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Stochastic Simulation of Daily Precipitation, Temperature, and Solar Radiation

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Long samples of weather data are frequently needed to evaluate the long-term effects of proposed hydrologic changes. The evaluations are often undertaken using deterministic mathematical models that require daily weather data as input. Stochastic generation of the required weather data offers an attractive alternative to the use of observed weather records. This paper presents an approach that may be used to generate long samples of daily precipitation, maximum temperature, minimum temperature, and solar radiation. Precipitation is generated independently of the other variables by using a Markov chain–exponential model. The other three variables are generated by using a multivariate model with the means and standard deviations of the variables conditioned on the wet or dry status of the day as determined by the precipitation model. Daily weather samples that are generated with this approach preserve the seasonal and statistical characteristics of each variable and the interrelations among the four variables that exist in the observed data.

INTRODUCTION

Meteorological data are frequently needed to evaluate the long-term effects of proposed, man-made hydrologic changes. These evaluations are often undertaken using deterministic mathematical models of hydrologic processes. The models require weather data as input. In addressing hydrologic responses to weather inputs it is seldom sufficient to examine only the response to observed weather events. Use of observed sequences gives a solution based on only one realization of the weather process. What would be the result if another series with the same properties as the observed series were used? What is the range of results that may be obtained with other equally likely weather sequences? To answer these questions, it is desirable to generate synthetic sequences of weather data based on the stochastic structure of the meteorologic processes.

The meteorological variables needed for most hydrologic models include precipitation, maximum and minimum temperatures, solar radiation, or some related variable [*Knisel,* 1980]. These variables are usually recorded daily, and most deterministic models require daily values. The objective of this study therefore was to develop a technique for simulating daily values of precipitation, maximum and minimum temperatures, and solar radiation.

THE MODEL

To develop a simulation model of the meteorological variables of interest, one must develop a concept of the stochastic relationships that underlie the basic meteorological processes. The processes are time dependent within each variable and interdependent among the four variables. Both radiation and temperature, for example, are more likely to be below normal on rainy days than on dry days. Similarly, maximum temperature will likely be low on a cloudy day with low solar radiation. Maximum and minimum temperatures on a given day will obviously be related because of heat storage in the soil and surrounding atmosphere. Maximum temperature should be serially correlated because of heat storage from one day to the next. The processes exhibit seasonal oscillations for each variable. A model for simulation of daily weather variables should be able to account for these interrelations and seasonal variations.

The general approach followed in this study was to consider precipitation as the primary variable and then condition the other three variables for a given day on whether the day was wet or dry. For the mathematical development that follows, let $Y_{p,i}$ be the daily precipitation amount, $X_{p,i}(1)$ the maximum temperature, $X_{p,i}(2)$ the minimum temperature, and $X_{p,i}(3)$ the solar radiation for year p and day i.

Precipitation

Many models have been proposed for simulating daily precipitation [*Buishand,* 1978; *Chin,* 1977; *Gabriel and Neuman,* 1962]. Since precipitation was chosen as the primary variable and daily precipitation amounts were determined independently of the other variables, any precipitation model that produces daily precipitation values (subject to some criterion of goodness) could be used for the precipitation component. For this study a simple Markov chain–exponential model was used for the precipitation component to illustrate the concept for simulating the other weather variables. A first-order Markov chain [*Bailey,* 1964] was used to describe the occurrence of wet or dry days. On days with rain the exponential distribution was used to describe the amount of rain.

With a first-order Markov chain the probability of rain on a given day is conditioned on the wet or dry status of the previous day. The Markov chain model for daily precipitation occurrence has been studied extensively. *Gabriel and Neuman* [1962] found that a first-order Markov chain provided a satisfactory model for daily precipitation occurrence at Tel Aviv, Israel. A Markov chain was also used by *Caskey* [1963], *Weiss* [1964], and *Hopkins and Robillard* [1964] to describe the occurrence of sequences of wet or dry days. *Haan et al.* [1976] used a first-order Markov chain with six rainfall states to model both occurrence and amounts of precipitation. *Smith and Schreiber* [1973] found a first-order Markov chain to be superior to a Bernoulli model (sequential independence) for

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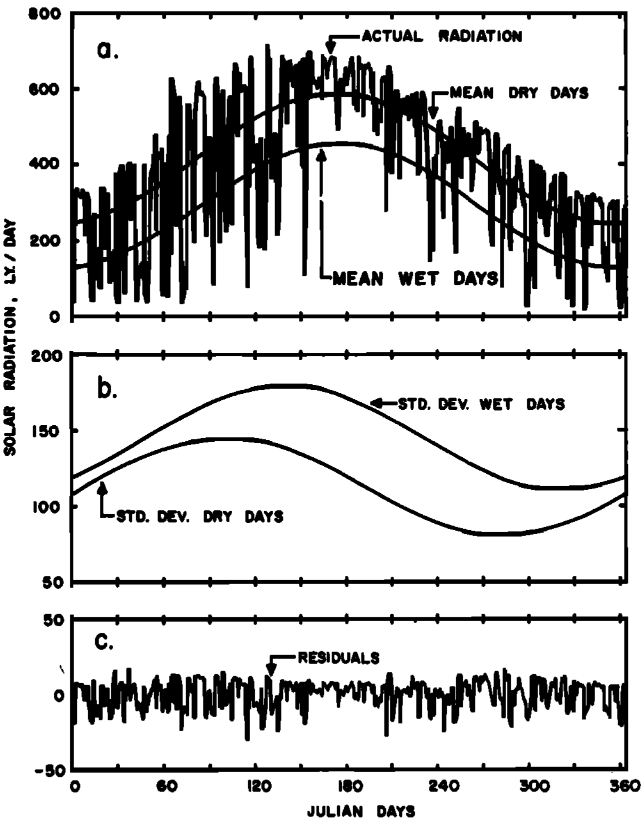

Fig. 1. Technique for reducing a daily solar radiation series to a series of residual elements, conditioned on the wet or dry status of the day.

describing the occurrence of wet or dry days in southeastern Arizona.

