A **system of linear equations** (or a **linear system**) is a collection of one or more linear equations involving the same variables--x1,...,xn.

A **solution** of a linear system is a list (s1, s2,.., sn) of numbers that makes each equation a true statement when s1,...,sn of numbers are substituted for x1,...,xn.

The set of all possible solutions is called the **solution set** of the linear system.

Two linear systems are called **equivalent** if they have the same solution set. Each solution of the first system is a solution of the second system and vica versa.

A system of linear equations has either:

1. No solution, or

2. Exactly one solution, or

3. Infinitely many solutions

A system of linear equations is said to be **consistent** if it has either one solution or infinitely many solutions; a system is **inconsistent** if it has no solution.

The essential information of a linear system can be recorded compactly in a rectangular array called a **matrix**. There are two types: a matrix of **coefficients** and an **augmented matrix of the system**.

The **size** of a matrix tells how many rows and columns it has.An **M x N matrix** is a rectangular array of numbers with M rows and N columns.

Two matrices are **row equivalent** if there is a sequence of elementary row operations that transforms one matrix into the other.

There exist two fundamental questions about a linear system:

1. Is the system consistent; does at least one solution **exist**?

2. If a solution exists, is it the only one; is the solution **unique**?

A rectangular matrix is in **echelon form** if it has the following properties:

1. All nonzero rows are above any rows of all zeros.

2. Each leading entry of a row is in a column to the right of the leading entry of the row about it.

3. All entries in a column below a leading entry are zeros.

If a matrix in echelon form satisfies the following additional properties, it is in **reduced echelon form**:

4. The leading entry in each nonzero row is 1.

5. Each leading 1 is the only nonzero entry in its column.

An **echelon matrix** is one that is in echelon form.

Theorem 1: Uniqueness of the Reduced Echelon Form

Each matrix is row equivalent to one and only one reduced echelon matrix.

A **pivot position** in a matrix A is a location in A that corresponds to a leading 1 in the reduced echelon form of A. A **pivot column** is a column of A that contains a pivot position.

A **pivot** is a nonzero number in a pivot position that is used as needed to create zeros via row operations.

Variables corresponding to pivot columns in the matrix are called **basic variables**. Other variables are called **free variables**.

The **general solution** of the system gives an explicit description of all solutions.

Theorem 2: Existence and Uniqueness Theorem

A linear system is consistent if and only if the rightmost column of the augmented matrix is not a pivot column--that is, if and only if an echelon form of the augmented matrix has no row of the form [0 ... 0 b] with b nonzero.

A matrix with only one column is called a **column vector**, or simply a **vector**. Entries are any real numbers.

The set of all vectors with -two- entries is denoted by R^2. R stands for the real numbers that appear as entries in the vectors, and the exponent indicates that the vectors each contain two entries.

Two vectors in R^2 are **equal** if and only if their corresponding entries are equal.

Vectors: Parallelogram Rule of Addition

If **u** and **v** in R^2 are represented as points in the plane, then **u + v** corresponds to the fourth vertex of the parallelogram whose other vertices are **u, 0,** and **v**.

If  **n** is a positive integer, R^n denotes the collection of all lists of **n** real numbers, usually written as n x 1 column matrices.

The vector whose entries are all zero is called the **zero vector** and is denoted by **0**.

Given vectors v1, v2,..., vp in R^n and given scalars c1, c2,...,cp the vector y defined by: y = c1v1 + ... + cpvp - is called a **linear combination** of v1,..., vp with **weights** c1,...,cp.

If v1,...,vp are in R^n, then the set of all linear combinations of v1,...,vp is denoted by span {v1,...,vp} and is called the **subset of** R^n **spanned** (or **generated**) **by v1,...,vp**. That is, Span {v1,...,vp} is the collection of all vectors that can be written in the form c1v1 + c2v2 + ... + cpvp, with c1,...,cp scalars.

If A is an m x n matrix, with columns **a1,...,an** and if **x** is in R^n, then the **product of A and x**, denoted **Ax**, is **the linear combination of the columns of A using the corresponding entries in x as weights.**

The form Ax = b is called the **matrix equation**.

If A is an mxn matrix, with columns