1 Background

In this section we will sketch out the most relevant background for our model formulation. We will shortly dive in to the theory regarding household decision-making models and outline the relevant institutional framework our model is set in.

1.1 Household decision-making and time allocation

Understanding how parental leave policies influence labor market behavior requires a framework that captures the complex and dynamic decision-making processes of individuals and households. A dynamic life-cycle model offers a structured approach to analyze how economic agents make intertemporal choices regarding labor supply and human capital accumulation in the presence of policy interventions, such as parental leave reforms. At the core of life-cycle models is the notion that individuals make forward-looking decisions, balancing current utility with future outcomes. This is also the case in the context of parental leave policies, regarding labor supply decisions. Parents must weigh the immediate benefits of taking leave—such as time spent with a newborn—against potential long-term costs, like wage penalties due to interrupted career trajectories (Adda et al., 2017); (Kleven et al., 2018). The following sections outlines the key theoretical considerations that inform the construction of our model, drawing on insights from labor economics, family economics, and human capital theory.

1.1.1 Human capital theory and specialization

A defining feature of life-cycle models in labor economics is the inclusion of human capital accumulation and its effect on wage trajectories (Mincer and Polacheck, 1974). A significant portion of the theoretical framework concerning human capital accumulation and household time allocation is grounded in the work of Becker (1981). His human capital theory combines the evolution of human capital with the allocation of resources to areas where individuals have comparative advantages. The basis of Becker's theory is that individuals can enhance their productivity by investing in human capital — e.g. through education or work experience — which increases their marginal productivity and, consequently, their earnings (Becker, 1962).

Parental leave policies can disrupt or delay this accumulation, leading to potential wage penalties upon re-entry into the labor force. Studies like Eckstein and Wolpin (1989) and Keane and Wasi (2016) emphasize the importance of modeling skill depreciation during periods of non-employment, particularly for parents who take extended leave. Research suggests that a more balanced distribution of parental leave could reduce the degree of specialization in household labor traditionally undertaken by women. For instance, Ekberg et al. (2013) argue that equitable parental leave could diminish women's comparative advantage in household work, leading to a more balanced division of tasks like childcare or housework. According to human capital theory, the more time fathers spend on childcare — e.g. during paternity leave — the more they build their human capital in domestic tasks (Becker, 1981). This shift would theoretically reduce men's relative labor market participation while increasing women's.

Recent studies have moved away from the sole focus on comparative advantage. With more women participating equally in the labor market and pursuing higher education, the traditional model fails to fully explain income disparities. Cortés and Pan (2020) show that the so-called "child penalty" affects women across all education levels, even those with higher qualifications than their partners. This indicates that factors beyond comparative advantage, such as gender role expectations and societal norms, continue to pressure women into assuming the majority of household responsibilities, including parental leave.

Such findings highlight the potential of reforms that challenge traditional gender norms. Encouraging fathers to take a more significant share of parental leave could help narrow the gender wage gap. However, the effectiveness of these reforms may be limited by inherited social norms. A critical challenge is ensuring that increased paternity leave translates into sustained paternal involvement in childcare over time. Studies also find that men who take parental leave often face discrimination, with a more pronounced negative impact on their wages compared to women Albrecht et al. (1999). Earmarked paternity leave could help counteract these biases and contribute to reducing gender inequality.

Research on gender norms further emphasizes the impact of societal expectations. For example, Bertrand et al. (2013) show that traditional views on gender identity — such as the belief that men should be the primary earners — negatively affect marriage patterns, women's labor market participation, income levels, marital satisfaction, and the distribution of household work. Furthermore, studies like Kleven et al. (2023) highlight how even generous parental leave policies can reinforce traditional gender roles if social norms discourage paternal participation. These findings stress the importance of challenging existing gender norms to strengthen the wage and human capital evolution of women. Reforms to parental leave policies can play a vital role in this process by signaling that childcare is a shared responsibility. By promoting greater paternal involvement from an early stage, such policies can contribute to increased gender equality both in the household and the labor market.

1.1.2 Intra-household decision making

A lot of economic theory relates to the choices and utility of individuals. Samuelson (1956) was the first to question this approach, since a wide range of consumption and labor choices are made together in the household, and not as individual agents. This is especially true in matters regarding how to allocate time in the presence of children. Here, the institutional settings regarding parental leave are especially relevant, as this directly affect the budget set of households. Changes to the rules regarding parental leave could thus be expected to have long-term implications for choices regarding time allocation between work and leave/leisure, which in turn may affect the relative distribution of income and the degree of specialization within the household.