For this study a first-order Markov chain with only two states, wet or dry, was used. A day with total rainfall of 0.2 mm or more was considered a wet day. Let $P_i(W/W)$ be the probability of a wet day on day i given a wet day on day $i-1$; let $P_i(W/D)$ be the probability of a wet day on day i given a dry day on day $i-1$. Then

$$P_i(D/W) = 1 - P_i(W/W) \quad (1)$$

$$P_i(D/D) = 1 - P_i(W/D) \quad (2)$$

where $P_i(D/W)$ and $P_i(D/D)$ are the probability of a dry day given a wet day on day $i-1$ and the probability of a dry day given a dry day on day $i-1$, respectively. Therefore the transition probabilities are fully defined given $P_i(W/W)$, $P_i(W/D)$, and the wet or dry state on day $i-1$.

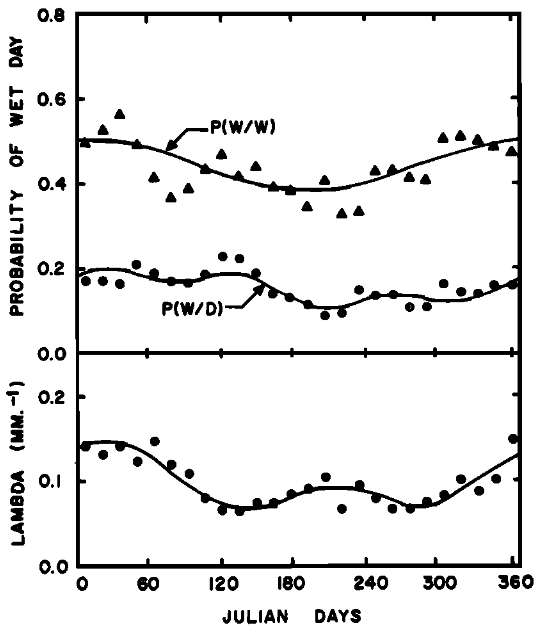

Fig. 2. Markov chain–exponential precipitation model parameters for Temple, Texas.

A number of models have been proposed for the distribution of precipitation amounts given the occurrence of a wet day. If the relative frequencies of daily precipitation amounts are plotted, the general shape resembles an exponential distribution with the smaller amounts occurring more frequently than the larger amounts. The exponential distribution has often been used as a first approximation of the distribution of rainfall amounts [*Todorovic and Woolhiser*, 1974]. The exponential distribution has a probability density function given by

$$f(Y) = \lambda e^{-\lambda Y} \quad (3)$$

where $f(Y)$ is the density function and λ is the distribution parameter. Other distributions that have been proposed include the two-parameter gamma distribution [*Buishand*, 1978] and the three-parameter mixed exponential distribution [*Woolhiser and Pegram*, 1979]. The gamma and mixed exponential distributions, in general, describe the distribution of precipitation amounts better than the exponential distribution because of the greater flexibility obtained with the large number

TABLE 1. Optimized Values of C_j and θ_j for Significant Harmonics of the Markov Chain–Exponential Precipitation Model Parameters

Parameter	C_0	C_1	θ_1	C_2	θ_2	C_3	θ_3
			Temple				
$P_i(W/W)$	0.445	0.058	−0.158	NS		NS	
$P_i(W/D)$	0.157	0.037	−1.060	NS		0.018	−0.836
λ_i, mm^{-1}	0.098	0.028	−0.339	0.021	−0.866	NS	
			Atlanta				
$P_i(W/W)$	0.467	NS		NS		NS	
$P_i(W/D)$	0.232	−0.051	1.435	0.052	−0.912	−0.028	−1.308
λ_i, mm^{-1}	0.094	−0.009	−1.097	0.009	−0.175	NS	
			Spokane				
$P_i(W/W)$	0.475	0.157	−0.326	NS		NS	
$P_i(W/D)$	0.226	0.107	−0.545	0.035	0.684	NS	
λ_i, mm^{-1}	0.282	−0.033	0.151	−0.029	0.873	NS	

NS means not significant at the 1% level by likelihood ratio test.

of parameters. The determination of the most appropriate distribution to use for precipitation amounts is beyond the scope of this study. The exponential distribution, because of its simplicity, was used to illustrate the technique for simulating the four meteorological variables.

The Markov chain transition probabilities $P_i(W/W)$ and $P_i(W/D)$ and the exponential distribution parameter λ_i are seasonal for most locations. To simulate precipitation throughout the year, the seasonal nature of these parameters may be described by using Fourier series or other periodic functions.

Maximum Temperature, Minimum Temperature, and Solar Radiation

Meteorological elements such as temperature and solar radiation are not as difficult to model statistically as precipitation. The problems caused by the high proportion of zero observations in daily rainfall data are absent with these meteorological variables, and the distributions of the variables are much less skewed than rainfall distributions.

Stochastic modeling of temperature, solar radiation, and other related weather variables has received little attention in the literature. *Kohler et al.* [1959] investigated class A pan evaporation records for stations in the United States and concluded that monthly and annual evaporation are approximately normally distributed. *Joseph* [1973] found significant serial correlation in maximum and minimum temperatures and significant cross correlation for daily temperature extremes for adjacent stations in mountainous terrain. *Goh and Tan* [1977] developed a stochastic model for forecasting solar radiation data and demonstrated a strong serial correlation in hourly solar radiation data.

Nicks [1975] developed a model for generating daily values of maximum and minimum temperatures and solar radiation. Each of the three variables was conditioned on the wet or dry state for day i and day $i - 1$. Each variable was generated independently of the other two. The technique preserved the lag 1 serial correlation for each variable.

