ECON324 Coursework Assignment

A Discrete-Time Infinite-Horizon Real Business Cycle Model Augmented for Investment Shocks

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Abstract

This investigation delves into the impacts of investment shocks on a Real Business Cycle (RBC) Model. After a review of the relevant literature, we develop this model step-by-step by first focusing on the household, then the firm and finally the economy as a whole. The model includes investment shocks, specifically Marginal Efficiency of Investment (MEI) shocks (μ_t), and Total Factor Productivity (TFP) shocks (A_t), the impact of both will be discussed in depth. Deriving the general equilibrium and steady state conditions facilitates the evaluation of this model's effectiveness with simulation and comparisons to real-world data from the Federal Reserve Economic Database (FRED).

Contents

1 Introduction

Real Business Cycles (RBC) have been formally studied for almost a century, with the seminal work of A.Burns and M.Wesley in 1946 [1] being expanded on throughout the latter years of the 20th Century. Their key takeaway was that many economic variables move in tandem, with deeply intertwined relationships highlighted during peaks and troughs in the business cycle. The economic community fell out of love with this model in the mid 1900's but M.Friedman and A.Swartz's conception of "Money and Business Cycles" in 1963 [2] brought the discussion back to life. From there on, the analysis of shocks on the RBC and real-world comparisons rose in academic popularity.

Fluctuations in RBC models are driven by exogenous and endogenous shocks. Shocks are unexpected economic events which directly influence the macro and micro economy. In ECON324 lecture notes [3] the role of TFP shocks is clearly outlined. Whereas, this paper seeks to introduce an investment shock into the aforementioned model. There are two main types of investment shocks; a Marginal Efficiency of Investment (MEI) shock and an Investment-Specific Technology (IST) shock. The former will be the investment shock we employ, mirroring that used in [4].

With large scale events such as the COVID-19 Pandemic and the 2008-9 Global Financial Crisis in the last two decades, the global economy has been an interesting economic environment. The interest of the writer is to look deeper into the transmission channels between key economic variables, such as output, consumption, and investment, within a RBC model. Introducing a TFP shock and a MEI shock sets the scene for an economy much like the one we see today in the UK. With studies finding that COVID-19 lowered TFP by up to 5% in the UK private sector [5], as well as other studies using MEI to simulate a COVID-19 shock in a DSGE model [6], it is clear to the writer that studying both is highly relevant for today's literature. It is with this sentiment that the writer opens this paper for the reader.

2 Literature Review

Whilst it has been discussed that RBC models originated with Burns and Wesley [1] in 1946, where they analysed co-movements between economic variables, Kydland and Prescott [7] derived a more robust model in 1982. The authors formalised a Dynamic Stochastic General Equilibrium (DSGE) model with pure production shocks. They found difficulty in obtaining accurate comparison data for output and employment, as well as identifying that their model suffered from over-simplicity, since they only employed a TFP shock. Despite those setbacks, the authors provided strong foundations for future RBC models in the fundamental theory behind multiple-period general equilibrium models explaining economic fluctuations. As a result of Kydland and Prescott's study, there have since been many studies focussing on the effects of different shocks, such as financial shocks in costly enforcement models [8], government spending shocks [9], and importantly, marginal efficiency of investment shocks [4].

Marginal efficiency of investment (MEI) shocks affect the efficiency of converting investment into future capital. Whereas, an IST shock affects the transformation of consumption into investment goods [10]. J.Greenwood has been involved in the study of both MEI and IST shocks, in studies [4] and [11]. Each of these studies found that investment shocks contribute highly to output fluctuations, with IST accounting for 30% of output fluctuations. MEI shocks are particularly interesting since they directly influence future output, whilst also influencing the marginal productivity of labour. Greenwood accepted that the modelled data in his paper did not have an advantage over previous modelled data, such as Hansen [12] in 1985, but argued that his model of a MEI shock that directly influenced *new* capital was more plausible and therefore had a greater real advantage over previous literature.

It arises, with the results of the studies above, that a combination of TFP and MEI shocks is a likely candidate for high modelling accuracy. Due to modern advancements in technologies such as Dynare and MatLab, solving complex DSGE models is no longer a challenge that earlier studies encountered. We are able to simulate the model with relative ease, as well as access US data from FRED for accurate comparisons. We therefore believe we are justified in expanding upon previous literature and will do so in the following study, starting with the derivation of the household.

3 The Household

3.1 The Household's Dynamic Optimization Problem

To formulate the household's dynamic optimisation problem, we first make some comments on households' behaviour and interactions. We measure the household's utility by the sum of its previous utility. In an infinite-horizon model with discrete time, households discount future utility at a rate we define as β, the discount factor. We set β as being $0 \leq \beta \leq 1$, meaning that future time periods, e.g. $t+1$, hold less weight than the period before, t. Consequently, a value of β which is close to 1 represents a patient household which would be willing to wait longer for future utility.

The utility function $U_t(C_t, N_t)$ (1) increases consumption and decreases labour hours worked as t gets larger, where consumption is C_t and labour hours worked is N_t. Households choose to supply labour N_t into the model at a price of a wage (W_t) and they also choose to lend capital (K_t) to firms at a price of a rental rate (r_t). They choose to spend their wage bill ($W_t N_t$) on consumption and/or investment (I_t), which is also known as savings. This means households derive utility from consumption C_t and experience dis-utility with labour N_t, so this model will use the following separable lifetime utility function:

$$U_t(C_t, N_t) = \mathbb{E}_0 \sum_{k=0}^{\infty} \beta^t \left[ln(C_t) - b\frac{N_t^2}{2} \right], \tag{1}$$

Where $\mathbb{E}_0$ represents the expectations operator and $b > 0$ is the constant weight attached to the dis-utility from labour. We also note that $\frac{N_t^2}{2}$ shows that the Frisch Inelasticity of Labour (φ) is equal to 1. This will cancel out in the following derivations, leaving a labour with a constant factor of inelasticity.

