UNIT-I

Probability & Random Variable

Short Answer questions

1. Define probability, Conditional probability & Bayes Theorem

Defination of probability:

In a Random Experiment, Let there be an 'n' mutually exclusive and equally likely elementary events. Let E be an event of experiment. The probability of E

$$P(E) = \frac{m}{n} = \frac{\text{Number of elementary events in } E}{\text{Total number of elementary events in the Random experiment}}$$

Probability - Axiomatic approach:-

Def: Let 'S' be a finite Sample Space. A real valued function P from the power set of 'S' into R is called a probability function on 'S' if the following three axioms are satisfied

i) Axiom of positivity: $P(E) \geq 0$ for every subset E of S

(ii) Axiom of Certainity:- $P(S) = 1$

iii) Axiom of Union:- If E_1 and E_2 are disjoint subset of 'S' Then

$$P(E_1 \cup E_2) = P(E_1) + P(E_2)$$

Conditional probability:-

Def:- If E_1 and E_2 are two events in a sample space 'S' and $p(E_1) \neq 0$, then the probability of E_2, after the event E_1 has occurred, is called the conditional probability of the event of E_2 given E_1 and is denoted by $p(E_2/E_1)$

$$p(E_2/E_1) = \frac{p(E_1 \cap E_2)}{p(E_1)}$$

Similarly we define $p(E_1/E_2) = \frac{p(E_1 \cap E_2)}{p(E_2)}$

Bayes Theorem

Statment:-

$E_1, E_2 \ldots E_n$ are n mutually exclusive and exhaustive events such that $p(E_i) > 0$ $(i = 1, 2 \ldots n)$ in a sample space 'S' and A is any other event in S intersecting with every E_i (i.e. A can only occur in combination with any one of the events $E_1, E_2 \ldots E_n$) such that $p(A) > 0$

if E_i is any of the events of $E_1, E_2 \ldots E_n$ where $p(E_1)\, p(E_2) \ldots p(E_n)$ and $p(A/E_1)\, p(A/E_2) \ldots p(A/E_n)$ are known then

$$p(E_k/A) = \frac{p(E_k) \cdot p(A/E_k)}{p(E_1)p(A/E_1) + p(E_2)p(A/E_2) + \cdots + p(E_n)p(A/E_n)}$$

(2) Define Random variable & Types of Random variables

Random variable :-

Def :- A real variable X whose value is determined by the outcome of a random experiment is called a random variable

Types of Random variable

i) Discrete Random variable

A random variable X which can take only a finite number of discrete values in an interval of domain is called a discrete Random variable

(ii) Continuous Random variable :

A Random variable 'X' which can take values continuously i.e which takes all possible values in a given Interval is called continuous random variable

(3) Define probability distribution function

Let 'x' be a random variable. Then the probability function associated with 'x' is defined as the probability that the outcome of an experiment will be one of the outcomes for which $X(s) \leq x, x \in R$

(4) A continous random variable has the p.d.f

$$f(x) = \begin{cases} K + \frac{x}{6} & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$ find 'k'

Sol:-

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{0}^{3} (K + x/6)\,dx = 1$$

$$\left[Kx + \frac{x^2}{12}\right]_0^3 = 1$$

$$3K + \frac{9}{12} = 1$$

$$3K = 1 - \frac{9}{12}$$

$$\cancel{3}K = \frac{\cancel{3}}{12} \Rightarrow K = \frac{1}{12}$$

(5) Throwing two dice, find the probability of getting sum is even

Throwing two die $6^2 = 36$ out comes $= n$

favorble out comes $m = 18$

$$\begin{bmatrix} (1,1) & (1,3) & (1,5) \\ (2,2) & (2,4) & (2,6) \\ (3,1) & (3,3) & (3,5) \\ (4,2) & (4,4) & (4,6) \\ (5,1) & (5,3) & (5,5) \\ (6,2) & (6,4) & (6,6) \end{bmatrix}$$

$$P(E) = \frac{18}{36} = \frac{1}{2}$$

(6) In a pack of Cards, find the probability of getting Ace (or) Spade

Sol:-

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(\text{Ace (or) Spade}) = P(\text{Ace}) + P(\text{Spade}) - P(\text{Ace} \cap \text{Spade})$$