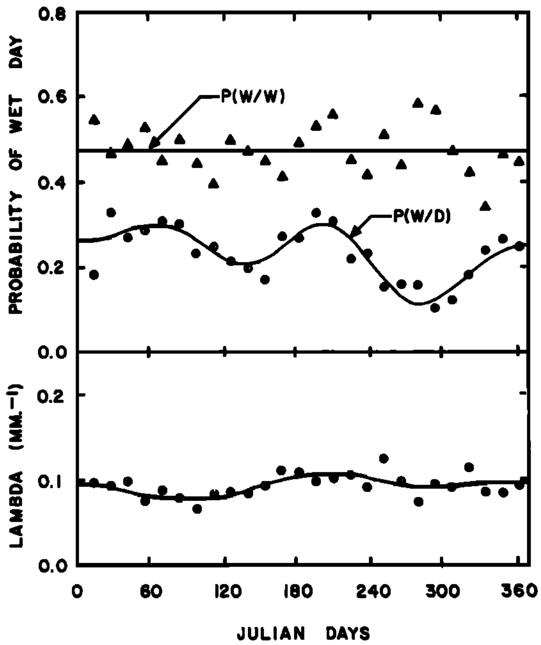

Fig. 3. Markov chain–exponential precipitation model parameters for Atlanta, Georgia.

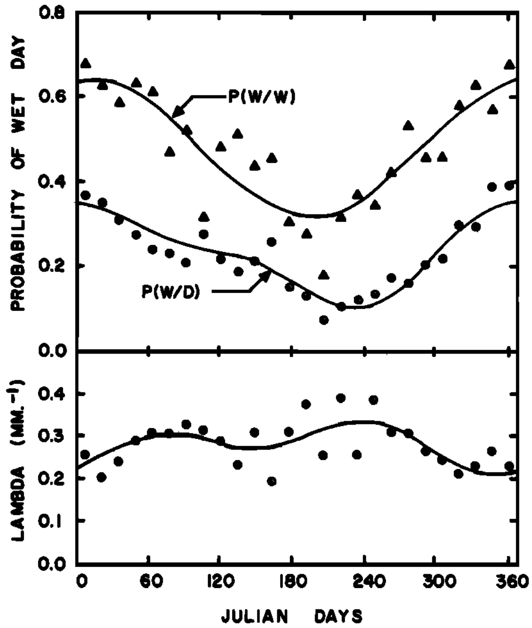

Fig. 4. Markov chain–exponential precipitation model parameters for Spokane, Washington.

The approach used here considers maximum temperature, minimum temperature, and solar radiation to be a continuous multivariate stochastic process with the daily means and standard deviations conditioned on the wet or dry state of the day. The technique presented by *Yevjevich* [1972] was used to analyze the process. The time series of each variable was reduced to a time series of residual elements by removing the periodic means and standard deviations. These elements were analyzed to determine the time dependence (serial correlation) within each series and the cross correlation between each pair of variables.

The technique for reducing the time series for each variable to residual elements may be illustrated by reference to Figure 1. The daily means and standard deviations of solar radiation were determined for wet days and for dry days using 5 years of data. Fourier series were used to smooth the seasonal means and standard deviations. The smoothed seasonal patterns of the means for wet days and dry days are shown superimposed on daily values of actual radiation in Figure 1*a*. Figure 1*b* shows the smoothed seasonal patterns of the standard deviations for wet and dry days. As would be expected, the mean was larger and the standard deviation smaller on dry days than on wet days. The residual elements were calculated by removing the periodic mean and standard deviation by using the equations

$$\chi_{p,i}(j) = \frac{X_{p,i}(j) - \bar{X}_i^0(j)}{\sigma_i^0(j)} \qquad Y_{p,i} = 0 \tag{4a}$$

or

$$\chi_{p,i}(j) = \frac{X_{p,i}(j) - \bar{X}_i^1(j)}{\sigma_i^1(j)} \qquad Y_{p,i} > 0 \tag{4b}$$

where $\bar{X}_i^0(j)$ and $\sigma_i^0(j)$ are the mean and standard deviation for a dry day ($Y_{p,i} = 0$), $\bar{X}_i^1(j)$ and $\sigma_i^1(j)$ are the mean and standard deviation for a wet day ($Y_{p,i} > 0$), and $\chi_{p,i}(j)$ is the residual component for variable j. The residual elements are shown in Figure 1*c*. The same technique was used to reduce daily maximum and minimum temperature to residual elements.

TABLE 2. Values of C_j and θ_j for Daily Maximum Temperature, Minimum Temperature, and Solar Radiation Conditioned on the Precipitation Status of the Day: Temple, Texas

Statistic	Wet/Dry	C_0	C_1	θ_1
	Maximum Temperature, °C			
Mean	dry	25.6	−9.7	−0.584
Mean	wet	22.3	−10.1	−0.581
Standard deviation	dry	4.4	2.5	−0.522
Standard deviation	wet	4.9	2.0	−0.261
	Minimum Temperature, °C			
Mean	wet or dry	13.9	−10.1	−0.608
Standard deviation	wet or dry	3.9	2.1	−0.582
	Solar Radiation, Ly			
Mean	dry	418.0	−171.1	−0.109
Mean	wet	294.5	−163.6	−0.152
Standard deviation	dry	113.9	−32.0	1.141
Standard deviation	wet	145.9	−34.6	0.460

Removing the mean and standard deviation from the maximum temperature, minimum temperature, and solar radiation series resulted in a residual series for each variable that was stationary in the mean and standard deviation with a mean of zero and a standard deviation of unity. The residual series for each variable was, in general, dependent in time, and the three series were mutually interdependent. The serial correlation coefficients and the cross-correlation coefficients of the series were calculated as descriptors of the time dependence and the interdependence, respectively.

