Supplementary Material: ***Hilbert Huang Transform***

One way of improving the time-frequency resolution is to seek an instantaneous value for the frequency at every time instant for a signal. Such instantaneous frequency (IF) values can be derived from phase derivative of the analytic form of a signal, obtained using the Hilbert transform (Cohen, 1995). However, the IF values calculated this way may not be always meaningful for a physical signal formed of many contributing components (Huang et al., 1998). Some appropriate pre-processing of the raw signal may be required to yield valid and relevant IF values. The Hilbert-Huang Transform is a method that breaks down the signal into components whose IF values can be meaningfully determined and then the overall time-frequency spectrum of the signal can be constructed from them.

The Hilbert-Huang Transform (Huang et al., 1998) consists of two essential steps – the Empirical Mode Decomposition (EMD) of the signal to obtain Intrinsic Mode Functions (IMF) constituting the signal, followed by Hilbert Spectrum Analysis (HSA) of these IMFs. These IMFs are extracted directly from the signal by an iterative sifting process so that they satisfy the following conditions:

1) The number of extrema (maxima and minima) must be the same as the number of zero crossings or they must differ by at most one.
2) The mean value of the envelopes defined by the local maxima and minima must be zero at any point.

The detailed procedure to extract IMFs of a signal is given in (Huang et al., 1998, 2003). The signal $X(t)$ can be expressed as a sum of these IMFs (c_i's) and a leftover residue (r_n , which is often ignored if it is insignificant).

$$X(t) = \sum_{i=1}^{n} c_i(t) + r_n(t) \tag{S1}$$

By their construction, the IMFs are effectively amplitude-modulated (AM) and frequency-modulated (FM) oscillatory modes (Huang et al., 2003). Thus the signal is decomposed into its intrinsic mode functions instead of sinusoidal basis functions as in the Fourier based methods. Since the constituent IMFs depend on the signal itself, this method is more adaptive to the signal and thought to represent a more physically relevant breakup of the signal. Next, the IF of each IMF is computed using Hilbert Transform. The Hilbert spectrum is then obtained from the time-frequency distribution of the amplitude (or squared amplitude) of all the IMFs for the signal.

Some Considerations on HHT Implementation

Despite the promising features of HHT, there are a few issues related to its usage and interpretation. The theoretical framework defining this method still needs some development and right now it is largely empirically driven. There are quite a few free parameters that control the implementation and the results of this method and variations of these parameters can give varying results and interpretations (Huang et al., 2003).

Ensemble Empirical Mode Decomposition (EEMD) is a variation of the original EMD in which IMFs are computed several times after adding some amount of white noise to the signal (Wu and Huang, 2009). The mean IMFs across such an ensemble gives the final set of IMFs. The averaging cancels out the effect of added noise but preserves better the signal characteristics in the IMFs. The amount of noise added and the number of samples in the ensemble have to be chosen appropriately to obtain satisfactory results.

Filtering Characteristics of HHT

EMD operation has been shown to have characteristics similar to dyadic filtering when it is used to decompose white noise (Flandrin et al., 2004; Wu and Huang, 2004, 2010). This means that EMD is similar to having a series of overlapping band-pass filtering operations wherein the bandwidth of the filter is approximately halved as we move from one extracted mode to the next (similar to the WT decomposition shown in Figure 2D). The filter for the first mode is like a high pass filter spanning the higher half width of the band. The modes with smaller scales (or higher frequencies) are therefore extracted first, followed by the larger scales. This filtering behaviour of EMD, however, depends on the sifting number (the number of iterations used while extracting the IMFs; see (Huang et al., 1998) for more details) and as it approaches infinity, it starts losing the dyadic nature (the ratio of bandwidths for successive filter windows shifts from 2 (for dyadic) to 1 (Wu and Huang, 2010)). It has been suggested that the number of sifting steps to obtain more physically meaningful IMFs should be kept low, since too many sifting operations take away the AM component of the IMFs and EMD then may effectively start getting closer to Fourier expansion. The maximum sifting number should be ideally limited to 10 (Wang et al., 2010).

We implemented both EMD (Supplementary Figure 1, top row) and EEMD (bottom row) using the code available at 'http://rcada.ncu.edu.tw/research1_clip_program.htm'. There is some difference in the IMFs in the single-trial spectrum for EMD (Supplementary Figure 1A) and EEMD (Supplementary Figure 1D), with somewhat smoother IMFs in the EEMD case, highlighting the advantage of using ensemble averaging. In these plots, we observe IMFs indicating activity in the gamma band after stimulus onset. For average spectra (Supplementary Figure 1B and E) and the difference spectra (1C and F), the differences between EMD and EEMD are negligible because averaging across multiple trials might be lending the same effect to EMD as the effect of using the ensemble averaging in EEMD. While gamma rhythm and stimulus onset transients were well represented in the average and difference spectra, the 120 Hz component or the monitor refresh artifact at 100 Hz could not be resolved using either EMD or EEMD.

Figure Legends

Supplementary Figure 1: The time-frequency Hilbert power spectra using HHT in logarithmic scale, using EMD (top row) or EEMD (bottom row). The spectrum of (A),(D) single trial LFP signal, (B),(E) averaged across 186 trials and (C),(F) change in power from the baseline period.

References

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