$$= \frac{4C_1}{52C_1} + \frac{13C_1}{52C_1} - \frac{1C_1}{52C_1}$$

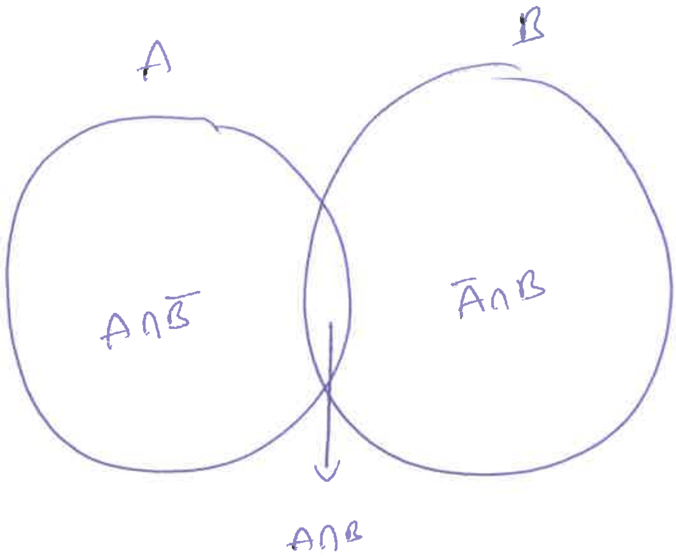
$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$$

Long Answer questions

(1) State and prove addition theorem on probability

Statment:- if S is a Sample space and A and B are any Two events

$$P(A \text{ or } B) = P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



Using above diagram

$A \cup B = (A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B)$

A and B disjoint sets by using Axiomatic def

$P(A \cup B) = P((A \cap \bar{B}) \cup (A \cap B) \cup (\bar{A} \cap B))$

$P(A \cup B) = P(A \cap \bar{B}) + P(A \cap B) + P(\bar{A} \cap B) \rightarrow 1$

$A = (A \cap \bar{B}) \cup (A \cap B)$

$P(A) = P(A \cap \bar{B}) + P(A \cap B)$

$P(A \cap \bar{B}) = P(A) - P(A \cap B) \rightarrow 2$

$B = (\bar{A} \cap B) \cup (A \cap B)$

$P(B) = P(\bar{A} \cap B) + P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B) \rightarrow 3$

2 & 3 sub in 1

$P(A \cup B) = P(A) - P(A \cap B) + \cancel{P(A \cap B)} + P(B) - \cancel{P(A \cap B)}$

So

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(2) Box 'A' Contain '5' red and '3' white marbles and box B contains '2' red and '6' white marbles. if marble is drawn from each box, what is The probability that they are both of same Colour.

Sol:- Suppose E_1 = the event that the marble is drawn from box 'A' and is red

$$P(E_1) = \frac{1}{2} \cdot \frac{5}{8} = \frac{5}{16}$$

and E_2 = The event that the marble from box 'B' and Red

$$P(E_2) = \frac{1}{2} \cdot \frac{2}{8} = \frac{1}{8}$$

The probability that both the marbles are red is

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2) = \frac{5}{16} \cdot \frac{1}{8} = \frac{5}{128}$$

Let E_3 = The event that the marble from box A and is white

$$P(E_3) = \frac{1}{2} \cdot \frac{3}{8} = \frac{3}{16}$$

Let E_4 = The event of marble from box B is white

$$P(E_4) = \frac{1}{2} \cdot \frac{6}{8} = \frac{3}{8}$$

and $P(E_3 \cap E_4) = \frac{3}{16} \cdot \frac{3}{8} = \frac{9}{128}$

The probability that the marbles are of same colour

$$= P(E_1 \cap E_2) + P(E_3 \cap E_4)$$

$$= \frac{5}{128} + \frac{9}{128} = \frac{14}{128} = \frac{7}{64} = 0.109$$

(3) Two factories produce identical clocks. The production of The first factory Consists of 10,000 clocks of which 100 are defective. The second factory produces 20,000 clocks of which '300' are defective. What is the probability that a particular defective clock was produced in the first factory?

Sol: Two factories is denoted by A and B

Output produced by first factory A = 10,000

Output produced by second factory B = 20,000

probability that items produced by A are defective

$$P(D/A) = \frac{100}{10000} = 0.01$$

Similarly $$P(D/B) = \frac{300}{20,000} = 0.015$$

$$P(A) = \frac{10000}{30000} = \frac{1}{3} = 0.33$$

$$P(B) = \frac{20000}{30000} = \frac{2}{3} = 0.66$$

find

$P(A/D)$ = prob of defective clock was produced by first factory

$$P(A|D) = \frac{P(A) \times P(D/A)}{P(A) \times P(D/A) + P(B) P(D/B)}$$

$$= \frac{(0.33) \times (0.01)}{(0.33)(0.01) + (0.66)\,0.015} =$$

4) $P(A) = \frac{2}{3}$ $P(B) = \frac{1}{5}$

Then P.T $\frac{2}{15} \leq P(A \cap B) \leq \frac{1}{5}$

proof:

Using Conditional probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow P(A) = \frac{P(A \cap B)}{P(B)}$$

$$P(A \cap B) = P(A) P(B)$$

$$P(A \cap B) = \frac{2}{3} \times \frac{1}{5}$$

$$P(A \cap B) = \frac{2}{15}$$

Since $\frac{2}{15} \leq \frac{1}{5}$ and $P(A \cap B) = \frac{2}{15}$

$$\Rightarrow \frac{2}{15} = P(A \cap B) \leq \frac{1}{5}$$

$$\frac{2}{15} \leq P(A \cap B) \leq \frac{1}{5}$$

5) From a lot of '10' items containing '3' defective. a Sample of 4 items is drawn at Random. Let the random variable X denote the number of defective items in the Sample. Find the probability distribution of 'X' when the Sample without Replacement

Sol:- obviously X takes the Values 0, 1, 2 or 3

Given total No of items = 10

No of good items = 7

No of defective items = 3

$$P(X=0) = P(\text{no defective}) = \frac{^7C_4}{^{10}C_4} = \frac{7!}{4!3!} \times \frac{4!6!}{10!} = \frac{1}{6}$$

$$P(X=1) = P(\text{one defective and 3 good items})$$

$$= \frac{^3C_1 \times {^7C_3}}{^{10}C_4} = \frac{3 \times 7!}{3!4!} = \frac{1}{2}$$

$$P(X=2) = P(\text{2 defective and 2 good items})$$

$$= \frac{^3C_2 \times {^7C_2}}{^{10}C_4} = \frac{3}{10}$$

$$P(X=3) = P(\text{3 defective and 1 good item})$$

$$= \frac{^3C_3 \times {^7C_1}}{^{10}C_4} = \frac{7}{^{10}C_4} = \frac{1}{30}$$

The probability distribution of Random Variable 'X' as follow

$X = x_i$	0	1	2	3
$P(X=x_i)$	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

⑥ Let $f(x) = 3x^2$ when $0 \le x \le 1$ be the probability density function of a continuous random variable X. Determine 'a' and 'b' such that

i) $P(X \le a) = P(X > a)$ (ii) $P(X > b) = 0.05$

Sol:-

Given data $f(x) = 3x^2 \quad 0 \le x \le 1$

i) $P(X \le a) = P(X > a)$

$$\int_0^a f(x)\,dx = \int_a^1 f(x)\,dx$$

$$\int_0^a 3x^2\,dx = \int_a^1 3x^2\,dx$$

$$\left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_0^a = \left[\frac{\cancel{3}x^3}{\cancel{3}}\right]_a^1$$

$$a^3 = [1 - a^3]$$

$$2a^3 = 1$$

$$a^3 = \frac{1}{2}$$

$$a = \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

(11)

$P(X>b) = 0.05$

$\int_b^1 3x^2 \, dx = 0.5$

$\left[\frac{3x^3}{3}\right]_b^1 = 0.05$

$1 - b^3 = 0.05$

$1 - 0.05 = b^3$

$b^3 = 0.05$

$b = (0.05)^{1/3}$

$\frac{}{P(B)}$

b) $P(A^c/B) = \frac{P(\bar{A} \cap B)}{P(B)} = \frac{P(B) - P(A \cap B)}{P(B)}$

$= \frac{1/3 - 1/3}{4/3}$ $= \frac{5-3}{15} \cdot \frac{2}{2/3}$

8) $P\left(\bigcap_{i=1}^{n} A_i\right) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

Methematic induction method is used to prove this
For any event, $E \in S$, $0 \leq P(E) \leq 1$

Consider the two event $(n=2)$ then
$P(A_1 \cup A_2) \leq 1$ $\because A_1 \cup A_2$ is an event

By using addition theorem
$P(A_1) + P(A_2) - P(A_1 \cap A_2) \leq 1$
$P(A_1) + P(A_2) \leq 1 + P(A_1 \cap A_2)$

we can write the above equation

$P(A_1 \cap A_2) \geq P(A_1) + P(A_2) - 1$

$\therefore$ The Statem is true for $n=2$
let us assume that statement is true for $n=k$
i.e $P\left(\bigcap_{i=1}^{k} A_i\right) \geq \sum_{i=1}^{k} P(A_i) - (k-1)$
then

then
$P\left(\bigcap_{i=1}^{k+1} A_i\right)$ $P\left(\bigcap_{i=1}^{k} (A_i \cap A_{k+1})\right) \geq$