Early life-cycle models treated the household as a single decision-making unit with a unified utility function, ignoring individual preferences and power dynamics (Chiappori and Mazzocco, 2017); (Samuelson, 1956). Known as unitary models, these frameworks assume that household income is pooled and allocated rationally to maximize collective utility. Under this model, individual incomes and preferences have no impact on decisions, leading to specialization based solely on comparative advantage. For example, longer maternity leave might reinforce the idea that women should specialize in household work while men focus on market labor.

A newer strand of models are the collective or "bargaining" models. These frameworks recognize individual preferences and assign bargaining power to each household member based on factors like income and human capital. The negotiations in the model will typically be based on Nash bargaining models or Pareto optimality. The collective decisions can thus be seen as the result of a bargaining process, where the individuals in the household attempt to maximize their own utility while taking the other members into account. Economic and cultural power dynamics thus significantly influence decisions about time and resource allocation (Chiappori and Mazzocco, 2017). As a result, achieving equality in income and human capital development is critical for promoting fairness within households. There is broad consensus that the "child penalty" is the most significant factor contributing to the gender wage gap, as noted by Kleven et al. (2018). Yet, there is limited research on how earmarked parental leave affects women's relative income *within* the household. This relative income reflects bargaining power, as highlighted by Browning et al. (1994), and shifts in income shares — not just income levels — have the potential to drive lasting changes in gender norms and reduce inequality. In our paper we will use a unitary model to simplify computation, but a collective model could be a relevant extension to investigate the within-household power dynamics.

Parental leave policies also directly alter the constraints and incentives faced by households. Paid leave modifies the effective budget constraint by providing income during non-employment periods, while earmarking constrains the choices regarding optimal allocation of leave for couples. Theoretical models must therefore account for these constraints and how they influence behavior. Moreover, the design of leave policies — such as the length of leave, the replacement rate, and the presence of incentives for fathers (e.g., earmarking) — can lead to heterogeneous behavioral responses. Empirical findings show that policies explicitly targeted at fathers increase paternal leave uptake, suggesting that preferences and social norms interact with policy structures in shaping outcomes (Druedahl et al., 2019); (Duvander and Johansson, 2012); (Ekberg et al., 2013); (Kotsadam and Finseraas, 2011).

In our model formulation we will therefore explicitly model human capital accumulation and how this impacts wage evolutions. We will further specify exactly how different aspects of the parental leave system (e.g. replacement rates, wage compensation, and earmarking) affects the budget set of households, and we will let males and females have different preferences for taking leave, reflecting different gender norms.

1.2 Institutional framework

In the following section we will provide a simple account of the main societal structures that we incorporate into our life-cycle model. We will describe the current parental leave regulations, the institutional costs related to having children and the Danish tax system.

1.2.1 Parental leave

We seek to account for the current regulations of the Danish parental leave system, as well as the rules that were valid before the 2022 reform. Building on earlier evidence, the Danish parental leave framework before the 2022 reform offered up to 52 weeks in total, including four weeks of pregnancy leave for the mother as well as two weeks of paternity leave immediately after childbirth. The remaining weeks were shared and were given without individual earmarking, meaning families had a high degree of flexibility when dividing leave. However, as we have shown, mothers tend to take most of the available weeks. Furthermore, compensation rules - combining public parental leave benefits (which typically only replaced part of earnings) with possible employer provided wage top-ups - often influence(s) how couples choose to allocate their leave. The allocation of leave hence largely reflected prevailing norms as well as potential income trade-offs within the household. The pre-reform setup serves as our reference point for evaluating the impact of the subsequent increase in earmarked leave via the 2022 reform.

Below, we briefly outline the current parental leave regulations in Denmark in line with Beskæftigelsesministeriet (2022) following the reform, as well as key differences from before. The rules apply to parents of children born from August 2, 2022.

- **Earmarked leave for each parent**: Each parent is entitled to 24 weeks of leave after childbirth. Of these 24 weeks, 11 weeks are earmarked. Earmarked leave cannot be transferred, meaning that if a parent does not use their earmarked leave, these weeks are lost.
- **Flexibility and transfer**: Parents have the flexibility to transfer up to 13 weeks of their non-earmarked leave to each other, depending on the family's needs.
- **Possibility of postponement**: Parents can choose to postpone parts of their leave for later use, though with certain restrictions, especially for the earmarked weeks.
- **Length of leave**: The mother is entitled to 4 weeks of pregnancy leave before childbirth and 10 weeks of mandatory maternity leave after childbirth. The father or co-parent can take their 2 weeks of paternity leave within the first 10 weeks after childbirth.

