CS M146 Midterm

Zipeng Fu

TOTAL POINTS

100 / 100

QUESTION 1

Short Questions 40 pts

1.1 True/False 21 / 21

✓ **- 0 pts all correct**

- 3 pts a incorrect

- 3 pts b incorrect

- 3 pts c incorrect

- 3 pts d incorrect

- 3 pts e incorrect

- 3 pts g incorrect

- 3 pts h incorrect

1.2 Model Evaluation 9 / 9

✓ **- 0 pts Correct**

- 3 pts One incorrect answer

- 6 pts Two incorrect answers

- 1 pts One incorrect explanation

- 2 pts Two incorrect explanations

- 0 pts Click here to replace this description.

1.3 Decision Boundaries 10 / 10

✓ **- 0 pts Correct**

- 5 pts one incorrect answer

- 10 pts Two incorrect answers

- 1 pts No mark which region is positive/negative

- 2 pts 1 minor mistake

QUESTION 2

Perceptron 20 pts

2.1 algorithm 12 / 12

- 12 pts 4 wrong answers

- 9 pts 3 wrong answers

- 6 pts 2 wrong answers

- 3 pts 1 wrong answer

✓ **- 0 pts Correct**

2.2 seperability 4 / 4

✓ **- 0 pts Correct (answer no)**

- 2 pts Answer true and show the data is linearly seperable

- 4 pts incorrect answer

The explanation is actually incorrect. The data is linearly separable. According to (1), the hyperplane is not crossing the origin even with a 2-dim weight. I give you full credit because you indicate "No".

2.3 data augmentation 4 / 4

✓ **- 0 pts Correct: either describe how to extend w and x; or provide a correct 3-d weight vector.**

- 2 pts minor error: either forget to describe how to extend either w or x; or provide an incorrect 3-d weight vector with some explanation

- 4 pts Incorrect answer: provide 2-d weight vector; or provide an incorrect 3-d weight vector with no explanation

QUESTION 3

Decision Tree 18 pts

3.1 H(Passed) 4 / 4

✓ **- 0 pts Correct**

- 2 pts minor mistake

- 4 pts incorrect

- 1 pts forget negative sign in the entropy

- 0 pts Correct formulations, but Incorrect calculation

3.2 G(passed, GPA) 4 / 4

✓ **- 0 pts Correct**

- 2 pts minor mistake

- 4 pts incorrect answer

- 1 pts tiny mistake

- 0 pts Correct formulations, but Incorrect calculation

3.3 G(passed, study) 4 / 4

✓ **- 0 pts** **Correct**

- 2 pts minor mistake

- 4 pts incorrect

- 1 pts tiny mistake

- 0 pts Correct formulation, but Incorrect calculation

3.4 Tree 6 / 6

✓ **- 0 pts** **Correct**

- 6 pts incorrect

- 2 pts split when labels are pure

- 2 pts Split on attribute with lower information gain

- 4 pts Only one split

QUESTION 4

Linear Regression 20 pts

4.1 application 6 / 6

✓ **- 0 pts** **Correct**

- 1 pts minor mistake

- 2 pts description is unclear

- 6 pts incorrect

4.2 optimization algorithm 6 / 6

✓ **- 0 pts** **Correct (GD,SGD, or analytic solution)**

- 2 pts missing or incorrect gradient

- 6 pts incorrect

- 4 pts no attempt at gradient

4.3 global optimality 8 / 8

✓ **- 0 pts** **Correct**

- 4 pts Incomplete answer, with arguements and derivation attempt

- 1 pts Almost correct (with a missing step)

- 6 pts Argues PSD or squared function

- 8 pts Incorrect

QUESTION 5

5 Name & ID 2 / 2

✓ **- 0 pts** **Correct**

- 2 pts No name and ID

CM146: Introduction to Machine Learning — Winter 2018

Midterm

Feb. 13^{th}, 2018

- This is a closed book exam. Everything you need in order to solve the problems is supplied in the body of this exam.
- This exam booklet contains **four** problems.
- You have 90 minutes to earn a total of 100 points.
- **Besides having the correct answer, being concise and clear is very important. For full credit, you must show your work and explain your answers.**

Good Luck!