Q. $P(A\cup B) = P(A) + P(B) - P(A\cap B)$.

S:- Let A and B be any two events. To write $A\cup B$ as the union of three mutually exclusive events: $A\cap B^c$, $A\cap B$ and $A^c\cap B$

$$A\cup B = (A\cap B^c)\cup(A\cap B)\cup(A^c\cap B) \quad --(1)$$

By axioms 3:

$$P(A\cup B) = P(A\cap B^c) + P(A\cap B) + P(A^c\cap B) \quad --(2)$$

Now,

$$A = (A\cap B^c)\cup(A\cap B)$$
$$B = (A^c\cap B)\cup(A\cap B)$$

Therefore

$$P(A) = P(A\cap B^c)\cup P(A\cap B) \quad --(3)$$

and $$P(B) = P(A^c\cap B)\cup P(A\cap B) \quad --(4)$$

By equation 3 and 4, we get

$$P(A) + P(B) = P(A\cap B^c) + P(A^c\cap B) + 2P(A\cap B) \quad --(5)$$

Compare eq (2) and (5),

$$P(A) + P(B) = P(A\cup B) + P(A\cap B)$$

So, $$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

52 - 26 - 13 - $\frac{13}{2}$ Card

Q. Explain Bayes theorem :-

→ Bayes theorem is a method to revise the probability of an event given additional information. Bayes theorem calculates a conditional probability called a posterior or revised probability.

- Bayes theorem is a result in probability theory that relates conditional probabilities. If A and B denote two events, P(A|B) denotes the conditional probability of A occuring, given that B occurs. The two conditional probabilities P(A|B) and P(B|A) are in general different.

- Bayes theorem gives a relation between P(A|B) and P(B|A). An important application of Bayes theorem is that it gives a rule how to update or revise the strength of evidence-based belief in light of new evidence a posteriori.

- A prior probability is an initial probability value originally obtained before any additional information is obtained.
- A Posterior probability is a probability value that has been revised by using additional information that is later obtained.

Proof:-

conditional probability

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad - (1)$$

$$P(B/A) = \frac{P(B \cap A)}{P(A)} \quad - (2)$$

from (1) & (2)

$$P(B/A) \cdot P(A) = P(A/B)\, P(B).$$

$$\therefore \boxed{P(A/B) = \frac{P(B/A) \cdot P(A)}{P(B)}}$$

(Q 43 > Q 44) Q 45

~~Q43)~~ Two boxes ~~B_1 and~~ B_2 ~~contain 100 and 200 light bulbs respectively.~~

Q. What is random variable? ~~what is a~~ Explain discrete random variable.

Ans:- The distribution function $F(x)$ or the density $f(x)$ completely characterizes the behaviour of a random variable x. The concept of a random variable will enable us to replace the original probability space with one in which event are set of numbers.

- Whenever you run and experiment, flip a coin, roll a die, pick a card, you assign a number to represent the value to the outcome that you get. This assign is called a random variable.

- A random variable is a variable x that assigns a real number $[x]$, for each and every outcome of a random experiment. If S is the sample space containing all the 'n' outcomes $[e_1, e_2, e_3, \dots e_i, e_n]$ of random experiment and x is a random variable defined as a function $X(e)$ on S, then for every outcome e_i (where $i = 1, 2, 3, \ldots, n$) that is in S the random variable $X(e_i)$ will assign a real value x_i

- Advantages of random variable is that user can define certain probability function that make it both convenient and easy to compute the probabilities of various events.

→ Discrete Random variable:-
The random variable is called a discrete random variable if it is defined over a sample space having a finite or a countable infinite number of sample points. In this case, random variable takes on discrete values and it is possible to enumerate all the values it may assume.

- A discrete random variable can only have a specific (or finite) number of numerical values

- We can have infinite discrete random variable if we think about things that we know have an estimated number. Think about the number of stars in the universe. we know that there are not a specific number that we have a way to count so this is an example of an infinite discrete random variable.

- Another example would be with investments with share market. If you were to invest ₹1 lakh at the start of year, you could only estimate the amount you would have at the end of year.

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Q. Explain difference between discrete and continu random variable

D	C
(1) It uses countable set	It uses set of interval on R
(2) F is set of all subset of Ω	F is made from sub-interv of Ω with set operations
(3) For a set $A \in F$ $P(A) = \sum_{\omega \in A} P(\omega)$	for a set $A \in F$, $P(A) = \int_{A}^{-A} f_X(x)\, dx$
(4) distribution function (cdf): $F_X(x) = \sum_{\omega \leq x} P(\omega)$	(4) distribution function (cdf): $F_X(x) = \int_{-\infty}^{x} f_X(x)\, dt$

Q. 60) 66) 67) 68

By equations (3) and (4), we get

$$P(A) + P(B) = P(A \cap B^c) + P(A^c \cap B) + 2P(A \cap B) \quad \dots (5)$$

Compare the equation (2) and (5),

$$P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

So, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Q.22 For any three arbitrary events A, B, C prove that :
$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$. [JNTU : Nov.-09, 10]

Ans. :

$$\begin{aligned} P(A \cup B \cup C) &= P[(A \cup B) \cup C] \\ &= P(A \cup B) + P(C) - P[(A \cup B) \cap C] \\ &= P(A) + P(B) - P(A \cap B) + P(C) \\ &\quad - P[(A \cap C) \cup (B \cap C)] \\ &= P(A) + P(B) + P(C) - P(A \cap B) \\ &\quad - P(A \cap C) - P(B \cap C) - P((A \cap C) \cap (B \cap C)) \\ &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ &\quad - P(B \cap C) + P(A \cap B \cap C) \end{aligned}$$

Q.23 If $P(A) = \frac{1}{5}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{15}$. Find a) $P(A \cup B)$ b) $P(A^c \cap B)$ c) $P(A \cap B^c)$.