The household's dynamic optimization problem seeks to maximise its utility with respect to the constraints:

$$C_t + I_t \leq W_t N_t + r_t K_t, \tag{2}$$

And;

$$K_{t+1} = (1 - \delta)K_t + \mu_t I_t, \tag{3}$$

Is the physical capital equation, with $K_t \geq 0; K_0 \geq 0$ as we assume this model began with some capital K_0. μ_t is an exogenous shock on the marginal efficiency of investments (MEI Shock) in the economy and follows an auto-regressive $AR(1)$ process:

$$\mu_t = (\mu_{t-1})^{\rho}_{\mu} exp(sd_\mu \cdot \epsilon^{\mu}_t), \tag{4}$$

With $0 < \rho_\mu < 1$ denoting a persistence (auto-correlation) parameter, and ϵ^{μ}_t are random mean-zero, serially-uncorrelated white-noise shocks with a constant standard deviation $sd_\mu > 0$.

The household's rearranged budget constraint, using (2), is:

$$0 \leq W_t N_t + r_t K_t - C_t - I_t. \tag{5}$$

The physical capital equality (3) can be rearranged to the following:

$$I_t = \frac{K_{t+1} - (1-\delta)K_t}{\mu_t}. \tag{6}$$

So that when combining (5) and (6), we find the household's budget constraint when taking into account MEI shocks:

$$0 \leq W_t N_t + r_t K_t - C_t - \frac{K_{t+1} - (1-\delta)K_t}{\mu_t}. \tag{7}$$

3.2 The Lagrange Multiplier

We define the Lagrange Multiplier, λ_t, on the household's budget constraint when taking into account MEI shocks (7). We construct the household's Lagrangian as follows:

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg(\ln(C_t) - b\frac{N_t^2}{2} + \lambda_t \Bigg[W_t N_t + r_t K_t - C_t - \frac{K_{t+1} - (1-\delta)K_t}{\mu_t} \Bigg] \Bigg), \tag{8}$$

Where we have taken the Lagrangian of the utility function (1) with respect to the household's rearranged budget constraint (7). Clearly the expectation, $\mathbb{E}_0$, of the Lagrangian will be trivial, so we omit it in any further calculations.

3.3 First-Order Conditions

We will take the first order conditions of the Lagrangian (8) by evaluating the partial derivatives with respect to C_t, K_{t+1} and N_t. Starting with respect to C_t, we find that:

$$\frac{\delta \mathcal{L}}{\delta C_t} = \frac{\beta^t}{C_t} - \beta^t \lambda_t = 0, \tag{9}$$

Where each β^t cancels out and rearranges to give:

$$\lambda_t = \frac{1}{C_t}. \tag{10}$$

Continuing with respect to K_{t+1}:

$$\frac{\delta \mathcal{L}}{\delta K_{t+1}} = \beta^{t+1} \lambda_{t+1} r_{t+1} - \beta^t \lambda_t \frac{1}{\mu_t} + \beta^{t+1} \lambda_{t+1} \frac{(1-\delta)}{\mu_{t+1}} = 0, \tag{11}$$

Where by rearranging, dividing by β^t, and using the property of $\lambda_t = \frac{1}{C_t}$ from (10) to substitute into the above, we find the marginal utility of consumption:

$$\frac{1}{C_t} = \beta\mu_t \frac{1}{C_{t+1}}\left(r_{t+1} + (1-\delta)\frac{1}{\mu_{t+1}}\right). \tag{12}$$

We conclude the partial first order derivatives with respect to N_t:

$$\frac{\delta \mathcal{L}}{\delta N_t} = -\beta^t bN_t + \beta^t \lambda_t W_t = 0, \tag{13}$$

Where rearranging, dividing by β^t and substituting (10) for λ_t finds the labour supply condition to be:

$$W_t = \frac{bN_t}{\lambda_t} = bN_t C_t, \tag{14}$$

And rearranging, we obtain the optimal labour supply condition:

$$bN_t = C_t^{-1} W_t. \tag{15}$$

From (15) we can see that the labour supply increases with wages W_t. This implies an intra-temporal substitution effect [4] between labour and leisure; higher wages increase incentives for households to supply more labour and give up leisure time as a consequence.

In contrast to this, (15) also implies that labour supply decreases with consumption, presenting a wealth effect between labour and leisure; higher wages mean households can afford higher consumption C_t, which in turn decreases the marginal utility of consumption, C_t^{-1}. This has an impact of households preferring more leisure time over additional wages, consequently decreasing the marginal benefit of labour supply when wages rise, resulting in a fall in labour supply N_t.

3.4 The Marginal Utility of Consumption

The marginal utility of consumption (MUC) was derived in section 3.3 as equation (13), but is referenced in (15) too. The MUC at time t is defined in this model as $\frac{1}{C_t}$ and it represents the additional utility a household gains from one more unit of consumption. Since this is a decreasing function in C_t, we can explain this using the economic principle of diminishing marginal utility, which leads to consumption smoothing.

In this model, the interest rate r_t directly influences household consumption decisions through it's effects on investment/savings returns. If r_{t+1} is high, then the return on future investments is high and the household has an incentive to sacrifice current consumption for investment. Consuming less in the current time period raises the current MUC, but also raises expected future consumption for households. The converse is true for low r_{t+1}; households will consume more in the current time period as the MUC is lower and the expected future returns on investments is diminished.