Name and ID: (2 Point) Zipeng Fu 004768139

Name		/2
Short Questions		/40
Perceptron		/20
Decision Tree		/18
Regression		/20
Total		/100

Short Questions [40 points]

1. [21 points] True/False Questions (Add 1 sentence to justify your answer if the answer is "False".)

(a) When the hypothesis space is richer, over-fitting is more likely.

True

~~False, by cross validation we can pick the best hypothesis among them, so discard the ones with overfitting or underfitting~~ on different hypotheses

(b) Nearest neighbors is more efficient at training time than logistic regression.

True

(c) Perception algorithms can always stop after seeing γ^2/R^2 number of examples if the data is linearly separable, where γ is the size of the margin and R is the size of the largest instance.

False should be $\left(\frac{R}{\gamma}\right)^2$

(d) Instead of maximizing a likelihood function, we can minimizing the corresponding negative log-likelihood function.

True

(e) If data is not linearly separable, decision tree can not reach training error zero.

False, counter example: XOR function

(g) If data is not linearly separable, logistic regression can not reach training error zero.

True

(h) To predict the probability of an event, one would prefer a linear regression model trained with squared error to a classifier trained with logistic regression.

False, linear regression only ~~fits~~ labels true or false

2. [9 points] You are a reviewer for the International Conference on Machine Learning, and you read papers with the following claims. Would you accept or reject each paper? Provide a one sentence justification if your answer is "reject".

   - **accept/reject**] "My model is better than yours. Look at the training error rates!"

     reject. low training error rates may be due to the overfitting of hypothesis, so may lack generalization of

   - **accept/reject** "My model is better than yours. After tuning the parameters on the test set, my model achieves lower test error rates!"

     out-of-sample data set

     reject, test set should not be used during the training of model.

   - **accept/reject** "My model is better than yours. After tuning the parameters using 5-fold cross validation, my model achieves lower test error rates!"

     accept, (if and only if only hyperparameters are tuned)

3. [10 points] On the 2D dataset of Fig. 1, draw the decision boundaries learned by logistic regression and 1-NN (using two features x and y). Be sure to mark which regions are labeled positive or negative, and assume that ties are broken arbitrarily.

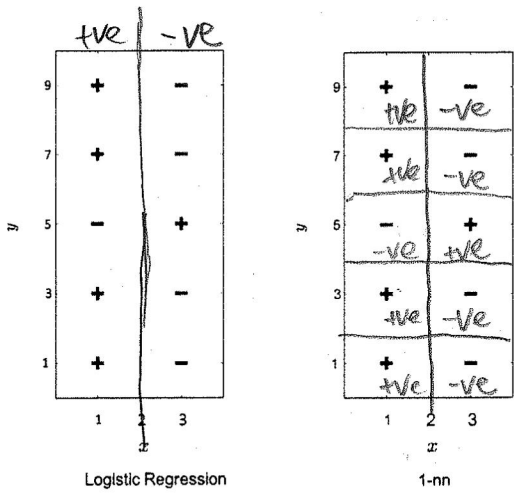


Figure 1: Example 2D dataset for question

Perceptron [20 points]

Recall that the Perceptron algorithm makes an updates when the model makes a mistake. Assume now our model makes prediction using the following formulation:

$$y = \begin{cases} 1 & \text{if } \quad w^T x \geq 1, \\ -1 & \text{if } \quad w^T x < 1. \end{cases} \tag{1}$$

1. [12 points] Finish the following Perceptron algorithm by choosing from the following options.

   (a) $w^T x_i \geq 0$ (b) $y_i = 1$ (c) $w^T x \geq 0$ and $y_i = 1$ (d) $w^T x \geq 0$ and $y_i = -1$
   (e) $w^T x_i < 0$ (f) $y_i = -1$ (g) $w^T x < 0$ and $y_i = 1$ (h) $w^T x < 0$ and $y_i = -1$
   (i) x_i (j) - x_i (k) $w + x_i$ (l) $w - x_i$
   (m) $y_i(w + x_i)$ (n) $-y_i(w + x_i)$ (o) $w^T x_i$ (p) $-w^T x_i$

   Given a training set $D = \{x_i, y_i\}_{i=1}^m$

   Initialize $w \leftarrow 0$.