The rules are depicted in Figure 1. The new regulations reflect an unchanged total entitlement to parental leave with parental benefits for parents (i.e., 52 weeks, of which 48 are after childbirth), but a redistribution of the weeks. This means that the father is now earmarked an additional 9 weeks of leave, so that 11 weeks of leave after childbirth are earmarked for both the mother and the father. The earmarked weeks cannot be transferred, meaning that if they are not taken, they are lost. The reform has thus reduced the flexibility in planning parental leave in an attempt to promote a more equal distribution of the leave. Parents are still eligible for the same amount of parental leave weeks with pay as before.

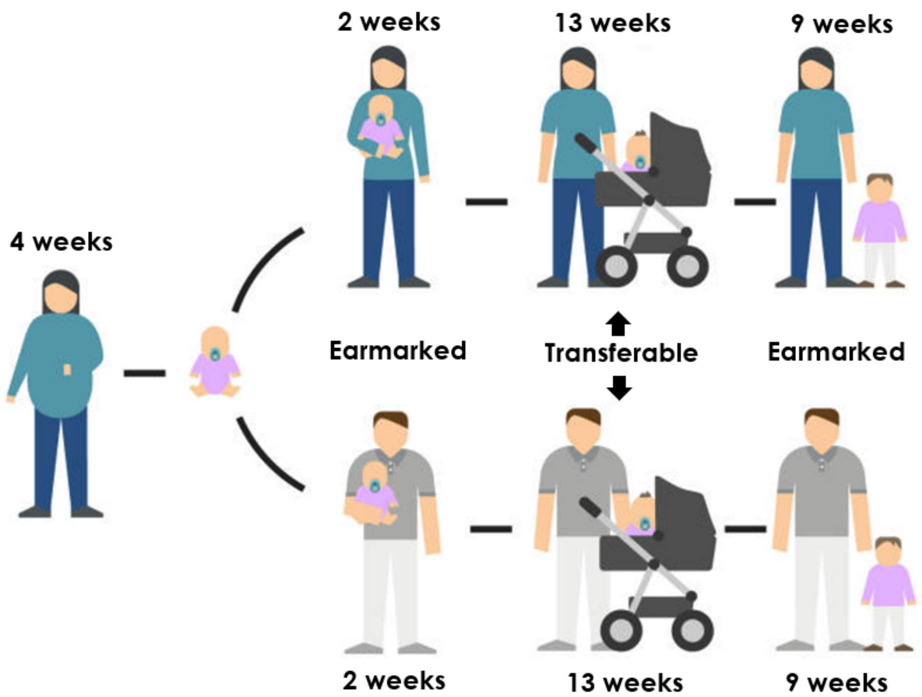

Figure 1: Current parental leave regulations (Danmarks Radio)

1.2.2 Costs of having children

Institutionsomkostninger - mangler

2 Model and Methodology

In this section, we present the formulation of our dynamic model of household labor supply and parental leave. The model is solved using dynamic programming and simulated forward for a heterogeneous sample of households.

2.1 Model formulation

We model opposite-sex couples who jointly make dynamic decisions over labor supply and parental leave. The model incorporates stochastic fertility, dynamic human capital accumulation, progressive taxation, and a parental leave system inspired by the Danish context. For simplicity we only model same age couples and couples are not allowed to divorce. The model is set in a unitary framework, in which couples jointly make optimal decisions, placing a constant pareto weight on the utility of each member in the household. We thus do not allow for intra-household bargaining.

2.1.1 Household agents and time allocation

The household consists of two members: a female (f) and a male (m). Time allocation is divided into two categories: Labor: $h_{j,t}$, $j \in \{f, m\}$, and parental leave: $\ell_{j,t}$, $j \in \{f, m\}$. The household's problem is to choose the allocation of the two agents' labor supply and parental leave. Agents jointly choose labor supply with parental leave being the residual. By doing this, we assume that all time not spent on labor is spent on parental leave, to make sure that parental leave crowds out labor supply. In the periods where the household has not conceived a child, any time spent not working (e.g. on leave) will be interpreted as leisure. We thus get the following time constraint: $\ell_{j,t} = 1 - h_{j,t}$ where parents can only obtain benefits from the allocation of time to parental leave following the birth of a child. Labor supply is a continuous choice, whereby the values of variables represent a fraction of working full time in a year. In this case, 1 represents full-time work for the duration of the period, here one year. There is no job insecurity in the model. The agents make their decisions given the state space $\Omega = (k_{f,t}, k_{m,t}, n_t)$. This includes the human capital level of both agents $k_{f,t}, k_{m,t}$, and whether they have a child, n_t.