Multivariate Generation Model

The model that is proposed for generating residual series of maximum temperature, minimum temperature, and solar radiation is the weakly stationary generating process suggested by *Matalas* [1967]. The equation is

$$\chi_{p,i}(j) = A\chi_{p,i-1}(j) + B\epsilon_{p,i}(j) \tag{5}$$

where $\chi_{p,i}(j)$ and $\chi_{p,i-1}(j)$ are (3×1) matrices for days i and $i - 1$ of year p whose elements are residuals of maximum temperature ($j = 1$), minimum temperature ($j = 2$), and solar radiation ($j = 3$); $\epsilon_{p,i}(j)$ is a (3×1) matrix of independent random components that are normally distributed with a mean of zero and a variance of unity; and A and B are (3×3) matrices whose elements are defined such that the new sequences have the desired serial correlation and cross-correlation coefficients. The use of (5) implies that the residuals of maximum temperature, minimum temperature, and solar radiation are normally distributed and that the serial correlation of each variable may be described by a first-order linear autoregessive model [*Matalas*, 1967]. The accuracy of these implications will be examined in a later section.

The A and B matrices may be determined from the matrix equations

$$A = M_1 M_0^{-1} \tag{6}$$

$$BB^T = M_0 - M_1 M_0^{-1} M_1^T \tag{7}$$

where the superscripts −1 and T denote the inverse and transpose of the matrix, respectively, and M_0 and M_1 are the lag 0 and lag 1 covariance matrices. In this study the $\chi_{p,i}(j)$ series had unity variances; therefore M_0 and M_1 are in effect matrices containing the lag 0 and lag 1 cross-correlation coefficients. The matrices may be written

$$M_0 = \begin{bmatrix} 1 & \rho_0(1,2) & \rho_0(1,3) \\ \rho_0(2,1) & 1 & \rho_0(2,3) \\ \rho_0(3,1) & \rho_0(3,2) & 1 \end{bmatrix} \tag{8}$$

$$M_1 = \begin{bmatrix} \rho_1(1) & \rho_1(1,2) & \rho_1(1,3) \\ \rho_1(2,1) & \rho_1(2) & \rho_1(2,3) \\ \rho_1(3,1) & \rho_1(3,2) & \rho_1(3) \end{bmatrix} \tag{9}$$

where $\rho_0(j, k)$ is the lag 0 cross-correlation coefficient between variables j and k, $\rho_1(j, k)$ is the cross-correlation coefficient between variables j and k with variable k lagged 1 day in relation to variable j, and $\rho_1(j)$ is the lag 1 serial correlation for variable j. Since $\rho_0(j, k) = \rho_0(k, j)$, M_0 is a symmetric matrix. However, $\rho_1(j, k) \neq \rho_1(k, j)$, and each element in M_1 must be defined separately.

TABLE 3. Values of C_j and θ_j for Daily Maximum Temperature, Minimum Temperature, and Solar Radiation Conditioned on the Precipitation Status of the Day: Atlanta, Georgia

Statistic	Wet/Dry	C_0	C_1	θ_1
	Maximum Temperature, °C			
Mean	dry	21.5	−9.8	−0.480
Mean	wet	20.0	−9.6	−0.536
Standard deviation	dry	4.2	1.9	−0.645
Standard deviation	wet	4.6	1.3	−0.664
	Minimum Temperature, °C			
Mean	wet or dry	9.2	−10.7	−0.528
Standard deviation	wet or dry	4.1	1.6	−0.654
	Solar Radiation, Ly			
Mean	dry	439.0	−171.1	0.070
Mean	wet	245.1	−163.4	−0.149
Standard deviation	dry	102.6	−25.4	0.242
Standard deviation	wet	122.5	−41.5	0.114

TABLE 4. Values of C_j and θ_j for Daily Maximum Temperature, Minimum Temperature, and Solar Radiation Conditioned on the Precipitation Status of the Day: Spokane, Washington

Statistic	Wet/Dry	C_0	C_1	θ_1
	Maximum Temperature, °C			
Mean	dry	14.9	−14.6	−0.515
Mean	wet	12.2	−11.0	−0.555
Standard deviation	dry	4.6	−0.1	−1.469
Standard deviation	wet	4.4	0.7	−0.006
	Minimum Temperature, °C			
Mean	wet or dry	2.6	−9.3	−0.590
Standard deviation	wet or dry	4.0	1.1	−0.262
	Solar Radiation, Ly			
Mean	dry	390.2	−296.2	−0.078
Mean	wet	236.4	−178.9	−0.081
Standard deviation	dry	84.3	−32.4	0.217
Standard deviation	wet	100.7	−70.0	−0.094

Use of (5) to generate new sequences of the residuals of maximum and minimum temperatures and solar radiation requires the definition of the three lag 0 cross-correlation coefficients in (8) and the three lag 1 serial correlation coefficients and six lag 1 cross correlation coefficients in (9). The daily values of the three weather variables are found by multiplying the residuals by the standard deviation and adding the mean. The mean and standard deviation are conditioned on the wet or dry status of the day determined by using the Markov chain model.

TESTS OF THE MODEL

Daily weather data for Temple, Texas; Atlanta, Georgia; and Spokane, Washington, were obtained for testing the weather simulation technique. These three widely separated locations were chosen to include different climatic conditions. Daily values of precipitation, maximum temperature, minimum temperature, and solar radiation data were obtained. For each station, 20 years of data were used to estimate the parameters of the precipitation model, and 5 years of data were used to estimate the parameters of the other three variables.