   For $(x_i, y_i) \in D$:

   if d

   $w \leftarrow$ l

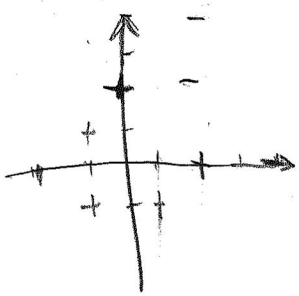
   if g

   $w \leftarrow$ k

   Return w

2. [4 points] Let w to be a two dimensional vector. Given the following dataset, can the function described in (1) separate the dataset?

Instance	1	2	3	4	5	6	7	8
Label y	+1	-1	+1	+1	+1	-1	-1	+1
Data (x_1, x_2)	(2, 0)	(2, 4)	(-1, 1)	(1, -1)	(-1, -1)	(4, 0)	(2, 2)	(0, 2)



No

Although the data are linearly separable, if w is 2D, the boundary line goes through origin. There's no way to separate whole dataset in this way.

Instance	1	2	3	4	5	6	7	8
Label y	+1	-1	+1	+1	+1	-1	-1	+1
Data (x_1, x_2)	(2, 0)	(2, 4)	(-1, 1)	(1, -1)	(-1, -1)	(4, 0)	(2, 2)	(0, 2)

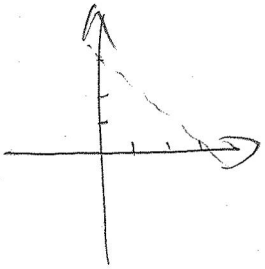
3. [4 points] If your answer to the previous question is "no", please describe how to extend w and data points x into 3-dimensional vectors, such that the data can be separable. If your answer to the previous question is "yes", write down the w that can separate the data.

Let $X_i = (1, x_{i1}, x_{i2})$

$W^T X = 1$

W is the normal vector of separating line

$W = (4, -1, -1)$



Decision Tree [18 points]

We will use the dataset below to learn a decision tree which predicts if people pass machine learning (Yes or No), based on their previous GPA (High, Medium, or Low) and whether or not they studied.

GPA	Studied	Passed
L	F	F
L	T	T
M	F	F
M	T	T
H	F	T
H	T	T

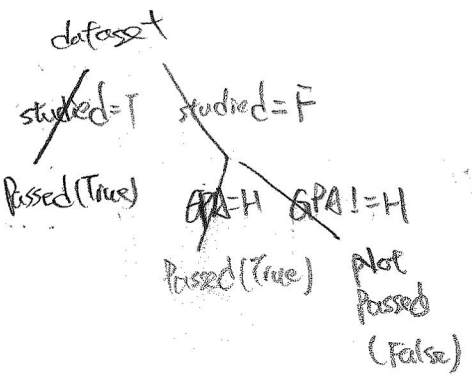
For this problem, you can write your answers using $\log_2$, but it may be helpful to note that $\log_2 3 \approx 1.6$ and entropy $H(S) = -\sum_{v=1}^{K} P(S = v) \log_2 P(S = v)$. The information gain of an attribute A is $G(S, A) = H(S) - \sum_{v \in Value(A)} \frac{|S_v|}{|S|} H(S_v)$, where S_v is the subset of S for which A has value v.

1. [4 points] What is the entropy H(Passed)?

2. [4 points] What is the entropy G(Passed, GPA)?

3. [4 points] What is the entropy G(Passed, Studied)?

4. [6 points] Draw the full decision tree that would be learned for this dataset. You do not need to show any calculations.



ID3 based on Information Gain

Linear Regression [20 points]

1. [6 points] Describe one application of linear regression. Please define clearly what are your input, output, and features.

