Software Requirements Specification for CVT Simulator

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Contents

Revision History

Date	Version	Notes
Oct 12	0.1	First Draft
Nov 12	0.2	Added references
Jan 10	0.3	Updated traceability matrices
Mar 18	0.4	Updated Standards(5.3), NFR:AC and Phase In Plan(8)
Mar 24	0.5	Updated functional requirements
Apr 04	1.0	Updated to Rev 1

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document SI (Système International d'Unités) is employed as the unit system. In addition to the basic units, several derived units are used as described below. For each unit, the symbol is given followed by a description of the unit and the SI name.

symbol	**unit**	**SI**
m	length	metre
kg	mass	kilogram
s	time	second
°C	temperature	centigrade
J	energy	joule
W	power	watt ($\mathrm{W} = \mathrm{J\,s^{-1}}$)
N	force	newtons ($\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$)
Nm	torque	newton meter ($\mathrm{Nm} = \mathrm{kg\,m^2\,s^{-2}}$)
rad	angle	radian

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made to be consistent with the heat transfer literature and with existing documentation for solar water heating systems. The symbols are listed in alphabetical order.

symbol	**unit**	**description**
A	m^2	area
a	$\mathrm{m/s}^2$	acceleration
a_{belt}	$\mathrm{m/s}^2$	belt acceleration
C_{d}		drag coefficient
C_{rr}		rolling resistance coefficient
d_{center}	m	center to center distance
d_{prim}	m	primary spring pre-compression distance
d_{sec}	m	secondary spring pre-compression distance
d_{shift}	m	shift distance

F	N	force
F_{c}	N	centrifugal force
F_{d}	N	air resistance force
F_{f}	N	friction force
F_{FW}	N	flyweight force
F_{g}	N	gravitational force
F_{H}	N	Helix side force
F_{n}	N	normal force
$F_{\mathrm{prim\_clamp}}$	N	primary spring clamp force
F_{rr}	N	rolling resistance force
F_{S}	N	spring side force
$F_{\mathrm{sec\_clamp}}$	N	secondary spring clamp force
f_{engine}	N m	engine torque function
$f_{\mathrm{prim\_angle}}$	rad	primary ramp geometry function
$f_{\mathrm{prim\_height}}$	m	primary ramp height function
$f_{\mathrm{sec\_ramp}}$	rad	secondary ramp geometry function
$f_{\mathrm{sec\_shift}}$	rad	secondary ramp rotation function
g	m/s^2	acceleration due to gravity
h_{belt}	m	belt height
h_{prim}	m	primary ramp output height
KE	J	kinetic energy
k	N/m	spring rate
k_{prim}	N/m	primary spring rate
$k_{\mathrm{sec\_comp}}$	N/m	secondary compression spring rate
$k_{\mathrm{sec\_tor}}$	N m/rad	secondary torsional spring rate
l_{belt}	m	belt length
m	kg	mass
m_{belt}	kg	belt weight
m_{d}	kg	driver weight
m_{fly}	kg	flyweight mass
m_{sheaves}	kg	sheave mass
m_{v}	kg	vehicle weight
P	W	power
R_{CVT}		CVT ratio
R_{gear}		gearbox reduction ratio

r	m	radius
r_{fly}	m	flyweight radius
r_{gear}	m	gear radius
$r_{\text{prim\_shaft}}$	m	primary shaft initial radius
$r_{\text{sec\_shaft}}$	m	secondary shaft initial radius
$r_{\text{sec\_ramp}}$	m	secondary ramp radius
t	s	time
V	m^3	volume
v	m/s	velocity
w_{belt}	m	belt width
β		helix angle
μ_{air}		air resistance coefficient
μ_{belt}		belt friction coefficient
μ_{s}		static friction coefficient
μ_{traction}		traction coefficient
ω	rad/s	angular velocity
ω_{engine}	rad/s	engine angular velocity
ϕ_{sheave}	rad	angle between sheaves
ρ	kg/m^3	density
τ	N m	torque
τ_1	N m	first side torque
τ_2	N m	second side torque
τ_{CVT}		CVT torque
τ_{eng}	N m	engine torque
τ_{load}	N m	load torque
τ_{input}	N m	input torque
τ_{output}	N m	output torque
θ_{inc}	rad	angle on incline
θ_{prim}	rad	primary ramp output angle
θ_{sec}	rad	secondary spring pre-torsional rotation
θ_{shift}	rad	secondary shift rotation
θ_{ramp}	rad	secondary ramp output angle

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
COMP	Compression
DD	Data Definition
ENG	Engine
GD	General Definition
GS	Goal Statement
FW	Flyweight
IM	Instance Model
INC	Incline
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
TM	Theoretical Model
TOR	Torsion
RPM	Revolutions Per Minute
CVT	Continuous Variable Transmission
SEC	Secondary
PRIM	Primary

1.4 Mathematical Notation

N/A

2 Introduction

The McMaster Baja engineering team is facing challenges in tuning their Continuous Variable Transmission (CVT). The current process of tuning the CVT is extensive, time-consuming and requires testing of multiple physical components leading to possible inconsistencies due to weather and wear. These inconsistencies then complicate the tuning process of the CVT's torque transfer directly affecting the performance of the Baja vehicles. To address these issues, our team aims to develop a CVT simulation tool that uses mathematical models, a rendering engine and a user-friendly interface. Our solution aims to streamline the optimization process allowing Baja members to simulate real-world factors and virtually test various tuning parameters. Our simulation tool will be validated against data collected by the McMaster Baja Data Acquisition team to ensure reliability and accuracy.

This introduction will outline the key objectives of this document to provide a structured guide for the development of the CVT tool and outline of the system's requirements. Additionally, the scope will explore technical and functional assumptions and will intend to streamline the complex tuning process of a CVT. A detailed overview of this document will provide guidance to the reader regarding the structure and flow of the document.

2.1 Purpose of Document

The purpose of this Software Requirements Specification (SRS) is to provide an outline of the system requirements given for the development of a CVT simulation tool. This ensures that all stakeholders including members of the Baja team, Dr. Smith and the developers understand the projects objectives and constraints. The document will serve as a communication tool to align expectations, provide guidance during the design of this tool and aid in the development and testing phases of this project. This SRS will be referenced throughout the development of the system to ensure the tool is within the scope and meets the outlined requirements.

2.2 Scope of Requirements

This model will simulate as a closed system and will not consider environmental factors such as temperature, humidity, and wind. It also will not consider any unintended damage, forces or effects on the components of the CVT system. The system will not account for any relative displacement between the components of the CVT system due to driver input or external forces. Further, the system will be delimited to 2 spatial dimensions, being forwards/backwards and up/down, and will not consider any lateral movement of the vehicle.

2.3 Characteristics of Intended Reader

Readers or reviewers of this SRS document should have a solid understanding of the principles of Newtonian physics and basic calculus. Specifically, it is recommended that they have completed courses in first-year mechanics, Calculus I, Calculus II, and Calculus III. Knowledge of the components in a Continuous Variable Transmission (CVT) and a basic understanding of how these parts work together will also be beneficial. Proficiency in these areas will be sufficient to comprehend the material in this document.

2.4 Organization of Document

This document is structured as follows:

- Section 3 discusses the general context and description of the system
- Section 4 details the specific system description, goals, and definitions
- Section 5 covers the system requirements
- Section 6 outlines the likely changes for the system
- Section 7 discusses the unlikely changes for the system
- Section 8 covers the traceability of the requirements

3 General System Description

This section provides general information about the system. It identifies the interfaces between the system and its environment, describes the user characteristics and lists the system constraints.

3.1 System Context

Figure 1 shows the system context. A circle represents an external entity outside the software. A rectangle represents the software system itself (CVT Simulator). Arrows are used to show the data flow between the system and its environment.

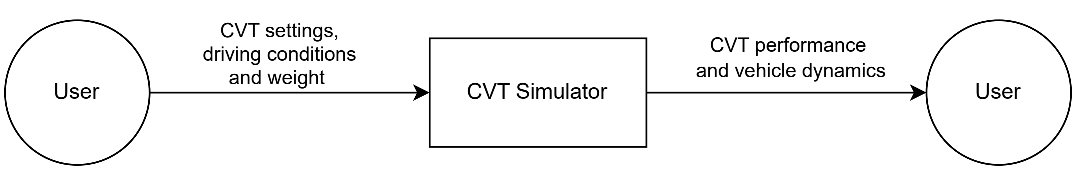

Figure 1: System Context

- User Responsibilities:
  - Provide the input data related to the tunable components of the CVT system.
  - Provide the input data related to the vehicle and driver, as well as the limited environment.
- CVT Simulator Responsibilities:
  - Accept user input data to simulate the CVT system, validating against set constraints.
  - Calculate the output data based on the input data and the CVT system model.
  - Display the results of the simulation.

3.2 User Characteristics

The expected user of this application meets the following criteria:

- Familiarity with drivetrains and CVTs.
- Completed a secondary school education, completing final year calculus and advanced functions classes.
- Basic understanding of physics and calculus.
- Ability to navigate a GUI and input data into the software.

Additional criteria, which may provide the user with more context to the data outputted, includes:

- First-year mechanics.
- Calculus I.
- Calculus II.
- Calculus III.

3.3 System Constraints

The system should be able to run on any machine that supports Python 3.9 or later as well as run basic Unity projects.

4 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models.