In each period households maximize the expected discounted sum of utility from the remainder of their lives. Agents accumulate human capital in each period if they work and wages are a function of the level of human capital. A period in the model corresponds to one year, and we will model agents from age 25 to 64, thus focusing only on their working life, assuming all agents retire at the age of 65. We will consider age 25-44 the fertile period, such that agents can only conceive a child in this period. The model timeline can be seen in figure 2.

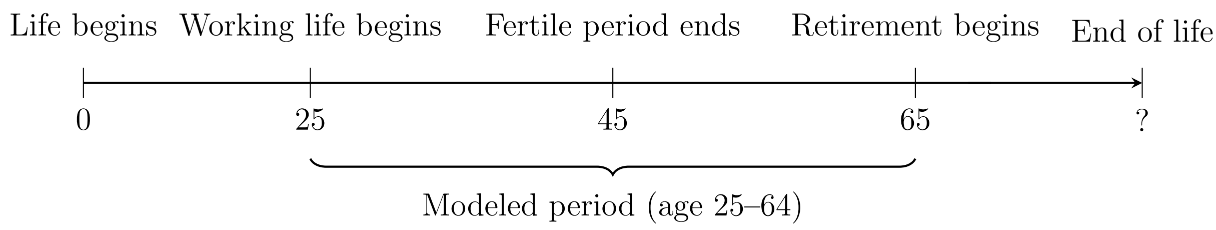

Figure 2: Model timeline. Only the working and parental leave period (ages 25–64) is modeled explicitly.

2.1.2 Children and fertility

We assume households can have at most one child. Let $n_t \in \{0, 1\}$ denote whether the couple has a child. Births occur probabilistically during the fertile period (age 25–45), with age-specific probabilities p_t:

$$p(n_t, a) = \begin{cases} p_n(a) & \text{if } n_t = 0 \text{ and } t \leq 44 - 25 \\ 0 & \text{if } n_t = 1 \end{cases}$$

Where a denotes the age of the household. We define a dummy to measure if a woman gives birth in a given period:

$$b_{t+1} = \begin{cases} 1 & \text{with prob. } p(n_t, a) \\ 0 & \text{with prob. } 1 - p(n_t) \end{cases}$$

We abstract from the option of having children moving out dynamically. The number of children hence evolve according to:

$$n_{t+1} = \begin{cases} n_t + b_{t+1} & \text{if } t \leq 44 - 25 \\ n_t & \text{otherwise} \end{cases}$$

2.1.3 Preferences

The household maximizes a unitary utility function in which they derive utility from consumption and disutility from labor supply. We will use the Constant Relative Risk Aversion (CRRA) utility function for both consumption, c_t and labor supply, $h_{j,t}$ in line with the methodology used by Keane (2011).

Preferences are the weighted sum of individual utility:

$$U(c_t, h_{f,t}, h_{m,t}) = \lambda u_f(c_t, h_{f,t}, h_{m,t}) + (1 - \lambda)u_m(c_t, h_{f,t}, h_{m,t})$$

Where u_f refers to the utility of the female agent and u_m refers to the utility of the male agent in the household. λ is thus the relative weight placed on the utility of the female in the household. Since λ is fixed we do not allow for intra-household bargaining, and instead model the unitary utility function in line with Chiappori and Mazzocco (2017). Individual utility is given by:

$$u_j(c_t, h_{j,t}) = \frac{(c_t/v_t)^{1-\rho}}{1-\rho} - \nu_i \frac{h_{j,t}^{1+\epsilon}}{1+\epsilon}$$

The parameter ν_i is a disutility scaling factor for time spent on labor. This factor is different across all agents in the model, and thus serve as a source of heterogeneity in the model. It is given by:

$$\nu_i = \nu_{0,j} + \nu_{1,j} n_t + \nu_{2,i} b_t$$

Where $\nu_{0,j}$ measure baseline disutility from working, which is just gender specific. $\nu_{1,j}$ measure gender specific disutility from work in all periods where a child is present. Lastly, $\nu_{2,i}$ is the heterogenous term, which is different across all agents, which measure disutility from working in birth periods. We assume all parameters greater than zero, such that agents face a larger disutility from working when a child is present, and possibly even larger in birth periods. The $\nu_{2,i}$ parameters are drawn at initialization from a log-normal distribution and kept fixed for all periods:

$$\nu_{2,i} \sim \log \mathcal{N}(\mu_j, \sigma_j)$$

The last parameters, $\rho > 0$ and $\epsilon > 0$, are the relative risk aversion for consumption and labor supply respectively. Consumption is scaled by v_t, which is defined as $v_t(n_t) = 1.5 + 0.3n_t$ according to OECD equivalence scales.

