Table 1
Critical Indices of Ions, Electrons and Photons in Space Plasmas

Stationary state(s)	Space Plasma	Reference	γ/κ
	Pulsating X-ray source Cygnus X-1	Schreier et al. (1971)	γ, P
q-frozen state	X-rays spectra of accretion disks	Syunyaev et al. (1994); Chakrabarti & Titarchuk (1995)	γ, P
$\kappa_{\mathrm{q-F}} \cong 1.5$	Io plasma torus	Steffl et al. (2004)	κ
	Solar wind, inner heliosphere	Fisk & Gloeckler (2006)	γ
	Soft X-rays spectra of Seyfert galaxies	Arnaud et al. (1985)	γ
Fundamental state	Hard X-rays flares	Crosby et al. (1993)	γ
$\kappa_{\mathrm{Fund}} \cong 1.63$	Active region transient brightenings	Shimizu (1995)	γ
	Blazar gamma rays	Kifune et al. (1995)	γ
	Soft γ-rays repeater bursts	Göğüş et al. (1999)	γ
	Solar wind, inner heliosheath	Decker et al. (2005)	κ
	Cosmic-ray iron nucleus	Sato et al. (1985)	γ
Escape state	X-rays from accretion disks	Ilovaisky (1986); Miyamoto et al. (1991)	γ
$\kappa_{\mathrm{Esc}} \cong 2.45$	Blazar gamma rays	Krennrich et al. (1999); Schroedter et al. (2005); Aharonian et al. (2006)	γ
	Stellar flares	Kashyap et al. (2002)	γ
	Io plasma torus	Steffl et al. (2004)	κ
	Heliospheric tail (ENAs)	McComas et al. (2009)	γ
Far-equilibrium	Blazar γ-rays	Montigny et al. (1995); Venters & Pavlidou (2007)	γ
region,	Solar flares	Aschwanden et al. (2000); Charbonneau et al. (2001); Norman et al. (2001)	γ
$\kappa_{\mathrm{Esc}} \geqslant \kappa > \frac{3}{2}$	Interplanetary shocks	Desai et al. (2004)	γ
	CIR's	Mason et al. (2008)	γ
	Quiet times of solar wind, inner heliosphere	Dayeh et al. (2009)	γ
	X-rays from accretion disks	Schreier et al. (1971)	γ
Near-equilibrium	Uranian magnetosphere	Mauk et al. (1987)	γ, P
region,	Solar wind, inner heliosphere	Collier et al. (1996)	κ
$\infty \geqslant \kappa > \kappa_{\mathrm{Esc}}$	Possible nanoflares	Benz (2004); Pauluhn & Solanki (2007); Bazarghan et al. (2008)	γ
	Saturnian magnetosphere	Dialynas et al. (2009)	κ

Notes.
γ: A power law was fitted and the spectral index refers to the H-E region.
κ: A kappa distribution was fitted (either first or second kind (Livadiotis & McComas 2009)).
P: Possible coexistence of more than one stationary state. The steeper law reveals in law energies while the flatter law dominates in high energies (see the text).

Interestingly, these values have also been frequently observed in space plasmas: (1) $\kappa_{\mathrm{EQ}} = \infty$ (e.g., Hammond et al. 1996), corresponding to the specific stationary state at equilibrium; (2) $\kappa_{\mathrm{q-F}} \cong 1.5$, which seems to act as an attractor of states, corresponding to the q-frozen state; (3) $\kappa_{\mathrm{Fund}} \cong 1.63$, detected in inner heliosheath, beyond the termination shock; and (4) $\kappa_{\mathrm{Esc}} \cong 2.45$ that has the unique role of separating the stationary states near equilibrium, with $\kappa > \kappa_{\mathrm{Esc}}$, from those near the q-frozen state, with $\kappa_{\mathrm{Esc}} \geqslant \kappa$. (For the references on these critical κ-indices of stationary states out of equilibrium, see Table 1 and Section 4.3).

The paper is organized as follows. In Section 2, we discuss the permissible values of the kappa indices. These are extracted from the convergence of the integral that gives the second statistical moment and depends on the dimensionality of the system. In Section 3, we examine the furthest possible stationary state from equilibrium, the q-frozen state. In Section 4, we develop the measure of the "thermodynamic distance" of stationary states from equilibrium, called q-metastability, which fulfills specific conditions and is expressed in terms of the Tsallis entropic index q. We demonstrate its role in the spectrum-like arrangement of stationary states. We also show how the q-metastability measure characterizes the identity of each stationary state. With the help of this q-metastability measure, we characterize the various stationary states that have been detected by the observations of the solar wind, planetary, or other space plasmas. In Section 5, we construct the Tsallis entropy and its expression in terms of the q-index and present how the spontaneous entropic procedures affect the values of q-index and the transition of stationary states. In Section 6, we deal with the dynamics of stationary state transitions and show the detailed paths by which the transition of stationary states evolves toward equilibrium. Finally, Section 7 summarizes the primary results of this study and provides some conclusions.