4.1 Problem Description

The CVT simulator is intended to address the challenges faced by the McMaster Baja racing team when they tune there CVT. The current tuning process is time-consuming and requires physical testing of multiple components and configurations. This leads to inconsistencies in the tuning process due to weather and wear. The CVT simulator will allow the Baja team to virtually test different tuning parameters and simulate real-world factors that affect the CVT's performance.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- RPM - Revolutions Per Minute, a measure of rotational speed.
- CVT - Continuous Variable Transmission
- Torque - The rotational force produced by the engine or seen as load by the wheels.
- Primary RPM - The RPM of the primary CVT.
- Secondary RPM - The RPM of the secondary CVT.
- Belt - The rubber V-belt that connects the primary and secondary CVT.
- Primary ramp - The ramp that the fly weights in the primary CVT push against. Highlighted green in Figure 2.
- Flyweight - The weights that experience centrifugal force and push against the primary ramp. Highlighted yellow in Figure 2.
- Helix - The helical ramp that the secondary CVT pushes against. Highlighted bronze in Figure 4.
- CVT System - This is the entire system that includes the primary and secondary CVT and the belt. The primary CVT is connected to the engine and the secondary CVT is connected to the wheels.
- Sheave - This is a grooved pulley used in the CVT system to transfer power between the engine and drivetrain. The Primary Pulley has both movable and fixed sheaves that adjust based on engine RPM, and the belt position to increase or decrease the gear ratio. The Secondary Pulley has movable and fixed sheaves, which respond to the belt's tension and centrifugal force, transferring power to the drivetrain. These sheaves are the circular disk shaped in 2 and 4.

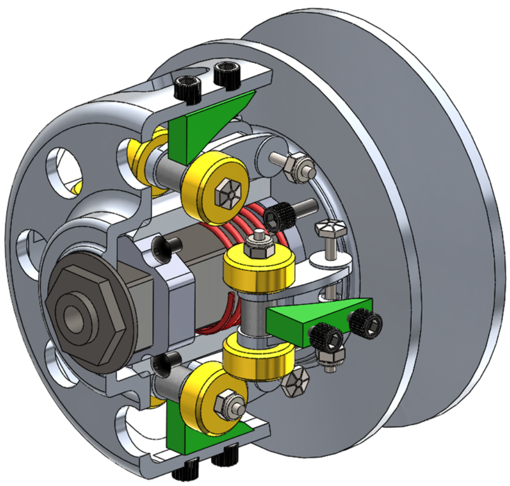

Figure 2: Primary CVT

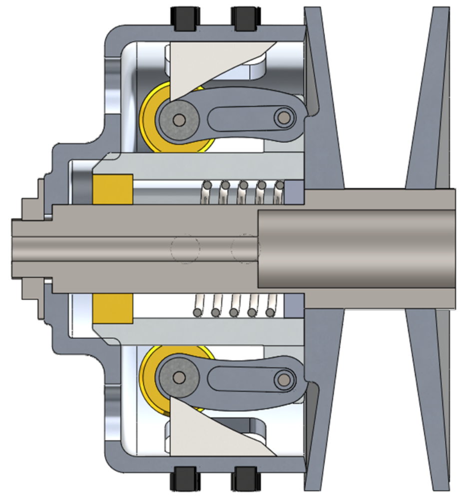

Figure 3: Primary CVT Side View

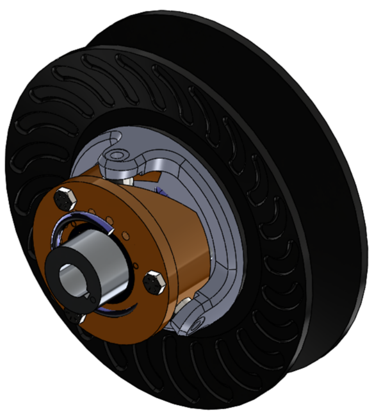

Figure 4: Secondary CVT

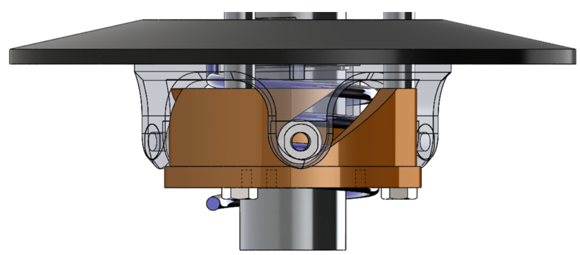

Figure 5: Secondary CVT Side View

Note: While the McMaster Baja Team's CVT differs slightly from these figures, its mechanics remain widely equivalent.

4.1.2 Physical System Description

The physical system of CVT Simulator, as shown in Figures 2-5, includes the following elements:

PS1: Flyweights

PS1a: mass (m_{fly}) in kg.

PS1b: initial radius (r_{fly}) in m.

PS2: Primary spring

PS2a: spring constant (k_{prim}) in N/m.

PS2b: pre-compression distance (d_{prim}) in m.

PS3: Primary ramp geometry function ($f_{\mathrm{prim\_angle}}$)

PS3a: shift distance (d_{shift}) in m.

PS3b: output angle (θ_{prim}) in radians.

PS4: Primary ramp height function ($f_{\mathrm{prim\_height}}$)

PS4a: shift distance (d_{shift}) in m.

PS4b: output height (h_{prim}) in m.

PS5: Secondary spring

PS5a: compression spring constant ($k_{\mathrm{sec\_comp}}$) in N/m.

PS5b: torsional spring rate ($k_{\mathrm{sec\_tor}}$) in Nm/rad.

PS5c: pre-compression distance (d_{sec}) in m.

P55d: pre-torsional rotation (θ_{sec}) in radians.

PS6: Secondary ramp geometry function ($f_{\mathrm{sec\_ramp}}$)

PS6a: shift distance (d_{shift}) in m.

PS6b: output angle (θ_{ramp}) in radians.

PS7: Secondary ramp rotation function ($f_{\mathrm{sec\_shift}}$)

PS7a: shift distance (d_{shift}) in m.

PS7b: output angle (θ_{shift}) in radians.

PS8: Weight

PS8a: driver weight (m_{d}) in kg.

PS8b: vehicle weight (m_{v}) in kg.

PS9: Engine torque function (f_{engine})

PS9a: input angular velocity (ω_{engine}) in rad/s.

PS9b: output torque (τ_{eng}) in Nm.

PS10: Belt

PS18a: belt length (l_{belt}) in m.

PS18b: belt width (w_{belt}) in m.

PS18c: belt height (h_{belt}) in m.

PS18d: belt weight (m_{belt}) in kg.

PS10b: friction coefficient (μ_{belt}), unitless.

PS11: Total reduction

PS11a: gearbox reduction ratio (R_{gearbox}), unitless.

PS11b: wheel radius (r_{wheel}) in m.

PS12: Coefficient of air resistance (μ_{air}), unitless.

PS13: Center to Center distance (d_{center}) in m.

PS14: Angle on Incline (θ_{inc}) in radians.

PS15: Secondary Ramp Radius ($r_{\text{sec\_ramp}}$) in m.

PS16: Traction (μ_{traction}), unitless.

PS16: CVT Geometry

PS16a: Primary shaft initial radius ($r_{\text{prim\_shaft}}$) in m.

PS16b: Secondary shaft initial radius ($r_{\text{sec\_shaft}}$) in m.

PS17: Angle between sheaves (ϕ_{sheave}) in radians.

4.1.3 Goal Statements

GS:IF : Given the material properties, initial conditions, and boundary conditions, predict the kinematics of the vehicle as a function of time, including its position, velocity, acceleration, and orientation.

GS:E : Given the engine characteristics, load torque and time simulate the output of the engine as a function of time, predicting engine torque and angular velocity.

GS:F : Given the flyweight specifications, spring coefficients, torque, input parameters, material properties, simulate the Continuous Variable Transmission (CVT) system over time, predict the output such as clamping forces, sheave acceleration, belt acceleration and system response to varying loads and engine outputs.

4.2 Solution Characteristics Specification

Figure 6 outlines the key components and interactions within the system's architecture, including theoretical models, general definitions, data definitions, assumptions, and instanced models. This specification defines the relationships between these elements, showing how they may reference, refine, or depend on each other.