2.1.4 Parental leave benefits

We model parental leave benefits to match the Danish parental leave system as described in section 1.2.1. Households receive parental leave benefits based on the duration of leave taken by each agent, possibly subject to an earmarking constraint. Each parent can receive full wage compensation for a fraction of their leave, after which benefits are paid at a reduced replacement rate. The total parental leave benefits available to the household cannot exceed 48 weeks, corresponding to a 0.92 fraction of a period. We let D_j be the maximum duration for which parent j receives full wage compensation and μ_j be the earmarked parental leave for agent j, which reduces the available leave for the other parent. We let π denote the replacement rate after D_j is exceeded. The effective maximum leave that agent j can take while still receiving benefits is:

$$\ell_j^{\max} = 0.92 - \mu_{-j} - \max(0, \ell_{-j} - \mu_{-j}). \tag{2.1.1}$$

This ensures that the earmarked leave for the other agent in the household, μ_{-j}, reduces the available leave for agent j, regardless of whether it is used. If agent $-j$ takes leave exceeding their earmarked portion μ_{-j}, this further reduces the available leave for agent j.

The parental leave benefits received by agent j are thus given by:

$$PL_j(b_t, w_{j,t}, \ell_{j,t}) = \begin{cases} 0, & \text{if } b_t = 0 \\ w_{j,t}\ell_{j,t}, & \text{if } b_t = 1, \ell_{j,t} \leq D_j, \ell_{j,t} \leq \ell_j^{\max} \\ w_{j,t}D_j + \pi w_{j,t}(\ell_{j,t} - D_j), & \text{if } b_t = 1, D_j < \ell_{j,t} \leq \ell_j^{\max} \\ w_{j,t}D_j + \pi w_{j,t}(\ell_j^{\max} - D_j), & \text{if } b_t = 1 \text{ and } \ell_{j,t} \geq \ell_j^{\max} \end{cases}$$

This formulation ensures that each parent can receive full wage benefits for a period up to D_j, after which benefits are reduced according to the replacement rate π. If they exhaust their maximum period with replacement they receive zero benefits. Finally, parents can only receive benefits in a period where they have conceived a child. We implement a dynamic reallocation algorithm which ensures compliance with caps and earmarking rules.

2.1.5 Cost of child care

We introduce a monetary cost when neither parent allocates time to parental leave in a birth period. This cost reflects the household's need to arrange external child care immediately following the birth, such as formal day care, private care, or other paid services. We implement this as a penalty applied in any period where a child is born ($b_t = 1$) but the combined leave taken by the parents is less than a full period. However, unlike a flat fee, we model this cost in line with the Danish system of *økonomisk friplads*, which reduces daycare payments for lower-income families through income-dependent subsidies. We therefore let the household pay a fraction $\mu_t(Y_t)$ of the full annual cost of daycare, κ, where μ_t is defined as:

$$\mu_t(Y_t) = \begin{cases} 0 & \text{if } Y_t < Y_{\min} \\ \min\{1, \varrho_0 + \varrho_1 \cdot (Y_t - Y_{\min})\} & \text{if } Y_t \geq Y_{\min} \end{cases}$$

where Y_t is household income in period t, and $Y_{\min}$ is the income threshold for subsidy eligibility. We do not account for sibling discounts or reductions for single parents, as we only model couples with one child. The parameters ϱ_0 and ϱ_1 control the slope and starting point of the payment schedule. The total child care cost in period t is then given by:

$$\text{Cost}_{\text{no-leave}}(b_t) = \kappa \cdot \mu_t(Y_t) \cdot (1 - \ell_{f,t} - \ell_{m,t}) \cdot \mathbf{1}(b_t = 1)$$

where $\ell_{j,t} = 1 - h_{j,t}$ denotes leave, and κ is the per-period cost of external infant care. This setup ensures that the household pays a cost proportional to the amount of uncovered time dependent on their household income. If household income is sufficiently low, the cost is fully subsidized. This approach captures the structure of the Danish daycare subsidy system, where municipal payments cover up to 100% of childcare costs for low-income households, while higher-income households pay a larger share up to a capped maximum, typically 25% (The Danish Ministry of Children and Education, 2025). Rather than modeling discrete income brackets, we follow (Jakobsen et al., 2024) in using a continuous approximation that retains computational tractability while reflecting observed policy incentives.

This formulation ensures that the model captures an important trade-off: parents can avoid expenses by staying home with the newborn, but doing so comes at the cost of foregone wages and lost human capital. If neither parent takes sufficient leave, the household must accept a child care cost. The utility-maximizing behavior thus reflects a realistic substitution between market-provided and parent-provided care.

