**University of Texas at Austin**

McCombs School of Business

RM 294: Optimization I

**Project 1 Report – Linear Programming**

**Portfolio Optimization using Conditional Value-at-Risk (CVaR)**

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**EXECUTIVE SUMMARY**

This report details the construction and analysis of investment portfolios optimized to minimize Conditional Value-at-Risk (CVaR), a more robust risk measure than traditional Value-at-Risk (VaR). Using a linear programming model, we determined optimal portfolio allocations based on the 100 stocks from the NASDAQ index using historical daily returns from 2019 and evaluated their performance against 2020, a year marked by the COVID-19 pandemic with volatile market conditions.

Our analysis explored several risk management strategies. We found that a static portfolio optimized with 2019 data performed poorly out-of-sample in 2020's volatile market. We also determined that adjusting risk tolerance (β) and the choice between minimizing average daily versus maximum monthly risk significantly altered portfolio composition and resilience. A dynamic monthly re-optimization strategy using a rolling one-year window of historical data proved most effective at managing risk and more adaptive to changing market conditions compared to the static approach, but it also introduced portfolio instability, highlighting a key trade-off.

This analysis recommends a dynamic, monthly re-optimization approach as the most effective strategy for mitigating tail risk in a fluctuating market. However, it also suggests that for practical implementation, constraints must be considered to ensure portfolio stability and costs. This project provides a quantitative framework for building resilient, low-risk portfolios adaptable to changing market conditions.

**METHODOLOGY OVERVIEW**

**Part 1: Data Cleaning and preparation**

The first step in our analysis was to prepare the historical stock data. We utilized two datasets corresponding to the daily prices of the 100 stocks from the NASDAQ index for the years 2019 and 2020. This data, loaded from provided CSVs with dates parsed as the index, was converted into daily returns using the percentage change method. This transformation normalizes the data and represents the daily performance of each stock. Note that first row is dropped since there is no previous day to calculate the percentage change for the daily returns.

Additionally, the 'NDX' (NASDAQ 100 index) column was remove, as our goal was to optimize a portfolio of individual stocks, not include the index itself (correlation). This process yielded two clean dataframes: 2019 returns for trained model optimization and 2020 returns for out-of-sample testing.

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**Part 2: Static Base CVaR Optimization & Out-of-Sample Analysis**

**Optimization Model ( =95%, R=0.02%)**

To construct the initial portfolio, we developed a linear programming model to minimize the 95% CVaR based on the 2019 daily return data. The objective is to find a portfolio allocation that minimizes the average loss on the worst 5% of trading days, subject to a set of constraints.

Objective Function

Minimize the 95% Conditional Value-at-Risk (CVaR).

* confidence level (e.g. 95%) - given
* number of return days
* dummy/helper variable representing the amount by which loss in given day k exceeds the VaR

Decision variables

* the 95% Value-at-Risk (VaR), representing the loss threshold
* the weight allocated to each stock j in the portfolio.
* a helper variable representing the "excess loss" for each trading day t (the loss that exceeds the VaR)

Constraints

1. Minimum return:The portfolio's average daily return must be at least R=0.02%
2. Excess loss constraint:for each trading day k, we need to ensure that correctly captures excess loss
3. Non-negative excess loss:for each excess loss variable cannot be negative
4. Full investment:The weights of all stocks must sum to 1
5. No short selling:The weight of any individual stock cannot be negative

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**Insights**

The model was first solved using 2019 data to create an optimal "in-sample" portfolio. This static portfolio was then tested against 2020's market data to evaluate its "out-of-sample" performance. The results are summarized below, comparing the optimized portfolio against the NASDAQ 100 (NDX) index as a benchmark.

|  |  |  |  |
| --- | --- | --- | --- |
|  | Metric | Optimized Portfolio | NDX Index |
| 2019 (In-Sample) | CVaR | 1.11% | 2.44% |
| VaR | 0.85% | 1.60% |
| 2020 (Out-of-Sample | CVaR | 4.58% | 5.59% |
| VaR | 2.53% | 3.90% |

1. In-Sample: The optimization was highly effective within the 2019 dataset. The optimized portfolio's CVaR of 1.11% was less than half that of the NDX index (2.44%), which demonstrates the model's ability to significantly reduce tail risk. Looking at the graph, you can see that the optimized portfolio is smoother and less volatile to the upsides/downsides than the NDX but following the same trend movement.