Parameter Evaluation

The three parameters of the precipitation model ($P_i(W/W)$, $P_i(W/D)$, and λ_i) were determined for each station. The year was partitioned into 26 14-day periods, and a maximum likelihood estimate of each parameter was calculated for each period. The seasonal variation of each parameter was then described by using a finite Fourier series given by

$$v_i = C_0 + \sum_{j=1}^{m} C_j \cos\left(\frac{ji}{T} + \theta_j\right) \tag{10}$$

where v_i is the value of the parameter for day i, C_0 is the mean of v_i, $T = 365/2\pi$, and C_j is the amplitude and θ_j the phase angle of the jth harmonic. The significant harmonics for each parameter and the maximum likelihood estimates of C_j and θ_j for each significant harmonic were determined by using the technique described by *Woolhiser and Pegram* [1979]. The values of C_j and θ_j for the significant harmonics are given for each parameter in Table 1. The number of significant harmonics varied from zero to three depending on the parameter and the station location. The estimated parameter values for the 14-day periods are shown in Figures 2–4 for the three locations. The Fourier series representation of the parameters, using the C_j and θ_j values given in Table 1, are also shown on the figures.

The daily means and standard deviations of maximum temperature, minimum temperature, and solar radiation were calculated conditioned on the wet or dry status of each day. The means and standard deviations for both maximum temperature and solar radiation were distinctly different on wet days than on dry days. There were no detectable differences in the means and standard deviations for minimum temperature on wet or dry days. Each mean and standard deviation displayed an annual cycle similar to that shown in Figure 1 for solar radiation. The annual cycle for the means and standard deviations for maximum temperature, minimum temperature, and solar radiation contained only one harmonic. The C_j and θ_j values for each variable are given in Tables 2–4 for the three stations.

TABLE 5. Mean, Standard Deviation, Skewness Coefficient, and Kurtosis Coefficient of the Residuals of Maximum Temperature, Minimum Temperature, and Solar Radiation

Variable	Mean	Standard Deviation	Skewness Coefficient	Kurtosis Coefficient
		Temple		
Maximum temperature	−0.001	1.020	−0.534	3.43
Minimum temperature	0.003	1.000	−0.161	2.90
Solar radiation	−0.006	1.000	−0.761	3.18
		Atlantic		
Maximum temperature	−0.005	1.000	−0.363	2.97
Minimum temperature	0.008	0.949	−0.189	3.15
Solar radiation	0.010	1.010	−0.550	3.46
		Spokane		
Maximum temperature	−0.002	1.030	−0.257	3.79
Minimum temperature	−0.003	1.010	−0.453	3.89
Solar radiation	0.009	0.965	−0.551	3.43

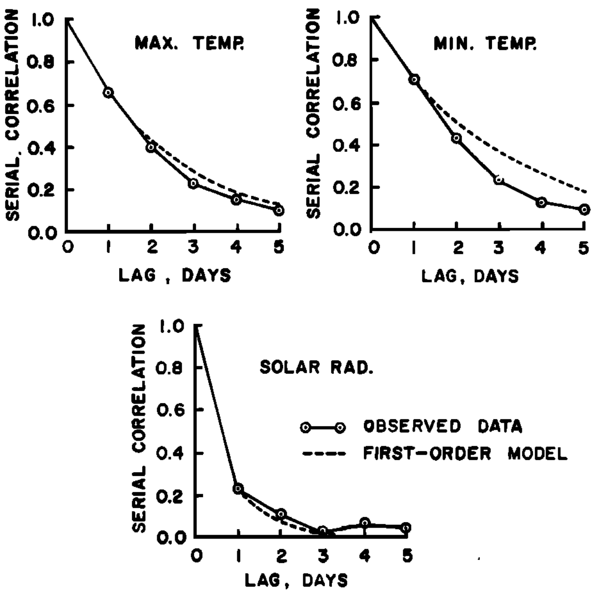

Fig. 5. Serial correlation coefficients of maximum temperature, minimum temperature, and solar radiation for Atlanta, Georgia, compared to a first-order linear model given by $\rho_k = \rho_1{}^k$.

The residuals for each variable, $\chi_{p,i}(j)$, were calculated by using (4). The means and standard deviations used in (4) were obtained from the Fourier series representation. Use of (4) resulted in a new time series of residuals for each variable that by virtue of the construction should be stationary with a mean of zero and a standard deviation of unity.

For (5) to be appropriate for generating new sequences of the residuals the residuals should be approximately normally distributed, and the serial dependence of the residuals should approximate a first-order linear autoregressive model. The serial dependence of a first-order autogressive model is given by $\rho_k = \rho_1{}^k$, where ρ_k is the serial correlation for lag k. These two requirements were examined by analyzing the residual series.

The mean, standard deviation, skewness coefficient, and kurtosis coefficient were calculated for each $\chi_{p,i}(j)$ series and are shown in Table 5. The results show that the use of (4) did successfully reduce the series to residuals with a mean near zero and a standard deviation near unity. The skewness coefficients of the residual series were negative but were near a value of zero that should be obtained from normally distributed samples. Similarly, the kurtosis coefficients were all near a value of 3, which is indicative of a normal distribution.

TABLE 7. Average Rainfall Amount and Number of Wet Days by 28-Day Period: Temple, Texas

	Rainfall Amount, mm		Number of Wet Days	
Period	Observed	Generated	Observed	Generated
1	58.4	50.8	8.0	7.1
2	57.9	54.4	8.4	6.7
3	52.8	65.3	6.5	6.9
4	74.9	101.9	6.6	7.9
5	126.5	100.1	8.3	6.8
6	86.4	68.3	6.2	4.7
7	49.8	35.6	4.6	3.6
8	36.3	47.8	3.4	5.1
9	61.0	67.1	5.0	5.9
10	66.8	85.1	4.3	5.8
11	64.0	59.7	5.4	5.0
12	68.8	54.9	6.7	6.5
13	58.4	54.4	7.0	7.0
Annual	862.3	845.8	80.4	79.0

The serial correlation coefficients were calculated for each residual series for lags up to 5 days. The serial correlation coefficients were compared to a first-order autoregressive model with $\rho_1 = r_1$, where r_1 is the lag 1 serial correlation coefficient from the residual series. The results for the Atlanta data are shown in Figure 5. For all three of the variables the serial correlation coefficient approximates that for a first-order model. Both the distribution of the residuals and the serial dependence structure of the residual elements suggest that (5) is appropriate for generating sequences of maximum temperature, minimum temperature, and solar radiation.

