UNIT-V

Stochastic processes and Markov Chains

Short Answer questions

① Define Stochastic process, Markov Chain & Transition Matrix

A:- Stochastic process:- The family of all the random variables at particular time 't' is known as Stochastic process.

Ex:- A Queuing system, turblent fluid flow.

Markov chain:- A Stochastic process is said to be Markov process (or) chain if it satifies markov property. i.e if occurance future state is depends on present state.

i.e $P\{X_{n+1} = x_{n+1} \mid X_n = x_n\}$.

Transition matrix:- The probability of future state is depends on present state is known as Transition matrix.

i.e $P\{X_{n+1} = j \mid X_n = i\} = P_{ij}$

② If the transition probability matrix is $\begin{bmatrix} 0 & 0.2 & x \\ x & 0.1 & y \\ 0.1 & 0.2 & z \end{bmatrix}$ find x, y, z

A:- The matrix is said to be Transition probability

Matrix if it satifies following conditions

(a) It is a square matrix with non-negative elements.

(b) Sum of each row is equal to '1'

$\therefore \rightarrow 0+0.2+x=1 \Rightarrow x=0.8$

$\rightarrow x+0.1+y=1$

$0.8+0.1+y=1$

$y=0.1$

$\rightarrow 0.1+0.2+z=1$

$z=0.7$

$\therefore x=0.8,\ y=0.1,\ z=0.7$

(3) Define Regular Stochastic process matrix with Example.

Ans: A Matrix is said to be regular stochastic if some powers of 'p' becomes non-zero elements of matrix.

Ex:- $A=\begin{bmatrix} 0 & 1 \\ 1/4 & 3/4 \end{bmatrix}$, $A^2=\begin{bmatrix} \frac{1}{4} & 3/4 \\ 3/16 & 13/16 \end{bmatrix}$

$\therefore$ A is regular Matrix

(4) Find the equilibrium vector of $\begin{bmatrix} 1/4 & 3/4 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$

Sol:- Given that $P = \begin{bmatrix} 1/4 & 3/4 \\ 1/2 & 1/2 \end{bmatrix}$

Let X be the probability vector. We want to find X such that $XP = X$ & $\boxed{x_1 + x_2 = 1}$ — (3)

$$[x_1 \ x_2] \begin{bmatrix} \frac{1}{4} & 3/4 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = [x_1 \ x_2]$$

$$\frac{x_1}{4} + \frac{x_2}{2} = x_1 \Rightarrow -\frac{3}{4}x_1 + \frac{x_2}{2} = 0 \quad \text{— (1)}$$

$$\frac{3x_1}{4} + \frac{x_2}{2} = x_2 \qquad \frac{3x_1}{4} - \frac{x_2}{2} = 0 \Rightarrow -\frac{3x_1}{4} + \frac{x_2}{2} = 0 \quad \text{— (2)}$$

eqn's (1) & (2) same.

$$\therefore -\frac{3x_1}{4} + \frac{x_2}{2} = 0 \Rightarrow -3x_1 + 2\cancel{x_2} = 0$$
$$x_1 + x_2 = 1 \qquad 2x_1 + \cancel{2x_2} = 2$$
$$-5x_1 = -2$$
$$x_1 = 2/5$$

$$\therefore \frac{2}{5} + x_2 = 1$$
$$x_2 = 1 - \frac{2}{5} = \frac{3}{5}$$

$$\therefore [x_1 \ x_2] = [2/5, 3/5]$$

(5)

(5) Recurrent State:- The state is said to be recurrent, if any time that we leave that state we will return to that state in the future with probability one.

Long Answer questions

① Define classification of states.

Ans:- **Classification of states:-**

① **Absorbing State:-** If $P_{ii}=1$ then 'i' is said to be absorbing state.

Ex:- $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 1 \end{bmatrix}$

Here absorbing states are 1, 3, because $P_{11}=1$, $P_{33}=1$

② **Transient State:-** If $P_{ii}<1$ then 'i' is said to be transient state

Ex:- $P = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$

∴ Here transient states are 1, 3. because $P_{11}<1$, $P_{33}<1$

③ **Return State:-** If $P_{ii}^{(n)}>0$ for some 'n' then 'i' is called return state.

∴ Ex:- $P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & \frac{1}{2} \end{bmatrix}$

∴ Here Return States are 1, 2, 3

because $P_{11}>0$, $P_{22}>0$, $P_{33}>0$

Irreducible State:- If $P_{ij}^{(n)} > 0$ for some 'n' then it is irreducible State

$$P = \begin{bmatrix} 0 & 1 \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$P_{12}^{(1)} > 0$, $P_{21}^{(1)} > 0$, $P_{22}^{(1)} > 0$ But $P_{11} \neq 0$

$$P^2 = \begin{bmatrix} 1/3 & 2/3 \\ 2/9 & 7/9 \end{bmatrix}$$

$\therefore \quad P_{11} > 0$

$\therefore$ It is irreduciable State

Periodic State:- The Periodic of return state is defined as the GCD of all n Such that

$P_{ij}^{(n)} > 0$, $d_i = GCD\{n, P_{ii}^{(n)} > 0$

if $d_i > 1$ then State 'i' is Called Periodic State

Ex:- $P = \begin{bmatrix} 0 & 1 \\ 1/3 & 2/3 \end{bmatrix}$, $P^2 = \begin{bmatrix} 1/3 & 2/3 \\ 2/9 & 7/9 \end{bmatrix}$

$P_{11}^{(2)} > 0 \quad \therefore \quad d_i = 1$

$P_{22}^{(1)} > 0$, $P_{22}^{(2)} > 0$

$\therefore \quad d_i = GCD\{1, 2\} = 1$

$\therefore \quad i = 1, 2$ are aperiodic States.