Linear Regression can be used to predict house price based on size of house

Input : instances (house) with its size

output : label (price) of the house

feature : the price of the house

2. [6 points] Given a dataset $\{(x^{(i)}, y^{(i)})\}_{i=1}^{M}$ in a two dimensional space. The objective function of linear regression with square loss is

$$J(w_1, w_2) = \frac{1}{2}\sum_{i=1}^{M}(y_i - (w_1 x_1^{(i)} + w_2 x_2^{(i)}))^2, \tag{2}$$

where w_1 and w_2 are feature weight to be learned. Write down one optimization procedure that can learn w_1 and w_2 from data. Please be as explicit as possible.

$$\frac{\partial J}{\partial w_1} = -\sum_{i=1}^{M}\left[(y_i - (w_1 x_1^{(i)} + w_2 x_2^{(i)}))\, x_1^{(i)}\right]$$

$$\frac{\partial J}{\partial w_2} = -\sum_{i=1}^{M}\left[(y_i - (w_1 x_1^{(i)} + w_2 x_2^{(i)}))\, x_2^{(i)}\right]$$

$$\vec{\nabla} J = \left(\frac{\partial J}{\partial w_1}, \frac{\partial J}{\partial w_2}\right)$$

$k = 0$, $\mu = 0.001$

while J is not converged (same value for several iterations)

$$w_1^{(k+1)} = w_1^{(k)} - \mu\frac{\partial J}{\partial w_1} = w_1^{(k)} + \mu\sum_{i=1}^{M}\left[(y_i - (w_1^{(k)} x^{(i)} + w_2^{(k)} x_2^{(i)}))\, x_1^{(i)}\right]$$

$$w_2^{(k+1)} = w_2^{(k)} - \mu\frac{\partial J}{\partial w_2} = w_2^{(k)} + \mu\sum_{i=1}^{M}\left[(y_i - (w_1^{(k)} x^{(i)} + w_2^{(k)} x_2^{(i)}))\, x_2^{(i)}\right]$$

3. [8 points] Prove that Eq. (2) has a global optimal solution. (Full points if the proof is mathematically correct. 4 points if you can describe the procedure for proving the claim.)

Get Hessian matrix,

$$H = \begin{pmatrix} \frac{\partial J}{\partial w_1^2} & \frac{\partial J}{\partial w_1 \partial w_2} \\ \frac{\partial J}{\partial w_2 \partial w_1} & \frac{\partial J}{\partial w_2^2} \end{pmatrix} = \begin{pmatrix} \sum_{i=1}^{M} (x_1^{(i)})^2 & \sum_{i=1}^{M} x_1^{(i)} x_2^{(i)} \\ \sum_{i=1}^{M} x_1^{(i)} x_2^{(i)} & \sum_{i=1}^{M} (x_2^{(i)})^2 \end{pmatrix}$$

Then let $Z = \begin{pmatrix} z_1 \\ z_2 \end{pmatrix}$

$$Z^T H Z = (z_1 \; z_2) \begin{pmatrix} \left(\sum_{i=1}^{M} (x_1^{(i)})^2\right) z_1 + \left(\sum_{i=1}^{M} x_1^{(i)} x_2^{(i)}\right) z_2 \\ \left(\sum_{i=1}^{M} x_1^{(i)} x_2^{(i)}\right) z_1 + \left(\sum_{i=1}^{M} (x_2^{(i)})^2\right) z_2 \end{pmatrix}$$

$$= \left(\sum_{i=1}^{M} (x_1^{(i)})^2\right) z_1^2 + \left(\sum_{i=1}^{M} x_1^{(i)} x_2^{(i)}\right) z_1 z_2 + \left(\sum_{i=1}^{M} x_1^{(i)} x_2^{(i)}\right) z_1 z_2 + \left(\sum_{i=1}^{M} (x_2^{(i)})^2\right) z_2^2$$

$$= \sum_{i=1}^{M} \left[(x_1^{(i)})^2 z_1^2 + 2 x_1^{(i)} x_2^{(i)} z_1 z_2 + (x_2^{(i)})^2 z_2^2 \right]$$

$$= \sum_{i=1}^{M} \left(x_1^{(i)} z_1 + x_2^{(i)} z_2 \right)^2 \geq 0$$

semi-positive

So, $J(\vec{w})$ is convex, so the global optimal can be found by gradient descent