To add to that, the MEI shock μ_t in the current period and μ_{t+1} in the next dictates how productive household's investments will be. A higher μ_t increases incentives to invest or save in the current time period as it directly increases the household's expectations of utility. A higher μ_{t+1} produces a more subtle effect. It inversely effects the return on future investments, meaning households would prefer to consume today rather than investing for tomorrow. The combined effects of a positive MEI shock therefore increases the MUC in the current period through providing incentives for households to invest for the future rather than consuming today.

3.5 The Euler Equation

The Euler equation with respect to the physical capital stock can be written, using equation (13), as:

$$U'_{C,t} = \beta\mu_t U'_{C,t+1}\left(r_{t+1} + (1-\delta)\frac{1}{\mu_{t+1}}\right). \tag{16}$$

The Euler equation links consumption in adjacent time periods and represent short-run optimality conditions for the model. Our interpretation of this Euler equation is that if households decrease consumption by x today, the households lose $x \cdot U'_{C,t}$ utility. When a household invests today, they will receive $x \cdot \left(r_{t+1} + (1-\delta)\frac{1}{\mu_{t+1}}\right)$ tomorrow. Since $\left(r_{t+1} + (1-\delta)\frac{1}{\mu_{t+1}}\right)$ is the expected gross real return on capital net of depreciation, we can say that the physical capital stock is negatively impacted by a positive MEI shock μ_{t+1}. This is because a high positive μ_{t+1} lowers the effectiveness of current capital in the next time period. This discourages household's investment into current physical capital as they know it's value will be less in the next time period. The effect is even greater with a highly negative MEI shock, where the rental rate in $t+1$ is eroded.

4 The Firm

4.1 The Profit Function

We introduce a standard representative firm in a perfectly competitive market which produces output according to a constant returns to scale production function:

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha} \tag{17}$$

Where A_t is a Total Factor Productivity (TFP) shock which is $A = 1$ during a steady state. It follows an AR(1) process, defined as $A_t = (A)^{1-\rho_A}(A_{t-1})^{\rho_A} exp(\epsilon_t^A)$, with $0 < \rho_A < 1$ denoting a persistence parameter. We also note that α is the output elasticity of capital and $0 \leq \alpha \leq 1$, implying diminishing marginal products. The firm aims to maximise it's profit function:

$$\Pi_t = A_t K_t^{\alpha} N_t^{1-\alpha} - W_t N_t - r_t K_t, \tag{18}$$

Where we can see that $[A_t K_t^{\alpha} N_t^{1-\alpha}]$ is equal to total revenue, Y_t, $[W_t N_t]$ is labour expenses, and $[r_t K_t]$ is capital expenses. Clearly, profit is determined by total revenue minus total expenses in a perfectly competitive market, so this profit function makes logical sense.

4.2 First-Order Conditions

To find the firm's optimum operating levels, we will take the first order conditions of the profit function (18) by evaluating the partial derivatives with respect to K_t and N_t, taking W_t and r_t as given. Starting with respect to N_t, we find that:

$$\frac{\delta\Pi}{\delta N_t} = (1-\alpha)A_t K_t^{\alpha} N_t^{-\alpha} - W_t = 0, \tag{19}$$

Which rearranges to give the Marginal Product of Labour (MPN):

$$W_t = (1-\alpha)A_t\left(\frac{K_t}{N_t}\right)^{\alpha} \equiv MPN_t. \tag{20}$$

Concluding with respect to K_t, we find that:

$$\frac{\delta \Pi}{\delta K_t} = \alpha A_t K_t^{\alpha-1} N_t^{1-\alpha} - r_t = 0, \tag{21}$$

Which rearranges to give the Marginal Product of Capital (MPK):

$$r_t = \alpha A_t\left(\frac{N_t}{K_t}\right)^{1-\alpha} \equiv MPK_t. \tag{22}$$

4.3 Intuition Behind the Optimality Conditions

In order to maximise their profits, the firm will increase labour and capital units up until the additional cost of each unit equals it's marginal contribution to output. Consequently, we set $W_t = MPN_t$ so each additional unit of labour has a cost that is equal their wage, and we also set $r_t = MPK_t$ so each additional unit of capital that is rented has a cost equal to the rental rate on that capital. This lays the foundations for labour and capital to be paid their marginal products.

4.4 Zero Profits in Equilibrium

This is modelled in a perfectly competitive market where there exists perfect information, a large number of firms with little market share, constant returns to scale, and homogenous goods. These conditions erode profit to zero, as a firm who raises their prices will loose all their customers to a competitor charging a lower price. Therefore, the zero profits is the maximum in equilibrium in this model.

Given that this firm experiences constant returns to scale with its production function, we can start by looking at the total expenses for the firm in terms of $W_t = MPN_t$ and $r_t = MPK_t$, as illustrated in (20) and (22):

$$\begin{aligned} W_tN_t + r_tK_t &= \left[\alpha A_t\left(\frac{N_t}{K_t}\right)^{1-\alpha}\right]K_t + \left[(1-\alpha)A_t\left(\frac{K_t}{N_t}\right)^{\alpha}\right]N_t \\ &= \alpha A_t K_t^{\alpha} N_t^{1-\alpha} + (1-\alpha)A_t K_t^{\alpha} N_t^{1-\alpha} \end{aligned} \tag{23}$$

Where we can notice that the CRS production function (17) can be substituted in to give:

$$\begin{aligned} (23) &= \alpha Y_t + (1-\alpha)Y_t \\ &= Y_t \end{aligned} \tag{24}$$

Leaving the zero profit condition:

$$W_tN_t + r_tK_t = Y_t \tag{25}$$

(25) shows that total income is equal to total expenses; the sum of labour and capital payments. From this, we find the demand functions for capital and labour respectively to be:

$$r_t = \alpha \frac{Y_t}{K_t} \equiv MPK_t, \tag{26}$$

$$W_t = (1-\alpha)\frac{Y_t}{N_t} \equiv MPN_t, \tag{27}$$

We interpret these as implying that a firm will hire capital/labour up until the point where they equal their MPK/MPN. If the firm has a level of capital/labour above their MPK/MPN then they are producing outside of their efficient frontier and would greatly benefit from offloading some units of capital/labour.