The lag 1 serial correlation coefficients for a given variable were similar for all three stations. The serial correlation coefficients for maximum temperature averaged about 0.67, the serial correlation for minimum temperature averaged about 0.70, and the serial correlation for solar radiation averaged about 0.24.

The interdependence among the variables was determined by calculating the lag cross-correlation coefficients of the $\chi_{p,i}(j)$ series for each station. These cross-correlation coefficients are shown in Table 6. Similar results were obtained for all three stations. Maximum and minimum temperatures on the same day (lag 0) were strongly correlated with a cross-correlation coefficient of about 0.70. Maximum temperature and radiation were positively correlated with a cross-correlation coefficient of about 0.25, indicating that incoming solar radiation has a significant direct influence on maximum temperature. Minimum temperature and solar radiation were found

TABLE 6. Cross-Correlation Coefficients Between the Residuals of Maximum Temperature, Minimum Temperature, and Solar Radiation

	Lag Cross Correlation		
Variables	$r_1(j, i)$	$r_0(i, j)$	$r_1(i, j)$
Temple			
Maximum temperature–minimum temperature	0.499	0.672	0.577
Maximum temperature–solar radiation	0.122	0.320	0.090
Minimum temperature–solar radiation	−0.080	−0.153	−0.060
Atlanta			
Maximum temperature–minimum temperature	0.537	0.687	0.635
Maximum temperature–solar radiation	0.076	0.235	0.040
Minimum temperature–solar radiation	−0.145	−0.248	−0.106
Spokane			
Maximum temperature–minimum temperature	0.559	0.732	0.683
Maximum temperature–solar radiation	0.108	0.192	0.043
Minimum temperature–solar radiation	−0.049	−0.176	−0.058

TABLE 8. Average Rainfall Amount and Number of Wet Days by 28-Day Period: Atlanta, Georgia

Period	Rainfall Amount, mm		Number of Wet Days	
	Observed	Generated	Observed	Generated
1	99.8	93.5	9.5	7.8
2	116.6	106.7	10.1	9.8
3	124.5	128.0	10.3	9.4
4	111.0	99.8	8.3	7.9
5	94.7	89.4	8.1	8.3
6	74.9	88.4	7.8	8.9
7	103.6	80.3	10.7	9.4
8	94.7	98.8	9.8	10.1
9	71.4	81.5	7.5	7.7
10	81.8	66.3	6.8	4.9
11	58.7	71.4	5.5	7.1
12	72.1	93.0	7.1	8.7
13	100.1	87.9	9.0	9.2
Annual	1203.9	1190.2	110.5	109.2

to have a negative cross correlation of about −0.20. The negative correlation reflects the fact that low minimum temperatures occur after nights with clear skies and high long-wave radiation. These conditions tend to be followed by days with high incoming solar radiation, because conditions are clear.

Weather Variable Simulation

The first step in generating synthetic sequences of weather data at a location was to generate the precipitation data by using the Markov chain–exponential model. The residuals of the other three variables were then generated by using (5). The daily values of the variables $X_{p,i}(j)$ were obtained by multiplying the generated $\chi_{p,i}(j)$ by $\sigma_i(j)$ and adding $\bar{X}_i(j)$, conditioned on the wet or dry status of each day.

The parameters of the precipitation model were obtained from the Fourier series description of $P_i(W/W)$, $P_i(W/D)$, and λ_i. The C_j and θ_j values for significant harmonics of these three parameters are given in Table 1 for the three test locations. The serial correlation and cross-correlation coefficients given in Tables 5 and 6 completely define matrices M_0 and M_1, which are required for the multivariate generation of the residuals of maximum temperature, minimum temperature, and solar radiation. The seasonal means and standard deviations of the three variables for wet and dry days were obtained from the Fourier series description of these parameters by using the C_j and θ_j values given in Tables 2–4.

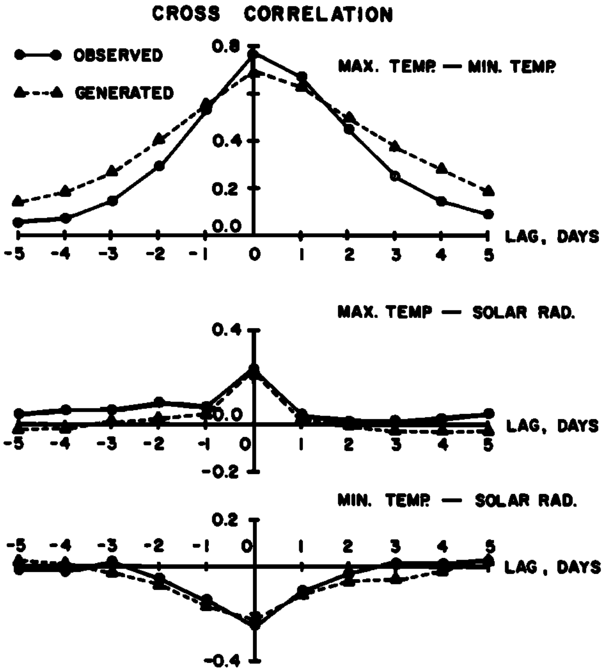

Fig. 6. Cross-correlation coefficients obtained from observed and generated weather variables for Atlanta, Georgia.

Twenty years of daily precipitation, maximum temperature, minimum temperature, and solar radiation data were generated for the three test locations. The generated data were compared with the observed data.

