Continuous probability Distribution & Fundamental Sampling Distributions

Unit-3

Short Answer questions

(1) Define normal distribution

Ans:- A random Variable X is Said to have a normal distribution, if its density function is given by

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}; \quad -\infty < x < \infty, \ -\infty < \mu < \infty, \ \sigma > 0$$

Where μ = mean, σ = S.D

(2) Define Gamma & Exponential distribution

Ans:- Let X be a Continuous random variable, assuming only non-negative values, distributed according to Gamma probability density function given by

$$f(x) = \begin{cases} \frac{\beta}{\Gamma(\alpha)} (\beta x)^{\alpha-1} e^{-\beta x}, & 0 < x < \infty, \ \alpha > 0, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Exponential:- A Continuous random Variable X having the range $0 < x < \infty$ is Said to have exponential distribution if it has a probability density is given by

$$f(x) = \begin{cases} \alpha e^{-\alpha x}, & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

(3) A random Sample of size 80 is taken from a Population whose S.D is 15. Find the Standard error of means.

Sol:-

$n = 80, \quad \sigma = 15$

Standard error of mean $= \frac{\sigma}{\sqrt{n}} = \frac{15}{\sqrt{80}} = 1.68.$

(4) Find the Value of the finite Population Correction factor for $n=10$ & $N=100$

Sol:-

$N = 1000, \quad n = 10$

Correction factor $= \frac{N-n}{N-1} = \frac{1000-10}{1000-1} = 0.991$

(5) Define Simple, random, Purposive Sample.

Simple Sample:- Simple Sample is defind as every item in the population has equal Chance.

Random Sample:- Random Sampling is Sample that it has an equal probability of being Choosen.

Purposive Sampling:- It is a non probability sampling Method, the elements selected for the sample are Choosen by the judgment of the researcher.

Long Answer question

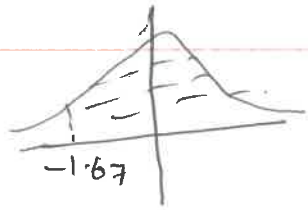
① If x is normally distributed with Mean 1 and S.D 0.6 obtain $P(x>0)$ & $P(-1.8 \le x \le 2.0)$

Sol:

Given that $\mu = 1$, $\sigma = 0.6$

① When $x=0$, $z = \frac{x-\mu}{\sigma} = \frac{0-1}{0.6} = -1.67$

$\therefore P(x>0) = P(z>-1.67)$



$= 0.5 + A(1.67)$

$= 0.5 + 0.4525$

$= 0.9525$

② $P(-1.8 < x \le 2.0)$

When $x=-1.8$, $z = \frac{x-\mu}{\sigma} = \frac{-1.8-1}{0.6} = -4.67$

$x = 2.0$, $z = \frac{x-\mu}{\sigma} = \frac{2.0-1}{0.6} = 1.67$

$\therefore P(-1.8 < x \le 2) = P(-4.7 < z < 1.7)$

$= A(1.67) + A(-4.67)$

$= 0.4525 + 0.0691$

$= 0.5216$

(2) The marks obtained by 500 students is normally distributed with mean 65% & S.D 8%. Determine how many get more than 80%

Sol:

$$\mu = 0.65, \quad \sigma = 0.08$$

$P(X > 0.8)$

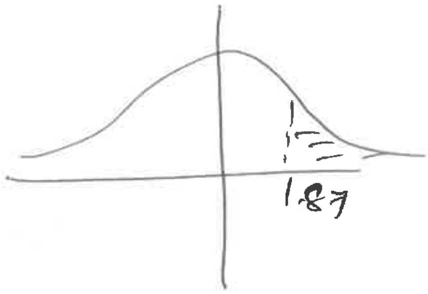
when $x = 0.8$, $z = \frac{x-\mu}{\sigma} = \frac{0.8 - 0.65}{0.08} = 1.87$

$\therefore P(X > 0.8) = P(z > 1.87)$

$= 0.5 - A(1.87)$

$= 0.5 - 0.4693$

$= 0.0307$



(3) Given that the mean heights of students in a class is 158 Cms with a S.D of 20cms. Find how many students height lie between 150 & 170 Cms.

Sol:

$$\mu = 158, \quad \sigma = 20$$

when $x = 150$, $z = \frac{x-\mu}{\sigma} = \frac{150-158}{20} = -0.4$

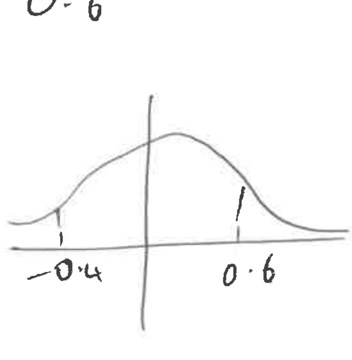
$x = 170$, $z = \frac{x-\mu}{\sigma} = \frac{170-158}{20} = 0.6$

$\therefore P(150 < X < 170) = P(-0.4 < z < 0.6)$

$= A(0.6) + A(-0.4)$

$= 0.2258 + 0.1554$

$= 0.3812$



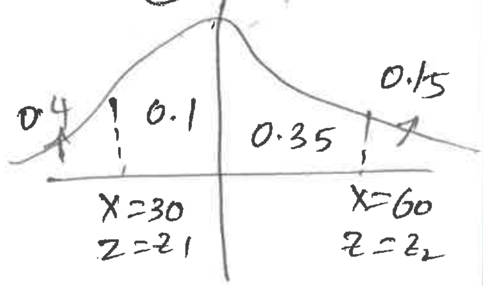
(3) The marks obtained in statistics in a certain exam found to be normally distributed. If 15% of the students ≥ 60 marks, 40% < 30 marks, find mean & S.D

Sol:-

$$P(x<30) = 0.4 \quad , \quad P(x \geq 60) = 0.15$$

when $x = 30$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{30-\mu}{\sigma} = -z_1$ — (1)

when $x = 60$, $z = \dfrac{x-\mu}{\sigma} = \dfrac{60-\mu}{\sigma} = z_2$ — (2)



$\therefore \quad P(0<z<z_1) = P(-z_1<z<0)$

$= 0.5 - 0.4$

$= 0.1$

& $P(0<z<z_2) = 0.5 - 0.15 = 0.35$

from normal tables $z_1 = 0.25$ & $z_2 = 1.04$

$\therefore$ (1) $\Rightarrow$ $\dfrac{30-\mu}{\sigma} = -0.25$ & $\dfrac{60-\mu}{\sigma} = 1.04$

$30-\mu = -0.25\sigma \qquad 60-\mu = 1.04\sigma$

Solve above eqn's

$\therefore \quad \mu = 35.81, \quad \sigma = 23.26$

(5) The mean Voltage of a battery is '15' and standard deviation 0.2. Find the probability that four such batteries connected in series will have a combined voltage of 60.8 (or) more Volts

Sol: Given that

$$\mu = 15 \qquad \sigma = 0.2$$

~~To find~~ $\cancel{p(x \geq 60.8)}$

~~When~~ $\cancel{x = 60.8 \quad z = \frac{60.8 - 15}{0.2}}$

Let mean voltage of a batteries 1, 2, 3, 4 be $\bar{x}_1, \bar{x}_2, \bar{x}_3, \bar{x}_4$

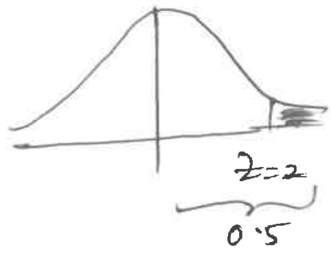
The mean of the series of the four batteries connected is

$$\mu(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4) = \mu(\bar{x}_1) + \mu(\bar{x}_2) + \mu(\bar{x}_3) + \mu(\bar{x}_4) = 15+15+15+15 = 60$$

$$\sigma(\bar{x}_1 + \bar{x}_2 + \bar{x}_3 + \bar{x}_4) = \sqrt{\sigma^2(\bar{x}_1) + \sigma^2(\bar{x}_2) + \sigma^2(\bar{x}_3) + \sigma^2(\bar{x}_4)} = \sqrt{4(0.2)^2} = 0.4$$

Let 'x' be the combined voltage of the series

When $x = 60.8$ $\quad z = \frac{\bar{x} - \mu}{\sigma} = \frac{60.8 - 60}{0.4} = 2$



So $P(x \geq 60.8) = P(z \geq 2)$

$= 0.5 - A(2)$

$= 0.5 - 0.4772$

$= 0.0228.$

Mean of Normal distribution

Consider the Normal distribution with μ, σ as the parameters

then $f(x, \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $\quad -\infty < x < \infty$

$$\text{Mean} = \mu = E(X) = \int_{-\infty}^{\infty} x \, f(x) \, dx$$

$$= \int_{-\infty}^{\infty} x \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$=$

$$\frac{x-\mu}{\sigma} = z$$

$$x = \mu + \sigma z$$

$$dx = \cancel{0} + \sigma dz$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) \, e^{-\frac{1}{2}(z)^2} \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu \, e^{-\frac{z^2}{2}} dz + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma z \, e^{-\frac{z^2}{2}} dz$$

$e^{-\frac{z^2}{2}}$ is even function $\qquad z e^{-\frac{z^2}{2}}$ odd function

$$\left[\int_{-\infty}^{\infty} f(x) dx = 2 \int_{0}^{\infty} f(x) dx \quad f(x) \text{ is even} \atop \qquad = 0 \qquad f(x) \text{ is odd} \right]$$

$\cancel{\int_{-\infty} f(x) dx = 2\int}$

$$= \frac{\mu}{\sqrt{2\pi}} \cdot 2 \int_0^\infty e^{-\frac{z^2}{2}} dz + 0$$

$$\frac{z^2}{2} = t$$

$$z^2 = 2t$$

$$z = \sqrt{2}\sqrt{t}$$

$$dz = \sqrt{2}\,\frac{1}{2} t^{\frac{1}{2}-1} dt$$

$$dz = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{t}} dt$$

$$= \frac{\mu}{\sqrt{2\pi}} 2 \int_0^\infty e^{-t} \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{t}} dt$$

$$= \frac{2\mu}{\cancel{\sqrt{2}}\sqrt{\pi}\cancel{\sqrt{2}}} \int_0^\infty e^{-t} t^{-\frac{1}{2}} dt$$

$$= \frac{\mu}{\sqrt{\pi}} \left(\Gamma\tfrac{1}{2} \right)$$

$$n-1 = -\frac{1}{2}$$

$$n = 1 - \frac{1}{2}$$

$$= \frac{1}{2}$$

$$\left[\int_0^\infty e^{-x} x^{n-1} dx = \Gamma\tfrac{1}{2} \right)$$

$$= \frac{\mu}{\cancel{\sqrt{\pi}}} \cancel{\sqrt{\pi}}$$

$$= \mu.$$

Variance of Normal distribution

$$Var(X) = E(X^2) - [E(X)]^2 \qquad (E(X) = \mu)$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$\frac{x-\mu}{\sigma} = z$$

$$x = \mu + \sigma z$$

$$dx = \sigma dz$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu+\sigma z)^2 e^{-\frac{1}{2}(z^2)} \sigma dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu^2 + \sigma^2 z^2 + 2\mu\sigma z) e^{-\frac{z^2}{2}} dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mu^2 e^{-\frac{z^2}{2}} dz + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma^2 z^2 e^{-\frac{z^2}{2}} dz + \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} 2\mu\sigma z e^{-\frac{z^2}{2}} dz$$

$e^{-\frac{z^2}{2}}$ is even function

$z^2 e^{-\frac{z^2}{2}}$ even function

$z\, e^{-z^2/2}$ odd function.

$$= \frac{\mu^2}{\sqrt{2\pi}} 2\int_0^\infty e^{-\frac{z^2}{2}} dz + \frac{\sigma^2}{\sqrt{2\pi}} 2\int_0^\infty z^2 e^{-\frac{z^2}{2}} dz + 0$$

Let $\frac{z^2}{2} = t$

$$z^2 = 2t$$

$$z = \sqrt{2}\sqrt{t}$$

$$dz = \sqrt{2}\,\frac{1}{2} t^{\frac{1}{2}-1} dt$$

$$dz = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{t}} dt$$

$$= \frac{\mu^2}{\sqrt{2\pi}} 2\int_0^\infty e^{-t} \frac{1}{\sqrt{2}}\frac{1}{\sqrt{t}} dt + \frac{\sigma^2}{\sqrt{2\pi}} 2\int_0^\infty 2t\, e^{-t} \frac{1}{\sqrt{2}}\frac{1}{\sqrt{t}} dt$$

$$= \frac{\cancel{2}\mu^2}{\cancel{\sqrt{2}}\cancel{\sqrt{2}}\sqrt{\pi}} \int_0^\infty e^{-t} t^{-\frac{1}{2}} dt + \frac{\cancel{4}\sigma^2}{\cancel{\sqrt{2}}\cancel{\sqrt{2}}\sqrt{\pi}} \int_0^\infty e^{-t} t^{\frac{1}{2}} dt$$

$n-1 = -\frac{1}{2}$ $\qquad$ $n-1 = \frac{1}{2}$

$n = \frac{1}{2}$ $\qquad$ $n = \frac{3}{2}$

$$= \frac{\mu^2}{\sqrt{\pi}} \left(\Gamma\tfrac{1}{2}\right) + \frac{2\sigma^2}{\sqrt{\pi}} \left(\Gamma\tfrac{3}{2}\right)$$

$$= \frac{\mu^2}{\cancel{\sqrt{\pi}}} (\cancel{\sqrt{\pi}}) + \frac{\cancel{2}\sigma^2}{\cancel{\sqrt{\pi}}} \frac{1}{\cancel{2}} \cancel{\sqrt{\pi}}$$

$$E(x^2) = \mu^2 + \sigma^2$$

$$var(x) = (\mu^2 + \sigma^2) - \mu^2$$

$$Var(x) = \sigma^2$$

→ A population consists of five numbers 2, 3, 6, 8 and 11. Consider all possible samples of size two which can be drawn with replacement from this population. Find

i) the mean of the population.