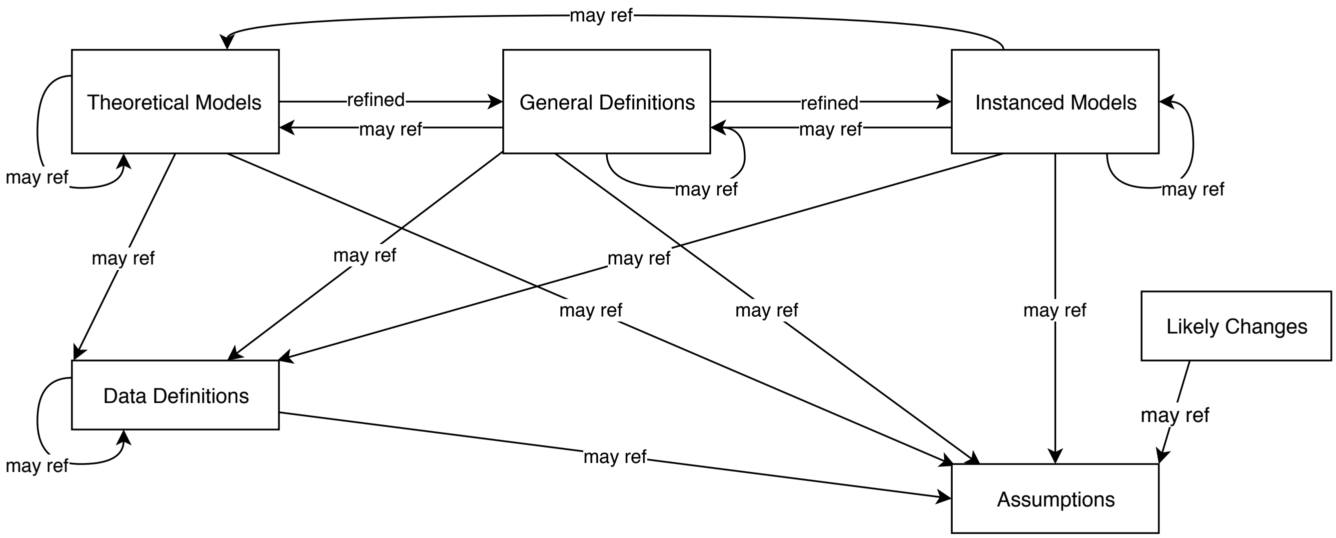

Figure 6: Solution Characteristics

The instance models that govern CVT Simulator are presented in Subsection 4.2.9. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

4.2.1 Types

N/A

4.2.2 Scope Decisions

N/A

4.2.3 Modelling Decisions

N/A

4.2.4 Assumptions

A:NT : **No Material Variations By Temperature** Ignore the effect of temperature on the material properties, excluding the belt.

A:NG : **Negligible Gravitational Influence**: The orientation of the CVT system and gravitational effects are considered negligible due to the high rotational speeds involved in the system's operation.

A:RB : **Rigidity of Belts**: Elasticity of belts are assumed negligible through its length, and the components are considered ideally rigid under normal operating conditions. The belt may still rotate about the pulleys.

A:NV : **No Vibrational Effects**: Vibrational effects from the vehicle or external environment are assumed to be minimal and are not factored into the transmission's operation in this model.

A:WT : **Ignore Wear and Tear**: Component wear over time, including belt/chain wear and pulley degradation, is assumed negligible in the short-term operational model.

A:NF : **No Frictional Losses in Non-critical Components**: Frictional losses in non-critical components (e.g., bearings, shafts) are assumed negligible, focusing only on friction relevant to the CVT system's core mechanics.

A:FT : **Full Throttle Consistent Power Engine:** The engine is modeled to operate at full throttle with consistent power curves and peak performance, maintaining a uniform response throughout the analysis. Load feedback will be integrated to adjust RPM and power output as necessary.

A:AA : **Perfect Axis Alignment:** The CVT's axes are aligned with one another.

A:NS : **No Belt Slippage:** The CVT will not slip relative to the sheaves.

A:IF : **Ignore Irrelevant Forces:** Only forces that directly contribute to the car's dynamics or the CVT's shifting behaviors are considered, allowing the simplification of dimensions where applicable.

A:PV : **Positive Velocity Only:** The model assumes that the vehicle will only move forward, meaning that velocity will never be negative. Any dynamics related to reversing or backward movement are not considered in this analysis.

4.2.5 Theoretical Models

This section focuses on the general equations and laws that CVT Simulator is based on.

General Theories

This portion will specify the existing theories that are used in the development of the CVT simulation tool.

RefName: **TM:HL**

Label: Hooke's Law (Compressional)

Equation: $F = k\Delta x$

Description: F is the force (N)
k is the spring constant (N/m)
Δx is the displacement (m)

Notes: In the context of this project Hooke's law is used in both a torsional and linear way.

Source: Encyclopaedia Britannica (2024)

Ref. By: IM:PCF, IM:SCF

Preconditions for TM:HL: None

Derivation for TM:HL: Not Applicable

RefName: **TM:CF**

Label: Centrifugal Force

Equation: $F_c = m\omega^2 r$

Description: F is the force (N)
m is the mass (kg)
ω is the angular velocity (rad/s)
r is the radius (m)

Notes: None.

Source: Bogna Szyk and Steven Wooding (2024)

Ref. By: IM:PCF

Preconditions for TM:CF: None

Derivation for TM:CF: Not Applicable

RefName: **TM:AR**

Label: Air Resistance

Equation: $F_d = \frac{1}{2}\rho v^2 C_d A$

Description: F is the force (N)
ρ is the density of the fluid (kg/m^3)
v is the velocity of the object (m/s)
C_d is the drag coefficient
A is the cross-sectional area of the object (m^2)

Notes: None.

Source: SoftSchools (2020)

Ref. By: IM:A

Preconditions for TM:AR: None

Derivation for TM:AR: Not Applicable

RefName: **TM:ET**

Label: Engine Torque

Equation: $\tau = \frac{P}{\omega}$

Description: τ is the torque (Nm)
P is the power (W)
ω is the angular velocity (rad/s)

Notes: None.

Source: 2024 Power Test, LLC (2024)

Ref. By: IM:A, IM:SCF, IM:SA, IM:BA

Preconditions for TM:ET: None

Derivation for TM:ET: Not Applicable

RefName: **TM:GR**

Label: Gearing

Equation: $\tau_{\text{output}} = \tau_{\text{input}} \times R_{\text{gear}}$

Description: τ_{output} is the output torque (Nm)
τ_{input} is the input torque (Nm)
R_{gear} is the gear reduction (unitless)

Notes: None.

Source: Brain (2023)

Ref. By: IM:A, IM:SCF

Preconditions for TM:GR: None

Derivation for TM:GR: Not Applicable

RefName: **TM:FR**

Label: Static Friction Formula

Equation: $F_f = \mu_s F_n$

Description: μ_s is the coefficient of friction (unitless)
F_f is the force of friction (N)
F_n is the normal force (N)

Notes: None.

Source: Wikipedia contributors (2024)

Ref. By: IM:A

Preconditions for TM:FR: None

Derivation for TM:FR: Not Applicable

RefName: **TM:DE**

Label: Density

Equation: $\rho = \frac{m}{V}$

Description: ρ is the density ($\mathrm{kg/m^3}$)
m is the mass (kg)
V is the volume ($\mathrm{m^3}$)

Notes: None.

Source: The Editors of Encyclopaedia Britannica (2024)

Ref. By: IM:A

Preconditions for TM:DE: None

Derivation for TM:DE: Not Applicable

RefName: **TM:CE**

Label: Capstan Equation

Equation: $\tau_1 = \tau_2 e^{\mu\theta}$

Description: τ_1 is the tension on the first side of the belt (N)
τ_2 is the tension on the second side of the belt (N)
μ is the coefficient of friction
θ is the angle of wrap of the belt (radians)

Notes: None.

Source: Hackaday (2021)

Ref. By: IM:BA

Preconditions for TM:CE: None

Derivation for TM:CE: Not Applicable

RefName: **TM:N2**

Label: Newtons 2nd Law

Equation: $F = ma$

Description: F is the force (N)
m is the mass (kg)
a is the acceleration (m/s^2)

Notes: Newtons 2nd law is used repeatedly in several Instance models

Source: The Physics Classroom - Newton's Second Law (2024)

Ref. By: IM:A, IM:SA, IM:BA

Preconditions for TM:N2: None

Derivation for TM:N2: Not Applicable

RefName: **TM:HT**

Label: Hooke's Law (Torsional)

Equation: $F = \frac{k \Delta\theta}{r}$

Description: F is the force (N)
k is the torsional spring constant (Nm/rad)
$\Delta\theta$ is the rotation (rad) r is the radius of the applied torque (m)

Notes: None.

Source: Dr. Templeman (2024)

Ref. By: IM:SCF

Preconditions for TM:HT: None

Derivation for TM:HT: Not Applicable

RefName: **TM:KE**

Label: Kinetic Energy

Equation: $KE = \frac{1}{2}mv^2$

Description: KE is the kinetic energy (J)
m is the mass (kg)
v is the velocity (m/s)

Notes: None.

Source: The Physics Classroom - Kinetic Energy (2024)

Ref. By: IM:A

Preconditions for TM:KE: None

Derivation for TM:KE: Not Applicable

RefName: **TM:GE**

Label: Gavitational Potential Energy

Equation: $GE = mgh$

Description: GE is the gravitational potential energy (J)
m is the mass (kg)
g is the gravitational constant (9.81 m/s^2)
h is the height (m)

Notes: None.

Source: Online Math Learning (n.d.)

Ref. By: IM:A

Preconditions for TM:GE: None

Derivation for TM:GE: Not Applicable

Helper Theories

This portion will define auxiliary theories to aid in further derivation below.