We restrict this cost to birth periods ($b_t = 1$) and do not impose any ongoing cost in later periods where $n_t = 1$. This assumption is made since dynamic extension of the cost would require detailed modeling of evolving child care subsidies and preferences over time, keeping track of the age of children. To keep the model tractable and focused on the intensive decision of who takes leave around birth, we therefore concentrate the cost only in the birth period. This matches the empirical reality that child care burdens — and gendered leave responses — are most prominent during the first months of a child's life.

2.1.6 Wages, human capital, and income

We define labor supply as a fraction of working full time in a year, such that individual income is given by the product of their full-time wage rate, $w_{j,t}$, and their labor supply choice, $l_{j,t}$. We abstract from the possibility of unemployment benefits. Additionally, if the female gives birth in a given period, the household can receive parental leave benefits dependent on their leave duration. Hence, total labor income is given as:

$$y_{j,t}(w_{j,t}, PL_{j,t}) = \sum_{j \in (f,m)} (w_{j,t} l_{j,t} + PL_{j,t})$$

Households receive parental leave benefits based on each other's leave duration based on the definition above.

Human capital, $k_{j,t}$, evolves according to:

$$k_{j,t+1} = [(1-\delta)k_{j,t} + l_{j,t}]\varepsilon_{j,t},\ j \in \{f, m\}$$

Human capital depreciates each period by a factor δ, as outlined in Keane and Wasi (2016). This depreciation reflects the value of remaining active in the labor market. When an agent is on parental leave and not working, no additional human capital is accumulated, causing it to depreciate more rapidly.

To capture unobserved heterogeneity and persistent uncertainty in earnings profiles, we introduce multiplicative shocks to human capital accumulation, captured by the parameter $\varepsilon_{j,t}$. This is a log-normal shock with mean one, following Keane and Wasi (2016) and Jakobsen et al. (2024). Specifically:

$$\log \varepsilon_{j,t} \sim \mathcal{N}\left(-\tfrac{1}{2}\sigma^2_{\varepsilon,j},\ \sigma^2_{\varepsilon,j}\right)$$

This ensures that $\mathbb{E}[\varepsilon_{j,t}] = 1$, preserving the expected level of human capital. These shocks are independent across individuals, time, and gender, and they directly affect next-period wages via the human capital channel. Wage evolves according to:

$$w_{j,t} = \alpha_{j,0} + \alpha_{j,1} k_{j,t},\ j \in \{f, m\}$$

2.1.7 Taxes

The Danish tax system operates with a relatively high and progressive tax margin, where individual earnings determine personal tax obligations. There is both a lower and an upper tax bracket alongside a labor market contribution that all earners pay. Additionally, various individual allowances and deductions affect how much each person ultimately pays. This means that the overall tax burden can differ significantly across individuals, depending on their income and deductions.

In our model we implement a tax system in line with Jakobsen et al. (2024), mimicking a simplified version of the Danish tax system. Every individual j with income $y_{j,t}$ has an after-tax income in each period, that can be calculated based on the following equations:

$$\begin{aligned}
\tau_{\max} &= \tau_l + \tau_u + \tau_c - \bar{\tau}, \\
\text{personal income} &= \left(1 - \mathbf{1}(l_{j,t} > 0)\tau_{LMC}\right) \cdot y_{j,t}, \\
\text{taxable income} &= \text{personal income} - \min\{WD \cdot y_{j,t}, \overline{WD}\}, \\
\underline{y_l} &= \underline{y} + \max\{0, \underline{y} - y_{-j,t}\}, \\
T_c &= \max\{0, \tau_c \cdot (\text{taxable income} - \underline{y})\}, \\
T_l &= \max\{0, \tau_l \cdot (\text{personal income} - \underline{y})\}, \\
T_u &= \max\{0, \min \tau_u, \tau_{\max} \cdot (\text{personal income} - \underline{y_u}), \\
\mathcal{T}_{j,t} &= \mathbf{1}(l_{j,t} > 0)\tau_{LMC} \cdot y_{j,t} + T_c + T_l + T_u,
\end{aligned}$$

Where τ_l is the tax rate in the lowest tax bracket, τ_u is the tax rate in the upper tax bracket, τ_c is the average municipality tax, $\bar{\tau}$ is the maximum tax rate, τ_{LMC} is the labor market contribution, WD is the working deductible, and $\overline{WD}$ is the maximum working deductible. $\underline{y}$ refers to the amount deductible from bottom and municipality tax, and $\underline{y_u}$ is the amount deductible from the top tax bracket.