5 The Economy

5.1 The Economy's Market Clearing Condition

Combining the household budget constraint (2), the capital accumulation equation (3), the firm's production function (17), and the zero profit condition (25) derived in section 4.4, we can determine the economy's market clearing condition. The market clearing condition seeks a balance between households and firms by ensuring all goods produced are consumed.

We find it is important to note to the reader that the inequality in the household's budget constraint (2) does not follow into the market clearing conditions below. This is because whilst $C_t + I_t$ cannot exceed total expenditures in the household's budget constraint (2), in a market clearing environment, all expenditures are maximised by consumption (or investments). Therefore, we can construct the market clearing condition by starting with combining the zero profit condition (25) and the household's budget constraint (2) in the following way:

$$\begin{aligned} Y_t &= W_t N_t + r_t K_t \\ &= C_t + I_t \end{aligned} \tag{28}$$

Since we know $I_t = \frac{K_{t+1}-(1-\delta)K_t}{\mu_t}$ from equation (3), we give the result that:

$$Y_t = C_t + \frac{K_{t+1}-(1-\delta)K_t}{\mu_t} \tag{29}$$

And using the firm's production function (17), we find the market clearing condition to be as follows:

$$A_t K_t^{\alpha} N_t^{1-\alpha} = C_t + \frac{K_{t+1}-(1-\delta)K_t}{\mu_t} = Y_t \tag{30}$$

Where total output Y_t is exactly equal to the sum of consumption C_t and investment $\frac{K_{t+1}-(1-\delta)K_t}{\mu_t}$ in the economy at a time period t.

6 The General Equilibrium

6.1 General Equilibrium Conditions with TFP and MEI shocks

The general equilibrium conditions of an economy with TFP and MEI shocks, μ_t, can be described in the following four conditions. The Euler Equation (13) is the first condition:

$$\frac{1}{C_t} = \beta\mu_t \frac{1}{C_{t+1}}\left(r_{t+1} + (1-\delta)\frac{1}{\mu_{t+1}}\right), \tag{13}$$

Combining equations (20) and (14), we find the second condition, the optimal labour supply equation:

$$bN_t = \frac{1}{C_t}(1-\alpha)A_t\left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{31}$$

Using the firm's production function (17) with (28), we find the third condition, the market clearing equation:

$$A_t K_t^{\alpha} N_t^{1-\alpha} = Y_t = C_t + I_t, \tag{32}$$

Finally, for the fourth condition, we repeat the physical capital equation for the readers convenience:

$$K_{t+1} = (1-\delta)K_t + \mu_t I_t, \tag{3}$$

Where it is given that $\mu_t = (\mu_{t-1})^{\rho}_{\mu} exp(sd_{\mu} \cdot \epsilon^{\mu}_t)$ (4). The MEI shock μ_t shows up in (13) and (3), highlighting the deep effect MEI shocks have on this model's equilibrium.

7 The Steady State

7.1 Steady State Equilibrium Conditions

The steady state is the point at which the equilibrium converges to without the influence of shocks. To derive the steady state equilibrium conditions of the model, we assume that there are no MEI shocks, $\mu_t = \mu = 1$, and we also assume that there are no TFP shocks, $A_t = A = 1$, as given. We also set $C_t = C$, $K_t = K$, $N_t = N$, $Y_t = Y$, $W_t = W$, and $r_t = r$ in a steady state equilibrium.

From the Euler Equation (13) in steady state and using $r = \alpha(\frac{N}{K})^{1-\alpha}$ from (22), we can substitute r into (33):

$$\beta^{-1} = \mu r + (1-\delta), \tag{33}$$

$$\beta^{-1} = \mu A\alpha\left(\frac{N}{K}\right)^{1-\alpha} + (1-\delta), \tag{34}$$

Which, after re-arranging, gives us the capital-to-labour ratio in the steady state:

$$\left(\frac{K}{N}\right) = \left[\frac{\mu A\alpha}{\beta^{-1} - (1-\delta)}\right]^{\frac{1}{1-\alpha}}, \tag{35}$$

After finding steady state investment using the physical capital equation (3), and using $K_{t+1} = K_t = K$, we find:

$$K = (1-\delta)K + \mu I,$$

$$I = \frac{\delta K}{\mu}. \tag{36}$$

From the production function in steady state $Y = AK^\alpha N^{1-\alpha}$, we can write (32) as:

$$A\left(\frac{K}{N}\right)^\alpha N = C + \frac{\delta K}{\mu}, \tag{37}$$

Which, after re-arranging, results in the steady state consumption level:

$$C = A\left(\frac{K}{N}\right)^\alpha N - \frac{\delta K}{\mu} \tag{38}$$

So (35), (36), and (38) provide solutions for K, I and C, each of which are determined by $A, \mu, \alpha, \beta, \delta$ and N. Writing the optimal labour supply equation (31) in its steady state form:

$$bN = A\frac{1}{C}(1-\alpha)\left(\frac{K}{N}\right)^\alpha, \tag{39}$$

Which can be solved for N:

$$N = Ab^{-1}C^{-1}(1-\alpha)\left(\frac{K}{N}\right)^\alpha, \tag{40}$$

In summary, combining the equations (35), (36), (38), and (40), the steady state conditions of this model are:

$$K = \left[\frac{\alpha}{\beta^{-1} - (1-\delta)}\right]^{\frac{1}{1-\alpha}} \cdot N, \tag{41}$$