(ii) The standard deviation of the population

(iii) The mean of the sampling distribution of means

(iv) The standard deviation of the sampling distributions of means (i.e the standard error of means)

Sol:-

a) Mean of the population is given by

$$\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6$$

b) Variance of the population (σ^2) is given by

$$\sigma^2 = \Sigma \frac{(x_i - \bar{x})^2}{n}$$

$$= \frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}$$

$$= \frac{16+9+0+4+25}{5} = 10.8$$

$$\sigma = \sqrt{10.8} =$$

$$= 3.29$$

c) Sampling with replacement (infinite population)

The total no. of samples with replacement is

$N^n = 5^2 = 25$ sample of size $= 2$

Here $N =$ population size

$n =$ sample size   The 25 samples

$$\begin{Bmatrix} (2,2) & (2,3) & (2,6) & (2,8) & (2,11) \\ (3,2) & (3,3) & (3,6) & (3,8) & (3,11) \\ (6,2) & (6,3) & (6,6) & (6,8) & (6,11) \\ (8,2) & (8,3) & (8,6) & (8,8) & (8,11) \\ (11,2) & (11,3) & (11,6) & (11,8) & (11,11) \end{Bmatrix}$$

Now mean of each of these 25 samples

The sample means are

$$\begin{Bmatrix} 2 & 2.5 & 4 & 5 & 6.5 \\ 2.5 & 3 & 4.5 & 5.5 & 7 \\ 4 & 4.5 & 6 & 7 & 8.5 \\ 5 & 5.5 & 7 & 8 & 9.5 \\ 6.5 & 7 & 8.5 & 9.5 & 11 \end{Bmatrix}$$

$$\mu_{\bar{x}} = \frac{\text{Sum of all sample means}}{25} = \frac{150}{25} = 6$$

So $\mu_{\bar{x}} = \mu$

d) $\sigma_{\bar{x}}^2 = \frac{(2-6)^2 + \cdots + (11-6)^2}{25} = \frac{135}{25} = 5.40$

and $\sigma_{\bar{x}} = \sqrt{5.40} = 2.32$

Clearly for finite population involving sampling with replacement (Infinite population)

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n} = \frac{(3.29)^2}{2} = 2.32$$

Solution of above problem with out Replacement (finite population)

Sol

i) $\mu = \frac{2+3+6+8+11}{5} = \frac{30}{5} = 6$

ii) $\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n}$

$$= \frac{(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2}{5}$$

$\sigma^2 = 10.8$ so $\sigma = 3.29$

iii) Sampling with out replacement

The total no of samples with out replacement is $^{N}C_{n} = {^5C_2} = 10$

The 10 samples are

$$\left\{\begin{matrix} (2,3) & (2,6) & (2,8) & (2,11) \\ (3,6) & (3,8) & (3,11) & \\ (6,8) & (6,11) & & \\ (8,11) & & & \end{matrix}\right\}$$

The Corresponding Sample means are

$$\begin{Bmatrix} 2.5 & 4 & 5 & 6.5 \\ 4.5 & 5.5 & 7 & \\ 7 & 8.5 & & \\ 9.5 & & & \end{Bmatrix}$$

The mean of the Sampling distribution of means

$$\mu_{\bar{x}} = \frac{2.5+4+5+6.5+4.5+5.5+7+7+8.5+9.5}{10} = 6$$

So $\mu_{\bar{x}} = \mu$

iv) The variance of Sampling distributions of means

$$\sigma_{\bar{x}}^2 = \frac{(2.5-6)^2+(4-6)^2+ \cdots +(9.5-6)^2}{10} = 4.05$$

$$\sigma_{\bar{x}} = 2.01$$

$$\sigma_{\bar{x}}^2 = \frac{\sigma^2}{n}\left(\frac{N-n}{N-1}\right) = \frac{10.8}{2}\left(\frac{5-2}{5-1}\right) = 4.05$$

for Sampling without Replacement

UNIT - III

Continous probability distribution :-

Normal distribution :-

Normal distribution was discovered by Karl Friedrich gauss so that it is also called Gaussian distribution.

The Normal distribution is limiting case of Binomial distribution under the conditions.

1) No. of trials n is very large. i.e, $n \to \infty$

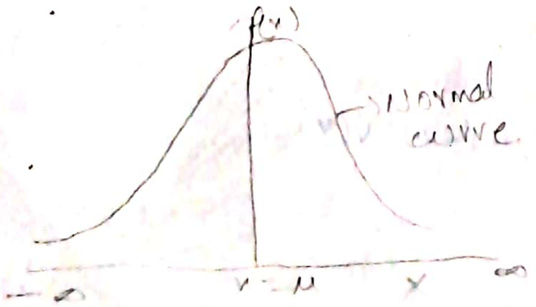
2) Neither p nor q is very small.

A continous R.V 'x' is said to have normal distribution if its density function is given by

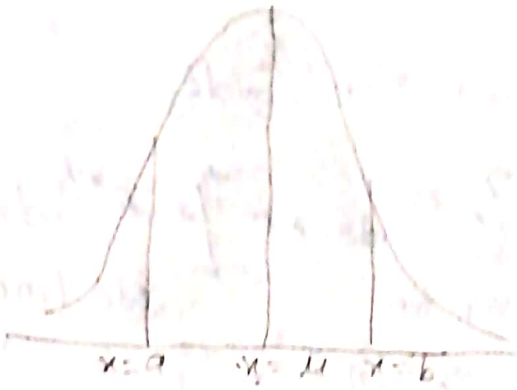
$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

where μ, σ are mean & standard deviations of x

The graph of the R.V 'x' and density function $f(x)$ is called Normal curve. The total area bounded by Normal curve and x axis is equal to 1.



$$\int_{-\infty}^{\infty} f(x)\,dx = 1 = 100\%.$$



$P(a \le x \le b) = \int_a^b f(x)\,dx$ = Area under the curve b/w lines $x=a$, $x=b$.

Normal Distribution case of B.D :-

Formulae :-

1) $\int_{-a}^{a} f(x)\,dx = \begin{cases} 2\int_0^a f(x)\,dx & , f(x) \text{ even} \\ 0 & , f(x) \text{ odd} \end{cases}$

2) if $\int_a^b f(x)\,dx = 0$, then $a=b$ $\{f(x) \neq 0\}$

3) $\int_0^\infty e^{-x^2/2}\,dx = \sqrt{\pi/2}$

4) $\int_{-\infty}^{\infty} e^{-x^2/2}\,dx = \sqrt{2\pi}$.

5) i) $\Gamma n = \int_0^\infty e^{-x} \cdot x^{n-1}\,dx$.

ii) $\Gamma n = (n-1)\,\Gamma(n-1)$ if n is +ve function.

iii) $\Gamma 1/2 = \sqrt{\pi}$ $\Gamma 1 = 1$.

Mean of Normal distribution :-

$E(x) = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$.

$E(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x \cdot e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$.

let $\frac{x-\mu}{\sigma} = z \Rightarrow \boxed{x = \mu + \sigma z}$

$\frac{dx}{\sigma} = dz$ $\boxed{\therefore dx = \sigma dz}$

$$E(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\mu + \sigma z) e^{-z^2/2} \sigma\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \left\{ \mu \underbrace{\int_{-\infty}^{\infty} e^{-z^2/2} dz}_{\text{even func}} + \sigma \underbrace{\int_{-\infty}^{\infty} z \cdot e^{-z^2/2} \cdot dz}_{\text{odd func.}} \right\}.$$

$$E(x) = \frac{1}{\sqrt{2\pi}} \left\{ 2\mu \int_{0}^{\infty} e^{-z^2/2} dz + 0 \right\}.$$

$$E(x) = \frac{2\mu}{\sqrt{2\pi}} \cdot \sqrt{\frac{\pi}{2}}$$

$$= \frac{2\mu\sqrt{\pi}}{\sqrt{2}\cdot\sqrt{2}\sqrt{\pi}}$$

$$\boxed{\therefore E(x) = \mu.}$$

Variance of N.D :-

$$V(x) = E[(x-\mu)^2] = E(x^2) - \mu^2$$

$$E[(x-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx.$$

$$V(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)^2 e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

let $z = \frac{x-\mu}{\sigma} \Rightarrow x - \mu = \sigma z$

$$dz = \frac{dx}{\sigma}.$$

$$dx = \sigma dz.$$

$$V(x) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z)^2 \cdot e^{-z^2/2} \sigma\, dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 \cdot e^{-z^2/2} dz$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \int_{0}^{\infty} z^2 \cdot e^{-z^2/2} dz.$$

let $z^2/2 = t$

$$z^2 = 2t.$$

$z = \sqrt{2}\sqrt{t}$.

$dz = \sqrt{2} \cdot \frac{1}{2\sqrt{t}} dt$

$dz = \frac{1}{\sqrt{2}\sqrt{t}} dt$.

$V(X) = \frac{2\sigma^2}{\sqrt{2\pi}} \int_0^\infty e^{-t} 2t \cdot \frac{1}{\sqrt{2}\sqrt{t}} dt$

$= \frac{4\sigma^2}{2\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{1-1/2} dt$.

$= \frac{2\sigma^2}{\sqrt{\pi}} \int_0^\infty e^{-t} t^{1/2} dt$.

$\Rightarrow \frac{2\sigma^2}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{3/2-1} dt$.

$\Rightarrow \frac{2\sigma^2}{\sqrt{\pi}} \Gamma 3/2$

$\Rightarrow \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \Gamma 1/2$

$\Rightarrow \frac{2\sigma^2}{\sqrt{\pi}} \cdot \frac{1}{2} \sqrt{\pi}$

$\therefore V(X) = \sigma^2$.

Mode of N.D :-

Mode is a value of 'x' for which $f(x)$ is maximum.

we get mode by $f'(x)=0$, $f''(x) < 0$ at mode.

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$

d.w.r.t to 'x'

$f'(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \cdot \frac{d}{dx}\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$

$f'(x) = -\frac{1}{2}[f(x)] \cdot 2\left(\frac{x-\mu}{\sigma}\right)\frac{1}{\sigma}$

$f'(x) = -f(x)\left(\frac{x-\mu}{\sigma^2}\right) \rightarrow (1)$

$\therefore f'(x) = 0$.

$$f(x)\left(\frac{x-\mu}{\sigma^2}\right)=0$$

$$\frac{x-\mu}{\sigma^2}=0$$

$$\boxed{\therefore x=\mu}$$

d. w.r.t to 'x'

$$f''(x) = -\left[f'(x)\left(\frac{x-\mu}{\sigma^2}\right)+f(x)\frac{1}{\sigma^2}\right]$$

at $x=\mu$,

$$f''(\mu) = -\left[0+f(\mu)\frac{1}{\sigma^2}\right]$$

$$f''(\mu) = -\frac{1}{\sigma\sqrt{2\pi}}\cdot\frac{1}{\sigma^2} < 0$$

$\therefore$ Mode $=\mu$.