2. THE MARGINAL STATIONARY STATES

The permissible values of the kappa indices are determined from the requirements that the integrals of normalization, mean value, and second statistical moment converge. The latter implies stronger conditions; hence, the following integral must have a finite value:

$$\int_0^\infty w^2 \cdot P(w; \theta_{\mathrm{eff}}; \kappa) g_{\mathrm{V}}(w) dw < +\infty, \tag{1}$$

where the (spherically symmetric) kappa distribution

$$P(w; \theta_{\mathrm{eff}}; \kappa) \sim \left[1 + \frac{1}{\kappa - \frac{f}{2}} \cdot \left(\frac{w}{\theta_{\mathrm{eff}}}\right)^2\right]^{-\kappa-1}, \tag{2}$$

and the density of velocity states,

$$g_{\mathrm{V}}(w) \sim w^{f-1}, \tag{3}$$

stand for an f-dimensional space of velocities. We defined $\vec{w} \equiv \vec{u} - \vec{u}_b$, that is the particle velocity measured in the reference frame of the plasma's bulk flow; $\vec{u}$ and $\vec{u}_b$ stand for the particle and bulk flow velocities, measured in the observing spacecraft's reference frame. The speed-scale parameter $\theta_{\rm eff}$ is connected with the physical temperature, namely, $\theta_{\rm eff} \equiv \sqrt{2k_B T_q/\mu}$, where μ is the particle mass. Its role is simply to express the temperature of the system (T_q) in velocity dimensions.

In the classical case of the stationary state at equilibrium, i.e., $\kappa = \infty$ (or $q = 1$), the probability distribution is given by a Maxwellian, which decays exponentially, and thus, the relevant integrals converge for any power-like expression of the density of velocity states, $g_V(u)$. However, in the case of the power-law-like decay of the kappa distribution (for $\kappa < +\infty$), the convergence is not trivial, as it depends on the value of the κ-index and the dimensionality *f*. In particular, the integrals converge as soon as the relevant integrants in the high-energy limit of $w \to \infty$ attain at least a power-law decay of $1/w^r$, with $r > 1$ (Ferri et al. 2005). At this limit, the kappa distribution (2) has the asymptotic behavior of $P(w; \theta_{\rm eff}; \kappa) \sim w^{-2(\kappa+1)}$, so that

$$w^2 \cdot P(w; \theta_{\rm eff}; \kappa) g_V(w) \sim w^{-2\kappa+f-1} \Rightarrow 1 < r = 2\kappa - f + 1. \tag{4}$$

Hence, in order for the integral of the second statistical moment to converge, we obtain the condition

$$\kappa > \frac{f}{2}, \quad \text{or } q < \frac{f+2}{f}, \tag{5}$$

(where we utilized $q = 1 + 1/\kappa$). The requirement of (5) is general for any *f*-dimensional system. For the three-dimensional case that characterizes space plasmas amongst many other systems, we obtain

$$\kappa > \frac{3}{2}, \quad \text{or } q < \frac{5}{3}, \tag{6}$$

and thus, all stationary states that can be detectable within the framework of Tsallis statistical mechanics must have values of the κ-index within the interval $\kappa \in (\frac{3}{2}, \infty]$, or, values of the q-index within the interval $q \in [1, \frac{5}{3})$. The first boundary value, i.e., $\kappa = \infty$ (or $q = 1$), defines equilibrium, while the second boundary value, i.e., $\kappa \to \frac{3}{2}$ (or $q \to \frac{5}{3}$), gives the furthest stationary state from equilibrium that can be approached. All the attainable stationary states lie between these extreme states.

3. FURTHEST FROM EQUILIBRIUM: THE q-FROZEN STATIONARY STATE

In this section, we focus on the concept of the stationary state furthest from equilibrium, the q-frozen state, in three-dimensional systems; however, generalization to other dimensions, *f*, is straightforward, following a parallel development. We have seen that for a three-dimensional system, the permissible (positive) values of κ-index span the interval $\kappa \in (\frac{3}{2}, \infty]$, or equivalently, $q \in [1, \frac{5}{3})$. Let us now focus on the two extreme stationary states. The marginal value of $\kappa = \infty$ (or $q = 1$) is the best-known stationary state at equilibrium that has been already widely studied within the framework of BG statistical mechanics, and its Maxwellian distribution of velocities is observed across a wide variety of physics, including some space plasmas (e.g., Hammond et al. 1996).