Ans. : a) $P(A \cup B)$

$$\boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$$

$$= \frac{1}{5} + \frac{2}{3} - \frac{1}{15}$$

$$= \frac{3 + 2 \times 5}{15} - \frac{1}{15}$$

$$= \frac{13 - 1}{15} = \frac{12}{15}$$

- A posterior probability is a probability value that has been revised by using additional information that is later obtained.
- Suppose that $B_1, B_2, B_3 ... B_n$ partition the outcomes of an experiment and that A is another event. For any number, k, with $1 \le k \le n$, we have the formula :

$$P(B_k/A) = \frac{P(A/B_k) \cdot P(B_k)}{\sum_{i=1}^{n} P(A/B_i) \cdot P(B_i)}$$

Q.43 Two boxes B_1 and B_2 contain 100 and 200 light bulbs respectively. The first box (B_1) has 15 defective bulbs and the second 5. Suppose a box is selected at random and one bulb is picked out.

a) What is the probability that it is defective ?

b) Suppose we test the bulb and it is found to be defective. What is the probability that it came from box 1 ?

Ans. :

a) **Probability that it is defective :** Box B_1 has 85 good and 15 defective bulbs. Similarly box B2 has 195 good and 5 defective bulbs.

Let D = "Defective bulb is picked out".

Then,

$$P(D/B_1) = \frac{15}{100} = 0.15, \; P(D/B_2) = \frac{5}{200} = 0.025.$$

Since a box is selected at random, they are equally likely.

$$P(B_1) = P(B_2) = \frac{1}{2}$$

Thus B_1 and B_2 form a partition and using above equation, we obtain

$$P(D) = P(D/B_1)\, P(B_1) + P(D/B_2)\, P(B_2)$$

$$= 0.15 \times \frac{1}{2} + 0.025 \times \frac{1}{2} = 0.0875$$

Thus, there is about 9% probability that a bulb picked at random is defective.

b) Probability that it came from box 1 :

$$P(B_1/D) = \frac{P(D/B_1)\, P(B_1)}{P(D)} = \frac{0.15 \times 1/2}{0.0875} = 0.8571$$

Q.44 A mechanical factory production line is manufacturing bolts using three machines, A, B and C. The total output, machine A is responsible for 25%, machine B for 35% and machine C for the rest. The machines that 5% of the output from machine A is defective, 4% from machine B and 2% from machine C. A bolt is chosen at random from the production line and found to be defective. What is the probability that it came from

I. machine A II. machine B III. machine C ?

Ans. : Let

D = {bolt is defective},

A = {bolt is from machine A},

B = {bolt is from machine B},

C = {bolt is from machine C}.

Given data : P(A) = 0.25, P(B) = 0.35, P(C) = 0.4.

P(D|A) = 0.05, P(D|B) = 0.04, P(D|C) = 0.02.

From the Bayes' Theorem :

$$P(A/D) = \frac{P(D/A) \times P(A)}{P(D/A) \times P(A) + P(D/B) \times P(B) + P(D/C) \times P(C)}$$

$$= \frac{0.05 \times 0.25}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4}$$

$$= \frac{0.0125}{0.0125 + 0.014 + 0.008}$$

P(A/D) = 0.3621

Similarly :

$$P(B/D) = \frac{P(D/B) \times P(B)}{P(D/A) \times P(A) + P(D/B) \times P(B) + P(D/C) \times P(C)}$$

$$= \frac{0.04 \times 0.35}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4}$$

$$= \frac{0.014}{0.0125 + 0.014 + 0.008} = \frac{0.014}{0.0345}$$

P(B/D) = 0.4057

$$P(C/D) = \frac{P(D/C) \times P(C)}{P(D/A) \times P(A) + P(D/B) \times P(B) + P(D/C) \times P(C)}$$

$$= \frac{0.02 \times 0.4}{0.05 \times 0.25 + 0.04 \times 0.35 + 0.02 \times 0.4}$$

$$= \frac{0.008}{0.0125 + 0.014 + 0.008} = \frac{0.008}{0.0345}$$

P(C/D) = 0.2318

Q.45 At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favour of women. If a student is selected at random from among all those over six feet tall, what is the probability that the student is a woman ?