(2) The transition probability matrix of a Markov Chain is given by $\begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}$. Is this matrix irreducible?

Sol:-

Given that $P = \begin{bmatrix} 0.3 & 0.7 & 0 \\ 0.1 & 0.4 & 0.5 \\ 0 & 0.2 & 0.8 \end{bmatrix}$

Here $P_{11}^{(1)} > 0$, $P_{12}^{(1)} > 0$, $P_{21}^{(1)} > 0$, $P_{22}^{(1)} > 0$

$P_{23}^{(1)} > 0$, $P_{32}^{(1)} > 0$, $P_{33}^{(1)} > 0$

But $P_{13} = 0$, $P_{31} = 0$

Then $P^2 = \begin{bmatrix} 0.16 & 0.49 & 0.35 \\ 0.07 & 0.35 & 0.6 \\ 0.02 & 0.24 & 0.74 \end{bmatrix}$

Here $P_{13}^{(2)} > 0$, $P_{31}^{(2)} > 0$

. So it is irreducible

(3) A fair die tossed repeatedly. If X_n denotes the maximum of the number occuring in the first n tosses, find the transition probability matrix. Find also P^2.

Sol:-

State space $= \{1, 2, 3, 4, 5, 6\}$

Let $X_n =$ max of the number occurring in the first n trials $= 3$ (say)

Then $x_{n+1} = 3$, if $(n+1)^{th}$ trial results is 1, 2, or 3

$= 4$, if $(n+1)^{th}$ trial results is 4

$= 5$, if $(n+1)^{th}$ trial results is 5

$= 6$, if $(n+1)^{th}$ trial results is 6.

$\therefore P\{X_{n+1} = 3 / x_n = 3\} = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = 3/6$

$P\{X_{n+1} = i / x_n = 3) = 1/6$ when $i = 4, 5, 6$

$\therefore$ The tpm of chain is

$$P = \begin{bmatrix} \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & \frac{2}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 3/6 & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 4/6 & \frac{1}{6} & \frac{1}{6} \\ 0 & 0 & 0 & 0 & 5/6 & 1/6 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 1 & 3 & 5 & 7 & 9 & 11 \\ 0 & 4 & 5 & 7 & 9 & 11 \\ 0 & 0 & 9 & 7 & 9 & 11 \\ 0 & 0 & 0 & 16 & 9 & 11 \\ 0 & 0 & 0 & 0 & 25 & 11 \\ 0 & 0 & 0 & 0 & 0 & 36 \end{bmatrix}$$

④ If the transition probability matrix is $\begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$

and the initial probabilities are $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ then find the probabilities after three periods. ⓑ Equilibrium Vector

Sol: G/T $P = \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$

$$P_0 = \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

After one period $P_1 = P_0 . P$

$$= \begin{bmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 0.25 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.42 & 0.17 & 0.42 \end{bmatrix}$$

After two periods. $P_2 = P_1 . P$

$$= \begin{bmatrix} 0.42 & 0.17 & 0.45 \end{bmatrix} \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.4 & 0.21 & 0.4 \end{bmatrix}$$

After three Periods $P_3 = P_2 . P$

$$= \begin{bmatrix} 0.4 & 0.21 & 0.4 \end{bmatrix} \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$= \begin{bmatrix} 0.405 & 0.2 & 0.405 \end{bmatrix}$$

Equilibrium vector:-

let $X = [x_1 \ x_2 \ x_3]$ then $X = XP$ and

$x_1 + x_2 + x_3 = 1$

$$\therefore [x_1 \ x_2 \ x_3] = [x_1 \ x_2 \ x_3] \begin{bmatrix} 0.5 & 0.25 & 0.25 \\ 0.5 & 0 & 0.5 \\ 0.25 & 0.25 & 0.5 \end{bmatrix}$$

$$x_1 = 0.5x_1 + 0.5x_2 + 0.25x_3$$
$$x_2 = 0.25x_1 + 0.25x_3$$
$$x_3 = 0.25x_1 + 0.5x_2 + 0.5x_3$$

$$\Rightarrow \quad -0.5x_1 + 0.5x_2 + 0.25x_3 = 0 \quad \text{—} (1)$$
$$0.25x_1 - x_2 + 0.25x_3 = 0 \quad \text{—} (2)$$
$$0.25x_1 + 0.5x_2 - 0.5x_3 = 0 \quad \text{—} (3)$$
$$\& \quad x_1 + x_2 + x_3 = 1 \quad \text{—} (4)$$

Sub $x_1 = 1 - x_2 - x_3$ in (1) & (2)

$$\therefore -0.5(1 - x_2 - x_3) + 0.5x_2 + 0.25x_3 = 0$$

(1) $\Rightarrow$ $-0.5 + x_2 + 0.75x_3 = 0 \Rightarrow x_2 + 0.75x_3 = 0.5$ — (5)

(2) $\Rightarrow$ $0.25(1 - x_2 - x_3) - x_2 + 0.25x_3 = 0$

$$0.25 - 1.25x_2 \cancel{- 0.25x_3} + \cancel{0.25x_3} = 0$$

$$\therefore \boxed{x_2 = \frac{0.25}{1.25} = \frac{1}{5}}$$

(5) $\Rightarrow$ $\frac{1}{5} + 0.75x_3 = 0.5 \Rightarrow \boxed{x_3 = 2/5}$

(1) $\Rightarrow$ $-0.5x_1 + \frac{0.5}{5} + 0.25(2/5) = 0$

$x_1 = 2/5$

(5) Suppose there are two market products of brands A and B respectually. let each of these two brands have exactly 50% of the total market in Same Period and let the market be of a fixed size. The transition matrix is given as follows

	A	B
A	0·9	0·1
B	0·5	0·5

If the initial market share breakdown is 50% for each brand, then determine their market shares in the steady state.