$$I = \delta K = \delta\left[\frac{\alpha}{\beta^{-1} - (1-\delta)}\right]^{\frac{1}{1-\alpha}} \cdot N, \tag{42}$$

$$C = \left[\left(\frac{K}{N}\right)^\alpha - \delta\left(\frac{K}{N}\right)\right] \cdot N = \left(\left[\frac{\alpha}{\beta^{-1} - (1-\delta)}\right]^{\frac{\alpha}{1-\alpha}} - \delta\left[\frac{\alpha}{\beta^{-1} - (1-\delta)}\right]^{\frac{1}{1-\alpha}}\right) \cdot N, \tag{43}$$

$$N = b^{-1}C^{-1}(1-\alpha)\left(\frac{K}{N}\right)^\alpha = \left[\frac{(1-\alpha)b^{-1}}{1 - \delta\left(\frac{K}{N}\right)^{1-\alpha}}\right]. \tag{44}$$

$$Y = K^\alpha N^{1-\alpha} \tag{45}$$

8 Model Simulation and Comparison

8.1 Simulation Using Given Parameter Values

To simulate our model we set $\rho_t = 0.95$ and $sd_\mu = 0.01$ for a 1% drop in μ_t, the same for A. This involved setting the standard error for μ to be -1. All other parameters were kept the same as the *RBC_Basic.mod* file, as required. The impulse response functions were plotted with the simulated data and are shown graphically in 1 below for the reader's convenience.

The Roles of a TFP Shock and a MEI shock

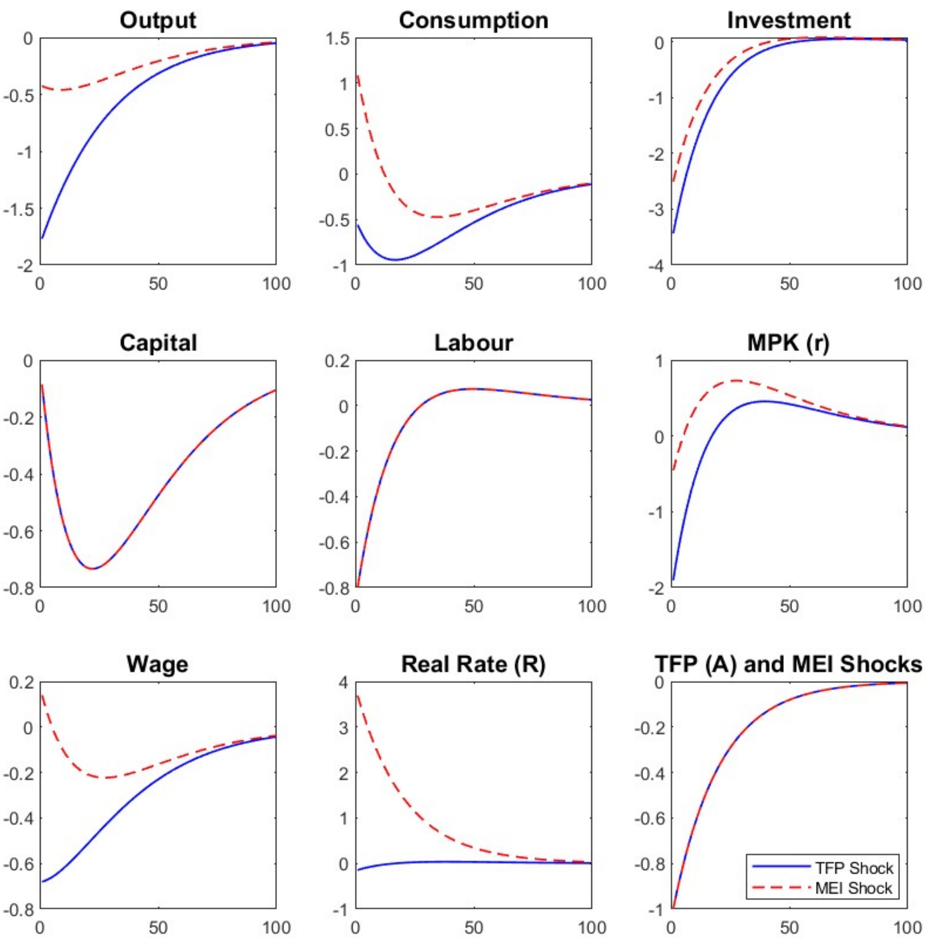

Figure 1: An illustration of the roles of TFP and MEI shocks on the economy using an RBC model. The code used to produce the above response functions is provided for the reader's review in B, along with the code used to calculate the data in A.

A negative MEI shock directly lowers the marginal efficiency of investment by -1%, increasing the cost of converting investment into capital and consequently raising the real rate by 4%, a much larger impact than TFP. Higher real rates initially mean the cost of investment increases, making investment less attractive than consumption for households, as they know the future return of investment is less than spending. Therefore, households allocate more of their income towards consumption, crowding out investment and leading to a 1% rise in consumption. This initial increase in consumption is slowly faded out as the real interest rate recovers and investment appears more attractive again, leaving consumption back at it's equilibrium.

Investment immediately sees a large fall of 2.5% as households allocate their income towards consumption. The fall in the capital stock is from reduced investment slowing capital accumulation. As the capital stock further depreciates, we see that the marginal product of capital starts to recover, which in turn creates incentives for households to increase investment, eventually bringing investment back to equilibrium.