Median of N.D :-

Let 'M' be median of N.D' $x \in [-\infty, \infty]$.

$$\int_{-\infty}^{M} f(x)dx = \int_{M}^{\infty} f(x)dx = 1/2 .$$

$$\int_{-\infty}^{m} f(x)dx = 1/2 .$$

Suppose, $-\infty < \mu < m$,

$$\int_{-\infty}^{\mu} f(x)dx + \int_{-\mu}^{m} f(x)dx = 1/2 \rightarrow (1)$$

consider,

$$\int_{-\infty}^{\mu} f(x)dx = \frac{1}{\sigma\sqrt{2\pi}}\int_{-\infty}^{\mu} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}dx .$$

let $\frac{x-\mu}{\sigma}=z$ — if $x=-\infty \Rightarrow z=-\infty$

$\frac{dx}{\sigma}=dz$. — if $x=\mu \Rightarrow z=0$.

$dx=\sigma dz$

$$\Rightarrow \int_{-\infty}^{\mu} f(x)\,dx = \frac{1}{\sigma\sqrt{2\pi}} \cdot \int_{-\infty}^{0} e^{-z^2/2} (\sigma\, dz)$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-z^2/2} dz$$

$$\int_0^{\infty} e^{-x^2/2} dx = \sqrt{\frac{\pi}{2}}$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-z^2/2} dz$$

$$\Rightarrow \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2}} \Rightarrow 1/2.$$

9, fr from ①,

$$\text{if } \int_a^b f(x)\,dx = 0 \quad (a=b)$$

$$\frac{1}{2} + \int_{\mu}^{m} f(x)\,dx = 1/2.$$

$$\int_{\mu}^{m} f(x)\,dx = 1/2 - 1/2 = 0$$

then $m = \mu$.

$$\boxed{\therefore \text{median} = \mu.}$$

Mean deviation :-

$$M.D = \int_{-\infty}^{\infty} |x-\mu|\, f(x)\,dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} (x-\mu)\, e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

let $\frac{x-\mu}{\sigma} = z \Rightarrow x-\mu = \sigma z$

$\frac{dx}{\sigma} = dz \Rightarrow dx = \sigma\, dz.$

$$\Gamma n = \int_0^{\infty} e^{-x} x^{n-1} dx$$

$$M.D = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} |\sigma z|\, e^{-z^2/2} (\sigma dz)$$

$$\Gamma\tfrac{1}{2} = \sqrt{\pi}, \quad \Gamma 1 = 1$$

$$= \frac{\sigma}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \underbrace{|z| \cdot e^{-z^2/2}}_{\text{even fn.}} dz.$$

$$= \frac{2\sigma}{\sqrt{2\pi}} \int_{0}^{\infty} |z| \cdot e^{-z^2/2} dz.$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty z \cdot e^{-z^2/2} \, dz$$

let $z^2/2 = t \Rightarrow z^2 = 2t$

$$z = \sqrt{2} \cdot \sqrt{t}$$

$$dz = \sqrt{2} \cdot \frac{1}{2\sqrt{t}} \, dt$$

$$dz = \frac{1}{\sqrt{2} \cdot \sqrt{t}} \, dt$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty \sqrt{2} \cdot \sqrt{t} \cdot e^{-t} \cdot \frac{1}{\sqrt{2}\sqrt{t}} \, dt.$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^0 \, dt.$$

$\Gamma\frac{1}{2} = \sqrt{\pi}$

$\Gamma 1 = 1$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \int_0^\infty e^{-t} \cdot t^{1-1} \, dt.$$

$$\Rightarrow \frac{\sqrt{2}\sigma}{\sqrt{\pi}} \cdot \Gamma(1)$$

$$\Rightarrow \sqrt{\frac{2}{\pi}}\sigma \Rightarrow \frac{4}{5}\sigma.$$

Chief characteristics of N.D :-

1) The graph of Normal distribution $y = f(x)$ in XY plane is called normal curve.

ii) The curve is Bell shapped and symmetrical about mean line $x = \mu$. and the 2 tails on the right and left sides of mean extends to infinity.

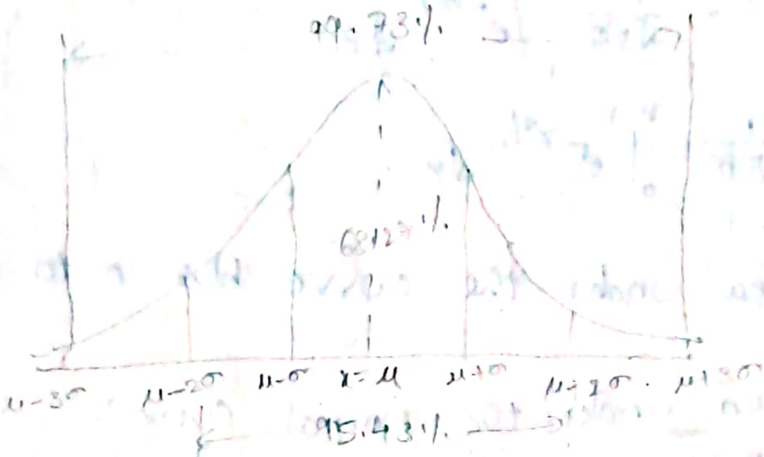
3) i) The area under the Normal curve represents total population.

ii) Mean, median & mode of distribution are coincide at $x = \mu$.

i.e; mean = median = mode.

4) X-axis is asymptote to the curve. And normal curve is unimodel.

5)



Area b/w $\mu-\sigma$ & $\mu+\sigma$ is 68.27%

i.e; $P(\mu-\sigma < x < \mu+\sigma) = 68.27\%$

$P(\mu-2\sigma < x < \mu+2\sigma) = 95.43\%$

$P(\mu-3\sigma < x < \mu+3\sigma) = 99.73\%$.

6) Normal Variable:-

The variable $\boxed{z = \frac{x-\mu}{\sigma}}$ is called normal variable.

Standard Normal Variables:-

Normal variable z is called standard normal variable if $\mu=0$, $\sigma=1$.

Area under the Normal Curve:-

If the R.V 'x' lies b/w μ & x_1, then probability of 'x' lies b/w μ & x_1 is

$$P(\mu < x < x_1) = \int_{\mu}^{x_1} f(x)\,dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{\mu}^{x_1} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

let $z = \frac{x-\mu}{\sigma}$ | if $x=\mu \Rightarrow z=0$

$dz = \frac{dx}{\sigma}$ | if $x=x_1 \Rightarrow z_1 = \frac{x_1-\mu}{\sigma}$.

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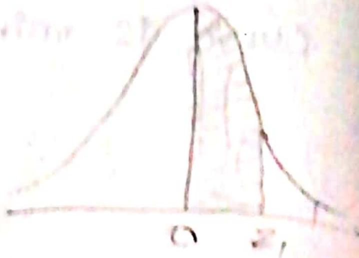
$$P(0<z<z_1) = \frac{1}{\sigma\sqrt{2\pi}}\int_0^{z_1} e^{-z^2/2}(\sigma dz).$$



$$P(0<z<z_1) = \frac{1}{\sqrt{2\pi}}\int_0^{z_1} e^{-z^2/2}\,dz$$

$$A(z_1) = \frac{1}{\sqrt{2\pi}}\int_0^{z_1} e^{-z^2/2}\,dz.$$

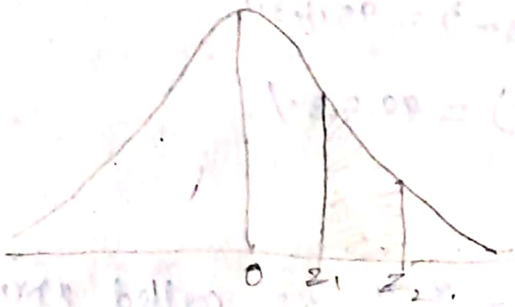
$A(z_1)$ is Area under the curve b/w 0 to z_1

Finding the area under the Normal curve :-

1) for $P(z_1<z<z_2)$:-

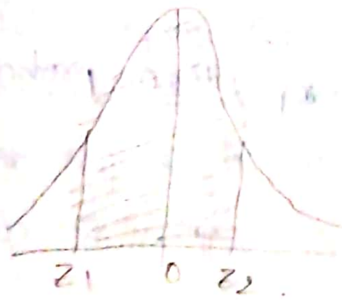
case - 1 :- if $z_1>0$, $z_2>0$.

for $P(z_1<z<z_2)$:-



$$P(z_1<z<z_2) = P(0<z<z_2) - P(0<z<z_1)$$
$$= A(z_2) - A(z_1);$$

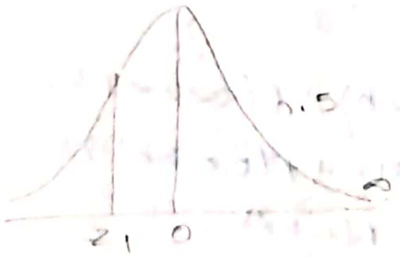
case - 2 :- if $z_1<0$, $z_2>0$



$$P(z_1<z<z_2) = P(z_1<z<0) + P(0<z<z_2)$$
$$= P(0<z<z_1) + P(0<z<z_2)$$
$$= A(z_1) + A(z_2)$$

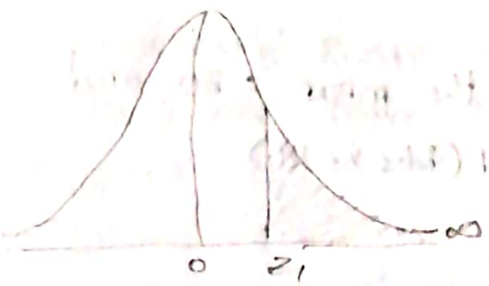
2) for $P(z > z_1)$:-

Case - I :- if $z_1 < 0$.



$$P(z > z_1) = 0.5 + P(z_1 < z < 0)$$
$$= 0.5 + P(0 < z < z_1)$$
$$= 0.5 + A(z_1).$$

case - II :- if $z_1 > 0$



$$P(z > z_1) = 0.5 - P(0 < z < z_1)$$
$$= 0.5 - A(z_1).$$

1) Normally distributed variable with mean $(\mu) = 1$, standard deviation $(\sigma) = 3$. Find (i) $P(3.43 \le x \le 6.19)$
(ii) $P(-1.43 \le x \le 6.19)$.

i) $P(3.43 \le x \le 6.19)$

$$z = \frac{x - \mu}{\sigma}$$

if $x = 3.43 \Rightarrow z = \frac{3.43 - 1}{3} = 0.81$

if $x = 6.19 \Rightarrow z = \frac{6.19 - 1}{3} = \frac{5.19}{3} = 1.73.$

$$P(0.81 \le z \le 1.73) = P(0 < z < 1.73) - P(0 < z < 0.81)$$
$$= A(1.73) - A(0.81)$$
$$= 0.4582 - 0.2910$$
$$= 0.1672$$

ii) $P(-1.43 \le x \le 6.19)$.

if $x = -1.43 \Rightarrow z = \dfrac{-1.43-1}{3} \Rightarrow \dfrac{-2.43}{3} \Rightarrow -0.81$.

if $x = 6.19 \Rightarrow z = \dfrac{6.19-1}{3} \Rightarrow \dfrac{5.19}{3} \Rightarrow 1.73$.

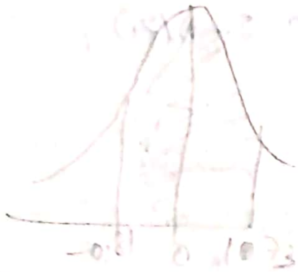
$P(-0.81 < z < 1.73) = P(-0.81 < z < 0) + P(0 < z < 1.73)$

$= P(0 < z < 0.81) + P(0 < z < 1.73)$

$= A(0.81) + A(1.73)$

$= 0.2910 + 0.4582$

$= 0.7492$



2) If x is normal variate if the mean $= 30$ and Std. deviation 5 then find i) $P(26 \le x \le 40)$ ii) $P(x \ge 45)$ iii) $P(x \le 45)$

1) Mean $(\mu) = 30$

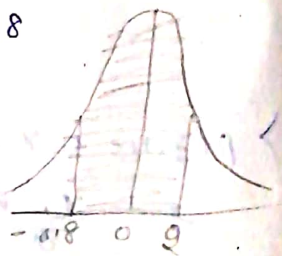
S.D $(\sigma) = 5$.