A graph of a stock market

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1. Concentrated Portfolio: The optimal 2019 portfolio was not broadly diversified, with only 13 of the 100 available stocks receiving an allocation. Over 56% of the capital was allocated to just two stocks: XEL (30.4%) and CHTR (26.5%). This high concentration suggests these assets had very strong historical risk-return profiles in 2019, but it also introduces concentration risk.

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1. Static Model Failure in Volatile Markets: The primary finding is the dramatic divergence between in-sample and out-of-sample performance. The portfolio's CVaR quadrupled from 1.11% in 2019 to 4.58% in 2020. This underscores a critical weakness of static optimization: a portfolio optimized for a calm market period (2019) is not prepared for a systemic shock and high volatility (the 2020 COVID-19 market).
2. Relative Outperformance: Despite its significant underperformance relative to expectations, the optimized portfolio was still less risky than the benchmark in 2020. Its out-of-sample CVaR of 4.58% was better than the NDX index's 5.59%, indicating that the stock selection logic provided some downside protection even during the market crash, but not as much as what we see in 2019 cut by half. Also, note when looking at the graph the optimized portfolio with 2019 weights deviates from the NDX index at the end of 2020.

A graph of a stock market

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The significant difference between the 2019 in-sample and 2020 out-of-sample CVaR reveals a core challenge in financial modeling: ***market non-stationarity***. Because the statistical properties (mean and variance) of stock returns change unpredictably, a portfolio optimized for one period does not remain optimal for the next. This result demonstrates that a ***static strategy is insufficient for managing risk in dynamic market environments***, especially during periods of high volatility/market stress.

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**Part 3: Impact of Beta (Risk Tolerance) on Portfolio Construction**

To understand how risk tolerance affects the portfolio, we re-ran the optimization using different confidence levels () of 0.90, 0.95, and 0.99. A higher indicates a more risk-averse stance, focusing the optimization on more extreme, less frequent tail-risk events.

***As the confidence level increases, the optimal CVaR of the portfolio also increases.*** This is because a higher forces the model to average losses from a more extreme/higher-loss tail of the distribution.

* CVaR ( = 0.90): 0.89%
* CVaR ( = 0.95): 1.11%
* CVaR ( = 0.99): 1.25%

The most significant impact of changing is on the ***portfolio's composition and diversification***. As risk aversion increases (higher ), the portfolio becomes more concentrated in fewer assets.

* At β = 0.90, the portfolio is the most diversified, with 16 stocks holding non-zero weights. The top holdings are CHTR (24.1%) and XEL (21.3%).
* At β = 0.95, the portfolio becomes slightly more concentrated, holding 13 stocks. The weights in XEL (30.4%) and CHTR (26.5%) increased.
* At β = 0.99, the portfolio is highly concentrated in just 9 stocks. The allocation to XEL surges to 44.7%, indicating that the model identifies this stock as the most effective for mitigating the worst 1% of potential losses.

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The cumulative return plot for 2019 shows that despite the significant differences in composition, the in-sample performance of the portfolios optimized with different values was very similar. No single portfolio consistently outperformed the others, and all tracked the general movement of the NDX index, but with less volatility.

This analysis reveals the trade-off of increasing risk aversion (a higher ) and a less diversified portfolio that is theoretically better protected against extreme events. However, this concentration also makes the portfolio more sensitive to the performance of its few holdings. The choice of is therefore a critical decision that must align with the investor's specific risk appetite and view on diversification.