Lower investment reduces firms incentives to hire, causing an initial decline in labour productivity and wages. Lower labour demand prompts households to reduce their labour supply, explaining the -0.8% fall in labour. A recovery of output due to the capital stock slowly accumulating restores demand for labour and encourages wages to rise back up. As wages rise, households have increased incentives to supply labour. But, as the marginal product of capital starts to recover and wages are on the rise, the demand for extra units of labour diminishes. This causes labour to briefly exceed pre-shock equilibrium levels, then stabilise at the equilibrium due to the marginal product of labour aligning with reduced investment.

Output experiences a short fall due to the reduction in investment and labour supply, but this is far smaller than the TFP shock's impact on output. It experiences a slow but steady incline towards the equilibrium as investment and capital recover due to time frictions associated with increasing the capital stock.

Overall, the negative MEI shock caused pro-cyclical movements in key economic variables, with the largest effect being on investment. As the shock fades, the economy is brought back to equilibrium and households and firms return to their pre-shock activities.

A negative TFP shock declines consumption on impact, since households perceive lower productivity as reducing future income; a negative wealth effect. Lower productivity leads to reduced marginal product of labour and marginal product of capital, leading to lower wages and reduced real return. As a consequence, households feel poorer and spend and save less than before. Despite spending and saving less, they still prioritise consumption over investment or savings, since there is a fall in the return on capital. Consumption begins to recover once the real interest rate reaches equilibrium, but this recovery is slow.

An immediate decline in investment can be seen due to the lower return on capital after the negative TFP shock. This leads to the accumulation of capital stock slowing down, leading to a reduction of new capital in the short term. As capital depreciates and is not supplemented by new capital, the capital stock reduces even more. Eventually, the capital stock begins to recover due to rising MPK, encouraging more investment and bringing it back to equilibrium.

The marginal product of labour is hit directly by the negative TFP shock, causing a reduction in demand for labour as well as a fall in wages of approximately -0.7%, far more negative than the impact of a negative MEI shock. Households can decide between working less due to lower payoff, the substitution effect, or working more to keep their disposable income and standard living the same, the income effect. We find that initially the income effect is the main driver, leading to increased labour supply. But, as the TFP shock fades, households reduce their supply of labour and labour returns to its equilibrium.

Since output is directly composed of consumption and investment, as well as being influenced by capital and labour, we see an almost -1.75% decline in output. This is a significant fall, which is almost double the size of the negative TFP shock.

Overall, a negative TFP shock causes mostly pro-cyclical downward movements in the key economic variables. As the shock fades, the economy recovers to it's pre-shock levels, with some variables, such as consumption, recovering slower than others.

To conclude, we can see that the TFP shock has a larger initial impact on output, wages, consumption and MPK as it directly impacts productivity. Whereas, a MEI shock directly impacts investment and the real rate, with more delayed impacts on the other variables. Whilst a MEI shock temporarily causes a fall in investment and rise in consumption, a TFP shock eventually increases consumption through output despite some crowding out effects. Both shocks lead to to an initial decline in labour, but also to a subsequent rise above equilibrium.

8.2 Comparison to FRED Data

Standard deviations of output, investment and consumption have been included in the attached Table 1 for the reader's review. Federal Reserve Economic Data (FRED) is the source for the real-world comparison to this model. GNP, GPDI and PCE were selected as appropriate comparators to output, investment, and consumption respectively, each of which were converted to percentage changes, to match the log-linearised modelling, before computing the standard deviations.

	Standard Deviation		
(log)Variable	TFP	TFP & MEI	source: FRED
Output	1.72	2.38	0.98
Investment	4.83	5.55	3.83
Consumption	0.50	1.69	0.53

Table 1: Standard deviations of key variables. Comparison between modelled outcomes and US data counterparts from 1970 Q1 to 2020 Q1, found on the Federal Reserve of Economic Data (FRED), provided by the Federal Reserve Bank of St. Louis. FRED data sources: Output, Investment, Consumption. TFP and MEI shock figures are pulled from the Dynare program in A.

From Table 1, several interesting features can be noticed. Firstly, output in the US, using GNP, is significantly less volatile than the modelled outcomes. Output with a TFP shock is almost double as volatile, whereas output with a TFP and a MEI shock is even more volatile.

Modelled investment also has higher volatility than GPDI, with modelled TFP and MEI shocks combined returning the largest fluctuations.

Consumption in the US is only marginally more volatile than modelled TFP shocks, implying that TFP shocks are more representative of US PCE. However, modelled TFP and MEI shocks have triple the PCE volatility.

Overall, the TFP model better explains key variations in US FRED measures than a TFP and MEI shocks combined model. This may be due to the set parameters for MEI shocks, so to further analyse the effect of standard deviations on MEI shocks, we ran simulations for a range of smaller standard deviations. We plotted these below in Figure 2 for the reader's review.