$P(26 \le x \le 40)$

if $x = 26 \Rightarrow z = \dfrac{26-30}{5} \Rightarrow -4/5 \Rightarrow -0.8$

if $x = 40 \Rightarrow z = \dfrac{40-30}{5} \Rightarrow 10/5 \Rightarrow 2$.

$P(26 \le x \le 40) \Rightarrow P(-0.8 \le z \le 2)$



$\Rightarrow P(-0.8 \le z \le 0) + P(0 \le z \le 2)$

$\Rightarrow P(0 \le z \le 0.8) + P(0 \le z \le 2)$

$\Rightarrow A(0.8) + A(2)$

$\Rightarrow 0.2881 + 0.4772$

$\Rightarrow 0.7653$.

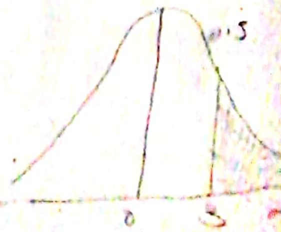
ii) $P(x \ge 45)$

if $x = 45 \Rightarrow z = \dfrac{45-30}{5} \Rightarrow \dfrac{15}{3} \Rightarrow 3$.

$P(x \ge 45) = P(z \ge 3)$

$\Rightarrow 0.5 - P(0 \le z \le 3) \Rightarrow 0.5 - A(3)$

$\Rightarrow 0.5 - 0.4987 \Rightarrow 0.0013$



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iii) $P(x \le 45)$

if $x = 45 \Rightarrow z = \frac{45-30}{5} \Rightarrow 3$

$P(x \le 45) \Rightarrow 0.5 + P(0 \le z \le 3)$

$\Rightarrow 0.5 + A(3)$

$\Rightarrow 0.5 + 0.4987$

$\Rightarrow 0.9987$

3) In a sample of 1000 cases, mean ~~is obtai~~ of certain test is 14 and std deviation 2.5. Assume the distribution to be normal. Find i) how many students scored b/w 12 & 15.

ii) How many score above 18.

iii) How many score below 18.

Mean$(\mu) = 14$

S.D $(\sigma) = 2.5$

Let x = score of student.

i) $P(12 < x < 15)$

$z = \frac{x-\mu}{\sigma}$

if $x = 12 \Rightarrow z = \frac{12-14}{2.5} \Rightarrow \frac{-2}{2.5} \Rightarrow -0.8$

if $x = 15 \Rightarrow z = \frac{15-14}{2.5} \Rightarrow \frac{1}{2.5} \Rightarrow \frac{10}{25} \Rightarrow 0.4$

$P(-0.8 < z < 0.4) = P(-0.8 < z < 0) + P(0 < z < 0.4)$

$= A(-0.8) + A(0.4)$

$= 0.2881 + 0.1554$

$= 0.4435$

No. of students score b/w 12 & 15 is

$\Rightarrow 0.4435 \times 1000$

$\Rightarrow 443.5$

$\Rightarrow 444$ students.

ii) $P(x > 18)$

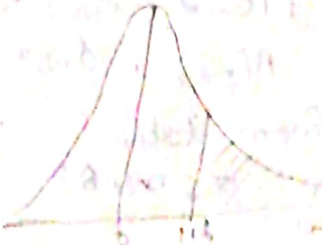
if $x = 18 \Rightarrow z = \frac{18-14}{2.5} \Rightarrow \frac{4}{2.5} \Rightarrow 1.6$

$P(z > 1.6) = 0.5 - P(0 < z < 1.6)$

$= 0.5 - A(1.6)$

$= 0.5 - 0.4452$

$\Rightarrow 0.0548$



No. of students score above 18 $\Rightarrow 0.0548 \times 1000$

$\Rightarrow 54.8$

$\Rightarrow 55.$

iii) $P(X<18)$.

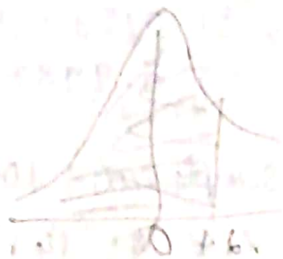
if $X=18 \Rightarrow z=1.6$.

$P(z<1.6) = 0.5 + P(0<z<1.6)$

$= 0.5 + A(1.6)$

$\Rightarrow 0.5 + 0.4452$

$\Rightarrow 0.9452.$



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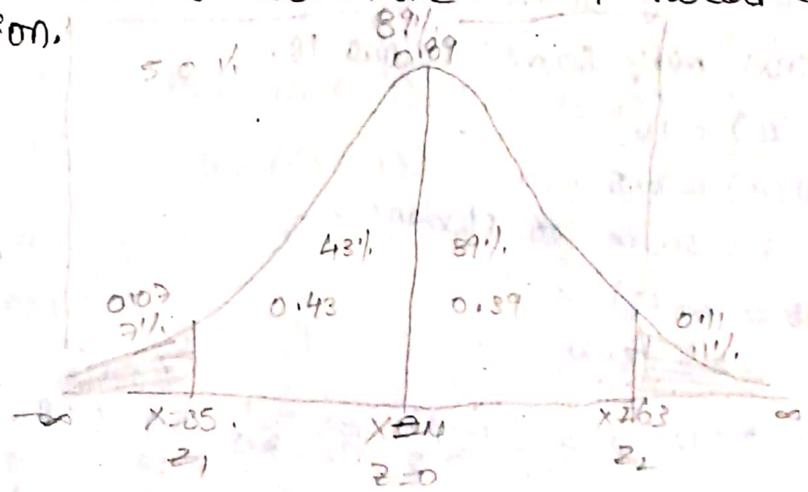
4) In a Normal dist. 7% of items under 35 and 89% of items under 63. Determine mean & variance of distribution.



$P(X<35) = 7\% = 0.07$

$P(X<63) = 89\% = 0.89$

$P(X \geq 63) = 1 - P(X<63)$

$= 1 - 0.89 \Rightarrow 0.11$

$z = \frac{x-\mu}{\sigma}$.

if $x=35 \Rightarrow \frac{35-\mu}{\sigma} \Rightarrow -z_1$ (say)

if $x=63 \Rightarrow \frac{63-\mu}{\sigma} \Rightarrow z_2$ (say) $\Big\} \rightarrow (1)$

from diagram,

$P(0<z<z_2) = 0.39$

$A(z_2) = 0.39$

from table

$z_2 = 1.23$

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$P(-z_1 < z < 0) = 0.43.$

$A(-z_1) = 0.43.$

$A(z_1) = 0.43.$

from table

$z_1 = 1.48.$

from ①.

$\frac{35}{\sigma} - \frac{\mu}{\sigma} = -1.48.$

$-\frac{63}{\sigma} - \frac{\mu}{\sigma} = 1.23$

$\frac{-28}{\sigma} = -2.71.$

$\boxed{\therefore \sigma = 10.33.}$

from ①

$\frac{35-\mu}{\sigma} = -1.48.$

$\frac{35-\mu}{10.33} = -1.48$

$35-\mu = -1.48 \times 10.33.$

$\boxed{\mu = 50.28.}$

5) In Normal distr 31% of items under 45 & 8% of items over 64. Find Mean & variance of distribution.

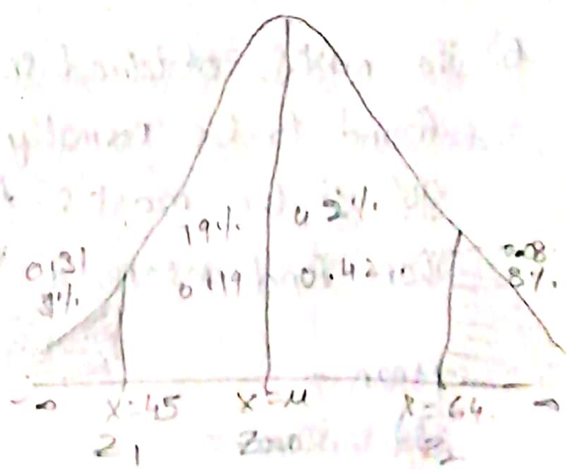
$P(x<45) = 0.31$

$P(x>64) = 8\% = 0.08$

$z = \frac{x-\mu}{\sigma}$

if $x = 45 \Rightarrow \frac{45-\mu}{\sigma} = -z_1$ (say).

if $x = 64 \Rightarrow \frac{64-\mu}{\sigma} \Rightarrow z_2$ (say) $\Big\} \to ①$



from diagram

$P(0 < z < z_2) = 0.42$

$A(z_2) = 0.42$

from table

$\boxed{z_2 = 1.41.}$

$P(-z_1 < Z < 0) = 0.19$

$A(-z_1) = 0.19$

$A(z_1) = 0.19$

From table

$z_1 = 0.5.$

From ①.

$$\frac{45}{\sigma} - \frac{\mu}{\sigma} = -0.50$$

$$\frac{64}{\sigma} - \frac{\mu}{\sigma} = 1.41$$

$$\frac{-19}{\sigma} = -1.91$$

$$\sigma = \frac{\cancel{-}19}{\cancel{-}1.91}$$

$$\boxed{\sigma = 9.94.}$$

From ①

$$\frac{45-\mu}{\sigma} = -0.5.$$

$$\frac{45-\mu}{9.94} = -0.5.$$

$$45-\mu = -0.5 \times 9.94.$$

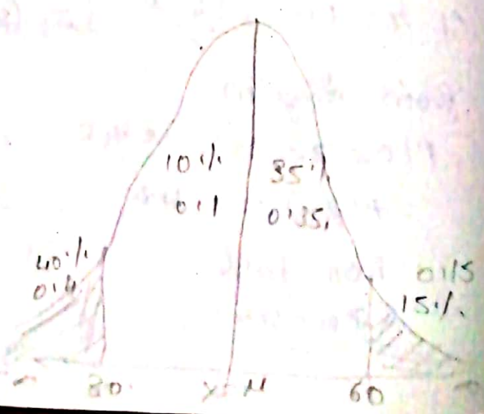
$$\mu = 45 + 0.5 \times 9.94.$$

$$\boxed{\mu = 55.4}$$

6) The marks obtained in Statistics in a certain examination found to be normally distributed. If 15% of student got ≥ 60 marks, 40% of students got < 30 marks. Then find Mean & Variance of distribution.

Mean =

Std variance =



7) The marks obtained in mathematics by 1000 students is normally distributed with mean 78% and std-deviation 11%. determine (i) how many students got marks above 90%. (ii) what was the highest mark obtained by lowest 10% of students (iii) within what limits did middle of 90% of students lie.

8) Given that the mean height of students in a college is 155 cm and S.D 15. what is the probability that mean height of 36 students is less than 157 cm.

1) If X is normally distributed with mean '2' & variance S.D is 0.1, then find $P(|x-2| \geq 0.01)$

7) Let x = Marks percentage of student.

Mean (μ) = 78% = 0.78.

S.D (σ) = 11% = 0.11.

i) $P(x > 90\%)$

if $x = 90\% = 0.9 \Rightarrow z = \frac{x-\mu}{\sigma} \Rightarrow \frac{0.9-0.78}{0.11} \Rightarrow 1.09.$

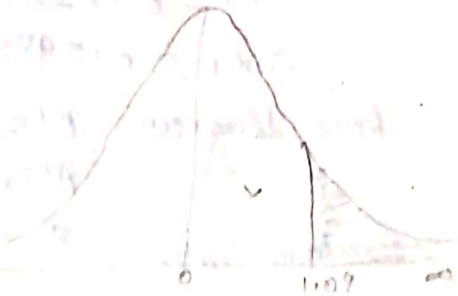
$P(x > 0.9) = P(z > 1.09)$

$\Rightarrow 0.5 - P(0 < z < 1.09)$

$\Rightarrow 0.5 - A(1.09)$

$\Rightarrow 0.5 - 0.3621$

$\Rightarrow 0.138.$



No. of student got marks above 90% $\Rightarrow 0.138 \times 1000$

$\Rightarrow$ 138 students.

