

# CSCI 411 - Advanced Algorithms and Complexity

## Assignment 8

April 23, 2023

Solutions to the written portion of this assignment should be submitted via PDF to Canvas before **April 30th at 11:59 pm**. Make sure to **show your work**. There are a total of 105 points on this assignment, 5 of which are extra credit.

There will likely be time in class to discuss these problems in small groups and I highly encourage you to collaborate with one another outside of class. However, you must write up your own solutions **independently** of one another. Feel free to communicate via [Discord](#) and to post questions on the appropriate forum on Canvas. Do not post solutions. Also, please include a list of the people you work with at the top of your submission.

### Written Problems

1. Show derivations (using the  $\Rightarrow$  notation) of the strings below given the following context-free grammar.

$$S \rightarrow SS$$

$$S \rightarrow (S)$$

$$S \rightarrow \epsilon$$

(a) (5 pts)  $((()()))$

(b) (5 pts)  $()(())$

2. Show derivations (using the  $\Rightarrow$  notation) of the strings below given the following context-free grammar.

$$S \rightarrow 0A \mid 1B \mid \epsilon$$

$$A \rightarrow 0S \mid 1C$$

$$B \rightarrow 1S \mid 0C$$

$$C \rightarrow 0B \mid 1A$$

(a) (5 pts) 000110

(b) (5 pts) 11001001

3. Show derivations (using the  $\Rightarrow$  notation) and parse trees for the strings below given the following context-free grammar.

$$\begin{aligned} S &\rightarrow S + S \mid S * S \mid (S) \mid N \\ N &\rightarrow 0.FD \mid PD.FD \mid PD \\ D &\rightarrow 0D \mid 1D \mid 2D \mid 3D \mid 4D \mid 5D \mid 6D \mid 7D \mid 8D \mid 9D \mid \epsilon \\ F &\rightarrow 0 \mid 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \\ P &\rightarrow 1 \mid 2 \mid 3 \mid 4 \mid 5 \mid 6 \mid 7 \mid 8 \mid 9 \end{aligned}$$

- (a) (5 pts)  $12.3 + 3 * 0.22$   
 (b) (5 pts)  $4 * (9 + 13.0) * 75.89$
4. Design CFGs for the languages described below.
- (a) (5 pts) The language of all binary strings ending in 11.  
 (b) (5 pts) The language of all binary strings containing 11.  
 (c) (5 pts) The language of all palindromes from  $\{a, b, c\}^*$ .  
 (d) (10 pts)  $\{a^i b^j c^k \mid i, j, k \geq 0, i = j \text{ or } i = k\}$ .  
 (e) (10 pts) The language of all strings from  $\{a, b, c\}^*$  where the number of  $a$ s is not divisible by three.  
 (f) (10 pts)  $\{a^i b^j c^k \mid i, j, k \geq 0, i + j = k\}$ .  
 (g) (10 pts) The language of all regular expressions over a binary alphabet where  $e$  is used in place of  $\epsilon$ . For example,  $0^*11 + (110)^* + (e + 1 + 10)$  and  $10011 + (100^*)^*(00)$  are in this language while  $* + 110)(ee$  is not. Note that  $e$  does not need to be treated differently from the other terminals, 0 and 1. So, for example,  $e + e$  is also in the language.
5. Convert the CFGs below to CNF.

- (a) (10 pts)

$$\begin{aligned} S &\rightarrow aAc \mid bbB \mid aa \\ A &\rightarrow cSc \mid C \\ B &\rightarrow A \mid Bbb \\ C &\rightarrow C \mid a \mid b \mid \epsilon \end{aligned}$$

- (b) (10 pts)

$$\begin{aligned} S &\rightarrow 0AB0 \mid 1C1 \mid C0A \mid \epsilon \\ A &\rightarrow C \mid AB \mid 1 \mid 001 \\ B &\rightarrow SB1 \mid 0 \\ C &\rightarrow SAC \mid C1SSC \end{aligned}$$