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Research Article

Do Small-Mass Neutrinos Participate in Gauge Transformations?

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Neutrino oscillation experiments presently suggest that neutrinos have a small but finite mass. If neutrinos have mass, there should be a Lorentz frame in which they can be brought to rest. This paper discusses how Wigner's little groups can be used to distinguish between massive and massless particles. We derive a representation of the $SL(2,c)$ group which separates out the two sets of spinors: one set is gauge dependent and the other set is gauge invariant and represents polarized neutrinos. We show that a similar calculation can be done for the Dirac equation. In the large-momentum/zero-mass limit, the Dirac spinors can be separated into large and small components. The large components are gauge invariant, while the small components are not. These small components represent spin-1/2 non-zero-mass particles. If we renormalize the large components, these gauge invariant spinors represent the polarization of neutrinos. Massive neutrinos cannot be invariant under gauge transformations.

1. Introduction

Whether or not neutrinos have mass and the consequences of this relative to the Standard Model and lepton number are the subject of much theoretical speculation [1, 2], as well as cosmological [3–5], nuclear reactor [6, 7], and high energy experimentation [8–11]. Neutrinos are fast becoming an important component of the search for dark matter and dark radiation [12, 13]. Their importance within the Standard Model is reflected in the fact that they are the only particles which seem to exist with only one direction of chirality; that is, only left-handed neutrinos have been confirmed to exist thus far. It was speculated some time ago that neutrinos in constant electric and magnetic fields would acquire a small mass and that right-handed neutrinos would be trapped within the interaction field [14]. Additionally there are several physical problems which right-handed neutrinos might help solve [15–17]. Solving generalized electroweak models using left- and right-handed neutrinos has also been discussed [18]. Today right-handed neutrinos which do not participate in weak interactions are called "sterile" neutrinos [19]. A comprehension discussion of the place of neutrinos in the present scheme of particle physics has been given by Drewes [12].

In this paper, we use representations of the Lorentz group to understand the physical implications of neutrinos having mass. In Section 2, two-by-two representations of the Lorentz group are presented. In Section 3, the internal symmetries of massive and massless particles are derived. A representation of the $SL(2,c)$ group, which separates out the two sets of spinors contained therein, is presented in Section 4. One set of spinors is gauge dependent and represents massive particles. The other is gauge invariant and represents polarized neutrinos. In Section 5, we show how, in the large-momentum/zero-mass limit, the Dirac spinors can be separated into two components, one of which can represent a spin-1/2 non-zero-mass particle. The question of gauge invariance is then discussed. In Section 6, we discuss the zero-mass limit and gauge invariance in the Lorentz transformation framework. Some concluding remarks are made in Section 7.

2. Representations of the Lorentz Group

The Lorentz group starts with a group of four-by-four matrices performing Lorentz transformations on the four-dimensional Minkowski space of (t, z, x, y) which leaves the quantity $(t^2 - z^2 - x^2 - y^2)$ invariant. Since there are three generators of rotations and three boost generators, the Lorentz group is a six-parameter group.

Einstein observed that the Lorentz group is also applicable to the four-dimensional energy and momentum space of (E, p_z, p_x, p_y). He derived the Lorentz-covariant energy-momentum relation commonly known as $E = mc^2$. As this transformation leaves $(E^2 - p_z^2 - p_x^2 - p_y^2)$ invariant, the particle mass is a Lorentz invariant quantity.

In his 1939 paper [20], Wigner studied the symmetry properties of free particles by using operators which commute with the specified four-momentum of the particle. His "little groups" were defined to be those transformations that do not change this four-momentum. For massive particles, the little group is isomorphic to $O(3)$; indeed the $O(3)$-like little group's kinematics is well understood. Massless particles are isomorphic to the Euclidean group commonly known as $E(2)$. Wigner noted that the $E(2)$-like subgroup of $SL(2,c)$ is isomorphic to the Lorentz group of transformations [21], but the kinematics of this group is not as well established as that of the $O(3)$-like little group as there is no Lorentz frame in which a massless particle is at rest.