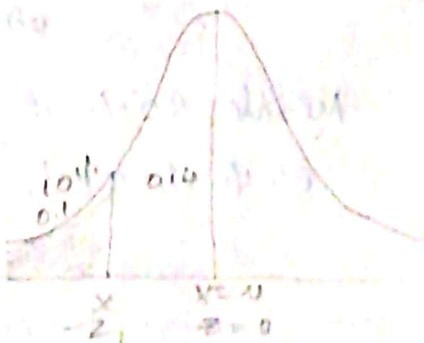
ii) $P(-z_1 < z < 0) = 0.4.$

$A(-z_1) = 0.4$

from tables,

$-z_1 = 1.29$

$\boxed{z_1 = -1.29}$



$\frac{x_1-\mu}{\sigma} = -1.29.$

$\frac{x_1-0.78}{0.11} = -1.29.$

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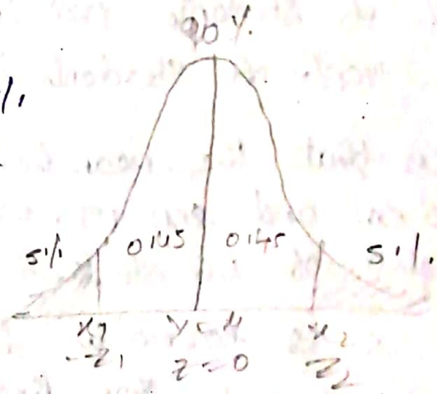
$x_1 - 0.78 = -1.29 \times 0.11$

$x_1 = 0.6381 \Rightarrow 63.81\% \cong 64\%$

Highest marks obtained by lowest 10% of student is 64%.

iii) Middle 90% means leaving 5% area on both sides of normal curve.



From diagram:-

$P(-z_1 < z < 0) = 0.45$

$A(-z_1) = 0.45.$

$-z_1 = 1.65$

$z_1 = -1.65.$ (from Tables).

$\frac{x_1 - \mu}{\sigma} = -1.65$

$\frac{x_1 - 0.78}{0.11} = -1.65$

$x_1 - 0.78 = -1.65 \times 0.11$

$x_1 = 0.78 - (1.65 \times 0.11)$

$\therefore x_1 = 0.5985 \Rightarrow 59.85\% \Rightarrow 60\%$

from diagram, $P(0 < z < z_2) = 0.45$

$A(z_2) = 0.45.$

From Tables $z_2 = 1.65$

$\frac{x_2 - \mu}{\sigma} = 1.65,$

$\frac{x_2 - 0.78}{0.11} = 1.65$

$\therefore x_2 = 0.78 + (1.65 \times 0.11).$

$\therefore x_2 = 0.9615 \Rightarrow 96.15\% \Rightarrow 96\%$

Middle 90% of student got marks b/w 60% & 90%.

8) Mean(μ) = 155 cm

S.D(σ) = 15 cm,

No. of students (n) = 36.

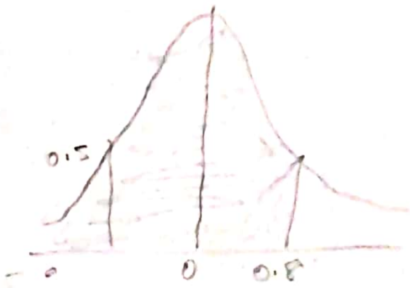
Sample mean ($\bar{x}$) = 157

$z = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

To find $P(\bar{x} < 157)$

if $\bar{x} = 157 \Rightarrow z = \frac{157-155}{15/\sqrt{36}}$

$\Rightarrow \frac{2\times 6}{15} \Rightarrow \frac{12}{15} \Rightarrow 0.8.$



$P(z < 0.8) = 0.5 + P(0 < z < 0.8)$

$\Rightarrow 0.5 + A(0.8)$

$\Rightarrow 0.5 + 0.2881$

$\Rightarrow 0.7881$

9)

$\mu = 2$

$\sigma = 0.1$

To find $P(|x-2| \geq 0.01)$.

$|x| < a \Leftrightarrow -a < x < a$

$|x| > a \Leftrightarrow x \in (-\infty, -a] \cup [a, \infty)$

* $P(|x-2| \geq 0.01) = 1 - P(|x-2| < 0.01) \rightarrow ①.$

$|x-2| < 0.01$

$-0.01 < (x-2) < 0.01$

$2 - 0.01 < x < 2 + 0.01$

$1.99 < x < 2.01$

if $x = 1.99 \Rightarrow z = \frac{1.99-2}{0.1} \Rightarrow \frac{-0.01}{0.1} \Rightarrow -0.1$

if $x = 2.01 \Rightarrow z = \frac{2.01-2}{0.1} \Rightarrow \frac{0.01}{0.1} \Rightarrow 0.1$

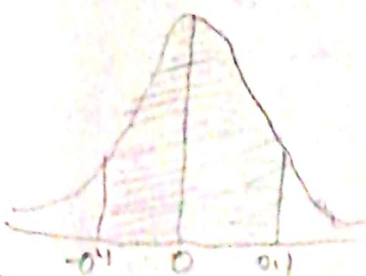
$P(|x-2| < 0.01) = P(1.99 < x < 2.01)$

$= P(-0.1 < z < 0.1)$

$= P(-0.1 < z < 0) + P(0 < z < 0.1)$

$= A(-0.1) + A(0.1)$

$= 2A(0.1) \Rightarrow 2(0.0398) \Rightarrow 0.0796.$



Normal approximation to Binomial distribution:-

Let X be a Random variable of Binomial distribution.

& n = no. of trails, p = prob. of success and

q = prob. of failure, $\mu = np$, $\sigma^2 = npq$

$\sigma = \sqrt{npq}$

To find prob. of x lies b/w x_1 & x_2

i.e., $P(x_1 \le x \le x_2)$

if $x = x_1 \Rightarrow z_1 = \frac{(x_1 - 1/2) - \mu}{\sigma}$

if $x = x_2 \Rightarrow z_2 = \frac{(x_2 + 1/2) - \mu}{\sigma}$

$P(x_1 < x < x_2) = P(z_1 < z < z_2)$.

Find the probability that out of 100 ~~students~~ patients b/w 84 & 95 inclusive will survive from heart operation given that the chances of survival is 0.9.

$p = 0.9 \quad n = 100$

$q = 1 - p \Rightarrow 1 - 0.9 = 0.1$

Let X = no. of survivals.

$\mu = np = (100)(0.9) = 90$

$\sigma = \sqrt{npq} \Rightarrow \sqrt{(90)(0.1)} \Rightarrow \sqrt{9} \Rightarrow 3$.

To find $P(84 \le x \le 95)$

if $x_1 = 84 \Rightarrow z_1 = \frac{(x_1 - 1/2) - \mu}{\sigma} \Rightarrow \frac{(84 - 1/2) - 90}{3} \Rightarrow -2.16$.

if $x_1 = 95 \Rightarrow z_2 = \frac{(x_2 + 1/2) - \mu}{\sigma} \Rightarrow \frac{(95 + 1/2) - 90}{3} = 1.83$.

$P(84 \le x \le 95) = P(-2.16 \le z \le 1.83)$

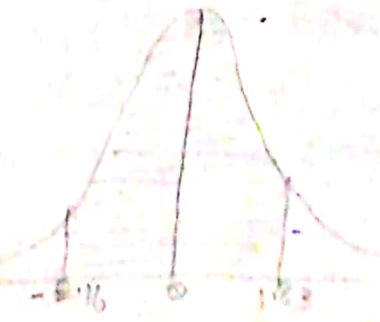
$\Rightarrow P(-2.16 \le z \le 0) + P(0 \le z \le 1.83)$.

$\Rightarrow A(-2.16) + A(1.83)$

$\Rightarrow A(2.16) + A(1.83)$

$\Rightarrow 0.4846 + 0.4664$

$\Rightarrow 0.952$.



2) 8 coins are tossed together find the probability that getting 1 to 4 heads in a single toss.

$p = 1/2$ $\quad n = 8$

$q = 1 - p \Rightarrow 1 - 1/2 = 1/2$

Let x = No. of heads.

$\mu = np = 8(1/2) = 4$

$\sigma = \sqrt{npq} \Rightarrow \sqrt{8(\frac{1}{2})(\frac{1}{2})} = \sqrt{2}$

To find $P(1 < x < 4)$.

3) Find the prob. of getting even number on the face in 3 to 5 times in throwing 10 dice together

$p = \frac{3}{6} \Rightarrow 1/2$ $\quad n = 10$

$q = 1 - 1/2 \Rightarrow 1/2$

Let x = no of times getting even number

$\mu = np =$

$\sigma = \sqrt{npq}$

$P(3 < x < 5)$.

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Exponential Distribution :-

If x is a continuous R.V probability density function.

$$f(x) = \begin{cases} \lambda . e^{-\lambda x}, & \text{if } x \geq 0, \lambda > 0. \\ 0 & , \text{elsewhere}. \end{cases}$$

1) Mean :-

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx$$

$\int uv dx = uv_1 - u'v_2 + u''v_3 - u'''v_4 \dots$

$v_1 = \int v dx \ , \ v_2 = \int\int v \, dx dx$

$$E(x) = 0 + \int_0^{\infty} \underset{u}{x} . \lambda . \underset{v}{e^{-\lambda x}} dx$$

$$E(x) = \lambda \left\{ x \left(\frac{e^{-\lambda x}}{-\lambda} \right) - 1 . \frac{e^{-\lambda x}}{\lambda^2} \right\}_0^{\infty}$$

$$E(x) = \lambda \{ [0-0] - [0 - 1/\lambda^2] \}.$$

$$E(x) = \text{mean} = 1/\lambda$$

2) Variance :-

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx.$$

$$E(x^2) = 0 + \lambda \int_0^{\infty} \underset{u}{x^2} \underset{v}{e^{-\lambda x}} dx$$

$$\Rightarrow \lambda \left\{ x^2 \left(\frac{e^{-\lambda x}}{-\lambda} \right) - (2x) \left(\frac{e^{-\lambda x}}{\lambda^2} \right) + 2 \left(\frac{e^{-\lambda x}}{-\lambda^3} \right) \right\}_0^{\infty}$$

$$\Rightarrow \lambda \{ [0+0+0] - [0+0-2/\lambda^3] \}.$$

$$E(x^2) = \lambda . 2/\lambda^3 \Rightarrow 2/\lambda^2$$

$$V(x) = E(x^2) - \mu^2 \Rightarrow 2/\lambda^2 - 1/\lambda^2$$

$$\therefore V(x) = 1/\lambda^2$$

1) The length of time a person speaks over phone follows exponential distribution with mean 6. What is the probability that person will talk for (i) More than 8 mins (ii) B/w 4 & 8 mins.

Mean $= 6 = 1/\lambda$

$\lambda = 1/6$.

$$f(x) = \begin{cases} \frac{1}{6} e^{-x/6}, & x \geq 0. \\ 0, & \text{otherwise}. \end{cases}$$

Let x = length of time.

i) $P(x > 8) = \int_8^\infty f(x)\,dx \Rightarrow \frac{1}{6}\int_8^\infty e^{-x/6}\,dx$

$\Rightarrow \frac{1}{6}\left[\frac{e^{-x/6}}{-1/6}\right]_8^\infty \Rightarrow -\{e^{-\infty} - e^{-8/6}\}. \Rightarrow e^{-4/3}$.

ii) $P(4 < x < 8) = \int_4^8 f(x)\,dx$

$\Rightarrow \frac{1}{6}\int_4^8 e^{-x/6}\,dx \Rightarrow \frac{1}{6}\left[\frac{e^{-x/6}}{-1/6}\right]_4^8$

$\Rightarrow e^{-4/6} - e^{-8/6}$

$\Rightarrow e^{-2/3} - e^{-4/3}$.

2) The milage which a car owners get with a certain kind of radiar tyre is R.V having exponenti dist. with mean 40000 km. Find the prob that one of these tyres will lost (i) atleast 20000 km.
ii) atmost 30,000 km

Mean $= 40000 = 1/\lambda$

$\lambda = \frac{1}{40000}$

$$\therefore f(x) = \begin{cases} \frac{1}{40000} e^{-x/40000}, & x \geq 0. \\ 0, & \text{otherwise}. \end{cases}$$

Let x = length of time.

i) $P(x \geq 2$

3> If $X \cong e^{\lambda}$ with $P(X \leq 1) = P(X > 1)$ find $var(X)$.

Exponential distribution.