RefName: **TM:D2R**

Label: Shift Distance to Pulley Radius

Equation: $\Delta r = \frac{d_{\text{shift}}}{2\tan(\frac{\phi}{2})}$

Description: d_{shift} is the axial shift distance, or sheave displacement (m)
Δr is the change in effective radius (m)
ϕ is the angle between sheaves (rad)

Notes: Derived based on geometry of the sheaves and belt

Source: See below for derivation

Ref. By: IM:PCF, IM:SCF

Preconditions for TM:D2R: None

Derivation for TM:D2R: Not Applicable

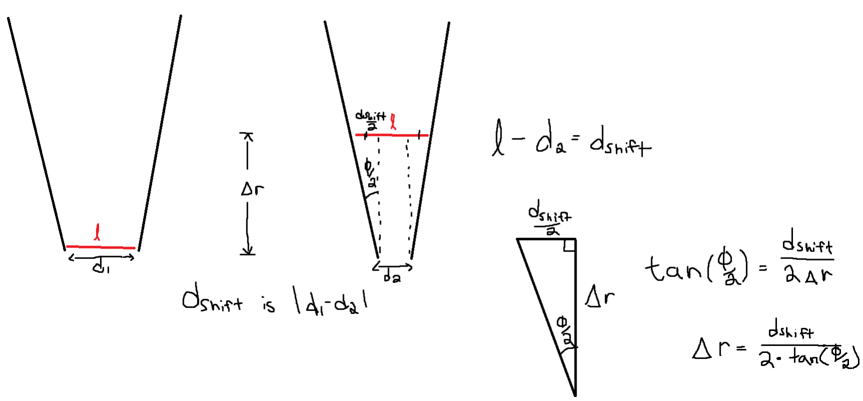

Figure 7: Sheave Geometry

RefName: **TM:D2RA**

Label: Shift Distance to Ramp Angle

Equation: $\theta_{\text{ramp}} = f_{\text{prim}}(d_{\text{shift}})$

Description: d_{shift} is the axial shift distance, or sheave displacement (m)
θ_{ramp} is the instantaneous ramp angle (rad)
$f_{\text{prim}}(d)$ is the function to express the ramp geometry (m $\rightarrow$ rad)

Notes: The user will input the geometry of the ramp, which will then be interpreted to generate f_{prim}

Source: N/A

Ref. By: IM:PCF, IM:SCF

Preconditions for TM:D2RA: None

Derivation for TM:D2RA: Not Applicable

4.2.6 General Definitions

RefName	**GD:HSF**
Label	**Helix Side Force**
SI Units	$\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$
Equation	$F_H = \frac{\tau_{\text{eng}} R_{\text{CVT}}}{2r\tan(\beta)}$
Description	F_H is the side force due to the helix (N) τ_{eng} is the engine torque (N-m) R_{CVT} is the CVT ratio (unitless) r is the radius of the ramps from the shaft (m) β is the helix angle (rad)
Source	-
Ref. By	IM:SCF

Derivation of Helix Side Force

The helix side force can be derived from using the torque across the CVT system, then by dividing out the radius to get a force.

$$\tau_{\text{CVT}} = \tau_{\text{eng}} \cdot R_{\text{CVT}} \qquad \text{(see TM:GR)}$$

Since this is a torque, we can convert it to a force by dividing by the radius of the ramps. Further, since the ramps are angled at β, we divide by $\tan(\beta)$ to get the force in the direction of the helix (see A:IF). Finally, since only one of the two pulleys is moveable, and the force is acting on both, we must divide by a factor of 2.

$$F_H = \frac{\tau_{\text{eng}} R_{\text{CVT}}}{2r\tan(\beta)}$$

RefName	**GD:SSSF**
Label	**Secondary Spring Side Force**
SI Units	$\mathrm{N} = \mathrm{kg\,m\,s^{-2}}$
Equation	$$F_S = \frac{k_{\text{tor}}(\theta_0 + \theta)}{2r\tan(\beta)} + k_{\text{comp}}(x_0 + x)$$
Description	F_S is the side force due to the spring (N) k_{tor} is the torsional spring constant (N m/rad) θ_0 is the initial torsion (rad) θ is the amount of angular torsion (rad) k_{comp} is the compression spring constant (N/m) x_0 is the initial compression (m) x is the amount of compression (m) r is the radius of the ramps from the shaft (m) θ is the amount of angular torsion (rad)
Source	-
Ref. By	IM:SCF

Derivation of Spring Side Force

This portion of the secondary side force can be split into two components, the torsional spring force and the compression spring force. Beginning with the compression spring force, we can use Hooke's Law to derive:

$$F_{\text{comp}} = k_{\text{comp}} \cdot x \qquad \text{(see TM:HL)}$$

Here, the displacement x is the change in the compression of the spring added with the precompression based on the tune.

$$F_{\text{comp}} = k_{\text{comp}} \cdot (x_0 + x) \qquad \text{(see TM:HL)}$$

Next, we can derive the torsional spring force. We can use Hooke's Law (Torsional) to derive:

$$\tau_{\text{tor}} = \frac{k_{\text{tor}} \cdot \theta}{r} \qquad \text{(see TM:HT)}$$

Similarly, we have an initial torsional compression θ_0 that is added to the current torsional compression θ.

$$F_{\text{tor}} = \frac{k_{\text{tor}} \cdot (\theta_0 + \theta)}{r}$$

Since this force is acting against the helix's angle, we must divide by $\tan(\beta)$ to get the force in the direction of the helix. Finally, since only one of the two pulleys is moveable, and the force is acting on both, we must divide by a factor of 2.

$$F_S = \frac{k_{\text{tor}}(\theta_0 + \theta)}{2r\tan(\beta)} + k_{\text{comp}}(x_0 + x)$$

Combining the two forces, we get the total side force acting on the secondary CVT created by the spring.

$$F_S = \frac{k_{\text{tor}}(\theta_0 + \theta)}{2r\tan(\beta)} + k_{\text{comp}}(x_0 + x)$$

RefName	**GD:AVE**
Label	**Angular Velocity of the Engine**
SI Units	rad/s
Equation	$$\omega_{\text{eng}} = \frac{v_{\text{car}} \cdot R_{\text{CVT}} \cdot R_{\text{gearbox}}}{r_{\text{wheel}}}$$
Description	ω_{eng}: Angular velocity of the engine v_{car}: Velocity of the car R_{CVT}: Ratio of the CVT R_{gearbox}: Gearbox ratio Due to the no slip assumption A:NS, the engine velocity is directly correlated to the speed of the vehicles through the gearbox ratio and CVT. Since the engine torque at each angular velocity is pre-defined, we can then get these at each point.
Source	-
Ref. By	IM:PCF

4.2.7 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

RefName	**DD:F**
Label	**Coefficient of Friction**
Symbol	μ
SI Units	unitless
Equation	$F_f = \mu F_n$
Description	μ is the coefficient of friction F_f is the force of friction F_n is the normal force
Sources	None.
Ref. By	IM:A

RefName	**DD:CSC**
Label	**Compressionional Spring Constant**
Symbol	k
SI Units	N/m
Equation	$F = k\Delta x$
Description	k is the spring constant (N/m) F is the force (N) Δx is the displacement (m)
Sources	None.
Ref. By	IM:PCF

RefName	**DD:TSC**
Label	**Torsional Spring Constant**
Symbol	k
SI Units	$\mathrm{N\,m/rad}$
Equation	$\tau = k\Delta\theta$
Description	k is the torsional spring constant ($\mathrm{N\,m/rad}$) τ is the torque ($\mathrm{N\,m}$) θ is the angle of rotation (radians)
Sources	None.
Ref. By	IM:SCF

RefName	**DD:GC**
Label	**Gravitational Constant**
Symbol	g
SI Units	$\mathrm{m/s^2}$
Equation	$F_g = mg$
Description	g is the gravitational constant ($\mathrm{m/s^2}$) F_g is the force of gravity m is the mass
Sources	None.
Ref. By	IM:A

RefName	**DD:DC**
Label	**Drag Coefficient**
Symbol	C_d
SI Units	unitless
Equation	$F_d = \frac{1}{2}\rho v^2 C_d A$
Description	C_d is the drag coefficient F_d is the force of drag ρ is the density of the fluid v is the velocity of the object A is the cross-sectional area of the object
Sources	None.
Ref. By	IM:A

RefName	**DD:RRC**
Label	**Rolling Resistance Coefficient**
Symbol	C_{rr}
SI Units	unitless
Equation	$F_{\text{rr}} = C_{\text{rr}} F_n$
Description	C_{rr} is the rolling resistance coefficient F_{rr} is the force of rolling resistance F_n is the normal force
Sources	None.
Ref. By	IM:A

RefName	**DD:ET**
Label	**Engine Torque**
Symbol	τ_{eng}
SI Units	Nm
Equation	$\tau_{\text{eng}}(\omega_{\text{eng}})$
Description	τ_{eng} is the engine torque (Nm) ω_{eng} is the engine angular velocity (rad/s)
Sources	This is defined by Baja SAE via Baja SAE (2022)
Ref. By	IM:A, IM:SCF, IM:SA, IM:BA

4.2.8 Data Types

N/A

4.2.9 Instance Models

This section transforms the problem defined in Section 4.1 into one which is expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.7 to replace the abstract symbols in the models identified in Sections 4.2.5 and 4.2.6.