Sol:

$$P = \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

The steady state vector is

$X = XP$ where $x_1 + x_2 = 1$

$$\begin{bmatrix} x_1 & x_2 \end{bmatrix} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix}$$

$x_1 = 0.9x_1 + 0.5x_2$

$x_2 = 0.1x_1 + 0.5x_2$

$\Rightarrow$ $-0.1x_1 + 0.5x_2 = 0$
$0.1x_1 - 0.5x_2 = 0$ } These two eqn's are same

$\therefore \; 0.1x_1 - 0.5x_2 = 0 \Rightarrow x_1 = \frac{0.5}{0.1}x_2$

$x_1 + x_2 = 1$

$$\frac{0.5}{0.1}x_2 + x_2 = 1$$

$$6x_2 = 1$$

$$x_2 = 1/6$$

$$x_1 = 5/6$$

$\therefore$ Vector is $[x_1 \; x_2] = \left[\frac{5}{6} \; 1/6\right]$.

(7) The tPm of a markov chain $\{x_n\}$, $n = 1, 2, 3 \ldots$ having 3 states 1, 2 & 3 $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ & the initial distribution is $P_0 = (0.7 \; 0.2 \; 0.1)$ Find (1) $P(x_2 = 3)$ (2) $P\{x_3 = 2, x_2 = 3, x_1 = 3, x_0 = 2\}$.

Sol:- Given that $P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 0.43 & 0.31 & 0.26 \\ 0.24 & 0.42 & 0.39 \\ 0.36 & 0.35 & 0.29 \end{bmatrix}$$

1. $P(x_2=3) = \sum_{i=1}^{3} P\{x_2=3 / x_0=i\} \cdot P[x_0=i]$

$$= P\{x_2=3 | x_0=1\} \cdot P(x_0=1) + P\{x_2=3/x_0=2\} \cdot P\{x_0=2\} + P\{x_2=3/x_0=3\} \cdot P\{x_0=3\}$$

$$= P_{13}^{(2)} \cdot (0.7) + P_{23}^{(2)}(0.2) + P_{33}^{(2)}(0.1)$$

$$= (0.26)(0.7) + (0.34)(0.2) + 0.29(0.1)$$

$$= 0.279$$

2. $P\{x_1=3, x_0=2\} = P\{x_1=3 | x_0=2\} \cdot P\{x_0=2\}$

$$= P_{23}^{(1)} \times 0.2$$

$$= 0.2 \times 0.2 = 0.04$$

$$P\{x_2=3, x_1=3, x_0=2\} = P\{x_2=3 | x_1=3, x_0=2\} \times P\{x_1=3, x_0=2\}$$

$$= P\{x_2=3 | x_1=3\} \times P\{x_1=3, x_0=2\}$$

$$= P_{33}^{(1)} \cdot \times 0.04$$

$$= 0.3 \times 0.04$$

$$= 0.012$$

$$P\{x_3=2, x_2=3, x_1=3, x_0=2\}$$

$$= P\{x_3=2 | x_2=3, x_1=3, x_0=2\} \times P\{x_2=3, x_1=3, x_0=2\}$$

$$= P\{x_3=2 | x_2=3\} \times 0.012$$

$$= P_{32}^{(1)} \times 0.012 = 0.4 \times 0.012$$

$$= 0.0048$$

UNIT – V

Stochastic process (or) Random process :-

A set of Random variable values $\{X_t\}$ (or) $\{X_1\}$ depending on some real parameters (time t, temprature etc..) is known as stochastic process.

States :-

The values assumed by R.V.

State space :-

The set of all possible values of any individual no. of random process is called state space.

It is denoted by $\{x_t, t \geq 1\} = I$ (or) S.

Ex: When a fair die is tossed, the no. of sixes is stochastic process

If the parameter set is discrete then the state space is discrete.

If the parameter set has infinite values, the state space is continous.

* Classification of stochastic process :-

1)

R.V x \ time t	Continous	Discrete
continous	continous stochastic process	continous stochastic sequence
Discrete	Discrete stochastic process	Discrete stochastic sequence

Stationary stochastic process:-

If the probability distribution do not depends on the time 't' then the random process is called stationary stochastic process.

Deterministic stochastic process:-

A Random process is called deterministic stochastic process if future values of any sample functions can be predicted from its past observations.

Non-Deterministic

A stochastic process is called non deterministic. if future values of any sample functions can't be predicted from its past observations

Markov process:-

A Random process X_n is called markov process if $P\{X(t_{n+1}) < x_{n+1} / X(t_n) < x_n, X(t_{n-1}) \leq x_{n-1}, \ldots,$

$X(t_0) \leq x_0\}$.

$\Rightarrow P\{X(t_{n-1}) < x_{n+1} / X(t_n) < x_n\}$

$\Rightarrow P(X_{n+1} / X_n)$.

$X_{n+1}, X_n, X_{n-1}, X_{n-2}, \ldots X_0$

All states of micro processor

(or)

$P\{X_n = k / X_{n-1} = j, X_{n-2} = j_1, \ldots X_0 = J_{n-1}\}$

$\Rightarrow P\{X_n = k / X_{n-1} = j\}$

$\Rightarrow P_{jk}^{(1)}$

Unit step transition probability :-

The probability $P_{jk}^{(1)}$ is called unit step transition probability.