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0, \lambda > 0 \\ 0, & \text{otherwise}. \end{cases}$$

Given,

$$P(X \leq 1) = P(X > 1)$$

$$P(X \leq 1) = 1 - P(X \leq 1)$$

$$2 \cdot P(X \leq 1) = 1.$$

$$P(X \leq 1) = 1/2.$$

$$\int_{-\infty}^{1} f(x) dx = 1/2$$

$$\lambda \int_0^1 e^{-\lambda x} dx = 1/2$$

$$\lambda \left[\frac{e^{-\lambda x}}{-\lambda}\right]_0^1 = 1/2$$

$$\Rightarrow \quad -[e^{-\lambda} - 1] = 1/2.$$

$$-e^{-\lambda} + 1 = 1/2$$

$$1 - 1/2 = e^{-\lambda}.$$

$$e^{-\lambda} = 1/2$$

Apply 'log'

$$\log_e e^{-\lambda} = \log(1/2)$$

$$-\lambda \log_e e = \log_e 2^{-1}$$

$$-\lambda = -\log 2$$

$$\lambda = \log 2$$

$$Var(X) = \frac{1}{\lambda^2} \Rightarrow \frac{1}{(\log 2)^2}$$

4> If X is exp. distributed with par. λ, find the val of k, there exists $\frac{P(X > k)}{P(X \leq k)} = a$

Given

$$P(X > k) = a \cdot P(X \leq k).$$

$$P(X > k) = a[1 - P(X > k)]$$

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$P(X > k) = a - a\,P(X > k)$

$(1+a)\,P(X > k) = a.$

$P(X > k) = \frac{a}{1+a}$

$\int_k^\infty f(x)\,dx = \frac{a}{1+a}$

$\lambda \int_k^\infty e^{-\lambda x}\,dx = \frac{a}{1+a}$

$\lambda \left[\frac{e^{-\lambda x}}{-\lambda}\right]_k^\infty = \frac{a}{1+a}.$

$-\left[e^{-\infty} - e^{-\lambda k}\right] = \frac{a}{1+a}.$

$e^{-\lambda k} = \frac{a}{1+a}.$

'$\log_e$' Apply on B.S.

$\log_e e^{-\lambda k} = \log\left[\frac{a}{1+a}\right]$

$-\lambda k \log_e e = \log\left(\frac{a}{1+a}\right)$

$\boxed{k = \frac{-1}{\lambda} \log\left(\frac{a}{1+a}\right)}$ $\boxed{\therefore\ k = \frac{1}{\lambda} \log\left(\frac{1+a}{a}\right)}$

Continous Uniform Distribution:-

If X is a continous R.V if it follows prob. density function.

$$f(x) = \begin{cases} k, & \text{if } a \le x \le b \\ 0, & \text{otherwise.} \end{cases}$$

w.k.t

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_a^b k\,dx = 1$$

$$k[x]_a^b = 1$$

$$k[b-a] = 1$$

$$k = 1/b-a.$$

$\therefore$ Density function:-

$$\boxed{f(x) = \begin{cases} \frac{1}{b-a}, & \text{if } a \le x \le b \\ 0, & \text{otherwise.} \end{cases}}$$

Mean:-

$$E(x) = \int_{-\infty}^{\infty} x f(x)\,dx.$$

$$\Rightarrow \int_a^b x \cdot \frac{1}{b-a}\,dx \Rightarrow \frac{1}{b-a}\left[\frac{x^2}{2}\right]_a^b \Rightarrow \frac{1}{2(b-a)}(b^2-a^2)$$

$$E(x) = \frac{(b+a)(b-a)}{2}$$

$$E(x) = \text{mean}(\mu) = \frac{b+a}{2}.$$

Variance:-

$$V(x) = E(x^2) - \mu^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx.$$

$$E(x^2) = \frac{1}{b-a}\int_a^b x^2\,dx.$$

$$= \frac{1}{b-a}\left[\frac{x^3}{3}\right]_a^b$$

$$E(x^2) = \frac{1}{3(b-a)}(b^3-a^3).$$

$$= \frac{(b-a)(b^2+ab+a^2)}{3(b-a)}$$

$$= \frac{b^2+ab+a^2}{3}.$$

$$V(x) = \frac{b^2+ab+a^2}{3} - \frac{(b+a)^2}{4}.$$

$$= \frac{1}{12}\left[4b^2+4ab+4a^2-3b^2-6ab-3a^2\right].$$

$$\Rightarrow \frac{1}{12}\left[b^2-2ab+a^2\right] \Rightarrow \frac{(b-a)^2}{12}.$$

$$V(x) = \frac{(b-a)^2}{12}.$$

~~Density~~ function:-

Distribution function:-

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx. \text{ if } a \le x \le b.$$

$$F(x) = \int_a^x f(x)\,dx.$$

$$= \int_a^x \frac{1}{b-a}\,dx \Rightarrow \frac{1}{b-a}[x]_a^x.$$

$$\Rightarrow \frac{[x-a]}{b-a}, \quad a \le x \le b.$$

$$F(x) = \begin{cases} 0 & , -\infty < x < a \\ \frac{x-a}{b-a} & , a \le x \le b \\ 1 & , b < x < \infty. \end{cases}$$

$F(-\infty) = 0$

$F(\infty) = 1.$

Note:-

if $x \in (-a, a)$

$$f(x) = \begin{cases} \frac{1}{2a} & , -a < x < a \\ 0 & , \text{otherwise.} \end{cases}$$

1) A electric trains on a certain lines run every half an hour b/w midnight & 6:00 in the morning. what is the prob. that a man entering the station at random time during this period will have to ~~weight~~ wait atleast 20 mins.

Let x = waiting time of a man.

$$f(x) = \begin{cases} \frac{1}{30-0}, & \text{if } 0 < x < 30 \\ 0, & \text{otherwise.} \end{cases}$$

prob of waiting time atleast 20 min.

$$P(x \geq 20) = \int_{20}^{30} f(x)dx \Rightarrow \int_{20}^{30} \frac{1}{30} \, dx \Rightarrow \frac{1}{30}[x]_{20}^{30}$$

$$\Rightarrow \frac{10}{30} \Rightarrow 1/3$$

2) Buses arrived to a specified bus stop at 15 mins intervals, starting at 7.am, ie., 7 am, 7:15 am, etc.. If a passenger arrives at same bustop at a random time which is uniformly distributed b/w 7:am and 7:30 am. Find the prob. that he waits (a) less than 5 mins (b) atleast 12 mins for a bus.

3) A passanger arrives at a local railway platform at 10 am knowing that the local train will arrive at some time uniformly distributed b/w 10 am and 10:30 am what is the prob. that (i) he will have to wait longer than 10 mins (ii) If at 10:15 am the train has not yet arrived, what is the prob. that he will have to wait atleast 10 additional mins.?

4) If x is uniformly distributed over $(0, 10)$ find the prob. that (i) $x < 2$ (ii) $x > 8$ (iii) $3 < x < 9$.

5) If x is uniformly distributed with mean 2 & variance $4/3$ then find $P(x < 0)$

6) If x is uniformly distributed over $(-a, a)$ where $a>0$. Find 'a'. such that $P(X \geq 1) = 1/3$.

7) A R.V 'x' has uniform distribution $(-3,3)$ find $|x+2|<2$ $\quad -2<(x-2)<2$

(i) $P(X<2)$. (ii) $P(|X|<2)$ iii) $P(|X-2|<2)$ add 2

iv) Find 'K'. such that $P(X>k) = 1/3$ $\quad P(0<x<4) = \int_0^4 f(x)dx$.

$\Rightarrow \int_0^3 f(x)dx$.

2) Let X = passenger arrives to bus stop at sometime to get bus.

$$f(x) = \begin{cases} \frac{1}{30-0}, & 0<X<30 \\ 0, & \text{otherwise} \end{cases}$$

i) prob. that he waits <5 mins = prob. that he arrives bus stop at b/w $7:10 - 7:15$ & $7:25 - 7:30$.

$\Rightarrow P(10<X<15) + P(25<X<30)$.

$\Rightarrow \int_{10}^{15} f(x)\,dx + \int_{25}^{30} f(x)dx \Rightarrow \int_{10}^{15} \frac{1}{30} dx + \int_{25}^{30} \frac{1}{30} dx$

$\Rightarrow \frac{1}{30}[x]_{10}^{15} + \frac{1}{30}[x]_{25}^{30} \Rightarrow \frac{1}{30}[15-10] + \frac{1}{30}[30-25]$

$\Rightarrow 5/30 + 5/30 \Rightarrow \frac{10}{30} \Rightarrow 1/3$.

ii) prob. that he waits ≥ 12 mins = prob. that he arrives bus stop b/w $7:00 - 7:03$ & $7:15 - 9:18$.

$\Rightarrow P(0<X<3) + P(15<X<18)$.

$\Rightarrow \int_0^3 f(x)\,dx + \int_{15}^{18} f(x)dx \Rightarrow \int_0^3 \frac{1}{30} dx + \int_{15}^{18} \frac{1}{30} dx$

$\Rightarrow \frac{1}{30}[x]_0^3 + \frac{1}{30}[x]_{15}^{18} \Rightarrow 3/30 + 3/30 \Rightarrow 6/30 \Rightarrow 1/5$.

3) Let X = passenger arrival time to railway station at some time.

$$f(x) = \begin{cases} \frac{1}{30-0}, & 0<X<30 \\ 0, & \text{otherwise.} \end{cases}$$

i) prob. that he waits ≥ 10 mins = prob. that he arrives railway station b/w $10:10 - 10:30$

$\Rightarrow P(10 < X < 30)$

$$\Rightarrow \int_{10}^{30} f(x)\,dx \Rightarrow \int_{10}^{30} \frac{1}{30}\,dx \Rightarrow \frac{1}{30}[30-10] \Rightarrow 2/3.$$

ii) To find prob. that the passenger has to wait atleast 10 additional mins given that the train has not arrived at 10:15 am.

P(he has to wait atleast 10 additional mins given that he has already waited 15 mins) = $P(X > (15+10) / X > 15)$.

$$\Rightarrow P(X > 25 / X > 15) = \frac{P(X > 25 \cap X > 15)}{P(X > 15)} \Rightarrow \frac{P(X > 25)}{P(X > 15)}$$

$$P(X > 25) = \frac{\int_{25}^{30} f(x)dx}{\int_{15}^{30} f(x)dx} \Rightarrow \frac{\frac{1}{30}[x]_{25}^{30}}{\frac{1}{30}[x]_{15}^{30}} \Rightarrow \frac{30-25}{30-15} \Rightarrow \frac{5}{15} \Rightarrow 1/3.$$

4) Given $(0,10) = (a,b)$.

$$f(x) = \begin{cases} \frac{1}{10} & , 0 < x < 10 \\ 0 & , \text{otherwise} \end{cases}$$

i) $P(x < 2) = \int_0^2 f(x)\,dx \Rightarrow \frac{1}{10}\int_0^2 1\,dx \Rightarrow \frac{1}{10}[x]_0^2 \Rightarrow 2/10 = 1/5.$

ii) $P(x > 8) = \int_8^{10} f(x)\,dx \Rightarrow \frac{1}{10}\int_8^{10} 1.dx \Rightarrow \frac{1}{10}[10-8] = 2/10 = 1/5.$

iii) $P(3 < x < 9) \Rightarrow \int_3^9 f(x)\,dx \Rightarrow \frac{1}{10}[9-3] \Rightarrow 6/10 \Rightarrow 3/5.$

5) Given

Mean = 1

$\frac{a+b}{2} = 1$

$a+b = 2$

Var = 4/3

$\frac{(b-a)^2}{12} = 4/3$

$(b-a)^2 = 16$

$b-a = \pm 4$

$b-a = 4$

$\Rightarrow b+a = 2$

$b-a = 4$

$2b = 6$

$b = 3$

$a = 2-3$

$a = -1$

$(-1, 3)$

$\Rightarrow b+a = 2$

$b-a = -4$

$2b = -2$

$b = -1$

$a = 2+1$

$a = 3.$

$$f(x) = \begin{cases} \frac{1}{b-a} & , a \le x \le b \\ 0 & , \text{otherwise} \end{cases}$$

$\therefore$ Interval $\Rightarrow (-1, 3)$

$$f(x) = \begin{cases} \frac{1}{3+1} , & -1 \le x \le 3 \\ 0 , & \text{otherwise.} \end{cases}$$

$$P(X<0) = \int_{-1}^{0} f(x)\,dx \Rightarrow \int_{-1}^{0} 1/4\,dx \Rightarrow 1/4[x]_{-1}^{0} \Rightarrow 1/4$$ 0.