Ans. : Let us assume following :

$$M = \{\text{Student is Male}\},$$
$$F = \{\text{Student is Female}\},$$
$$T = \{\text{Student is over 6 feet tall}\}.$$

Given data :

$$P(M) = 2/5,$$
$$P(F) = 3/5,$$
$$P(T|M) = 4/100$$
$$P(T|F) = 1/100.$$

We require to find $P(F|T)$?

Using Bayes' Theorem we have :

$$P(F/T) = \frac{P(T/F)\,P(F)}{P(T/F)\,P(F) + P(T/M)\,P(M)}$$

$$= \frac{\frac{1}{100}\times\frac{3}{5}}{\frac{1}{100}\times\frac{3}{5}+\frac{4}{100}\times\frac{2}{5}} = \frac{\frac{3}{500}}{\frac{3}{500}+\frac{8}{500}}$$

$$P(F/T) = \frac{3}{11}$$

Q.46 A pair of dice is rolled. If the sum of 9 has appeared, find the proabability that one of the dice shows 3.

Ans. : Let A = The event that the sum is 9

B = The event the one of dice shows 3.

Exhaustive cases $= 6^2 = 36$.

Favorable cases of the event A = (3, 6), (6, 3), (4, 5), (5, 4).

So $P(A) = 4.36$

$$P(A) = \frac{1}{9}$$

Favorable case for the event $A \cap B$ = (3, 6), (6, 3)

Hence $P(A \cap B) = \frac{2}{36} = \frac{1}{18}$

But $P(A \cap B) = P(A) \times P(B/A)$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$P(B/A) = \frac{1/18}{1/9} = \frac{1}{18} \times \frac{9}{1}$$

$$P(B/A) = 1/2$$

Q.47 If A_1 and A_2 are equally lik[e] exclusive and exhaustive events and P[...] $P(B/A_2) = 0.3$, find $P(A_1/B)$.

Ans. : Since A_1 and A_2 are equally [...] $P(A_1) = P(A_2)$. Further, they are mutu[...]

$$P(A_1) + P(A_2) = 1$$
$$P(A_1) = P(A_2) = 0.5$$

$$= \frac{0.5 \times 0.2}{0.5 \times 0.2 + 0.5 \times [...]}$$

$$= \frac{0.1}{0.1 + 0.15} = \frac{0.1}{0.2[...]}$$

$$P(A_1/B) = 0.4$$

Q.48 10% of the bulbs produced [...] of red colour and 2% are red and [...] bulb is picked up at random, [...] probability of its being defective if [...]

Ans. :

Let A and B be the events that the b[...] detective, respectively.

$$P(A) = 10/100 = 1/10$$
$$P(A \cap B) = 2/100 = 1/50$$
$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/50}{1/10}$$

Thus the probability of the pick[...] being defeftive, if it is red, is 1/5.

1.6 : Random Variables an[d] Distributions : Concep[t of] Variable

Q.49 Define discrete sample spa[ce]

Ans. : If a sample space contain[s] possibilities or an unending seq[uence of] elements as there are whole nu[mbers ...] discrete sample space.

om variable.

a continuous rando
in an interval

distribution for the

head side up.

il side up.

coin to land hea

e number of
. Determine

$$= \frac{1}{16}+\frac{4}{16}+\frac{6}{16}+\frac{4}{16}$$

$$= \frac{1+4+6+4}{16} = \frac{15}{16}$$

$$F(4) = f(0) + f(1) + f(2) + f(3) + f(4)$$

$$= \frac{1}{16}+\frac{4}{16}+\frac{6}{16}+\frac{4}{16}+\frac{1}{16}$$

$$= \frac{1+4+6+4+1}{16} = \frac{16}{16} = 1$$

Q.55 A shipment of 8 similar microcomputers to a. retail outlet contains 3 that are defective. If a school makes a random purchase of 2 of these computers, find the probability distribution for the number of defectives.

Ans. : Let X be a random variable whose values x are the possible numbers of defective computers purchased by the school. Then x can be any of the numbers 0, 1, and 2. Now

Number of ways of choosing any 2 :

$$^8C_2 = \frac{8!}{((8-2)!2!)} = 28$$

(This will be the denominator)

$$P[0] = {}^3C_0 \times {}^5C_2/28$$

$$= (3!/((3-0)!0!) \times (5! \times ((5-2)!2)))/28$$

$$= 5/14$$

$$P[1] = {}^3C_1 \times {}^5C_{1/28} = 15/28$$

$$P[2] = {}^3C_2 \times {}^5C_{0/28} = 3/28$$

Q.56 If a random variable X takes the values 1, 2, 3 and 4 such that 2P (X = 1) = 3P (X = 2) = P (X = 3) = 5P (X = 4), derive the probability distribution function of X.