In contrast to equilibrium, which is trivially achievable, the other extreme stationary state of $\kappa \to \frac{3}{2}$ (or $q \to \frac{5}{3}$) is not attainable, even though it can be approached arbitrarily closely. In Figure 2(a), we depict the kappa distribution $P(w; \theta_{\rm eff}; \kappa)$ in terms of the velocity $w \equiv |\vec{u} - \vec{u}_b|$ for various values of

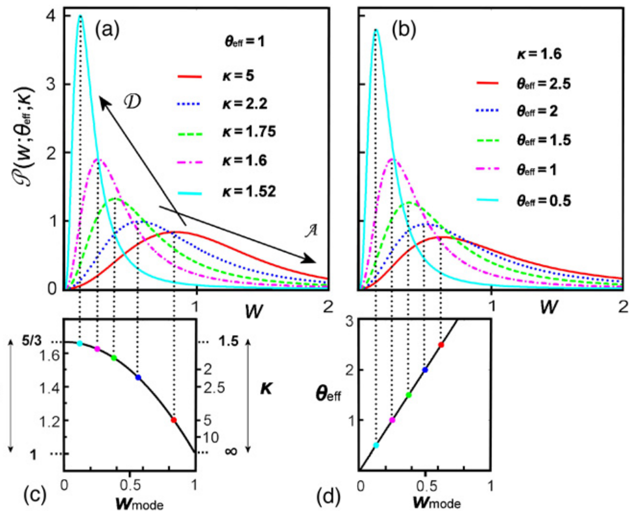

Figure 2. Kappa distribution is depicted for (a) constant temperature $\theta_{\rm eff} = 1$ and various values of the κ-index (isothermal procedure: fixed temperature T_q or $\theta_{\rm eff}$); and for (b) a constant index $\kappa = 1.6$ and various values of temperature $\theta_{\rm eff}$ (iso-metastability procedure: fixed value of the κ-index). In the former case, by decreasing κ, the distribution is shifted to smaller velocities (or energies) toward the q-frozen state, causing a phenomenological deceleration (along the indicated D array). By increasing κ, the distribution is shifted to larger velocities toward equilibrium (along the indicated A array). In the latter case, the distribution is shifted to smaller (or larger) velocities due to the more familiar procedure of decreasing (or increasing) the temperature and staying at the same stationary state. The shifting of distributions is also demonstrated by depicting the most possible velocity $w_{\rm mode}(\theta_{\rm eff}; \kappa)$: (c) For constant temperature, $\theta_{\rm eff} = 1$, and various values of κ-index; and (d) for constant index $\kappa = 1.6$ and various values of $\theta_{\rm eff}$.

the κ-index (and $\theta_{\rm eff} = 1$), showing the statistical behavior of the particles, as the extreme stationary state of $\kappa \to \frac{3}{2}$ is approached. The distribution is normalized by the density of velocity states $g_V(w) = 4\pi w^2$, namely, $\int_0^\infty P(w; \theta_{\rm eff}; \kappa) dw = 1$, with the whole distribution $P(w; \theta_{\rm eff}; \kappa) \equiv P(w; \theta_{\rm eff}; \kappa) g_V(w)$ to be given by

$$P(w; \theta_{\rm eff}; \kappa) = \frac{4}{\sqrt{\pi}} \cdot \frac{\left(\kappa - \frac{3}{2}\right)^{-\frac{3}{2}} \Gamma(\kappa+1)}{\Gamma\left(\kappa - \frac{1}{2}\right)} \cdot \theta_{\rm eff}^{-3} \cdot \left[1 + \frac{1}{\kappa - \frac{3}{2}} \cdot \left(\frac{w}{\theta_{\rm eff}}\right)^2\right]^{-\kappa-1} \cdot w^2. \tag{7}$$

The mode of the distribution can be easily found to be given by the most possible velocity

$$w_{\rm mode}(\theta_{\rm eff}; \kappa) = \sqrt{\frac{\kappa - \frac{3}{2}}{\kappa}} \cdot \theta_{\rm eff}, \tag{8}$$

attaining the maximum of the probability density, i.e.,

$$P_{\rm mode}(\theta_{\rm eff}; \kappa) = \frac{4}{\sqrt{\pi}} \cdot \frac{1}{\sqrt{\kappa - \frac{3}{2}} \cdot \theta_{\rm eff}} \cdot \frac{\Gamma(\kappa)}{\Gamma(\kappa - \frac{1}{2})} \cdot \left(\frac{\kappa}{\kappa+1}\right)^{\kappa+1}. \tag{9}$$

Note that $w_{\rm mode}$ is proportional to $\theta_{\rm eff}$, but depends also on the values of the κ-index. For any value of κ, it is always restricted to the interval $0 < w_{\rm mode} \leqslant \theta_{\rm eff}$, with the equality applying only at equilibrium ($\kappa \to \infty$). For smaller values of κ, the maximum $w_{\rm mode}$ is also smaller and shifted closer to zero. The suprathermal tails appear sufficiently far from this maximum, $w > w_{\rm mode}$, and for lower values of $w_{\rm mode}$ the tail is expected to become