6> Interval $\Rightarrow (-a, a)$; $a>0$

$$f(x) = \begin{cases} \frac{1}{a-(-a)} , & -a<x<a \\ 0 , & \text{otherwise} \end{cases} \Rightarrow \begin{cases} \frac{1}{2a} , & -a<x<a \\ 0 , & \text{otherwise.} \end{cases}$$

Given, $P(X \ge 1) = 1/3$

$$\int_{1}^{\infty} f(x)\,dx = 1/3$$

$$\int_{1}^{a} f(x)\,dx = 1/3 \Rightarrow \int_{1}^{a} \frac{1}{2a}\,dx = 1/3$$

$$\frac{1}{2a}[x]_1^a \Rightarrow 1/3$$

$$\frac{a-1}{2a} = 1/3$$

$$3a - 3 = 2a$$

$$\boxed{\therefore a = 3}$$

Gamma Distribution :-

The continous R.V 'x' is said to follow gamma distribution if the density function, .

$$f(x) = \begin{cases} \frac{\lambda^{a}}{\Gamma\alpha} \cdot e^{-\lambda x} \cdot x^{\alpha-1} , & \alpha ; \lambda > 0 , \ 0 < x < \infty . \\ 0 , & \text{otherwise.} \end{cases}$$

* $\Gamma(n) = \int_0^\infty e^{-x} \cdot x^{n-1}\,dx$

$\Gamma n = (n-1)\Gamma(n-1)$. if n is +ve function.

$\Gamma n = (n-1)!$ if n is +ve integral.

$\Gamma 1 = 1$

$\Gamma 1/2 = \sqrt{\pi}$

Mean :-

$$E(x) = \int_{-\infty}^{\infty} x \cdot f(x) dx$$

$$\Rightarrow \int_0^{\infty} x \frac{\lambda^{\alpha}}{\Gamma(\alpha)} e^{-\lambda x} \cdot x^{\alpha-1} dx$$

$$\Rightarrow \frac{\lambda^{\alpha}}{\Gamma\alpha} \int_0^{\infty} e^{-\lambda x} \cdot x^{\alpha} dx$$

Let $\lambda x = t$

$x = t/\lambda$, $dx = \frac{dt}{\lambda}$

$$E(x) = \frac{\lambda^{\alpha}}{\Gamma\alpha} \int_0^{\infty} e^{-t} \left(\frac{t}{\lambda}\right)^{\alpha} \cdot \frac{dt}{\lambda}$$

$$E(x) = \frac{1}{\lambda \Gamma\alpha} \int_0^{\infty} e^{-t} \cdot t^{\alpha} dt$$

$$= \frac{1}{\lambda \Gamma\alpha} \int_0^{\infty} e^{-t} \cdot t^{(\alpha+1)-1} dt$$

$$= \frac{1}{\lambda \Gamma\alpha} \Gamma\alpha+1 \Rightarrow \frac{1}{\lambda \Gamma\alpha} \alpha \cdot \Gamma\alpha$$

$$\boxed{\therefore \text{Mean} = \frac{\alpha}{\lambda}}$$

if $1/\lambda = \beta$, $\boxed{\therefore \text{Mean} = \alpha\beta}$

Variance :-

$$V(x) = E(x^2) - \mu^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$E(x^2) = \frac{\lambda^{\alpha}}{\Gamma\alpha} \int_0^{\infty} x^2 e^{-\lambda x} \cdot x^{\alpha-1} dx$$

$$E(x^2) = \frac{\lambda^{\alpha}}{\Gamma\alpha} \int_0^{\infty} e^{-\lambda x} \cdot x^{\alpha+1} dx$$

Let $\lambda x = t$ $\Rightarrow dx = \frac{dt}{\lambda}$

$x = t/\lambda$

$$E(x^2) = \frac{\lambda^{\alpha}}{\Gamma\alpha} \int_0^{\infty} e^{-t} \left(\frac{t}{\lambda}\right)^{\alpha+1} \cdot \frac{dt}{\lambda}$$

$$E(x^2) = \frac{\cancel{\lambda^{\alpha}}}{\Gamma\alpha} \frac{1}{\cancel{\lambda^{\alpha}} \cdot \lambda^2} \int_0^{\infty} e^{-t} \cdot t^{\alpha+1} dt$$

$$E(x^2) = \frac{1}{\lambda^2 \Gamma\alpha} \Gamma(\alpha+2) \quad \frac{1}{\lambda^2 \Gamma\alpha} \int_0^{\infty} e^{-t} \cdot t^{(\alpha+2)-1} dt$$

$$\Rightarrow \frac{1}{\lambda^2 \Gamma\alpha} \Gamma(\alpha+2)$$

$$\Rightarrow \frac{1}{\lambda^2 \Gamma\alpha} (\alpha+1)\Gamma(\alpha+1)$$

$$\Rightarrow \frac{1}{\lambda^2 \Gamma\alpha} (\alpha+1)\alpha\Gamma\alpha$$

$$E(x^2) = \frac{\alpha^2+\alpha}{\lambda^2}$$

$$V(x) = E(x^2) - \mu^2$$

$$= \frac{\alpha^2+\alpha}{\lambda^2} - \frac{\alpha^2}{\lambda}$$

$$\boxed{V(x) = \frac{\alpha}{\lambda^2}}$$

if $1/\lambda = \beta$ then $\boxed{V(x) = \alpha\beta^2}$

1) In a certain city, the dialy consumption of electric power in millions of kwh can be treated as a R.V. having Gamma distribution with parameters $\lambda = 1/2, \alpha = 3$. If the power plant of this city daily capacity of 12 mkwh, what is the prob. that this power supply will be inadequate on any given day?

Given $\lambda = 1/2$, $\alpha = 3$

X = power consumption in a city.

$$f(x) = \frac{\lambda^\alpha}{\Gamma\alpha} e^{-\lambda x} x^{\alpha-1}$$

$$f(x) = \left(\frac{1}{2}\right)^3 \cdot \frac{1}{\Gamma 3} e^{-x/2} \cdot x^2, \quad 0 < x < \infty$$

$$= \frac{1}{2^3} \cdot \frac{1}{2!} x^2 e^{-x/2}, \quad 0 < x < \infty$$

$$f(x) \Rightarrow \frac{x^2 e^{-x/2}}{16}, \quad 0 < x < \infty$$

$$P(x > 12) = \int_{12}^{\infty} f(x)\,dx \Rightarrow \int_{12}^{\infty} \frac{x^2 e^{-x/2}}{16}\,dx.$$

$$\Rightarrow \frac{1}{16}\left[x^2\left(\frac{e^{-x/2}}{-1/2}\right) - 2x\left(\frac{e^{-x/2}}{1/4}\right) + 2\left(\frac{e^{-x/2}}{-1/8}\right)\right]_{12}^{\infty}$$

$\Rightarrow \frac{1}{16}\{(0+0+0) - e^{-6}(-2(12)^2 - 4(24) + (-16)\}$

$\Rightarrow \frac{e^{-6}}{16}[288+96+16] \Rightarrow \frac{e^{-6}}{16}[400] \Rightarrow 25e^{-6}.$

2) The daily consumption of milk in a city, in excess of 20,000 litres is approximately distributed as a gamma variate with parameters $\alpha = 2$, $\lambda = \frac{1}{10,000}$. The city has daily stock of 30,000 litres, what is the prob. that the stock is insufficient on particular day?

$\lambda = \frac{1}{10,000}$, $\alpha = 2$.

x = excess of milk consumption. / y = daily consumption.

$y = x + 20,000$ then $\Rightarrow x = y - 20,000$.

$$f(x) = \frac{1}{(10,000)^2}\frac{1}{\lceil 2}e^{-x/10,000}\cdot x$$

$$f(x) = \frac{x}{(10,000)^2}\cdot e^{-x/10,000}, \; 0<x<\infty.$$

$$P(y>30,000) = P(x+20000>30000)$$
$$= P(x>10,000)$$
$$= \int_{10,000}^{\infty} f(x)\,dx \Rightarrow \int_{10,000}^{\infty}\frac{x}{(10,000)^2}\cdot e^{-x/10000}dx.$$

$$\Rightarrow \frac{1}{(10,000)^2}\int_{10,000}^{\infty} x\cdot e^{-x/10000}dx \Rightarrow \frac{1}{(10,000)^2}\left[x\left(\frac{e^{-x/10000}}{-1/10000}\right) - 1\left(\frac{e^{-x/10000}}{1/(10000)^2}\right)\right]_{10,000}^{\infty}$$

$$\Rightarrow \frac{(10,000)^2}{(10,000)^2}\left[10000xe^{-x/10000} - e^{-x/10000}\right]_{10,000}^{\infty}$$

$\Rightarrow \{(0+0) - ((-10000)^2e^{-1} - e^{-1})\}$

$\Rightarrow \frac{1}{e}[10000^2+1] \Rightarrow$

Sampling Distributions :-

Population :- The aggregate or totality of statistical data corresponding to observations whether it is either finite or infinite is called population.

Ex:- i) population of heights of Indians.

ii) No. of students in a college those are classified according to blood group.

Size of population :- The no. of observations in the population is known as size of population. It is denoted by 'N'.

Sample :- A small portion of population which gives population characteristics is known as sample (or) The subset of population is known as sample.

Size of sample :- The no. of observations in sample is known as ~~sam~~ size of sample. It is denoted by 'n'.

Large Sample :- If the size of sample $n \geq 30$, then sample is called large sample.

Small sample :- If the size of sample $n < 30$, then the sample is called small sample.

Sampling :- The process of selection of sample from the population is known as sampling.

* There are 2 types of sampling distributions

1) probability sampling distributions :-

   i) Random Sampling distribution
   ii) Stratified " "
   iii) Systematic " "

2) Non-probability sampling distributions :-

i) purposive sampling distribution

ii) Sequential " "

Note :-

i) No. of samples with replacement from population $= N^n$

ii) No. of samples without replacement from population $= {}^N C_n$

iii) Sampling with replacement considered as infinite population.

iv) sample without replacement finite population.

Symbols

population

1) population mean $= \mu$

$\mu = \frac{1}{N} \sum_1^N x_i$

ii) pop. variance

$\sigma^2 = \frac{1}{N} \sum_1^N (x_i - \mu)^2$

iii) N = size of population

sample.

1) Sample mean $= \bar{x}$

$\bar{x} = \frac{1}{n} \sum_1^n x_i$

ii) sample variance

$s^2 = \frac{1}{n-1} \sum_1^n (x_i - \bar{x})^2$

iii) n = size of sample.

Parameters :-

The statistical measurements of all observations of populations is known as parameters.

Ex:- μ, σ^2.

Statistics :-

The statistical measurements of all units selected in a sample is known as statistics.

Ex:- $\bar{x}, s^2$.

Sampling distributions of mean :-

Let $x_1, x_2, x_3, \dots x_n$ be 'n' random samples with size 'n' drawn from population of size 'N' with mean μ & variance σ^2, let $\bar{x}$ be mean of sample

1) Infinite population :- (Sampling with replacement) :-

i) mean of popln $= \mu = \frac{1}{N} \sum x_i$

ii) variance of pop $= \sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$

iii) Mean of sampling distribution of means $= \mu_{\bar{x}}$

$$\boxed{\mu_{\bar{x}} = \mu}$$

iv) variance of sampling distribution of means $= \sigma^2_{\bar{x}}$

$$\sigma_{\bar{x}}^2 = \frac{1}{\text{total samples}} \left[\sum (\bar{x}_i - \mu_{\bar{x}})^2 \right] \quad (\text{or})$$

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n}$$

2) Finite population :- (sampling without Replacement).

i) Mean of Pop. $= \mu$

ii) variance of pop $= \sigma^2$

iii) Mean of sampling dist. of means $= \mu_{\bar{x}}$

$$\mu_{\bar{x}} = \mu .$$

iv) Variance of sampling distr. of means $= \sigma^2_{\bar{x}}$

$$\sigma^2_{\bar{x}} = \frac{1}{\text{total}} \sum (\bar{x}_i - \mu_{\bar{x}})^2 \quad (\text{or}) \quad \sigma^2_{\bar{x}} = \left(\frac{N-n}{N-1} \right) \frac{\sigma^2}{n}$$

Finite population correction factor :-

$$\frac{N-n}{N-1} .$$

Central limit theorem :-

If $\bar{x}$ is ~~central limit~~ sample mean of with size n drawn from a population having mean μ & Standard deviation σ, then the sampling distribution of mean follows normal distribution by $\boxed{z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}}$

Standard error of sample mean :-

$$S.E.(\bar{x}) = \frac{\sigma}{\sqrt{n}}$$

Note:-

1) For infinite population, the sampling distribution of sum's of mean's has the mean $\mu_{\bar{x}_1+\bar{x}_2}$ and S.D $\sigma_{\bar{x}_1+\bar{x}_2}$ is defined by

$$\mu_{\bar{x}_1+\bar{x}_2} = \mu_{\bar{x}_1} + \mu_{\bar{x}_2}$$

$$\sigma_{\bar{x}_1+\bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

2) Sampling distribution of differences of means has the mean $\mu_{\bar{x}_1-\bar{x}_2}$ & S.D $\sigma_{\bar{x}_1-\bar{x}_2}$ are defined by

$$\mu_{\bar{x}_1-\bar{x}_2} = \mu_{\bar{x}_1} - \mu_{\bar{x}_2}$$

$$\sigma_{\bar{x}_1-\bar{x}_2} = \sqrt{\sigma_{\bar{x}_1}^2 + \sigma_{\bar{x}_2}^2} = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

1) A population consists of 5 numbers 2, 3, 6, 8 & 11. consider all possible samples of size '2' which can be drawn from this population. Then find (i) mean of population (ii) Standard deviation of population. (iii) mean of sampling distribution of means (iv) S.D of sampling distribution of mean.