Taxes paid on the individual level, $\mathcal{T}_{j,t}$, are thus the sum of the labor market contribution, the municipality taxes, and the taxes paid in the lower and upper tax brackets. This implies that overall taxes paid on the household level is the sum of individual tax payments:

$$\mathcal{T}_t(w_{f,t}, w_{m,t}) = \mathcal{T}_{f,t} + \mathcal{T}_{m,t}$$

2.1.8 Budget constraint

The household chooses consumption by taking into account labor income, taxes, and possibly the child care cost in birth periods, if they do not take leave. We abstract from the possibility of savings, such that households use all their income in a given period. This assumption may be quite restrictive, but it is often used in the literature regarding life-cycle labor supply of women (Eckstein and Wolpin, 1989); (Francesconi, 2002); (Keane and Wolpin, 2010); (Klaauw, 1996). The budget constraint is therefore given by:

$$c_t = y_{f,t}(w_{f,t}, PL_{f,t}) + y_{m,t}(w_{f,t}, PL_{f,t}) - \mathcal{T}_t(w_{f,t}, w_{m,t}) - \text{Cost}_{\text{no-leave}}(b_t)$$

2.1.9 Recursive formulation

The Bellman equation and the recursive formulation of the model is thus given by:

$$V_t(k_{f,t}, k_{m,t}, n_t) = \max_{h_{f,t}, h_{m,t}} U(c_t, h_{f,t}, h_{m,t}) + \beta E_t[V_{t+1}(k_{f,t+1}, k_{m,t+1}, n_{t+1}]$$

$$\text{s.t.}$$

$$c_t = y_t - \mathcal{T}_t(w_{f,t}, w_{m,t})$$

Where all variable definitions follow the aforementioned descriptions. The expectation of the next period value is dependent on the probability of giving birth, whereby the Bellman equation will be given by:

$$\begin{aligned} V_t(k_{f,t}, k_{m,t}, n_t) = \max_{h_{f,t}, h_{m,t}} & U(c_t, h_{f,t}, h_{m,t}) \\ & + \beta \left[p(n_t, a) V_{t+1}(k_{f,t+1}, k_{m,t+1}, n_{t+1} = 1,) \right. \\ & \left. + (1 - p(n_t, a)) V_{t+1}(k_{f,t+1}, k_{m,t+1}, n_{t+1} = 0) \right] \end{aligned}$$

After the age of 45, when the female is no longer assumed to be fertile, there is no uncertainty. However, for the following periods until retirement, labor and consumption choices will still depend on the presence of any children. Hence, the Bellman equation will be given by:

$$V_t(k_{f,t}, k_{m,t}, n_t) = \max_{h_{f,t}, h_{m,t}} U(c_t, h_{f,t}, h_{m,t}) + \beta V_{t+1}(k_{f,t+1}, k_{m,t+1}, n_{t+1})$$

2.2 Solving the model

Given the formulated model, we can then solve it using backwards induction. The problem of the agents are to maximize their expected discounted sum of utility. Agents are forward-looking, and we assume that they behave optimally in all periods. Therefore we start by solving the model in the last period. In this period the solution is given by maximizing the utility function, as agents do not need to take into account how their behavior affect their next period states. Hence, the problem is given by:

$$V_T(k_{f,T}, k_{m,T}, n_T) = \max_{h_{f,T}, h_{m,T}} U(c_T, h_{f,T}, h_{m,T})$$

Using the solution for the last period, we then know the expected value of discounted utility in the second-to-last period, whereby we can solve for policy functions. The problem in the second to last period will therefore be given by:

$$\begin{aligned} V_{T-1}(k_{f,T-1}, k_{m,T-1}, n_T - 1) = & \max_{h_{f,T-1}, h_{m,T-1}} U(c_{T-1}, h_{f,T-1}, h_{m,T-1}) \\ & + \beta \mathbb{E}[V_T(k_{f,T}, k_{m,T}, n_T) | c_{T-1}, h_{f,T-1}, h_{m,T-1}] \end{aligned}$$

We then continue this approach for all periods until the first, thereby getting the optimal policy functions for every time-period. We solve the model over all possible state combination. The state space is defined over the following dimensions:

- k_f, k_m: human capital levels on a non-linear grid (20 points each)
- n_t: child presence (0 or 1)
- b_t: birth event (0 or 1)
- ν_{2f}, ν_{2m}: preference types (5 grid points each)

This results in a 7-dimensional value and policy function array. Since the human capital level of each agent is a continuous variable, we solve the model on grids for human capital, where we for each agent use 20 grid points. We use a non-linear grid, which helps concentrate grid points where the policy functions exhibit the most curvature, thereby improving numerical accuracy without requiring a very large state space. We then interpolate between the grid points to find the optimal solution. Additionally, we use five grid points for the preference for taking parental leave for each parent, which we interpolate over in the simulation to allow for random preference draws. We only allow for values of labor supply to lie weakly between 0 and 1 and use the optimal solution from period t as an initial guess for period t-1.