The goal statement GS:IF are solved by Instance Models IM:A, IM:V and IM:D
The goal statement GS:E are solved by Instance Models IM:SD
The goal statement GS:F are solved by Instance Models IM:PCF, IM:SCF, IM:SA and IM:SV

RefName	**IM:A**
Label	**Acceleration**
Input	$r_{\text{wheel}}, \rho, C_d, A, \theta_{\text{inc}}, R_{\text{CVT}}, R_{\text{gearbox}}, \tau_{\text{eng}} m_v, m_d$
Equation	$$a_{\text{car}} = \frac{dv_{\text{car}}}{dt} = \frac{\frac{R_{\text{CVT}} R_{\text{gearbox}}}{r_{\text{wheel}}} \tau_{\text{eng}}(\frac{v_{\text{car}} R_{\text{CVT}} R_{\text{gearbox}}}{r_{\text{wheel}}})}{m_v + m_d} - g \sin\theta_{\text{inc}} - \frac{\rho C_D A}{2(m_v + m_d)} v_{\text{car}}^2$$
Output	The acceleration of the car $a_{\text{car}}(t)$ (m/s^2)
Description	r_{wheel} is the radius of the wheel (m) ρ: Density of the fluid (kg/m^3) C_d: Drag coefficient (unitless) A: Cross-sectional area of the vehicle (m^2) θ_{inc}: Angle of incline (rad) m_v: Mass of the vehicle (kg) m_d: Mass of the driver (kg) R_{CVT}: Current ratio of the CVT (unitless) R_{gearbox} is the gearbox reduction ratio (unitless) $\tau_{\text{eng}}(\omega_{\text{eng}})$ is the torque seen at the engine as a function of engine angular velocity (Nm)
Sources	-
Ref. By	IM:V

Derivation of Acceleration

Using equations for kinetic energy, gavitational potential energy, air resistance and engine power, the acceleration of the vehicle is derived using the law of conservation of energy.

$$P = \frac{dK}{dt} + \frac{dU}{dt} + \frac{dF_D}{dt} \qquad \text{(see TM:KE, TM:GE, TM:AR)}$$

Substituting the above equations in gives us:

$$P = \frac{d(m_v v_{\text{car}})}{dt} + \frac{d(m_v gh)}{dt} + \frac{d(\frac{1}{2}\rho v^2 C_D A)}{dt}$$

Where the power is defined as the engine torque multiplied by the angular velocity of the engine.

- The rate of change of the kinetic energy is defined as the mass of the vehicle multiplied by the acceleration of the vehicle.
- The rate of change of gravitational potential energy is defined as the mass of the vehicle multiplied by the gravitational constant multiplied by the sine of the incline angle.
- The power lost to air resistance is the drag forces times the velocity.

We can then write this as:

$$P = mv\frac{dv}{dt} + mgv\sin(\theta) + \frac{1}{2}\rho v^3 C_D A$$

Solving for $\frac{dv}{dt}$ gives us:

$$\frac{dv}{dt} = \frac{P}{mv} - g\sin(\theta) - \frac{\rho C_D A}{2m}v^2$$

Now we can use DD7, GD3 to substitute power in for the engine torque and the engine angular velocity.

$$\frac{dv}{dt} = \frac{\frac{R}{r}\tau(\frac{vR}{r})}{m} - g\sin(\theta) - \frac{\rho C_D A}{2m}v^2$$

RefName	**IM:V**
Label	**Velocity**
Input	a_{car}
Equation	$$\frac{dv_{\text{car}}}{dt} = a_{\text{car}}$$
Output	The velocity of the car v_{car} (m/s)
Description	a_{car}: Acceleration of the vehicle (m/s^2)
Sources	-
Ref. By	IM:D

RefName	**IM:D**
Label	**Distance**
Input	v_{car}
Equation	$$\frac{dd_{\text{car}}}{dt} = v_{\text{car}}$$
Output	The distance the car has traveled d_{car} (m)
Description	v_{car}: Velocity of the vehicle (m/s)
Sources	-
Ref. By	-

RefName	**IM:PCF**
Label	**Primary Clamping Force**
Input	d_{shift}, ω_{eng}, m_{fly}, r_{fly}, $f_{\text{prim\_height}}(d_{\text{shift}})$, $f_{\text{prim\_angle}}(d_{\text{shift}})$, k_{prim}, d_{prim}
Equation	$F_{\text{prim\_clamp}}(d_{\text{shift}}, \omega_{\text{eng}}) = m_{\text{fly}}(r_{\text{fly}} + f_{\text{prim\_height}}(d_{\text{shift}}))\omega_{\text{eng}}^2 \tan(f_{\text{prim\_angle}}(d_{\text{shift}})) + k_{\text{prim}}(d_{\text{prim}} + d_{\text{shift}})$
Output	The primary clamping force $F_{\text{prim\_clamp}}$ (N)
Description	d_{shift}: Displacement of the sheave as a function of time. (m) ω_{eng}: Angular velocity of the engine as a function of time. (rad/s) m_{fly}: Mass of the flyweight system. (kg) r_{fly}: Initial radius of the flyweights. (m) $f_{\text{prim\_height}}(d_{\text{shift}})$: Function that represents the height of the flyweights based on the shift distance. (rad) $f_{\text{prim\_angle}}(d_{\text{shift}})$: Function that returns the instantaneous angle of the ramps based on the shifting distance. (rad) k_{prim}: Primary spring coefficient. (unitless) d_{prim}: Initial compression distance of the primary spring. (m)
Sources	-
Ref. By	IM:SA

Derivation of Primary Clamping Force

The flyweight experiences a centrifugal force $F_c = mr\omega^2$ as it rotates. The ramp angle $f_{\text{prim\_angle}}(d_{\text{shift}})$ is where the force applied by the flyweight meets the inclined surface. The force can be broken into the radial force, which is perpendicular to the ramp, and the axial force which acts in the direction of the ramp. The axial force is what is responsible for moving the mechanism causing the flyweights to press against a ramp or surface, therefore the force we are interested in. We can ignore the radial force as it acts perpendicular to the ramp and does not contribute to the axial force. (see A:IF) To find the flywheel force in this axial direction the tangent of the angle $f_{\text{prim\_angle}}(d_{\text{shift}})$ is used. The flywheel force is derived as follows:

$$F_{FW} = mr\omega^2 \tan(f_{\text{prim\_angle}}(d_{\text{shift}})) \qquad \text{(see TM:CF)}$$

Where r can be represented by $r_{\text{fly}} + f_{\text{prim\_height}}(d_{\text{shift}})$. Then F_{FW} becomes

$$F_{FW} = m_{\text{fly}}(r_{\text{fly}} + f_{\text{prim\_height}}(d_{\text{shift}}))\omega^2 \tan(f_{\text{prim\_angle}}(d_{\text{shift}}))$$

The spring force is derived as follows:

$$-F_S = +k_{\text{prim}}(d_{\text{prim}} + d_{\text{shift}}) \qquad \text{(see TM:HL)}$$

Combining these two forces gives the total primary clamping force:

$$F_{FW} - F_S = m_{\text{fly}}(r_{\text{fly}} + f_{\text{prim\_height}}(d_{\text{shift}}))\omega^2 \tan(f_{\text{prim\_angle}}(d_{\text{shift}})) + k_{\text{prim}}(d_{\text{prim}} + d_{\text{shift}})$$

The final equation can be written as:

$$F_{\text{prim\_clamp}}(t) = m_{\text{fly}}(r_{\text{fly}} + f_{\text{prim\_height}}(d_{\text{shift}}(t)))(\omega_{\text{eng}}(t))^2 \tan(f_{\text{prim\_angle}}(d_{\text{shift}}(t))) + k_{\text{prim}}(d_{\text{prim}} + d_{\text{shift}}(t))$$

RefName	**IM:SCF**
Label	**Secondary Clamping Force**
Input	d_{shift}, τ_{eng}, R_{CVT}, $k_{\text{sec\_tor}}$, θ_{sec}, $r_{\text{sec\_ramp}}$, $f_{\text{sec\_ramp}}(d_{\text{shift}})$, $f_{\text{sec\_shift}}(d_{\text{shift}})$, $k_{\text{sec\_comp}}$, d_{sec}
Equation	$F_{\text{sec\_clamp}}(d_{\text{shift}}, \tau_{\text{eng}}, R_{\text{CVT}}) = \frac{k_{\text{sec\_tor}}(\theta_{\text{sec}} + f_{\text{sec\_shift}}(d_{\text{shift}})) + \tau_{eng} R_{\text{CVT}}}{2 r_{\text{sec\_ramp}} \tan(f_{\text{sec\_ramp}}(d_{\text{shift}}))} + k_{\text{sec\_comp}}(d_{\text{sec}} + d_{\text{shift}})$
Output	The secondary clamping force $F_{\text{sec\_clamp}}$ (N)
Description	d_{shift}: Displacement of the sheave as a function of time. (m) τ_{eng}: Torque seen at the engine as a function of time. (Nm) R_{CVT}: Ratio of the CVT as a function of time. (unitless) $k_{\text{sec\_tor}}$: Torsional spring rate. (Nm/rad) θ_{sec}: Initial torsional rotation. (rad) $f_{\text{sec}}(d_{\text{shift}})$: Amount of torsional rotation based on the distance shifted. (rad) $r_{\text{sec\_ramp}}$: Radius of the helix from the shaft. (m) $k_{\text{sec\_comp}}$: Compression spring rate. (N/m) d_{sec}: Initial compression distance. (m)
Sources	-
Ref. By	IM:SA

Derivation of Secondary Clamping Force

The secondary clamping force is derived as follows:

$$F_{\text{sec\_clamp}} = F_H + F_S$$

The helix force is derived as follows:

$$F_H = \frac{\tau_{eng}(t) R_{\text{CVT}}(t)}{2r\tan(\beta)} \qquad \text{(see GD:SSSF)}$$

Substituting in our variables gives us the following equation:

$$F_H = \frac{\tau_{eng}(t) R_{\text{CVT}}(t)}{2r_{\text{sec\_ramp}}\tan(f_{\text{sec\_ramp}}(d_{\text{shift}}(t)))}$$

The spring force is derived as follows:

$$F_S = \frac{k_{\text{tor}}(\theta_0 + \theta)}{2r\tan(\beta)} + k_{\text{comp}}(x_0 + x) \qquad \text{(see GD:HSF)}$$