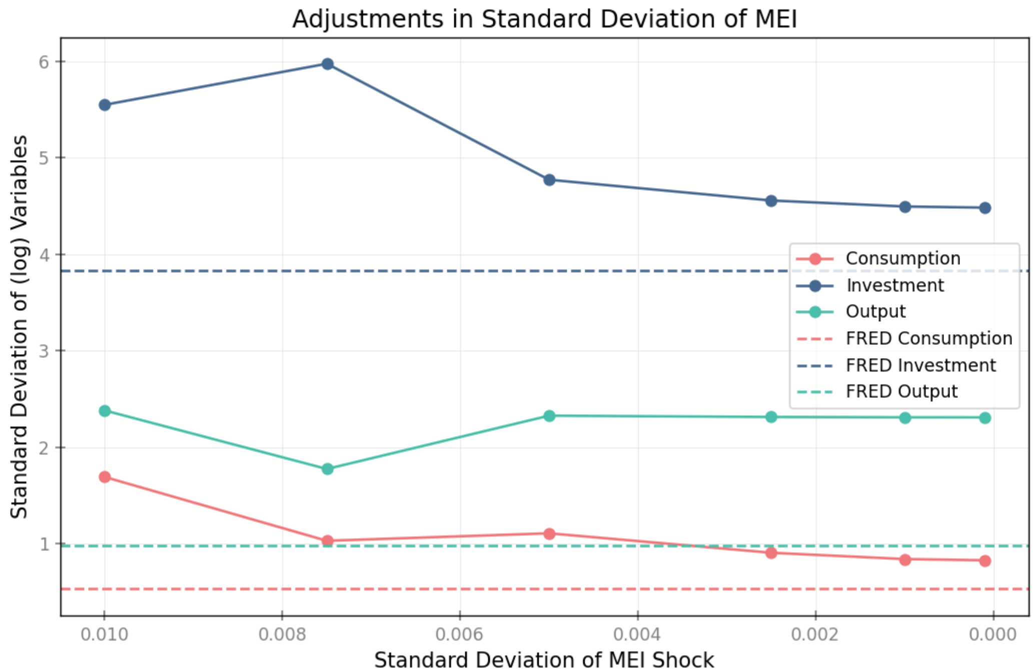

Figure 2: An illustration of adjustments in the standard deviation for a MEI shock. We used the Dynare code A to calculate the standard deviations of each (log) variable. We then used Matplotlib to plot the values, using the code in C.

The results from Figure 2 show that reductions in the standard deviation of the MEI shock lead to closer estimation of the FRED values. As the standard deviations of the MEI shock get closer to zero, the standard deviations of variables look set to converge to the FRED values. The exception to this is output, which remains very different to FRED GNP regardless of the standard deviation of MEI. With a standard deviation of 0.0001, the MEI & TFP shock estimates are almost exactly the same as the TFP shock estimates. This is due to the undermining of the MEI shock with a lower standard deviation. This leads the writer to believe that a model with just a TFP shock is a better representation of FRED data than a model with TFP & MEI.

9 Conclusion

This paper addressed the effects of negative MEI and TFP shocks on a RBC model. Through simulations, using Dynare, we were able to make comparisons between models and against US data with high computational accuracy. The transmission channels observed provide an insight into the inter-linkings between economic variables.

The results in this paper indicate that the combined TFP and MEI model is not the most accurate for US data. Although the writer finds this contrary to most literature, we understand that small adjustments in parameter values could lead to better results. Saying that, our findings have been that smaller standard deviations for MEI shocks help the model to adjust more accurately to real-world data. It is also found that TFP shocks are a more representative shock in immediate effects, whereas MEI shocks are a more considered approach with longer transmission effects.

A further extension could involve testing both TFP and MEI shocks with different values of standard deviation and persistence, as well as different sizes of shocks. This could significantly alter both models and would be a valuable comparison to the study presented above. A comparison of MEI and IST shocks could also be performed, an extension of that shown in [10].

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A Dynare Code for RBC Model with MEI and TFP Shocks

```
// // // // // The RBC Model with MEI shocks // // // // //
// Endogenous Variables
var c k n I y r w R a mu Prod log_y log_c log_I log_k log_n log_r
   log_R log_w log_a log_mu log_Prod;
// Exogenous Variables (Shocks)
varexo eps_a eps_mu;
// Parameters and Calibration
parameters beta alpha delta a_ss mu_ss b_ss varphi k_n_ss n_ss k_ss
   I_ss c_ss y_ss w_ss r_ss R_ss I_y_ss c_y_ss k_n_ss_annual
   k_y_ss_annual c_I_ss Prod_ss rho_a sd_a rho_mu sd_mu;
beta=0.995;
alpha=0.35;
delta=0.025;
a_ss=1;
mu_ss=1;
b_ss=3.67;
// Steady State Conditions // // // // // // // // // // // // // //
//SS capital to labour ratio
k_n_ss=((a_ss * alpha)/(((1/beta)-(1-delta)))^(1/(1-alpha)));
//SS labour
n_ss = ((1-alpha)*b_ss^(-1)/(1-delta*(k_n_ss)^(1-alpha)));
//SS capital
k_ss=k_n_ss*n_ss;
//SS investment
I_ss=delta*k_ss;
//SS consumption
c_ss=n_ss*(a_ss*((k_n_ss)^alpha)-delta*(k_n_ss));
//SS output
y_ss=a_ss*(k_ss^alpha)*(n_ss^(1-alpha));
//SS MPN
w_ss=a_ss*(1-alpha)*(k_ss/n_ss)^alpha;
//SS MPK
r_ss=a_ss*alpha*(n_ss/k_ss)^(1-alpha);
//SS Gross Real Return on Capital net of depreciation
R_ss=r_ss+(1-delta)*((1)/(mu_ss));
I_y_ss=I_ss/y_ss;
c_y_ss=c_ss/y_ss;
k_n_ss_annual=k_ss/4*n_ss;
k_y_ss_annual=k_ss/4*y_ss;
c_I_ss=c_ss/I_ss;
Prod_ss=y_ss/n_ss;
// Shock Calibration // // // // // // // // // // // // // // // //
sd_a=0.01;     //Standard Deviation of TFP Shock
```

```
rho_a=0.95;    //Persistence of TFP
rho_mu=0.95;    //Persistence of MEI Shock
sd_mu=0.01;     //Standard Deviation of MEI Shock
model;
// RBC Model with MEI Shock // // // // // // // // // // //
// NB: capital is a predetermined variable and is brought one period
    backwards in all equations
//Euler Equation
(1/c)=beta*mu*(1/c(1))*(r(1) + (1-delta)*((1)/(mu(1))));
//Labour Supply Condition
b_ss*n=(1/c)*a*(1-alpha)*(k(-1)/n)^alpha;
//Production Function
y=a*(k(-1)^alpha)*(n^(1-alpha));
//Resource Constraint
y=I+c;
//Capital Accumulation Constraint
k=(I*mu)+(1-delta)*(k(-1));
//Marginal Product of Capital - Demand for Capital
r=a*alpha*(n/k(-1))^(1-alpha);
//The Real Gross Return on Capital
R=r(1)+((1-delta)*(1/mu(1)));
//Marginal Product of Labour - Demand for Labour
w=a*(1-alpha)*(k(-1)/n)^alpha;
//Process for TFP A
a=((a_ss)^(1-rho_a))*((a(-1))^rho_a)*exp(-sd_a*eps_a);
// Process for MEI shock mu
mu=((mu(-1))^rho_mu)*exp(-sd_mu*eps_mu);
//Labour Productivity
Prod=y/n;
// Log-Linearising Model // // // // // // // // // // // // // //
//Annualized Log Variables
log_y=100*(y-y_ss)/y_ss;
log_c=100*(c-c_ss)/c_ss;
log_I=100*(I-I_ss)/I_ss;
log_k=100*(k-k_ss)/k_ss;
log_n=100*(n-n_ss)/n_ss;
log_r=100*(r-r_ss)/r_ss;
log_R=400*(R-R_ss)/R_ss;
log_w=100*(w-w_ss)/w_ss;
log_a=100*(a-a_ss)/a_ss;
log_mu=100*(mu-mu_ss)/mu_ss;
log_Prod=100*(Prod-Prod_ss)/Prod_ss;
end;
// NB: Below steady state values derived in a separate matlab file
initval;
```

```
c=c_ss;
k=k_ss;
n=n_ss;
I=I_ss;
y=y_ss;
r=r_ss;
R=R_ss;
w=w_ss;
a=a_ss;
mu=mu_ss;
Prod=Prod_ss;
end;
// Running Model Simulation // // // // // // // // // // // // //
steady;
check;
shocks;
var eps_a; stderr 1;
var eps_mu; stderr 1;
end;
stoch_simul(order=1,irf=100,hp_filter=1600) log_y log_c log_I log_k
   log_n log_r log_R log_w log_a log_mu log_Prod;
```