Given population is 2, 3, 6, 8 & 11

population size (N) = 5

Sample size (n) = 2

i) mean of population (μ):-

$$\mu = \frac{2+3+6+8+11}{5} \Rightarrow \frac{30}{5} \Rightarrow 6.$$

ii) variance of population (σ^2):-

$$\sigma^2 = \frac{1}{N} \sum (x_i-\mu)^2$$

$$\Rightarrow \frac{1}{5}\left[(2-6)^2 + (3-6)^2 + (6-6)^2 + (8-6)^2 + (11-6)^2\right]$$

$$\Rightarrow \frac{1}{5}[16+9+0+4+25]$$

$$\Rightarrow \frac{1}{5}(54)$$

$$\Rightarrow 10.8$$

$$\sigma = \sqrt{10.8} \Rightarrow 3.$$

iii) No. of samples with replacement $= N^2 = 5^2 = 25$

Samples = { (2,2), (2,3), (2,6), (2,8), (2,11)
(3,2), (3,3), (3,6), (3,8), (3,11)
(6,2), (6,3), (6,6), (6,8), (6,11)
(8,2), (8,3), (8,6), (8,8), (8,11)
(11,2), (11,3), (11,6), (11,8), (11,11) }

All samples are = { 2, 2.5, 4, 5, 6.5,
2.5, 3, 4.5, 5.5, 7,
4, 4.5, 6, 7, 8.5,
5, 5.5, 7, 8, 9.5,
6.5, 7, 8.5, 9.5, 11 }

Mean of sampling distributions of means

$$\mu_{\bar{x}} = \frac{2+2.5+4+5+ \dots +9.5+11}{25}$$

$$\mu_{\bar{x}} = 6.$$

iv) S.D of sampling distribution.

$$\sigma^2_{\bar{x}} = \frac{1}{total} \sum (\bar{x}_i - \mu_{\bar{x}})^2$$

$$= \frac{1}{25}\left[(2-6)^2 + (2.5-6)^2 + \dots + (11-6)^2\right].$$

$$\Rightarrow 5.4.$$

$$\sigma_{\bar{x}} = \sqrt{5.4} \Rightarrow 2.32.$$

(or)

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{10.8}{2} \Rightarrow 5.4. \quad (\text{w.k.t})$$

2) Solve above problem for without replacement :-

Given population is 2, 3, 6, 8 & 11.

i) (ii) questions are same as above

iii) No. of samples without replacement $= {}^{N}C_{n} = {}^{5}C_{2} = 10$

Samples = { (2,3), (2,6), (2,8), (2,11)
(3,6), (3,8), (3,11),
(6,8), (6,11)
(8,11) }

All Samples means = { 2.5, 4, 5, 6.5,
4.5, 5.5, 7,
7, 8.5, 9.5 }.

Mean of Sampling distribution of means = .

$$\mu_{\bar{x}} = \frac{2.5+4+5+\dots+8.5+9.5}{10} \Rightarrow 6.$$

iv) $\sigma_{\bar{x}}^2 = \frac{1}{total} \sum (\bar{x}_i - \mu_{\bar{x}})^2$

$\sigma_{\bar{x}}^2 = \frac{1}{10}[(2.5-6)^2 + (4-6)^2 + \dots + (9.5-6)^2]$.

$\sigma_{\bar{x}}^2 = 4.05$

$\sigma_{\bar{x}} = \sqrt{4.05} \Rightarrow 2.01$.

(or)

$$\sigma_{\bar{x}}^2 = \left(\frac{N-n}{N-1}\right)\frac{\sigma^2}{n} \Rightarrow \left(\frac{5-2}{5-1}\right)\frac{10.8}{2} = \left(\frac{3}{4}\right)\left(\frac{10.8}{2}\right) = 4.05$$

H.W

2) A population is 5, 10, 14, 18, 13, 24. consider all possible samples of size '2' which can be drawn without replacement from this population. Find (i) Mean of population (ii) S.D of population iii) Mean of sampling dist. of means (iv) S.D of sampling dist. of means.

3) If the population 3, 6, 9, 15 & 27. (i) Find all possible samples of size '3' that can be taken without replacement from the finite population. (ii) mean of sampling dist. of means (iii) S.D of all means

Given population is 3, 6, 9, 15, 27

Population size (N) = 5

Sample size (n) = 3

Mean of population (μ):-

$$\mu = \frac{3+6+9+15+27}{5} = \frac{60}{5} \Rightarrow 12$$

Variance of population.

$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$

$= \frac{1}{5}[(3-12)^2 + (6-12)^2 + (15-12)^2 + (27-12)^2]$

$\Rightarrow \frac{1}{5}[81+36+9+9+25] \Rightarrow \frac{1}{5}(360) = 72$

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$\sigma^2 = 72$

$\sigma = \sqrt{72} = 8.48$.

Corr

1

8) A Random Sample of size 64 is taken from a normal population with mean $\mu = 51.4$ and $\sigma = 6.8$. What is the probability that the mean of sample will
i) exceed 52.9 ii) fall b/w 50.5 & 52.3
iii) less than 50.6.

~~9) The mean height of students in a clg 155 cm~~

9) A Random sample of size 100 is taken from infinite population having mean $\mu = 76$ $\sigma^2 = 256$. What is the probability that $\bar{x}$ lies b/w 75 & 78.

10) The mean breaking strength of copper wire is 575 lbs with S.D 8.3 lbs. How large a sample must be used inorder that there will be one chance in 100 that mean v. strength of sample is less than 572 lbs

11) 3 masses are measured as 62.34, 20.48, 35.97 kg S.D = 0.54, 0.21, 0.46 kgs. find mean and S.D of sum of masses.

8)

Mean = 51.4. Sample size (n) = 64
S.D(σ) = 6.8

i) To find $P(\bar{x} > 52.9)$

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} \Rightarrow$$

If $\bar{x}$ 52.9 $\Rightarrow z = \frac{52.9 - 51.4}{6.8/\sqrt{64}} \Rightarrow 1.87$

$P(\bar{x} > 52.9) = P(z > 1.87)$
$= 0.5 - P(0 < z < 1.87)$
$= 0.5 - A(1.87)$
$= 0.5 - 0.4693 \Rightarrow 0.0307$

ii) $P(50.5 < \bar{x} < 52.3)$

if $\bar{x} = 50.5 \Rightarrow z = \dfrac{50.5 - 51.4}{6.8/\sqrt{64}} \Rightarrow \cancel{-1.125}\ -1.05 \Rightarrow \simeq -1.13$

if $\bar{x} = 52.3 \Rightarrow z = \dfrac{52.3 - 51.4}{6.8/\sqrt{64}} \Rightarrow \cancel{1.125}\ -1.06 \simeq \cancel{1.13}$.

$P(-1.06 < z < 1.06) = P(-1.06 < z < 0) + P(0 < z < 1.06)$

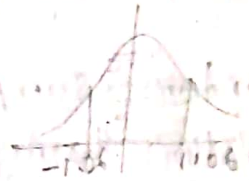
$\Rightarrow P(0 < z < 1.06) + A(1.06)$

$\Rightarrow A(1.06) + A(1.06)$

$\Rightarrow 2A(1.06)$

$\Rightarrow 2(0.3554)$

$\Rightarrow 0.7108$.



iii) $P(\bar{x} < 50.6)$

if $\bar{x} = 50.6 \Rightarrow z = \dfrac{50.6 - 51.4}{6.8/\sqrt{64}} = -0.94$

$P(\bar{x} < 50.6) = P(z < -0.94)$

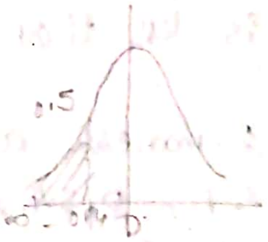
$\Rightarrow 0.5 - P(-0.94 < z < 0)$

$\Rightarrow 0.5 - P(0 < z < 0.94)$

$\Rightarrow 0.5 - A(0.94)$

$\Rightarrow 0.5 - 0.3264$

$\Rightarrow 0.1736$.



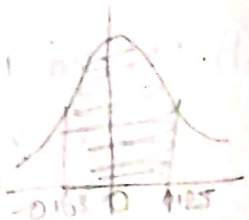
9) Mean $= \mu = 76$.

$S.D(\sigma) = \sqrt{256} = 16$. sample size $(n) = 100$.

i) $P(75 < \bar{x} < 78)$

if $\bar{x} = 75 \Rightarrow z = \dfrac{75 - 76}{16/\sqrt{100}} = -0.63$

if $\bar{x} = 78 \Rightarrow z = \dfrac{78 - 76}{16/\sqrt{100}} = 1.25$.



$P(-0.63 < z < 1.25) = P(-0.63 < z < 0) + P(0 < z < 1.25)$

$= A(-0.63) + A(1.25)$

$= 0.2357 + 0.3944$

$\Rightarrow$

10) Mean(μ) = 575 lbs

S.D(σ^2) = 8.3 lbs

if $\bar{x}$ = 572 lbs $\Rightarrow z = \frac{x-\mu}{\sigma/\sqrt{n}} \Rightarrow \frac{572-575}{8.3/\sqrt{n}} \Rightarrow \frac{-3\sqrt{n}}{8.3}$

$P(\bar{x} < 572) = \frac{1}{100} = 1\% \Rightarrow 0.01$.

from diagram.

$P(-z_1 < z < 0) = 0.49$

$A(-z_1) = 0.49$

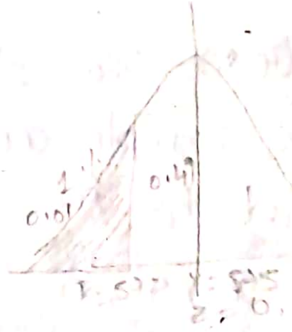
$z_1 = -2.33$

$\frac{-3\sqrt{n}}{8.3} = -2.33$

$\sqrt{n} = \frac{2.33 \times 8.3}{3}$

$n = \left(\frac{2.33 \times 8.3}{3}\right)^2$

$n = 42$.



11) Assume masses,

$\mu_{\bar{x}_1} = 62.34$

$\mu_{\bar{x}_2} = 20.48$

$\mu_{\bar{x}_3} = 35.97$.

Mean of Sum of Markes $\Rightarrow \mu_{\bar{x}_1} + \mu_{\bar{x}_2} + \mu_{\bar{x}_3} = \mu_{\bar{x}_1+\bar{x}_2+\bar{x}_3}$

$\Rightarrow 62.34 + 20.48 + 35.97$

$\Rightarrow$

S.D of sum of markes $\sigma_{\bar{x}_1+\bar{x}_2+\bar{x}_3} = \sqrt{\sigma^2_{\bar{x}_1} + \sigma^2_{\bar{x}_2} + \sigma^2_{\bar{x}_3}}$.

$= \sqrt{(62.34)^2 + (20.48)^2 + (35.97)^2}$

$=$

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