Substituting in our variables gives us the following equation:

$$F_S = \frac{k_{\text{sec\_tor}}(\theta_{\text{sec}} + f_{\text{sec\_shift}}(d_{\text{shift}}(t)))}{2r_{\text{sec\_ramp}}\tan(f_{\text{sec\_ramp}}(d_{\text{shift}}(t)))} + k_{\text{sec\_comp}}(d_{\text{sec}} + d_{\text{shift}}(t))$$

Combining these two forces gives the total secondary clamping force:

$$F_H + F_S = \frac{\tau_{eng}(t) R_{\text{CVT}}(t)}{2r_{\text{sec\_ramp}}\tan(f_{\text{sec\_ramp}}(d_{\text{shift}}(t)))} + \frac{k_{\text{sec\_tor}}(\theta_{\text{sec}} + f_{\text{sec\_shift}}(d_{\text{shift}}(t)))}{2r_{\text{sec\_ramp}}\tan(f_{\text{sec\_ramp}}(d_{\text{shift}}(t)))} + k_{\text{sec\_comp}}(d_{\text{sec}} + d_{\text{shift}}(t))$$

The final equation can be reached through simplifying the above equation by combining like terms:

$$F_{\text{sec\_clamp}} = \frac{k_{\text{sec\_tor}}(\theta_{\text{sec}} + f_{\text{sec\_shift}}(d_{\text{shift}}(t))) + \tau_{eng}(t) R_{\text{CVT}}(t)}{2r_{\text{sec\_ramp}}\tan(f_{\text{sec\_ramp}}(d_{\text{shift}}(t)))} + k_{\text{sec\_comp}}(d_{\text{sec}} + d_{\text{shift}}(t))$$

RefName	**IM:SA**
Label	**Shift Acceleration**
Input	$d_{\text{shift}}, \omega_{\text{eng}}, \tau_{\text{eng}}, R_{\text{CVT}}, m_{\text{sheaves}}$
Equation	$$a_{\text{shift}} = v_{\text{shift}} \frac{dv_{\text{shift}}}{dd_{\text{shift}}} = \frac{F_{\text{prim\_clamp}}(d_{\text{shift}}, \omega_{\text{eng}}) - F_{\text{sec\_clamp}}(d_{\text{shift}}, \tau_{\text{eng}}, R_{\text{CVT}})}{m_{\text{sheaves}}}$$
Output	The shift acceleration a_{shift}
Description	d_{shift}: Displacement of the shift as a function of time. (m) ω_{eng}: Angular velocity of the engine as a function of time. (rad/s) τ_{eng}: Torque seen at the engine as a function of time. (Nm) R_{CVT}: Ratio of the CVT as a function of time. (unitless) m_{sheaves}: Mass of the sheaves. (kg)
Sources	-
Ref. By	IM:SV

Derivation of Sheave Acceleration

One can derive the acceleration of the sheaves using Newton's Second Law. The forces acting on the built include only the primary and secondary clamping forces, which act in opposite directions as they act out radially. The acceleration of the sheave is derived as follows:

$$mv\frac{dv}{dt} = F_{\text{prim\_clamp}}(d_{\text{shift}}, \omega_{\text{eng}}) - F_{\text{sec\_clamp}}(d_{\text{shift}}, \tau_{\text{eng}}, R_{\text{CVT}})$$

Rearranging the equation for acceleration gives the final equation:

$$v\frac{dv}{dt} = \frac{F_{\text{clamp\_prim}}(t) - F_{\text{clamp\_sec}}(t)}{m}$$

RefName	**IM:SV**
Label	**Shift Velocity**
Input	a_{shift}
Equation	$$\frac{dv_{\text{shift}}}{dt} = a_{\text{shift}}$$
Output	The shift Velocity v_{shift}
Description	a_{shift}: Acceleration of the sheave in the shift direction. (m/s^2)
Sources	-
Ref. By	IM:SD

RefName	**IM:SD**
Label	**Shift Distance**
Input	v_{sheave}
Equation	$$\frac{dd_{\text{shift}}}{dt} = v_{\text{sheave}}$$
Output	The distance shifted d_{shift}
Description	v_{sheave}: Velocity of the sheave. (m/s)
Sources	-
Ref. By	IM:PCF, IM:SCF, IM:SA

RefName	**IM:BA**
Label	**Belt Acceleration**
Input	$\tau_{\text{eng}}, r_{\text{prim}}, m_{\text{belt}}$
Equation	$a_{\text{belt}} = \frac{\tau_{\text{eng}}}{m r_{\text{prim}}}$
Output	Belt Acceleration a_{belt}
Description	τ_{eng}: Engine torque r_{prim}: Primary radius m_{belt}: Mass of the belt.
Sources	-
Ref. By	IM:SA

Derivation of Belt Acceleration

The torque of the engine equation in terms of belt tension is derived as such:

$$\tau_{\text{eng}} = (\tau_T - \tau_s) r_{\text{prim}} \quad \Rightarrow \quad \tau_T - \tau_s = \frac{\tau_{\text{eng}}}{r_{\text{prim}}}$$

Similarly, for the load torque:

$$\tau_{\text{load}} = (\tau_T - \tau_s) r_{\text{sec}} \quad \Rightarrow \quad \tau_T - \tau_s = \frac{\tau_{\text{load}}}{r_{\text{sec}}}$$

Equating the two torque equations:

$$\frac{\tau_{\text{eng}}}{r_{\text{prim}}} = \frac{\tau_{\text{load}}}{r_{\text{sec}}} \qquad \text{(see A:NS)}$$

The equation for the torque of the taut side is derived using the capstan equation:

$$\tau_T = \tau_s e^{\mu\theta} \qquad \text{(TM:CE)}$$

Substituting our engine torque into this:

$$\frac{\tau_{\text{eng}}}{r_{\text{prim}}} + \tau_s = \tau_s e^{\mu\theta}$$

Solving for τ_s (slack side):

$$\tau_s = \frac{\tau_{\text{eng}}}{r_{\text{prim}}(e^{\mu\theta} - 1)}$$

Using Newton's 2nd law the final equation for belt acceleration is derived as:

$$m_{\text{belt}} a_{\text{belt}} = \frac{\tau_{\text{eng}} e^{\mu\theta}}{r_{\text{prim}}(e^{\mu\theta} - 1)} - \frac{\tau_{\text{eng}}}{r_{\text{prim}}(e^{\mu\theta} - 1)} \qquad \text{(TM:N2)}$$

Solving for acceleration:

$$a_{\text{belt}} = \frac{\tau_{\text{eng}}}{m_{\text{belt}} r_{\text{prim}}(e^{\mu\theta} - 1)}(e^{\mu\theta} - 1)$$

Finally, simplifying to get:

$$a_{\text{belt}} = \frac{\tau_{\text{eng}}}{m_{\text{belt}} r_{\text{prim}}}$$

4.2.10 Input Data Constraints

Table 2 shows the data constraints on the input output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The column for software constraints restricts the range of inputs to reasonable values. The software constraints will be helpful in the design stage for picking suitable algorithms. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise.

The specification parameters in Table 2 are listed in Table 4.

Table 2: Input Variables

Var	Physical Constraints	Software Constraints	Typical Value	Uncertainty
m_{fly}	$0 \leq m_{fly}$	$m_{fly} \leq 1$	0.2 kg	10%
ω_{engine}	$0 \leq \omega_{engine}$	$\omega_{engine} \leq \omega_{\text{max}}$	400 rad/s	10%
d_{shift}	$0 \leq d_{shift} \leq w_{belt}$		0.1 m	10%
θ_{prim_out}	$0 \leq \theta_{prim\_out} \leq \frac{\pi}{2}$		$\frac{\pi}{4}$ rad	10%
d_{prim}	$0 \leq d_{\text{prim}} \leq d_{\text{max\_prim\_pre}}$		0.1 m	10%
d_{sec}	$0 \leq d_{\text{sec}} \leq d_{\text{max\_sec\_pre}}$		0.1 rad	10%
m_{driver}	$0 \leq m_{\text{driver}}$	$m_{\text{driver}} \leq m_{\text{max\_human}}$	75 kg	10%
m_{car}	$0 \leq m_{\text{car}}$	$m_{\text{car}} \leq m_{\text{max\_car}}$	225 kg	10%
k_{prim}	$0 \leq k_{\text{prim}}$	$k_{\text{prim}} \leq k_{\text{max\_comp\_spring}}$	100 N/m	10%
$k_{\text{sec\_comp}}$	$0 \leq k_{\text{sec\_comp}}$	$k_{\text{sec\_comp}} \leq k_{\text{max\_comp\_spring}}$	100 N/m	10%
$k_{\text{sec\_tor}}$	$0 \leq k_{\text{sec\_tor}}$	$k_{\text{sec\_tor}} \leq k_{\text{max\_tor\_spring}}$	50 Nm/rad	10%

Table 4: Specification Parameter Values

Var	Value
L_{min}	0.1 m
ω_{max}	600 rad/s
w_{belt}	0.0222 m
$d_{\text{max\_prim\_pre}}$	0.0254 m
$d_{\text{max\_sec\_pre}}$	$\frac{5\pi}{4}$ rad
$m_{\text{max\_human}}$	200 kg
$m_{\text{max\_car}}$	350 kg
$k_{\text{max\_comp\_spring}}$	1000 N/m
$k_{\text{max\_tor\_spring}}$	750 Nm/rad
v_{max}	19.44 m/s
$f_{\text{belt\_max}}$	25000 N

4.2.11 Properties of a Correct Solution

Table 6: Output Variables

Var	Physical Constraints	unit
$v(t)$	$0 \leq v(t) \leq v_{\max}$	m/s
$F_{\text{clamp\_prim}}, F_{\text{clamp\_sec}}$	$F_{\text{clamp\_prim}} + F_{\text{clamp\_sec}} \leq f_{\text{belt\_max}}$	N
$v(t)$, $\tau_{\text{eng}}(t)$, $\omega_{\text{eng}}(t)$	$\frac{d}{dt}(\frac{1}{2}(m_{\text{car}} + m_{\text{driver}})v(t)^2) \leq \tau_{\text{eng}}(t)\omega_{\text{eng}}(t)$	N

5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the nonfunctional requirements, the qualities that the software is expected to exhibit. The priority for both sections 5.1 and 5.2 are in descending order, which is to say that the top requirement is the most important and the bottom requirement is the least important.

