MAT 444 Honors Contract

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Conjecture 1. *$\mathbb{Z}_n^*$ is cyclic iff $n < 8$, $n = p^k$, or $n = 2p^k$ where p is an odd prime and $k \geq 1$.*

Proposition 1. *If $n = p^k$ where p is an odd prime and $k \in \mathbb{Z}^+$, $\mathbb{Z}_n^*$ is cyclic.*

Proof. Suppose that p is an odd prime and $k \geq 1$. We will show that $\mathbb{Z}_{p^k}^* \cong \mathbb{Z}_{p^{k-1}(p-1)}$. From Euler's Theorem, we know that $\phi(p^k) = p^{k-1}(p-1)$, so $|\mathbb{Z}_{p^k}^*| = p^{k-1}(p-1) = |\mathbb{Z}_{p^{k-1}(p-1)}|$. Let's define a function $\phi : \mathbb{Z}_{p^k}^* \to \mathbb{Z}_{p^{k-1}} \times \mathbb{Z}_{(p-1)}$ by $pq + r \mapsto (q, r)$. $\square$

Proposition 2. *If $n = 4m$ where $m > 1$, $\mathbb{Z}_n^*$*