M-step transition probability :-

$$P\{X_{n+m}=k / X_n=j\} = P_{jk}^{(m)}.$$

Homogeneous Markov process :-

~~m-step~~ If the transition probability P_{jk} is independent of 'n', then the markov chain is called Homogeneous markov process :-

Non - Homogeneous

If the transition probability P_{jk} is dependent of 'n' then the step is called markov non - Homogeneous markov process.

probability distribution vector :-

A row or column matrix which consists of the probabilities of occurences of markov process then it is known as probability distribution vector.

if $P_1, P_2, P_3, \ldots P_n$ are probabilities.

then it is $[P_1, P_2 \ldots P_n]$

Transition probability matrix :-

The transition probabilities P_{jk} satisfies

i) $P_{jk} \geq 0$ i.e, non -ve elements.

ii) $\sum_{k=1}^{n} P_{jk} = 1 \ \forall j$. i.e, each row sum $= 1$

$$P = [P_{jk}]_{m\times n} = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1n} \\ P_{21} & P_{22} & \cdots & P_{2n} \\ \vdots & & & \\ P_{m1} & P_{m2} & \cdots & P_{mn} \end{bmatrix}$$

This is called Transition probability matrix.

Stochastic matrix:-

A transition probability matrix is called stochastic matrix if it is square matrix.

Regular matrix:-

A tpm is called Regular matrix if it satisfies

i> stochastic matrix

ii> diagonal element shouldn't equal to 1.

iii> All element of P^m, $m = 2, 3, \cdots$ are positive.

1> which of the following matrices are stochastic matrix

i> $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ ii> $\begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$ iii> $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

iv> $\begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}$ v> $\begin{bmatrix} 0 & 2 \\ 1/4 & 1/4 \end{bmatrix}$

i) stochastic

ii) stochastic

iii> Not stochastic. It is Rectangular matrix

iv> not stochastic. Negative element.

v) Row sum $\neq 1$

$\therefore$ Non stochastic

2) which of the following are regular matrices.

i) $A = \begin{bmatrix} 1/2 & 1/4 & 1/4 \\ 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \end{bmatrix}$ ii) $B = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

iii) $C = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$ iv) $D = \begin{bmatrix} 1/2 & 1/4 & 1 \\ 0 & 1/2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

i) It is not Regular. Since, 1 lies on the diagonal.

ii) $B^2 = B \cdot B = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 3/8 & 3/8 & 0 \end{bmatrix}$

$B^3 = B^2 \cdot B = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 3/8 & 3/8 & 0 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 1/4 & 1/4 & 1/2 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 1/2 & 1/2 & 0 \\ 7/16 & 7/16 & 1/8 \end{bmatrix}$

The elements B_{13}, B_{23} are non-zeroes.

iii)

$C^3 = C^2 \cdot C = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 0 & 1/2 & 1/2 \end{bmatrix}$

$C^4 = C^3 \cdot C = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}$

$C^5 = C^4 \cdot C = \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/8 & 1/2 & 3/8 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$

The all elements of P^5 is non-zero..

$\therefore$ It is Regular stochastic matrix.

iv) Not a Regular stochastic matrix.

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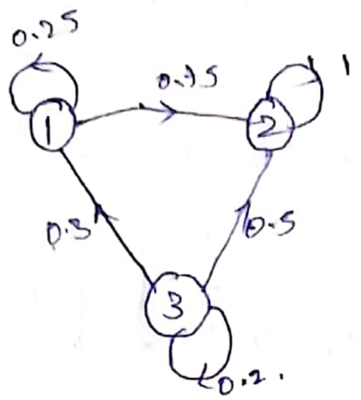
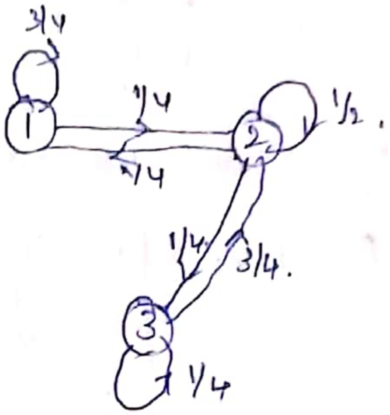
3) Represent the following ~~matrices as a~~ transition matrices as a diagraph.

i) $\begin{bmatrix} 0.25 & 0.75 & 0 \\ 0 & 1 & 0 \\ 0.3 & 0.5 & 0.2 \end{bmatrix}$. ii) $\begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix} \end{array}$





Steady state prob. Distribution :-

If $p^{(0)}$ is initial state probability distribution vector then after 1 step the distribution vector becomes $p^{(1)} = p^{(0)}.P$. Here $P =$ tpm.

