## Independent variables, y<sub>i</sub>

i			y(i)
1	$\omega_{\mathrm{A}1}$	Mass fraction of component A (solute) in lumen side	y(1)
2	$\omega_{ m A2}$	Mass fraction of component A (solute) in shell side	y(2)
3	$\omega_{\mathrm{B1}}$	Mass fraction of component B (solute) in lumen side	y(3)
4	$\omega_{ m B2}$	Mass fraction of component B (solute) in shell side	y(4)
5	$u_1$	Mass-averaged velocity of lumen-side stream	y(5)
6	$u_2$	Mass-averaged velocity of shell-side stream	y(6)
	$ ho_1$	Averaged concentration of mass-to-volume in lumen side	MassConc(1)
	$ ho_2$	Averaged concentration of mass-to-volume in shell side	MassConc(2)
	$j_{ m M,A}$	Transmembrane mass flux of component A from the shell side to lumen side	MassFlux(A)
	$j_{ m M,B}$	Transmembrane mass flux of component B from the lumen side to shell side	MassFlux(B)

## $f_i(y_i, t)$

i	lhs	rhs	Pseudo code	
1	$d\omega_{A1}$	$4~\omega_{\mathrm{A1}}j_{\mathrm{M,B}} + (1-\omega_{\mathrm{A1}})j_{\mathrm{M,A}}$	Circum(1)/CSA(1) * (y(1)*MassFlux(B)+y(3)*MassFlux(A)) /	
	dz	$\overline{d_1}$ $\rho_1 u_1$	(MassConc(1)*y(5))	
2	$d\omega_{A2}$	$4d_1  (\omega_{ ext{A2}}-1)j_{ ext{M,A}}-\omega_{ ext{A2}}j_{ ext{M,B}}$	- Circum(2)/CSA(2) * (y(4)*MassFlux(A)+y(2)*MassFlux(B)) /	
	dz	$\overline{d_2^2 - d_1^2} = \rho_2 u_2$	(MassConc(2)*y(6))	
3	$d\omega_{\mathrm{B1}}$	$4 (\omega_{\rm B1} - 1)j_{\rm M,B} - \omega_{\rm B1}j_{\rm M,A}$	- Circum(1)/CSA(1) * (y(1)*MassFlux(B)+y(3)*MassFlux(A)) /	
	dz	$\overline{d_1}$ $\rho_1 u_1$	(MassConc(1)*y(5))	
4	$\mathrm{d}\omega_{\mathrm{B2}}$	$4d_1  \omega_{ ext{B2}} j_{ ext{M,A}} + (1-\omega_{ ext{B2}}) j_{ ext{M,B}}$	Circum(2)/CSA(2) * (y(4)*MassFlux(A)+y(2)*MassFlux(B)) /	
	$\mathrm{d}z$	$d_2^2 - d_1^2 = \rho_2 u_2$	(MassConc(2)*y(6))	
5	$\mathrm{d}u_1$	$4 \left[ \rho_{\rm A}(\omega_{\rm A1} - \omega_{\rm B1}) + 2\rho_{\rm B}\omega_{\rm B1} \right] j_{\rm M,A} - \left[ 2\rho_{\rm A}\omega_{\rm A1} - \rho_{\rm B}(\omega_{\rm A1} - \omega_{\rm B1}) \right] j_{\rm M,B}$	Circum(1)/CSA(1) * (MassFlux(B)-	
	dz	$\frac{1}{d_1}$ $\rho_1^2$	MassFlux(A))/MassConc(1) -	
			y(5)/MassConc(1)*(Density(A)*rhs(1)+Density(B)*rhs(3))	
6	$du_2$	$4d_{1}  \left[ (\omega_{\rm A2} - \omega_{\rm B2}) \rho_{\rm A} + 2\omega_{\rm B2} \rho_{\rm B} \right] j_{\rm M,A} - \left[ 2\omega_{\rm A2} \rho_{\rm A} - (\omega_{\rm A2} - \omega_{\rm B2}) \rho_{\rm B} \right] j_{\rm M,B}$	Circum(2)/CSA(2) * (MassFlux(B)-	
	$\overline{\mathrm{d}z}$	$-\overline{d_2^2 - d_1^2}$ $\rho_2^2$	MassFlux(A))/MassConc(2) -	
			y(6)/MassConc(2)*(Density(A)*rhs(2)+Density(B)*rhs(4))	

i		y(i), where z = 0	
1	$\omega_{ m A1}(0) = \omega_{ m A10}$	y(1) = InitMassFrac(A,1)	
2	$\omega_{ m A2}(0) = \omega_{ m A20}$	y(2) = InitMassFrac(A,2)	
3	$\omega_{ m B1}(0) = \omega_{ m B10}$	y(3) = InitMassFrac(B,1)	
4	$\omega_{\mathrm{B2}}(0) = \omega_{\mathrm{B20}}$	y(4) = InitMassFrac(B,2)	
5	$u_1(0) = u_{10}$	y(5) = InitVelocity(1)	
6	$u_2(0) = u_{20}$	y(6) = InitVelocity(2)	

Jacobi matrix  $\partial f_i/\partial y_j$