5.1 Functional Requirements

R:MM : The system shall simulate the behavior of the McMaster Baja Team's Continuous Variable Transmission(CVT) using mathematical models.

R:AV : The system shall calculate the acceleration of the vehicle as a function of time using inputs: gear radius, wheel radius, density of the fluid, drag coefficient, cross-sectional area, angle of incline, torque transferred to the wheel, coefficient of rolling resistance, mass of the vehicle and driver.

R:VV : The system shall calculate the velocity of the vehicle as a function of time by taking the integral of acceleration with respect to time.

R:DV : The system shall calculate the distance as a function of time by taking the integral of the velocity with respect to time.

R:PCF : The system shall calculate the primary clamping force using inputs: mass of the flyweight, radius related to the flywheel, height of the primary system, force dependent of the shifting distance, stiffness of primary system and displacement or shift in the system.

R:SCF : The system shall calculate the secondary clamping force using inputs: torsional spring rate, initial torsional rotation, amount of torsional rotation based on the distance shifted, radius of the ramps from the shaft, compression spring rate, initial compression distance, displacement in system, torque of the engine and ratio of the CVT system.

R:SA : The system shall calculate the sheave acceleration using inputs: primary clamping force, secondary clamping force, coefficient of friction, torque of the taut side of the belt, torque of the slack side of the belt and mass of the system.

R:BA : The system shall calculate the belt acceleration using inputs engine torque, primary radius, friction force and mass of the system.

R:ENG : The system shall calculate RPM and Torque of the Engine using inputs time and load torque.

R:TP : The system shall allow users to adjust the following CVT tuning parameters: primary weight, primary ramp geometry, primary spring rate, primary spring pretension, secondary helix geometry, secondary spring rate, secondary spring pretension.

R:UA : The system shall allow users to adjust vehicle and driver weight, traction and angle of incline in addition to the CVT tuning parameters.

R:UI : The system shall provide users with an interface to input tuning parameters.

R:DG : The system shall display graphs of simulation output based on the user inputted tuning parameters.

R:ER : The system shall allow users to export outputted simulated graphical data.

R:AA : The system shall allow users to access the application through the use of a personal computer or laptop.

R:WM : The system shall be compatible with Windows machines.

5.2 Nonfunctional Requirements

NFR:AC : The system shall achieve an accuracy of at least 95 % in correctly predicting CVT outputs. See VnV Plan Section 4.2 for the test procedure. An accuracy rate of 95% ensures that the system is able to make reliable predictions, which are important to the performance optimization. Accuracy validation shall use Python's `assertAlmostEqual()` with a tolerance of delta = 1e-7, to account for minor numerical deviations.

NFR:US : 85 % of a representative user group shall be able to successfully input CVT parameters and receive CVT outputs on their first use. An 85% success rate was chosen for successfully inputting parameters and receiving outputs on their first use is realistic based on the complexity of the system. This 85 % acknowledges that some users may require trial and error, but the majority of users should still be able to effectively interact with the system.

NFR:MA : If a likely change is made to the finished software, it will take at most 10 % of the original development time, assuming the same development resources are available. This 10% figure is a reasonable expectation for maintainability. Given that our system will evolve, this 10% represents having a modular design that should be able to accommodate changes without significant extra effort and additions.

NFR:VE : The system shall output the resulting tunned CVT outputs which must be able to be cross-verified against data available through the McMaster Baja Team with a consistency rate of 95 %. This 95% reflects the need for a high level of precision and consistency of the system. This number was chosen as the system must be valuable to its users through producing accurate reliable outputs.

NFR:UN : The system shall be understood by at least 90 % of a representative user group with no more than 15 minutes of initial training, as measured by assessing ability to generate and export tunned CVT output where users must score at least 90 %. A target of 90% was chosen as this ensures that system is intuitive and user-friendly to a large majority of users. By providing users with 15 minutes of training this demonstrates that the learning curve is minimal.

NFR:RE : The system's components shall be designed for reuse in the case the McMaster Baja team acquires a new CVT. These modifications should be made with minimal modifications, defined as requiring modification to less than 30 % of the code to configure these changes. By choosing 30% this is a reasonable limit showing for flexibility and reusability of components. This 30% target ensures that future modifications such as accommodating for a new CVT, can be achieved without requiring a complete redesign of the system as majority if not all the mathematical calculations will remain the same.

5.3 Regulatory and Standards Compliance

The CVT Simulator software must adhere to relevant industry standards, legal regulations, and ensure accuracy, reliability, and compliance with safety and performance requirements. This section outlines key regulatory, legal, and industry compliance factors that influence the design and validation of the software.

5.3.1 Industry Standards

The CVT Simulator software shall comply with recognized software engineering and automotive control standards, including but not limited to: While no specific industry regulations directly relate to this software, practices such as software validation, numerical accuracy, and performance testing will be followed to ensure reliability and correctness. Testing methodologies, such as unit testing will be used to verify system accuracy.

5.3.2 Legal and Safety Regulations

The software must comply with applicable regulations concerning software integrity and data accuracy, as no personal information is used security requirements are considered out of scope.

- **ISO/IEC 25002:2024**: Software quality models ensuring reliability and maintainability International Organization for Standardization (ISO) (2024). This will be ensured via NFR:MA.

5.3.3 Software Development Constraints Due to Compliance

Compliance with these standards imposes specific requirements on the CVT software, including:

- **Precision in Computation**: Floating-point errors must be mitigated this will be achieved by NFR:AC.
- **Version Control and Documentation**: Compliance with International Organization for Standardization (ISO) (2024) mandates detailed software version control and traceability of changes. This will be shown at the top of each document's Revision History section.

Therefore, the CVT Simulator software will comply with these regulatory and industry standards.

5.4 Rationale

The assumptions made for the CVT (Continuously Variable Transmission) system model aim to simplify it's dynamics, facilitating the development of mathematical models to simulate the behavior of the CVTR:MM. Many key assumptions are made to meet the requirement of developing these mathematical models. The following assumptions are also factored in to the calculations of the CVT ratio, clamping force, RPM, torque, engine input, distance, speed and acceleration(R:AV).

The system excludes the effects of temperature on material properties, aside from the belt, which allows for a more straightforward focus on the mechanics of the interactions (A:NT). The influence of gravitational forces is negligible and the elasticity of belts is considered insignificant(A:NG, A:RB). Vibrational effects from external sources are also assumed to be minimal, avoiding the complexity of vibration-induced disturbances in transmission behavior (A:NV). Wear and tear of components, including the belt and pulley are disregarded and frictional losses in non-critical components, are deemed negligible (A:WT, A:NF). The engine is assumed to operate at full throttle with consistent power delivery, simplifying the model by maintaining uniform engine behavior (A:NV). Finally, the CVT axis are assumed to be in line and the belt slippage of the CVT is assumed as negligible (A:AA,A:NS).

These simplifications affect the process of creating our mathematical models and formulating equations governing velocity, distance, primary clamping force, secondary clamping force, CVT Ratio, Torque transfer and RPM and Torque of the Engine(IM:A-IM:SD)

6 Likely Changes

LC:TMP : The effect of temperature on material properties might be included in the future, (A:NT) Stakeholders would be interested in seeing this information in the future.

LC:EB : The elasticity of the belt might be considered in the future, (A:RB) This would provide a more accurate model of the CVT system.

LC:FL : Frictional losses in non-critical components might be considered in the future, (A:NF) This would provide a more accurate model of the CVT system.

LC:MM : Some of the math models will most likely change as the design is refined.

7 Unlikely Changes

ULC:AL : The assumption that the CVT axis are in line is unlikely to change because tuning is simply not valuable in a system as poor as this. (A:AA)

ULC:FT : The assumption that the engine operates at full throttle is unlikely to change as different behavior is simply not ideal. (A:FT)

ULC:GF : The assumption that the influence of gravitational forces is negligible is unlikely to change as it heavily reduces complexity and plays an incredibly small part on spinning objects at high rotational speeds. (A:NG)

8 Phase In Plan of Requirements

The implementation of the CVT simulation software will be divided into two phases:

1. **Phase 1 (February 3–February 14): Functional Requirements Implementation**. The functional requirements are aimed to be fully implemented by Rev0 which is the date given.

2. **Phase 2 (March 7): Nonfunctional Requirements Implementation**. The non functional requirements are aimed to be implemented before the VnV Report is due which is the date given.

8.1 Phase 1: Functional Requirements Implementation (February 3)

The primary focus of this phase is to implement the core functionalities of the system, ensuring that all mathematical computations and system mechanics are operational by Rev 0. These requirements follow the priority of the order given below, the first groups of the requirements are of a higher priority. The requirements of highest priority are 8.1.1, 8.1.2, 8.1.3.