After 2 steps:-

$$p^{(2)} = p^{(1)}.P = p^{(0)}.P^2.$$

After n steps:-

$$p^{(n)} = p^{(n-1)}P = p^{(0)}.P^n.$$

If a homogeneous markov chain is regular, every sequence of state probability distribution approaches a unique fixed prob. distribution is called stationary distribution (or) steady state distribution of markov chain.

when, $n \to \infty$

$$\lim_{n \to \infty} p^{(n)} = \pi$$

where $\pi = [\pi_1, \pi_2, \ldots \pi_n]$

and the steady state distribution satisfies

$\boxed{\pi P = \pi}$ and $\pi_1 + \pi_2 + \cdots + \pi_n = 1$

Champman - kolmogorov's Theorem:-

If P is tpm of homogeneous markov chain then n step tpm ($p^{(n)}$) is equal to P^n.

$$\left[p_{ij}^{(n)}\right] = \left[p_{ij}\right]^n$$

Classification of states:-

i) Irreducible :-

If $p_{ij}^{(n)} > 0$ for some 'n' and for every i and j., then every state can be reached from every other state.

i.e., All states are communicate themselves. Then the markov chain is said to be irreducible. The tpm of irreducible chain is called irreducible matrix. otherwise it is reducible.

ii) The state i of markov chain is called return state if $p_{ij}^{(n)} > 0$ for some 'n'.

iii) The period d_i of return state 'i' is defined as the greatest common divisor of all 'm'. there exists $p_{ij}^{(m)} > 0$.

i.e., $GCD\{m, p_{ij}^{(m)} > 0\} = d_i$

iv) The state 'i' is said to be periodic if $d_i > 1$ and aperiodic if $d_i = 1$.

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iii) The state 'i' is aperiodic if $p_{ii} \neq 0$

iv) The probability that the chain returns to state i having started from i, for the first time at n^{th} step is denoted by $f_{ii}^{(n)}$, it is called first return time prob, $\{n, f^{(n)}_{ii}, n=1,2,3 \ldots\}$ is called the distribution recurrence time state i.

** $F_{ii} = \sum_{i=1}^{\infty} f_{ii}^{(n)}$

$\mu_{ii} = \sum_{i=1}^{\infty} n \cdot f_{ii}^{(n)}$

if $F_{ii}=1$, then the state i is called recurrent (or) persistent ~~of~~ return to the state i it is ~~so~~ also called certain.

If $F_{ii}<1$, then the state i is called transient or uncertain.

μ_{ii} is called mean recurrence time of state i.

If μ_{ii} is finite then it is called non-null persistent.

If μ_{ii} is infinite then it is called null persistent.

v) A non null persistant and aperiodic state is called ergodic.

vi) If a markov chain is finite irreducible then all its states are non-null persistant.

vii) $P(X_3=a, X_2=b, X_1=c, X_0=a) = P(X_3=a/X_2=b)P(X_2=b/X_1=c) \cdot P(X_1=c/X_0=a) \cdot P(X_0=a)$

1) A mark

1) If the tpm of a markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$ then find steady state distribution of chain.

Let $\pi = [\pi_1 \ \pi_2]$ be steady state distr. vector.

$\pi P = \pi$, $\boxed{\pi_1 + \pi_2 = 1}$ $\rightarrow$ (1)

$[\pi_1 \ \pi_2] \begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} = [\pi_1 \ \pi_2]$

$\left[\frac{1}{2}\pi_2 \quad \pi_1 + \frac{1}{2}\pi_2\right] = [\pi_1 \ \pi_2]$

compare same position element.

$\frac{1}{2}\pi_2 = \pi_1$, $\pi_1 + \frac{1}{2}\pi_2 = \pi_2$.

$\pi_2 = 2\pi_1 \rightarrow$ (2) $\quad 2\pi_1 + \pi_2 = 2\pi_2 \rightarrow$ (3).

from (2), (1).

$\pi_1 + 2\pi_1 = 1$

$3\pi_1 = 1$

$\pi_1 = 1/3$

from (2), $\pi_2 = 2/3$.

steady state dist - $\pi = [1/3 \ \ 2/3]$.

2) If the initial state prob. distribution of a markov chain is $p^{(0)} = \left(\frac{5}{6} \ \frac{1}{6}\right)$ and the tpm of chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$ then find the prob. dist. of chain after 2 steps.

$p^{(0)} = \left(\frac{5}{6}, \frac{1}{6}\right)$

$P = \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$.

after 2 steps

$p^{(1)} = p^{(0)} \cdot P = \left(\frac{5}{6} \ \frac{1}{6}\right) \begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$

$p^{(1)} = \left(\frac{1}{12} \ \frac{11}{12}\right)$

$$p^{(2)} = p^{(1)} \cdot P = \left(\frac{1}{12} \quad \frac{11}{12}\right)\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$$

$$p^{(2)} = \left(\frac{11}{24} \quad \frac{13}{24}\right)$$

Note :-

The steady state distribution is also called as limiting probabilities, probability in the long run invarient probabilities, stationary probabilities, fraction, proportion, How often.

2) A students study habits are as follows. If he studies one night, he is 70% sure not to study next night. On the other hand the prob. that he does not study two nights in the succession is 0.6. In the long run, how often does he study.

S - study at night

T - Not study at night.

$$P = \begin{array}{c|cc} & S & T \\ S & 0.3 & 0.7 \\ T & 0.4 & 0.6 \end{array} \quad \text{Today}$$

let $\pi = [\pi_1 \quad \pi_2]$ be steady state dist,

$\pi P = \pi$ where $\pi_1 + \pi_2 = 1 \to (1)$

$$[\pi_1 \quad \pi_2] \begin{bmatrix} 0.3 & 0.7 \\ 0.4 & 0.6 \end{bmatrix} = [\pi_1 \quad \pi_2]$$

$$[0.3\pi_1 + 0.4\pi_2 \quad 0.7\pi_1 + 0.6\pi_2] = [\pi_1 \quad \pi_2]$$

compare same position element.