It is possible to construct the Lie algebra of the Lorentz group from the three Pauli spin matrices [22–25] as

$$J_i = \frac{1}{2}\sigma_i, \qquad K_i = \frac{i}{2}\sigma_i. \tag{1}$$

These two-by-two matrices satisfy the following set of commutation relations:

$$\begin{aligned} \left[J_i, J_j\right] &= i\epsilon_{ijk}J_k, \\ \left[J_i, K_j\right] &= i\epsilon_{ijk}K_k, \\ \left[K_i, K_j\right] &= -i\epsilon_{ijk}J_k, \end{aligned} \tag{2}$$

where the generators J_i represent rotations and the generators K_i represent boosts. There are six generators of the Lorentz group which satisfy the three sets of commutation relations given in (2). The Lie algebra of the Lorentz group consists of these sets of commutation relations.

These commutation relations are invariant under Hermitian conjugation; however, while the rotation generators are Hermitian, the boost generators are anti-Hermitian:

$$J_i^\dagger = J_i, \qquad \text{while } K_i^\dagger = -K_i. \tag{3}$$

Thus, it is possible to construct two representations of the Lorentz group, one with K_i and the other with $-K_i$. For this purpose, we will use the notation [24, 26, 27]

$$\dot{K}_i = -K_i. \tag{4}$$

To demonstrate that this set of generators do perform Lorentz transformations, let us consider a point X in four-dimensional space such as the Minkowskian four-vector (t, z, x, y). A Hermitian matrix of the form

$$X = \begin{pmatrix} t+z & x-iy \\ x+iy & t-z \end{pmatrix}, \tag{5}$$

with determinant

$$t^2 - z^2 - x^2 - y^2, \tag{6}$$

can be written where all the components of X are real. Indeed, every Hermitian matrix can be written this way with real components. Consider next a matrix of the form

$$G = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix}, \tag{7}$$

with four complex matrix elements, thus eight real parameters, and require that the determinant be equal to one. If

$$G^\dagger = \begin{pmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{pmatrix} \tag{8}$$

is the Hermitian conjugate of G, then

$$X' = GXG^\dagger \tag{9}$$

defines a linear transformation with real coefficients such that the determinant of X' is equal to the determinant of X. This constitutes a real Lorentz transformation. The transformation of (9) can be explicitly written as

$$\begin{aligned} &\begin{pmatrix} t'+z' & x'-iy' \\ x'+iy' & t'-z' \end{pmatrix} \\ &\quad = \begin{pmatrix} \alpha & \beta \\ \gamma & \delta \end{pmatrix} \begin{pmatrix} t+z & x-iy \\ x+iy & t-z \end{pmatrix} \begin{pmatrix} \alpha^* & \gamma^* \\ \beta^* & \delta^* \end{pmatrix}. \end{aligned} \tag{10}$$

It is important to note that the transformation of (9) is not a similarity transformation. In the $SL(2,c)$ regime, not all the matrices are Hermitian [25]. Moreover, since the determinants of G and $G^\dagger$ are one, the determinant of $GG^\dagger$ is also one. As

$$\operatorname{Tr}\left(GG^\dagger\right) = \left(\alpha\alpha^* + \beta\beta^* + \gamma\gamma^* + \delta\delta^*\right) \geq 1, \tag{11}$$

(9) is a proper Lorentz transformation [25, 28, 29].

Since the determinant of G is fixed and is equal to one, there are six independent parameters. This six-parameter group is commonly called $SL(2,c)$. As the Lorentz group has six generators, this two-by-two matrix can serve as a representation of the Lorentz group.

Likewise, the two-by-two matrix for the four-momentum of the particle takes the form

$$P = \begin{pmatrix} p_0 + p_z & p_x - ip_y \\ p_x + ip_y & p_0 - p_z \end{pmatrix} \tag{12}$$