B MATLAB Code for Impulse Response Functions

```
clear; close all;
load Negative_Shock.mat
oo1=oo_;
lag = (1:1:100);
F1=figure(1);
set(F1, 'numbertitle','off')
set(F1, 'name', 'Response Functions for a MEI shock and TFP Shock')
  h1 = area(1:30);
  set(h1,'FaceColor',[.9 .9 .9]);
subplot(3,3,1)
plot(lag,oo1.irfs.log_y_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_y_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('Output','fontsize',12)
subplot(3,3,2)
plot(lag,oo1.irfs.log_c_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_c_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
```

```
title('Consumption','fontsize',12)
subplot(3,3,3)
plot(lag,oo1.irfs.log_I_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_I_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('Investment','fontsize',12)
subplot(3,3,4)
plot(lag,oo1.irfs.log_k_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_k_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('Capital','fontsize',12)
subplot(3,3,5)
plot(lag,oo1.irfs.log_n_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_n_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('Labour','fontsize',12)
subplot(3,3,6)
plot(lag,oo1.irfs.log_r_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_r_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('MPK (r)','fontsize',12)
subplot(3,3,7)
plot(lag,oo1.irfs.log_w_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_w_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('Wage','fontsize',12)
subplot(3,3,8)
plot(lag,oo1.irfs.log_R_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_R_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('Real Rate (R)','fontsize',12)
subplot(3,3,9)
plot(lag,oo1.irfs.log_a_eps_a(:,[1: 100])','b','linewidth',1);
hold on
plot(lag,oo1.irfs.log_mu_eps_mu(:,[1: 100]),'--r','linewidth',1);
hold off
title('TFP (A) and MEI shock','fontsize',12)
```

C Matplotlib Code to Plot Standard Deviations for MEI

```
import matplotlib.pyplot as plt
import numpy as np

# Data
mei_sd = np.array([0.01, 0.0075, 0.005, 0.0025, 0.001, 0.0001])
consumption = np.array([1.69, 1.0276, 1.1053, 0.9034, 0.8382,
    0.8253])
investment = np.array([5.55, 5.9764, 4.7730, 4.5575, 4.4953,
    4.4835])
output = np.array([2.38, 1.7726, 2.3255, 2.3122, 2.3085, 2.3078])
fred_consumption = 0.53
fred_investment = 3.83
fred_output = 0.98

# Plot
plt.figure(figsize=(10, 6))
plt.plot(mei_sd, consumption, label='Consumption', color='#ef767a',
    marker='o')
plt.plot(mei_sd, investment, label='Investment', color='#456990',
    marker='o')
plt.plot(mei_sd, output, label='Output', color='#49beaa', marker='o'
    )
plt.axhline(fred_consumption, color='#ef767a', linestyle='--', label
    ="FRED Consumption")
plt.axhline(fred_investment, color='#456990', linestyle='--', label
    ="FRED Investment")
plt.axhline(fred_output, color='#49beaa', linestyle='--', label="
    FRED Output")
plt.gca().invert_xaxis()
plt.grid(True, which='both', axis='both', color='lightgray',
    linestyle='-', linewidth=0.5)
plt.tick_params(axis='both', which='both', length=6, width=1, colors
    ='gray', direction='inout', grid_color='lightgray', grid_alpha
    =0.5)
plt.xlabel("Standard Deviation of MEI Shock", fontsize=12)
plt.ylabel("Standard Deviation of (log) Variables", fontsize=12)
plt.title("Adjustments in Standard Deviation of MEI", fontsize=14)
plt.legend()

# Show plot
plt.show()
```