$0.3\pi_1 + 0.4\pi_2 = \pi_1$

$-0.7\pi_1 + 0.4\pi_2 = 0$,

$-7\pi_1 + 4\pi_2 = 0 \to (2)$

$0.7\pi_1 + 0.6\pi_2 = \pi_2$

$0.7\pi_1 - 0.4\pi_2 = 0$

$7\pi_1 - 4\pi_2 = 0 \to (3)$

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from ①, ②

② $\Rightarrow 7\pi_1 = 4\pi_2 \Rightarrow \pi_1 = 4/7\,\pi_2$.

① $\Rightarrow \pi_1 + \pi_2 = 1$

$4/7\,\pi_2 + \pi_2 = 1$

$4\pi_2 + 7\pi_2 = 7$

$11\pi_2 = 7$.

$\pi_2 = 7/11$

$\therefore \pi_1 = \frac{4}{7}\pi_2 = \frac{4}{7} \times \frac{7}{11} = \frac{4}{11}$

$\therefore \pi = \left[\frac{4}{11} \;\; \frac{7}{11}\right]$.

3) The tpm ~~of~~

3) Two boys B_1, B_2 and Two girls G_1, G_2 are throwing a ball from one to another each boys throws the ball to other boy with prob. 1/2. and to each girl with prob 1/4. On the other hand each girl throws the ball to each boy with prob. 1/2 and never to other girl. In the long run how often does each girl recieved the ball.

Boy → other Boy prob 1/2
→ each girl prob 1/4,

Girl → each boy prob. 1/2.
→ never to othe girl.

$$P = \begin{array}{c} \\ B_1 \\ B_2 \\ G_1 \\ G_2 \end{array} \begin{array}{c} \begin{array}{cccc} B_1 & B_2 & G_1 & G_2 \end{array} \\ \begin{bmatrix} 0 & 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix}_{4\times4} \end{array}$$

Let $\pi = [\pi_1 \; \pi_2 \; \pi_3 \; \pi_4]$ be steady state dist. Vector

$\pi P = \pi$

& $\pi_1+\pi_2+\pi_3+\pi_4 \rightarrow ①$

$$[\pi_1 \ \pi_2 \ \pi_3 \ \pi_4]\begin{bmatrix} 0 & 1/2 & 1/4 & 1/4 \\ 1/2 & 0 & 1/4 & 1/4 \\ 1/2 & 1/2 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \end{bmatrix} = [\pi_1 \ \pi_2 \ \pi_3 \ \pi_4]$$

$$\left[\frac{\pi_2}{2}+\frac{\pi_3}{2}+\frac{\pi_4}{2}, \ \frac{\pi_1}{2}+\frac{\pi_3}{2}+\frac{\pi_4}{2} \ \ \frac{\pi_1}{4}+\frac{\pi_2}{4} \ \ \frac{\pi_1}{4}+\frac{\pi_2}{4}\right] = [\pi_1 \ \pi_2 \ \pi_3 \ \pi_4]$$

compare same position element.

$\frac{1}{2}(\pi_2+\pi_3+\pi_4)=\pi_1$ $\quad \frac{1}{2}(\pi_1+\pi_3+\pi_4)=\pi_2$ $\quad \frac{1}{4}(\pi_1+\pi_2)=\pi_3$ $\quad \frac{1}{4}(\pi_1+\pi_2)=\pi_4$

$-2\pi_1+\pi_2+\pi_3+\pi_4=0$ $\quad \pi_1-2\pi_2+\pi_3+\pi_4=0$ $\quad \pi_1+\pi_2-4\pi_3=0$ $\quad \pi_1+\pi_2-4\pi_4=0$

$\pi_1 = 1/3$, $\pi_2 = 1/3$, $\pi_3 = 1/6$, $T_4 = 1/6$.

Steady state distribution:

$$\pi = \left[\frac{1}{3} \ \frac{1}{3} \ \frac{1}{6} \ \frac{1}{6}\right].$$

4) A Housewife buys 3 kinds of cereals A, B, C. she never buys the same cereal in successive weeks. If she buys cereal A, the next week she buys cereal B. However she buys B (or) C, next week she is 3 times as ~~lightly~~ likely to buy A as other cereal. How often she buys each of the 3 cereals.

$A \rightarrow B$.

$B \rightarrow$ 3 times as likely to buy A as other cereal C

$C \rightarrow$ " " " " " " " " B

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} \end{array}$$

let $\pi = [\pi_1 \ \pi_2 \ \pi_3]$ be steady state distribution vector.

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$\pi P = \pi$ & $\pi_1 + \pi_2 + \pi_3 = 1 \rightarrow (1)$

$$[\pi_1 \;\; \pi_2 \;\; \pi_3] \begin{bmatrix} 0 & 1 & 0 \\ 3/4 & 0 & 1/4 \\ 3/4 & 1/4 & 0 \end{bmatrix} = [\pi_1 \;\; \pi_2 \;\; \pi_3]$$

$$\left[\frac{3\pi_2}{4} + \frac{3\pi_3}{4} \quad \pi_1 + \frac{\pi_3}{4} \quad \frac{\pi_2}{4}\right] = [\pi_1 \;\; \pi_2 \;\; \pi_3].$$

$\frac{3\pi_2}{4} + \frac{3\pi_3}{4} = \pi_1$ $\quad \pi_1 + \frac{\pi_3}{4} = \pi_2$ $\quad \frac{\pi_2}{4} = \pi_3$.

