]	[[False], [False]]	[[False], [False]]	[[False], [False]]
[[True], [True]]	[[True], [True]]	[[False], [False]]	[[True], [True]]
[[True], [True]]	[[False], [True]]	[[False], [False]]	[[True], [True]]

可以看出最后在测试集上的精度为百分之九十七点多。在前八十个结果中预测错了三个。

注：具体代码和训练结果看附件