8.1.1 Clamping Force and Sheave Dynamics

- R:PCF
- R:SCF
- R:SA
- R:BA
- R:ENG

8.1.2 Mathematical Models

- R:MM
- R:AV
- R:VV
- R:DV

8.1.3 User Adjustments and Inputs

- R:TP
- R:UA
- R:UI

8.1.4 Output Visualization and Export

- R:DG
- R:ER

8.1.5 Platform Compatibility

- R:AA
- R:WM

8.2 Phase 2: Nonfunctional Requirements Implementation (March 7 and March 23)

This phase will focus on refining the system by ensuring accuracy, usability, maintainability, and verification. The priority of the Nonfunctional requirements is stated in the order below, with 8.2.1 and 8.2.2 being of highest priority. These two subsections of the Nonfunctional requirements will be phased in by March 7 which is the due date of the VnV Report. The set of Nonfunctional requirements 8.2.3 will be highlighted in the Useability Report which will be completed by March 23rd.

8.2.1 System Accuracy and Verification

- NFR:AC
- NFR:VE

8.2.2 Maintainability and Reusability

- NFR:MA
- NFR:RE

8.2.3 Usability and User Experience

- NFR:US
- NFR:UN

9 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" may have to be modified as well. Table 8 shows the dependencies of theoretical models, general definitions, data definitions, and instance models with each other. Table 12 shows the dependencies of instance models, requirements, and data constraints on each other. The table 9 shows the dependencies of theoretical models, general definitions, data definitions, instance models, and likely changes on the assumptions.

	A:NT	A:NG	A:RB	A:NV	A:WT	A:NF	A:FT	A:AA	A:NS	A:IF
TM:HL										
TM:CF										
TM:AR										
TM:ET										
TM:GR										
TM:FR										
TM:DE										
TM:CE										
TM:N2										
TM:HT										
TM:KE										
TM:GE										
TM:D2R										
TM:D2RA										
GD:HSF										X
GD:SSSF										
GD:AVE										
DD:F										
DD:CSC										
DD:TSC										
DD:GC										
DD:DC										
DD:RRC										
DD:ET										

Table 8: Traceability Matrix Showing the Connections Between Assumptions, Theoretical Models, General Definitions, Data Definitions

	A:NT	A:NG	A:RB	A:NV	A:WT	A:NF	A:FT	A:AA	A:NS	A:IF
IM:A										
IM:V										
IM:D										
IM:PCF										X
IM:SCF										
IM:SA										
IM:SV									X	
IM:SD										
IM:BA										
LC:TMP	X									
LC:EB			X							
LC:FL						X				
LC:MM										
ULC:AL								X		
ULC:FT							X			
ULC:GF		X								

Table 9: Traceability Matrix Showing the Connections Between Assumptions and Instance Models and Likely Changes

	IM:A	IM:V	IM:D	IM:PCF	IM:SCF	IM:SA	IM:SV	IM:SD	IM:BA
TM:HL				X	X				
TM:CF				X					
TM:AR	X								
TM:ET	X				X	X			X
TM:GR	X				X				
TM:FR	X								
TM:DE	X								
TM:CE									X
TM:N2	X					X			X
TM:HT					X				
TM:KE	X								
TM:GE	X								
TM:D2R				X	X				
TM:D2RA				X	X				
GD:HSF					X				
GD:SSSF					X				
GD:AVE				X					
DD:F	X								
DD:CSC				X					
DD:TSC					X				
DD:GC	X								
DD:DC	X								
DD:RRC	X								
DD:ET	X				X	X			X

Table 10: Traceability Matrix Showing the Connections Between Instance Models and Other Items

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 9 shows the dependencies of theoretical models, general definitions, data definitions, instance models, likely changes, and assumptions on each other. Figure 11 shows the dependencies of instance models, requirements, and data constraints on each other.

	IM:A	IM:V	IM:D	IM:PCF	IM:SCF	IM:SA	IM:SV	IM:SD	IM:BA
R:MM									
R:AV	X								
R:VV		X							
R:DV			X						
R:PCF				X					
R:SCF					X				
R:SA						X			
R:BA									X
R:ENG									
R:TP									
R:UA									
R:UI									
R:DG									
R:ER									
R:AA									
R:WM									

Table 11: Traceability Matrix Showing the Connections Between Instance Models and Functional Requirements

	IM:A	IM:V	IM:D	IM:PCF	IM:SCF	IM:SA	IM:SV	IM:SD	IM:BA
IM:A		X							
IM:V			X						
IM:D									
IM:PCF						X			
IM:SCF						X			
IM:SA							X		
IM:SV								X	
IM:SD				X	X	X			
IM:BA						X			

Table 12: Traceability Matrix Showing the Connections Between Instance Models

10 Development Plan

N/A

11 Values of Auxiliary Constants

N/A

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Appendix — Reflection

[Not required for CAS 741 —SS]

The information in this section will be used to evaluate the team members on the graduate attribute of Lifelong Learning.

The purpose of reflection questions is to give you a chance to assess your own learning and that of your group as a whole, and to find ways to improve in the future. Reflection is an important part of the learning process. Reflection is also an essential component of a successful software development process.

Reflections are most interesting and useful when they're honest, even if the stories they tell are imperfect. You will be marked based on your depth of thought and analysis, and not based on the content of the reflections themselves. Thus, for full marks we encourage you to answer openly and honestly and to avoid simply writing "what you think the evaluator wants to hear."

Please answer the following questions. Some questions can be answered on the team level, but where appropriate, each team member should write their own response:

1. What went well while writing this deliverable?

   We were successful in splitting up the work that could be done individually and then coming together to discuss and combine our work. We also did a good job in playing to each team members strengths and dividing the work accordingly. For example, we collaboratively worked on deriving our instance models by hand which required us as a team to also discuss our theoretical models, general definitions, data definitions and assumptions. This was crucial that each team member was on the same page at this stage as everything in this SRS document was centered around these concepts.

   Once these concepts were discussed we were then able to split up writing up these instance models, theoretical models, general definitions, data definitions and assumptions. We also split up the remaining sections such as the functional/non-functional requirements, problem description, and general description sections. By splitting the work up this way it allowed our team to be time effective but also ensured that each team member had a solid grasp on the key concepts the SRS was centered around.

2. What pain points did you experience during this deliverable, and how did you resolve them?

   The main pain points of this deliverable were trying to understand the mathematical models and physics behind the CVT system. We resolved this by meeting together as a team and having a group discussion on the topic. We went through the math and derived the equations together, which helped us understand the material better. We also reached out to our clients for clarifications, and they provided us with help and resources to better understand the topic. As a result our team was more confident in deriving the mathematical models required for this project.

3. How many of your requirements were inspired by speaking to your client(s) or their proxies (e.g. your peers, stakeholders, potential users)?

   One key requirement from our clients is the Verifiability requirement. During our first capstone meeting with Dr. Smith he stressed the importance of verifying our results. This resulted in us adding the requirement to our document. Additionally, the majority if not all of our functional requirements were inspired by the needs of the McMaster Baja team who we meet with regularly to gather feedback and insights.

4. Which of the courses you have taken, or are currently taking, will help your team to be successful with your capstone project.

   Many of the courses we have taken so far will prove helpful in completing our capstone project. SFWRENG 4HC3, will help us with the design of the GUI. SFWRENG 2AA4 and SFWRENG 3A04 will help us architect our code base. PHYSICS 1D03, SFWRENG 3MX3 and SFWRENG 3DX34, will help us understand and model the physics and math behind the CVT system. SFWRENG 3XB3 and SFWRENG 4X03, will help with creating efficient and reliable implementations. SFWRENG 3S03, will help us verify and validate our software.

5. What knowledge and skills will the team collectively need to acquire to successfully complete this capstone project? Examples of possible knowledge to acquire include domain specific knowledge from the domain of your application, or software engineering knowledge, mechatronics knowledge or computer science knowledge. Skills may be related to technology, or writing, or presentation, or team management, etc. You should look to identify at least one item for each team member.

   To successfully complete this capstone project our team will need to acquire several important skills. We will need to learn and understand Unity to create a user-friendly interface which will use our created mathematical models in Python. For Unity, we need to learn how to make a GUI interface, animate 3D models, create and display graphs, and learn how to interface with a Python script to run the mathematical models. For Python, we need to learn how to create and implement these complex mathematical models and learn how to make these Python models interface with Unity. It is important that each team member acquires a general knowledge of these skills as team members completing the Python code must be familiar with interfacing the code with Unity. As well as team members completing the models within Unity must understand the mathematical Python models to use Unity to simulate them.

6. For each of the knowledge areas and skills identified in the previous question, what are at least two approaches to acquiring the knowledge or mastering the skill? Of the identified approaches, which will each team member pursue, and why did they make choice?

   To familiarize ourselves with Unity projects, we will watch YouTube tutorials and read guides provided by Unity. The team members who chose to do the videos did so because they are more visual learners and prefer to see the process in action. Those that chose to read the guides did so because they prefer to read and also prefer that it is from an official source. For learning the Python required, we will do research on relevant libraries and functions that will help us implement our mathematical models. The team members who chose to do research did so because they are more comfortable researching and learning on their own.

This approach allows us to take into account the different learning styles of each team member so that everyone can learn in a way that suits them best. Additionally, this also allows our team to have a larger variety of resources and variation leading to a stronger final project.