$-4\pi_1 + 3\pi_2 + 3\pi_3 = 0 \rightarrow (2)$ $\quad 4\pi_1 - 4\pi_2 + \pi_3 = 0 \rightarrow (3)$ $\quad \pi_2 - 4\pi_3 = 0, \rightarrow (4)$

By solving (1), (2), (4),

$\pi_1 = 3/7$ $\quad \pi_2 = \frac{16}{35}$ $\quad \pi_3 = \frac{4}{35}$

5) The transition prob. matrix of markov chain $\{X_n\}, n = 1, 2, 3 \ldots$ having 3 states 1, 2 & 3 is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$ and the initial distribution $p^{(0)} = (0.7 \;\; 0.2 \;\; 0.1)$

Find i) prob. $P(X_2 = 3)$. ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$

Given $p^{(0)} = (\overset{1}{0.7} \;\; \overset{2}{0.2} \;\; \overset{3}{0.1})$

$P(X_0 = 1) = 0.7$ $\quad P(X_0 = 2) = 0.2$ $\quad P(X_0 = 3) = 0.1$

i) $P(X_2 = 3) = p^{(2)}(\text{at state } 3)$

$$p^{(1)} = p^{(0)} \cdot P = [0.7 \;\; 0.2 \;\; 0.1] \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$p^{(1)} = [0.22 \quad 0.43 \quad 0.35]$$

$$p^{(2)} = p^{(1)} \cdot P = [0.22 \;\; 0.43 \;\; 0.35] \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$

$$= [0.385 \quad 0.336 \quad 0.279]$$

$\therefore p^{(2)}(\text{at state } 3) = P(X_2 = 3) = 0.279.$

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ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2) = P(X_3 = 2 | X_2 = 3) P(X_2 = 3 | X_1 = 3)$
$P(X_1 = 3 | X_0 = 2) \cdot P(X_0 = 2)$

$\Rightarrow P_{32}^{(1)} \cdot P_{33}^{(1)} \; P_{23}^{(1)} \; (0.2)$

$\because P(X_{n+1} = k / X_n = j)$

$\Rightarrow (0.4)(0.3)(0.2)(0.2)$

$\Rightarrow 3/625.$

6) 3 Boys A, B, C are throwing a ball to each other. A always throws the ball to B and B always throws ball to C but C is just as likely to throw the ball to B as to A. Show that the process is markovian. Find the transition prob-matrix and classify the states.

$A \rightarrow B$

$B \rightarrow C$

$C \rightarrow B$, $C \rightarrow A$

$$P = \begin{array}{c c} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} A \\ B \\ C \end{array} & \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \end{array}$$

Future values depends on present values.
$\therefore$ The chain is markovian.

Irreducible :-

$$P^2 = P \cdot P = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \end{bmatrix}$$

$$P^2 = \begin{bmatrix} 0 & 0 & 1 \\ 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \end{bmatrix}$$

$$P^3 = P^2 \cdot P = \begin{bmatrix} 1/2 & 1/2 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 1/4 & 1/2 \end{bmatrix}$$

$$P^4 = P^3 \cdot 3 = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 1/2 & 1/2 \\ 1/4 & 1/2 & 1/4 \end{bmatrix}.$$

$$P^5 = P^4 \cdot P = \begin{bmatrix} 1/4 & 1/4 & 1/2 \\ 1/4 & 1/2 & 1/4 \\ 1/8 & 3/8 & 1/2 \end{bmatrix}$$

$\therefore P_{11}^{(3)} > 0,\ P_{13}^{(2)} > 0,\ P_{21}^{(2)} > 0,\ P_{22}^{(2)} > 0,\ P_{33}^{(2)} > 0.$

$\therefore P_{ij}^{(n)} > 0$ for some 'n'.

$\therefore$ All states are irreducible.

$\therefore$ Matrix is irreducible.

periodic :-

periodicity of state 'A' :-

$P_{11}^{(3)} > 0,\ P_{11}^{(5)} > 0,\ P_{11}^{(6)} > 0, \ldots$

G.C.D $= \{3, 5, 6 \ldots\} = 1 = d_i$.

if $d_i = 1$ then state 'A' is aperiodic.

periodic of B.

$P_{22}^{(2)} > 0,\ P_{22}^{(3)} > 0,\ P_{22}^{(4)}, \ldots$

G.C.D $= \{2, 3, 4 \ldots\} = 1$

if period $(d_1) = 1$

$\therefore$ state B is aperiodic

$\therefore$ all states are aperiodic.

~~state~~ periodic of C

$P_{33}^{(2)} > 0,\ P_{(33)}^{3} > 0, P_{33}^{(4)}, \ldots$

G.C.D $= \{2, 3, 4, \ldots\} = 1$

if period $(d_i) = 1$

$\therefore$ state C is aperiodic

$\therefore$ All states are aperiodic

It has finite states.

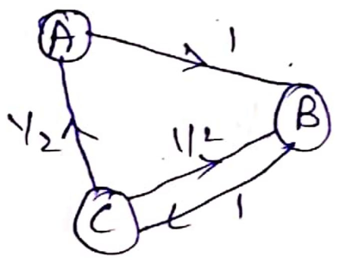
The finite irreducible matrix becomes non-null, persistent.

All states are non-null, persistent & aperiodic.

∴ All states are ergodic.

(or)

Diagraph:-



The state A is reachable to state B & C

The state B " " " " C & A

" " C " " " " A & B.

All states are reachable from all other states

∴ The chain is irreducible.

It has infinite states.

∴ finite irreducible matrix becomes non-null persistent.

period of A:-

G.C.D of {3, 5, 7 ...} = 1.