[JNTU : March-17, Marks 5]

Assume P (X = 3) = α By the given equation

$$1) = \frac{\alpha}{2} \quad P(X = 2) = \frac{\alpha}{3} \quad P(X = 4) = \frac{\alpha}{5}$$

a probability distribution (and mass function)

$$\sum P(x) = 1$$

$$P(1) + P \quad) + P(4) = 1$$

$$\frac{\alpha}{2} + \quad 1 \quad \Rightarrow \frac{61}{30}\alpha = 1 \quad \Rightarrow \alpha = \frac{30}{61}$$

$$P(X = 1) = \frac{15}{61};\ P(X = 2) = \frac{10}{61};\ P(X$$

$$P(X = 4) = \frac{6}{61}$$

- The probability distribution giv

X	1	2	
P(X)	15/61	10/61	30

1.7 : Continuous Probability Statistical Independence

Q.57 What is Continuous probabil

Ans. : Continuous probability dist of distribution that deals with co data or random variables. The co variables deal with different kinds o

Q.58 What is a discrete probabil What two conditions determine distribution?

Ans. : • Discrete distribution is distribution where sample space is random variable in such a case can values. For example: number of peop coins etc.

- Discrete Probability distribution values a random variable can a corresponding probabilities of the va
- The two conditions are

1. Sum of all probabilities should be
2. Each probability must be greater to zero.

Q.59 Explain difference between continuous random variable.

Ans. :

Sr. No.	Discrete	C
1.	It uses countable set	It uses on R.

2.	F is set of all subset of Ω.	F is made from sub-intervals of Ω with set operations.
3.	For a set $A \in F$, $P(A) = \sum_{\omega \in A} p(\omega)$	For a set $A \in F$, $p(A) = \int_{A}^{-A} f_X(x)\,dx$
4.	Distribution function (Cdf) : $F_X(x) = \sum_{\omega \le x} p\omega$	Distrinbution function (Cdf) : $F_X(x) = \int_{-\infty}^{x} f_X(x)\,dt$

Q.60 Explain statistical independence

Ans. : Two variates A and B are statistically independent if the probability $P(A|B)$ of A given B satisfies $P(A|B) = P(A)$

in which case the probability of A and B is just

$$P(AB) = P(A \cap B) = P(A)\,P(B)$$

- Statistical independence means one event conveys no information about the other; statistical dependence means there is some information.
- Statistically independent is not the same as mutually exclusive: if A and B are mutually exclusive, then they can't be independent, unless one of them is probability 0 to start with :

$P(r)(()A \cap B) = 0 = Pr(()A)\ Pr(()B)$

Q.61 Three cards are drawn in succession from a deck without replacement. Find the probability distribution for the number of spades.

Ans. : X = number of spades in the three draws.

Let S and N stand for a spade and not a spade, respectively.

$$P(X = 0) = P(NNN)$$
$$= (39/52)\ (38/51)\ (37/50)$$
$$= 703/1700$$

$$P(X = 1) = P(SNN) + P(NSN) + P(NNS)$$
$$= 3(13/52)(39/51)(38/50)$$
$$= 741/1700$$

$$P(X = 3) = P(SSS)$$
$$= (13/52)(12/51)(11/50)$$
$$= 11/850$$

$$P(X = 2) = 1 - \frac{703}{1700} - \frac{741}{1700} - \frac{11}{850} =$$

Q.62 From a box containing 4 black green balls, 3 balls are drawn in suc ball being replaced in the box be draw is made. Find the probability d the number of green balls.

Ans. : • Let X denotes the number of the three draws.

- Let G and B stand for the colors black, respectively.

Sample Event	x	
BBB	0	(2/3)3
GBB	1	(1/3)
BGB	1	(1/3)
BBG	1	(1/3)
BGG	2	(1/3)2
GBG	2	(1/3)2
GGB	2	(1/3)2
GGG	3	(1/3)3

Q.63 A traffic engineer is interested i of vehicles reaching a particular cros periods of relatively low traffic flow. finds that the number of vehicles X crossroads per minute is gover probability distribution:

x	0	1	2
P(X = x)	0.37	0.39	0.19

Calculate the expected value, the the standard deviation of the rand

Ans. : The expected value, the va standard deviation of the random vaia

x	x^2	P(X = x)
0	0	0.37
1	1	0.39
2	4	0.04
3	9	0.09
4	16	0.01

$\times 0.19+3\times 0.04+4\times 0.01$

$xP(X=x)\Big]^2$

$0.19+9\times 0.04+16$

ven by $\sigma = \sqrt{V(X)} =$

gram of the following
ch represents the
s from a video store

... be a random variable with the following probability distribution

X	−3	6	9
P(X = x)	1/6	1/2	1/3

then evaluate $E(2X+1)^2$

Ans. : We know that :

$$E(2X+1)^2 = E(4X^2+4X+1) = E4(X^2)+E4(X)+1$$

$$E(X) = (-3)\frac{1}{6}+(6)\frac{1}{2}+(9)\frac{1}{3} = \frac{11}{12}$$

$$E(X^2) = (-3)^2\frac{1}{6}+(6^2)\frac{1}{2}+(9^2)\frac{1}{3} = \frac{93}{2}$$

$$E(2X+1)^2 = (4)\times(93/2)+(4)(11/2)+1 = 209$$

Q.66 A class contains of 6 girls and 10 boys. If a committee of 3 is chosen at random from the class, find the probability that (i) 3 boys are selected, (ii) exactly 2 girls are selected.