To solve for optimal labor supply, we implement a coordinate descent algorithm, alternating golden-section searches over each spouse's labor supply in $[0,1]$ until convergence. The expected future value is computed using pre-computed expectations over the human capital shocks. To do so efficiently and accurately, we use Gauss-Hermite quadrature to discretize the log-normal distribution of shocks into $N = 10$ nodes with associated probabilities for each gender:

- Let $\{\varepsilon_f^{(i)}, p_f^{(i)}\}_{i=1}^{10}$ and $\{\varepsilon_m^{(j)}, p_m^{(j)}\}_{j=1}^{10}$ denote the quadrature nodes and weights for the female and male shocks.
- For each (k_f, k_m) grid point, we compute:

$$\mathbb{E}_\varepsilon \left[V_{t+1}(k_{f,t+1}, k_{m,t+1})\right] = \sum_{i=1}^{10}\sum_{j=1}^{10} p_f^{(i)} p_m^{(j)} \cdot V_{t+1}(k_f^{(i)}, k_m^{(j)})$$

- Where $k_j^{(i)} = [(1-\delta)k_j + h_j] \cdot \varepsilon_j^{(i)}$

This expectation is precomputed at each step of the value function iteration for all grid points and state combinations. The precomputed expected values are then used in the Bellman equation to avoid redundant computation.

To ensure tractability and speed, we use `Numba`'s `@njit` (no-Python JIT compilation) to compile performance-critical functions. In particular, the main solve loop and all inner functions are decorated with `@njit(fastmath=True)`. Where possible, we use parallelization via `prange`, enabling multithreaded execution across states and grid points. This significantly reduces runtime when solving the model for all combinations of human capital and preference heterogeneity. Since the model is solved via backward induction over a high-dimensional state space, we parallelize the innermost loops using `prange`, specifically over preference types (ν_{2f}, ν_{2m}) and human capital grid points (k_f, k_m), which span 5 and 10 grid points respectively, since these loops are fully independent. We deliberately avoid parallelization over periods, as backwards order of the solution is necessary, such that each solution depends on the previous one. The result is a full set of value and policy functions for all states and periods, stored in 7-dimensional arrays. With the solution for all possible state combinations in each time-period, we are then able to simulate agent behavior forwards.

2.3 Simulating the model

After having solved the model backwards we can now simulate behavior forwards, from age 25 (start of working live) to age 65 (retirement). Agents are heterogeneous in terms of their initial level of human capital, which we distribute between 0 and 5 following a random uniform distribution, and taking into account their preference for parental leave. This preference follow a log-normal distribution with different mean and standard deviation for male and female agents. This distribution is chosen, as we thereby do not allow for a negative preference for taking leave. We also initiate a fraction of agents with $b_t = 1$ corresponding to the age 25 fraction of mothers giving birth present in data.

We begin the simulation by drawing the preference for parental leave for each simulated agent as well as their initial human capital level. All agents are initiated with no children. We then simulate all time periods forwards using the solutions. To find the labor supply of each agent, we interpolate the policy functions over $(\nu_{2,f}, \nu_{2,m}, k_f, k_m)$ using 4D interpolation. We can then calculate the level of leave, wages, parental leave benefits, income and taxes given the model formulated in section 2. Given these choices we then store the next period values of human capital, which is updated based on realized shocks $\varepsilon_{j,t}$. Productivity shocks $\varepsilon_{j,t}$ are drawn independently from the corresponding log-normal distribution. These realized shocks introduce stochasticity into the wage and human capital paths across individuals and allow the model to generate realistic heterogeneity in life-cycle earnings and labor supply.

Lastly we evaluate if a household will give birth in the following period. We do this by drawing from a uniform distribution distributed between 0 and 1. If the draw is below the age-specific birth probability, a given household will give birth in the following period. The probability schedule reflects observed birth timing and is fixed across households. We use a simulated sample size of 10,000 households. The simulation stores full trajectories for labor supply, income, leave, taxes, and fertility outcomes for each household and period. These are used to compute average behavior and evaluate counterfactual policies. The average simulated behavior can be seen in figure ???.