∴ state A is aperiodic

period of B:-

G.C.D {2, 3, 4, 5} = 1

∴ state B is aperiodic

period of C:-

G.C.D = {2, 3, 4, 5, ...} = 1

state C is aperiodic

Note :-

1) Absorbing state :-

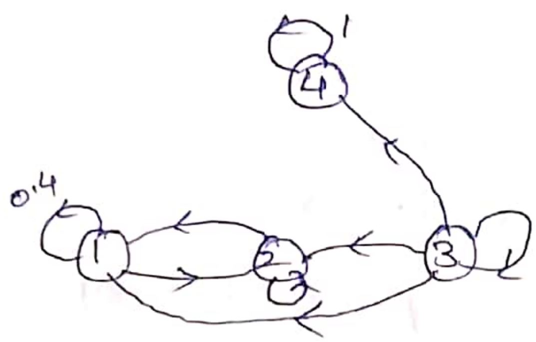
If a state i is called absorbing state if $P_{ii} = 1$.

2) If a matrix with absorbing state then it is not irreducible.

1) Construct the markov chain with transition prob matrix

$$P = \begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{bmatrix} 0.4 & 0.6 & 0 & 0 \\ 0.3 & 0.7 & 0 & 0 \\ 0.2 & 0.4 & 0.1 & 0.3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Is this matrix irreducible.

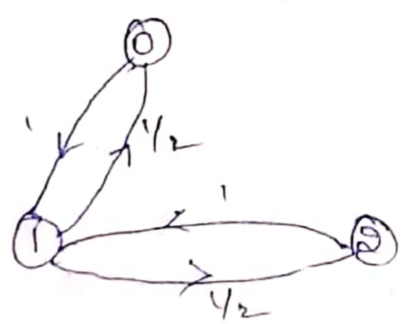


The state 4 is not reachable to state ①,②,③.

∴ The state 4 is absorbing state.

2) Find the nature of the states of the markov chain with the tpm $\begin{bmatrix} 0 & 1 & 0 \\ 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$

diagraph :-



The state '0' is reachable from the states 1 & 2

The state '1' is reachable from the states 0 & 2,

The " '2' " " " " " 1 & 0

every state is reachable from all other states.

∴ The chain is irreducible.

∴ the chain is finite.

periodicity:-

period of state '0':- G.C.D $\{2,4,6,8,\dots\} = 2 = d_i$

period of '0' is $= d_i = 2$.

period of state '1':-

G.C.D $= \{2,4,6,\dots\} = 2 = d_i$

period of state '1' $= d_i = 2$.

period of state '2':-

G.C.D $= \{2,4,6,\dots\} = 2 = d_i$.

∴ period of state '2' is $= 2 = d_i$.

∴ All states are non-null, persistent & aperiodic

∴ All states are ergodic.

3) Suppose that the probability of dry day following rainy day is 1/3 and the rainy day following is 1/2. Given that May 1st is dry day. Find that May 3rd is dry day and also May 5th is dry day.

Let D → Dry day

R → Rainy day.

$$P = \begin{array}{c c} & \begin{array}{cc} D & R \end{array} \\ \begin{array}{c} D \\ R \end{array} & \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix} \end{array}$$

Given May 1st is dry day

$$p^{(0)} = \begin{array}{c} \begin{array}{cc} D & R \end{array} \\ \begin{bmatrix} 1 & 0 \end{bmatrix} \end{array}$$

$$p^{(1)} = p^{(0)} \cdot P = [1 \quad 0] \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}.$$

$$p^{(1)} = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix}$$

$$p^{(2)} = p^{(1)} \cdot P = \begin{bmatrix} 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}.$$

$$p^{(2)} = \begin{bmatrix} \frac{1}{4} + \frac{1}{6} & \frac{1}{4} + \frac{2}{6} \end{bmatrix} = \begin{bmatrix} \frac{5}{12} & \frac{7}{12} \end{bmatrix}$$

$$p^{(3)} = p^{(2)} \cdot P = \begin{bmatrix} \frac{5}{12} & \frac{7}{12} \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 \\ 1/3 & 2/3 \end{bmatrix}.$$

$$p^{(3)} = \begin{bmatrix} 0.402 & 0.59 \end{bmatrix}$$

prob. of may 3rd dry day $= 0.402$.

$$p^{(4)} = p^{(3)} \cdot P.$$

$$p^{(5)} = p^{(4)} \cdot P.$$

4) A man either drives a car or catches the train to go to the office on each day. He never goes two days in a row by train but if he drives one day, the next day he is just as likely to drive again as he is to travel by train. Now suppose that, on the 1st day of week the man tossed a fair die and drove to if 6 appear. Find

i) prob. that he takes on the 3rd day.

ii) prob. that he drives to work in the long run.

Let $C \to$ car
$T \to$ Train

$$P = \begin{array}{c|cc} & T & C \\ \hline T & 0 & 1 \\ C & 1/2 & 1/2 \end{array}$$

He tossed a fair die i.e if it shows '6' then he goes by car.

$$p^{(1)} = \begin{array}{cc} T & C \\ \frac{5}{6} & \frac{1}{6} \end{array}$$

i) prob. of 3rd day by Train.

$$P^{(2)} = P^{(1)} \cdot P = \begin{bmatrix} \frac{5}{6} & \frac{1}{6} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{12} & \frac{5}{6} + \frac{1}{12} \end{bmatrix} = \begin{bmatrix} \frac{1}{12} & \frac{11}{12} \end{bmatrix}$$

$$P^{(3)} = P^{(2)} \cdot P = \begin{bmatrix} \frac{1}{12} & \frac{11}{12} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{11}{24} & \frac{1}{12} + \frac{11}{24} \end{bmatrix} = \begin{bmatrix} \frac{11}{24} & \frac{13}{24} \end{bmatrix}$$

prob. of 3rd day goes by Train is $= \frac{11}{24}$.