The mean precipitation amounts and the mean number of wet days for each 28-day period of the year and for the annual totals are shown in Tables 7–9 for the generated and observed data at the three test locations. The mean precipitation amounts from the generated data did not differ significantly from the values obtained from the observed data. The average number of wet days generated for each period was also a close approximation of that obtained from the observed data. The proper description of the occurrence of wet days by season is important in this study, because the generation of maximum temperature and solar radiation are conditioned on the occurrence of wet or dry days. Most of the differences between the observed and the generated data that are shown in Tables 7–9 were due to smoothing the precipitation model parameters by Fourier series rather than the Markov chain–exponential precipitation model.

The maximum and minimum temperature and solar radiation data generated with the model are compared with the observed data in Tables 10–12. The means for all three variables for each period of the year compared favorably with the observed means, indicating that the model gave a good description of the seasonal variation of the variables. The differences here were also thought to be due to Fourier series smoothing.

In order to evaluate the capability of the model for reproducing the distribution of annual extreme temperatures the mean annual maximum and minimum temperatures were calculated for both the generated and the observed data at each test location. The mean number of days per year with a maximum temperature greater than 35°C and the number of days with a minimum temperature less than 0°C were also determined. The results are given in Table 13. The mean annual

TABLE 9. Average Rainfall Amount and Number of Wet Days by 28-Day Period: Spokane, Washington

Period	Rainfall Amount, mm		Number of Wet Days	
	Observed	Generated	Observed	Generated
1	62.2	54.6	14.2	12.8
2	44.5	36.6	11.8	11.5
3	31.2	32.8	9.5	10.3
4	25.4	24.9	8.1	8.3
5	30.2	27.7	7.9	7.6
6	35.1	23.9	8.3	6.0*
7	13.2	12.4	4.5	4.3
8	9.1	11.2	2.9	3.4
9	14.5	14.0	4.5	4.2
10	21.1	26.9	6.5	6.6
11	30.2	33.8	7.7	8.0
12	53.1	46.2	11.8	11.6
13	57.4	65.2	14.1	13.0
Annual	427.0	412.5	111.8	107.6

* The observed mean and the generated mean are significantly different at the 1% level.

TABLE 10. Mean Daily Maximum Temperature, Minimum Temperature, and Solar Radiation by 28-Day Period and for the Year: Temple, Texas

	Maximum Temperature, °C		Minimum Temperature, °C		Solar Radiation, Ly	
Period	Observed	Generated	Observed	Generated	Observed	Generated
1	14.1	14.8	3.3	5.2	222	215
2	15.9	15.2	4.9	4.3	281	239
3	19.8	17.9	8.2	6.9	376	335
4	23.7	22.3	12.2	12.1	441	379
5	26.3	25.3	16.1	13.8	461	521
6	30.6	30.7	19.9	19.9	543	519
7	34.2	34.6	22.9	22.9	583	589
8	34.2	34.8	23.0	23.7	528	560
9	34.1	34.0	22.9	23.2	484	483
10	30.1	28.9	20.0	18.6	380	403
11	23.9	25.3	13.6	15.1	308	325
12	19.6	20.9	8.6	9.6	252	265
13	16.6	17.0	5.9	6.1	205	215
Year	24.9	24.7	14.0	14.0	390	388

TABLE 11. Mean Daily Maximum Temperature, Minimum Temperature, and Solar Radiation by 28-Day Period and for the Year: Atlanta, Georgia

	Maximum Temperature, °C		Minimum Temperature, °C		Solar Radiation, Ly	
Period	Observed	Generated	Observed	Generated	Observed	Generated
1	9.9	11.6	0.1	−0.4	202	218
2	10.8	8.9	−0.1	−2.7	270	235
3	16.0	14.2	3.4	2.0	394	320
4	22.4	20.4	9.9	7.7	451	426
5	24.9	23.6	13.0	11.6	508	482
6	26.8	27.1	16.4	15.8	518	528
7	29.3	30.7	19.3	19.6	510	530
8	29.9	30.6	19.8	19.7	487	507
9	28.9	28.9	18.7	18.3	451	451
10	25.7	24.3	14.6	13.1	391	406
11	20.7	20.6	8.6	8.9	310	319
12	15.9	15.8	4.5	4.1	234	234
13	12.3	11.6	1.2	0.3	202	195
Year	21.0	20.6	10.0	9.1	379	373

TABLE 12. Mean Daily Maximum Temperature, Minimum Temperature, and Solar Radiation by 28-Day Period and for the Year: Spokane, Washington

	Maximum Temperature, °C		Minimum Temperature, °C		Solar Radiation, Ly	
Period	Observed	Generated	Observed	Generated	Observed	Generated
1	0.3	0.6	−5.6	−6.2	99	90
2	3.8	−0.2	−4.0	−8.0	187	148
3	7.5	7.0	−2.5	−2.7	316	256
4	12.4	12.2	0.6	1.2	400	378
5	17.9	15.2	3.7	2.9	540	502
6	22.3	22.6	8.4	7.7	600	588
7	26.5	28.2	11.1	11.5	630	637
8	30.1	27.1	13.1	11.1	640	596
9	26.7	26.9	10.9	10.7	487	529
10	20.9	20.3	7.0	7.1	340	358
11	13.3	15.4	1.4	4.2	218	240
12	5.2	7.1	−1.4	−2.9	102	145
13	1.1	3.2	−4.4	−5.3	75	84
Year	14.5	14.3	2.9	2.4	356	350

TABLE 13. Mean Annual Maximum Temperature, Minimum Temperature, Number of Days per Year with Maximum Temperature Greater Than 35°C, and Number of Days per Year With Minimum Temperature Less Than 0°C for the Observed and Generated Data at the Three Test Locations

Variable	Annual Mean	
	Observed	Generated
Temple		
Maximum temperature, °C	39.0	40.1
Minimum temperature, °C	−8.9	−7.8
Days ≥ 35°C	47.1	42.9
Days ≤ 0°C	29.1	20.1*
Atlanta		
Maximum temperature °C	35.9	36.5
Minimum temperature °C	−12.3	−13.6
Days ≥ 35°C	7.9	3.9
Days ≤ °C	51.4	68.7*
Spokane		
Maximum temperature °C	37.1	38.9*
Minimum temperature °C	−21.3	−17.9
Days ≥ 35°C	6.0	7.6
Days ≤ 0°C	139.4	142.7

* The observed mean and the generated mean are significantly different at the 1% level.

maximum and minimum temperatures from the generated data were all within a few degrees of the value from the observed data. The generated data also contained about the same number of days with extreme temperatures as the observed data.