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**Part 4: CVaR Optimization on Monthly Returns**

**Part 5: Dynamic Monthly Re-optimization Strategy**

To address the shortcomings of the static portfolio, we implemented a dynamic strategy. At the beginning of each month in 2020, we re-optimized the portfolio using a rolling 12-month window of historical daily returns. This approach allows the portfolio to adapt to changing market conditions.

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Summary of Monthly Realized CVaRs:

* Mean: 2.94%
* Standard deviation: 2.09%
* Minimum: 0.81% (January 2020)
* Maximum: 8.93% (March 2020)

The dynamic strategy proved far ***more effective at managing risk*.** Its mean monthly CVaR of 2.94% is significantly lower than the static portfolio's out-of-sample CVaR of 4.58%. While the risk spiked in March 2020 during the market crash due to Covid-19, the strategy adapted in subsequent months, keeping the average risk well below the static alternative.

The cumulative performance graph provides a clear visual confirmation of the dynamic strategy's superiority. It not only generated a higher overall return by the end of 2020 but, more importantly, it experienced a significantly smaller drawdown during the March crash compared to both the static portfolio and the NDX index. This resilience is a direct result of its ***ability to re-optimize and adapt away from escalating risks***. Therefore, reoptimizing the portfolio across months is a better approach from both a risk and performance perspective.

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The standard deviation of 2.09% is relatively high compared to the mean CVaR of 0.0294. This indicates that while the ***strategy was effective on average, the realized risk was not stable, fluctuating significantly from month to month.*** This volatility underscores the unpredictable nature of 2020's market and the challenge of maintaining consistent risk levels.

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**Part 6: Portfolio Stability Analysis**

While the dynamic strategy demonstrated superior risk management, its practical feasibility depends on its stability. ***A strategy that requires constant, drastic changes incurs high transaction costs and may be difficult to implement***. To measure this, we analyzed the month-to-month changes in portfolio weights, defining a "stability violation" as any change in a single stock's weight greater than 5%.

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The analysis clearly shows that the **dynamic portfolio is not stable**. Over the course of 2020, there were a total of 43 instances where a stock's weight was adjusted by more than 5% in a single month. The instability peaked in March (9 violations) and August (8 violations). The March re-optimization, reacting to the COVID-19 crash, resulted in the largest single-asset change of the year: the weight of CTXS was drastically increased by over 45%. This shows the model making a massive defensive shift in response to the market turmoil. Conversely, the high number of violations in August likely reflects the model reacting to the sharp tech-led market recovery. ***Model is overfitting to noise in the most recent data rather than capturing true market trends.***

Proposed Constraint to Enforce Stability

The portfolio's instability highlights a critical trade-off between risk optimization and transaction costs. To create a more practical and stable portfolio, we can add stability constraints to the optimization model. For each rebalancing period, we would add an additional linear constraint for every stock in the portfolio:

We can limit how much the weight of any single stock j can change from the previous month (t-1) to the current month (t).

In Gurobi, we would implement this as two linear constraints for each stock j:

These two constraints would force the model to find the optimal portfolio while ensuring that no single asset's weight can change by more than our defined 5% threshold. It reduces transaction costs and makes the portfolio more stable, but it also restricts the model's ability to react to new information, which could slightly increase risk compared to the unconstrained dynamic model.

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**CONCLUSION/FINAL RECOMMENDATION**

This analysis confirms that a static, buy-and-hold strategy is inadequate for managing tail risk in a volatile and non-stationary market. A dynamic, monthly re-optimization strategy using a CVaR minimization model is demonstrably superior, reducing the average realized risk by over a third compared to the static approach during the turbulent 2020 period.

However, the unconstrained dynamic model is too unstable for practical implementation due to high turnover. Therefore, the final recommendation is to adopt a ***constrained dynamic CVaR optimization strategy****.* By adding stability constraints that limit the maximum allowable change in any single asset's weight each month, we can strike an effective balance, creating a portfolio that is both adaptive to changing market conditions and cost-efficient to manage.