COMPUTER SCIENCE AND ENGINEERING
Indian Institute of Technology, Palakkad
CS5016: Computational Methods and Applications

8 Feb, 2024

Coding Assignment 4

Numerical Differentiation and Integration

1. Write a program to visualize actual derivative ($f^{'}(x)$) and forward finite difference approximation ($\delta^{+}_{0.01}(x)$) of the function $\sin(x^2)$ in the internal $[0, 1]$. [10]

2. Write a program to visualize the absolute errors of approximation $\delta^{+}_{0.01}(x)$, $\delta^{-}_{0.01}(x)$ and $\delta^{c}_{0.01}(x)$ of function $\sin(x^2)$ in the internal $[0, 1]$. [10]

3. Write a program to visualize, as a function of h, the maximum absolute error of approximations $\delta^{+}_{h}(x)$ and $\delta^{c}_{h}(x)$ of function $\sin(x^2)$ in the internal $[0, 1]$. In the same figure, also plot the theoretical maximum absolute error of approximations $\delta^{+}_{h}(x)$ and $\delta^{c}_{h}(x)$. [20]

4. Write a program to visualize, as a function of M (number of intervals), area under the curve $y(x) = 2x \cdot e^{x^2}$ in the interval $[1, 3]$ computed using the trapezoidal formula. In the figure, also indicate the exact area. [20]

5. Write a program to visualize, as a function of u, area under the curve $y(x) = 2x \cdot e^{x^2}$ in the interval $[0, u]$ computed using various integration functions available in Python's `scipy.integrate` module. In the figure, also indicate the actual area under the curve. [20]

6. Enhance the class `Polynomial`, developed in the last coding assignment, as follows [10]

   - Add a method `derivative` that will return the polynomial's derivative.

     ```
     p = Polynomal([1, 2, 3])
     pd = d.derivative()
     print(pd)
     ```

     Expected output:

     ```
     Coefficients of the polynomial are:
     2 6
     ```

   - Add a method `area` that takes two arguments a and b, and returns the exact area under the polynomial in the interval $[a, b]$

     ```
     p = Polynomal([1, 2, 3])
     print(p.area(1,2))
     ```

     Expected output:

     ```
     Area in the interval [1, 2] is: 11
     ```

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7. Write a program that uses the enhanced `Polynomial` class to approximate area under the curve $y(x) = e^x \cdot \sin x$ in the interval $[0, 1/2]$ within a guaranteed error of 10^{-6}. [10]

   NOTE: Your code should not use any numerical integration techniques, and should not compute the actual area under the curve.