Ans. : Total number of student (n) = 16

Then n(s) = number of ways of choosing 3 from 16

$$= {}^{16}C_3$$

i) 3 boys are selected : We can write ${}^{10}C_3$ here.

$$n(E) = {}^{10}C_3 \text{ Therefore}$$

$$P(E) = \frac{{}^{10}C_3}{{}^{16}C_3} = \frac{10\times 9\times 8}{16\times 15\times 14} = \frac{3}{14}$$

ii) exactly 2 girls are selected

Then $n(E) = {}^{6}C_2 \times {}^{10}C_1$

$$P(E) = \frac{{}^{6}C_2 \times {}^{10}C_1}{{}^{16}C_3} = \frac{15}{56}$$

Q.67 Three groups of students contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys; one student is selected at random from each group. Find the probability of selecting 1 girl and 2 boys.

Ans. : If G denotes selection of girls and B denotes selection of a boy then the various cases of selection can be GBB respectively from the three groups, or BBG respectively from the three groups.

So, $P(GBB) = P(G \text{ from group 1}) \times P(B \text{ from group II}) \times P(B \text{ from group III})$

$$P(GBB) = \frac{{}^3C_1}{{}^4C_1} \times \frac{{}^2C_1}{{}^4C_1} \times \frac{{}^3C_1}{{}^4C_1} = \frac{3}{2} \times \frac{2}{4} \times \frac{3}{4} = \frac{9}{32}$$

Similarly,

$$P(BGB) = \frac{1}{4} \times \frac{2}{4} \times \frac{3}{4} = \frac{3}{32}$$

$$P(BBG) = \frac{1}{4} \times \frac{2}{4} \times \frac{1}{4} = \frac{1}{32}$$

Required probability $= P(GBB) + P(BGB) + P(BBG)$

$$= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$$

Q.68 A batch of 100 manufactured components is checked by an inspector who examines 10 components selected at random. If none of the 10 component's is defective, the inspector accepts the whole batch. Otherwise, the batch is subjected to further inspection. What is the probability that a batch containing 10 defective components will be accepted ?

Ans. : • Let the N denote the number of ways of indiscriminately selecting 10 components from a batch of 100 components. Then N is given by

$$N = C(100, 10)$$

$$= \frac{100!}{(100-10)! \times 10!} = \frac{100!}{90! \times 10!}$$

- Let E denote the event "the batch containing 10 defective components is accepted by the inspector".
- The number of ways that E can occur is the number of ways of selecting 10 components from the 90 non-defective components and no components from the 10 defective component's.
- This number, N(E) is given by

$$N(E) = C(90, 10) \times C(10, 0) = C(90, 10)$$

$$= \frac{90!}{(90-10)! \times 10!} = \frac{90!}{80! \times 10!}$$

The probability of event E is given by :

$$P(E) = \frac{N(E)}{N} = \frac{90!}{80! \times 90!} \times \frac{90! \times 10!}{100!} = \frac{90! \times 90!}{100! \times 80!}$$

$$P(E) = 0.3305$$

Q.69 Two coins are tossed. Let A d... "at most one head on the two to... denote the event "one head and o... tosses". Are A and B independent e...

Ans. : The sample space of the exper... HT, TH, TT}

Now events are defined as follows :

$A = \{HT, TH, TT\}$ B =

and $A \cap B = \{HT, TH\}$

Thus :

$P(A) = \frac{3}{4}$ P(B) =

$P(A \cap B) = \frac{2}{4} = \frac{1}{2}$

$P(A)\,P(B) = \frac{8}{3}$

Since $P(A \cap B)$ $P(A)\,P(B)$, we concl... and B are not independent.

Q.70 A die is thrown 3 times. I... considered as success find the... atleast 2 success.

Ans. :

$p = \frac{1}{6}, q = \frac{5}{6}, n = 3$

P (at least 2 success) $= P(X \geq 2) =$ P...

$= 3C_2 \left(\frac{1}{6}\right)^2 \frac{5}{6} + 3C_3$ (...

Fill in the Blanks for Mid ...

Q.1 A _____ variable takes ... which is determined by t... random experiment.

Q.2 A pictorial representation ... manipulations with events using _____ diagrams.

Q.3 Statistical data, generated in ... be very useful for studying ... the distribution if presente... tabular and graphic display ... plot