The cross-correlation coefficients of the observed and generated data are illustrated in Figure 6 for Atlanta. Only the cross-correlation coefficients for lags − 1, 0, and +1 were used in the generation technique. As is indicated in Figure 6, the cross-correlation coefficients for the generated data for all three pairs of variables compared closely with those from the observed data. These results show that the cross correlation structure that is inherent in the generation model closely matches the cross-correlation structure of the observed data.

SUMMARY AND CONCLUSIONS

A technique for generating daily values of precipitation, maximum temperature, minimum temperature, and solar radiation has been proposed. The basic approach was to generate precipitation independently of the other three variables and then generate the other variables conditioned on the wet or dry status of the day. A Markov chain–exponential model was used to describe precipitation, and a multivariate model was used to describe maximum temperature, minimum temperature, and solar radiation.

Tests of the model showed that the model was capable of representing many of the characteristics that existed in the observed data. The time dependence of each variable and the interdependence among the variables were closely described by the dependence structure that is inherent in (5). Data generated with the model compared closely with the observed data in rainfall amounts, occurrence of wet days, mean daily temperatures and solar radiation, and annual extreme temperatures.

Application of the model requires that about 15 parameters be defined for each day of the year. However, some of these parameters vary seasonally and may be described over the year by Fourier series with only a few harmonics. For the three locations used to test the model a set of about 40 coefficients was required to define the model completely. The 12 coefficients used to describe the correlation structure of temperature and solar radiation were approximately the same for all locations. If these coefficients are in fact constant for all locations, the number of coefficients that must be evaluated could be reduced to about 28. It may also be possible, as was suggested by *Woolhiser and Pegram* [1979], to map some of the coefficients to present a picture of the regional parameter variation. If this can be done, it may be possible to simulate weather sequences knowing only the location for which data are needed.

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REFERENCES

Bailey, N. T. J., *The Elements of Stochastic Processes*, p. 39, John Wiley, New York, 1964.

Buishand, T. A., Some remarks on the use of daily rainfall models, *J. Hydrol., 36*, 295–308, 1978.

Caskey, J. E., Jr., A Markov chain model for the probability of precipitation occurrence in intervals of various lengths, *Mon. Weather Rev., 91*, 298–301, 1963.

Chin, E. H., Modeling daily precipitation occurrence process with Markov chain, *Water Resour. Res., 13*(6), 949–956, 1977.

Gabriel, R., and J. Neuman, A Markov chain model for daily rainfall occurrence at Tel Aviv, Israel, *Q. J. R. Meteorol. Soc., 88*, 90–95, 1962.

Goh, T. N., and K. J. Tan, Stochastic modeling and forecasting of solar radiation data, *Sol. Energy, 19*, 755–757, 1977.

Haan, C. T., D. M. Allen, and J. O. Street, A Markov chain model of daily rainfall, *Water Resour. Res., 12*(3), 443–449, 1976.

Hopkins, J. W., and P. Robillard, Some statistics of daily rainfall occurrence for the Canadian prairie provinces, *J. Appl. Meteorol., 3*, 600–602, 1964.

Joseph, E. S., Spectral analysis of daily maximum and minimum temperature series on the east slope of the Colorado Front Range, *Mon. Weather Rev., 101*(6), 505–509, 1973.

Knisel, W. G. (Ed.), CREAMS: A field scale model for chemicals, runoff, and erosion from agricultural management systems, *Conserv. Res. Rep. 26*, 640 pp., U.S. Dep. of Agric., Washington, D. C., 1980.

Kohler, M. A., T. J. Nordenson, and D. R. Baker, Evaporation maps for the United States, *Tech. Pap. 37*, U.S. Weather Bur., Washington, D. C., 1959.

Matalas, N. C., Mathematical assessment of synthetic hydrology, *Water Resour. Res., 3*(4), 937–945, 1967.

Nicks, A. D., Stochastic generation of hydrologic model inputs, Ph.D. dissertation, 142 pp., Univ. of Okla., Norman, 1975.

Smith, R. E., and H. A. Schreiber, Point processes of seasonal thunderstorm rainfall, 1, Distribution of rainfall events, *Water Resour. Res., 9*(4), 871–884, 1973.

Todorovic, P., and D. A. Woolhiser, Stochastic model of daily rainfall, Proceedings of the Symposium on Statistical Hydrology, *Misc. Publ. 1275*, pp. 223–246, U.S. Dep. of Agric., Washington, D. C., 1974.

Weiss, L. L., Sequence of wet and dry days described by a Markov chain probability model, *Mon. Weather Rev., 92*, 169–176, 1964.

Woolhiser, D. A., and G. G. S. Pegram, Maximum likelihood estimation of Fourier coefficients to describe seasonal variations of parameters in stochastic daily precipitation models, *J. Appl. Meteorol., 8*(1), 34–42, 1979.

Yevjevich, V., Structural analysis of hydrologic time series, *Hydrol. Pap. 56*, 59 pp., Colo. State Univ., Fort Collins, 1972.

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