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ORIGINAL PAPER

Effect of general loss-cone distribution function on kinetic Alfvén wave in multi-ions plasma by kinetic approach

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Abstract: Kinetic Alfven waves with general loss-cone distribution function are investigated in multi-ions (H^+, He^+ and O^+) plasma. Dispersion relation and damping rate for the wave are derived using Vlasov equation. Variations in frequency and damping rate versus $k_\perp \rho_i$ (where $k_\perp$ is wave vector perpendicular to ambient magnetic field, ρ_i is ion Larmor radius and i denotes multi-ions) are investigated. Parameters relevant to plasma sheet boundary layer are used for graphical analysis. It is observed that wave frequency fluctuates with loss-cone distribution indices of ions. In comparison with Maxwellian plasma ($J = 0$), the wave frequency is enhanced for He^+ and O^+ and reduced for H^+ with the increase in J indices and the wave existence limit shift towards lower $k_\perp \rho_i$ for lighter ions. This may be due to difference in penetration of multi-ions through the cone. H^+ and He^+ ions show damping at lower $k_\perp \rho_i$ only, whereas O^+ ions exhibit damping over wide range of $k_\perp \rho_{O^+}$. The damping is reduced with the increase in loss-cone indices for all the ions which signify propagation of wave over long distances towards auroral ionosphere. The applications of these results are in understanding the effect of gyrating multi-ions in transfer of energy and in describing the generation of aurora.

Keywords: Kinetic Alfven waves; Kinetic approach; PSBL region; Loss-cone distribution function

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1. Introduction

Kinetic Alfven waves (KAW) are the low-frequency electromagnetic waves which propagate obliquely to the ambient magnetic field $\mathbf{B_0}$. These waves have a large wave number $k_\perp$ transverse to $\mathbf{B_0}$ [1]. KAWs play an important role in transport of energy, particle acceleration and heating, auroral currents and ultra-low-frequency (ULF) emissions in Earth's magnetosphere [2–4]. Alfven waves could couple to kinetic Alfven waves through interaction with the density gradient at the plasma sheet–tail lobe boundary [2]. Generation of these waves through velocity shear [3, 4] and due to gradient effects in inhomogeneous plasma [5] during substorms in plasma sheet boundary layer (PSBL) region have also been discussed in past. Recently, it has been found that the KAW can be produced by interaction of Alfven waves with inhomogeneous background plasma satisfying the condition $k_\perp \sim d_p - 1 > > k_\parallel$ where wavelengths of these waves are comparable to the proton inertial length d_p and wave vectors nearly perpendicular to the mean magnetic field [6]. Energy exchange between fluctuated electromagnetic field and charged particles in the presence of an undamped KAW has also been suggested based on MMS (Magnetospheric Multiscale) mission [7]. Observations from various satellites like Polar, Cluster and Geotail have shown existence of large amplitude KAW in the PSBL during substorm [8–10]. The existence of these waves has been shown in PSBL at an altitude of 4–6R_E having frequency varying from 0.17 to 4 Hz [8].

The PSBL region is located between the auroral acceleration region and the distant magnetotail. This region exists as a layer at interface of lobe and plasma sheet where ion data of energy range from 20eV to 40 keV [11]. This region is of great importance as strong energy transfer occurs between the distant tail and the auroral ionosphere during auroral substorm [4, 8, 12]. The plasma sheet boundary layer region also controls the transport between plasma trapped in high-β plasma sheet in the quasi-neutral magnetic field reversal layer and the low-β lobe plasma. This region is dynamically active region with high-

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streaming velocities [13]. Many studies included multi-ions effect in PSBL region to determine variation in wave properties associated with these ions in homogeneous and inhomogeneous plasma.

Observational study of H^+, He^+, He^{++} and O^+ beams was presented for PSBL region [12]. Distribution of H^+, He^+ and O^+ ions in the near-Earth plasma sheet and calculation of various properties of these ions are already investigated [14]. The existence of energetic and accelerated O^+ ions inside PSBL was predicted through simulation [15]. Cusp outflow also provides source of O^+ ions [16], and these heavy O^+ ions are accelerated towards plasma sheet during the substorm [17]. A review based on evidences from Polar, Cluster, Geotail, ISEE-1, Goes1 and 2, etc., satellite data has predicted the existence of various ions mainly H^+, He^+, He^{++}, O^+ ions at different altitudes in PSBL and demonstrated that heavy O^+ ions influence plasma sheet bulk properties [18]. Observational evidences from cluster satellite have also shown the presence of multi-ions in PSBL [19–21]. Cluster observation-based study was presented on H^+ and O^+ ions outflow in PSBL region and demonstrated that distribution functions of H^+ ion outflows are conical with varying cone angles which exhibits both parallel and perpendicular acceleration [22].

The non-Maxwellian distribution was given by Dory–Guest–Harris called loss-cone distribution [23]. Functional form of the loss-cone distribution shows variation of angular width of loss-cone or sharpness of loss-cone edge as practically geometrical loss-cone region which cannot be completely empty due to particle scattering and collisional processes. A loss-cone region has a distribution having a deficiency of particles. This region arises due to magnetic trapping of particles as is observed in planetary magnetospheres, solar atmosphere and laboratory plasma [24]. PSBL region observations have also shown that distribution of ions is found to be closer to crescent type rather than bi-Maxwellian [10]. A lot of studies using loss-cone distribution function have been done for waves like EMIC [25, 26] and KAW [27–30] in various Earth's magnetospheric regions. It has been reported that plasma particles imbedded with curved and converging field lines (magnetic trapping) have very high velocity anisotropies [8, 9, 27]. Loss-cone distribution index can alter the nature of propagating kinetic Alfven wave [30]. Simulation results have shown that velocity distribution function of protons departs from Maxwellian distribution in the presence of kinetic Alfven wave [6]. Recently, observational and theoretical studies have shown the presence of loss-cone or pitch-angle scattering in inner plasma sheet region based on THEMIS mission [31] and the inner magnetosphere [32]. So, in the present paper, KAWs propagating from PSBL region towards ionosphere through multi-ions plasma are studied using loss-cone distribution function. In past, the studies of KAWs are done using loss-cone distribution considering the presence of protons only. The presence of multi-ions and their effects on various electromagnetic waves are mainly known based on observational data. This motivated us to investigate KAW using kinetic approach in multi-ions plasma with loss-cone effect considering gyroradius of each ion.

We have derived the dispersion relation and damping rate of KAW and have evaluated wave frequency and damping rate numerically over wide range of $k_\perp \rho_i$. The results and discussion are presented by incorporating loss-cone index variation for PSBL region. The organisation of paper is as follows: Sect. 1 deals with introduction, basic mathematical model is discussed in Sect. 2, and dispersion relation and damping rate expressions are derived in Sects. 3 and 4, respectively. The results are discussed in Sect. 5 followed by the conclusion in Sect. 6.

2. Basic model

The kinetic approach is used for the study of kinetic Alfven waves in plasma sheet boundary layer region. The plasma is assumed to be homogeneous with externally applied magnetic field $\mathbf{B}_0 = B_0\hat{e}_z$. So basic collisionless Vlasov–Maxwell's equation [33, 34] is considered,

$$\frac{\partial f_i(x,v,t)}{\partial t} + \mathbf{v}\cdot\nabla f_i(x,v,t) + \left[\frac{q_i}{m_i}\left(\mathbf{E}(x,t) + \frac{\mathbf{v}}{c}\times\mathbf{B}(x,t)\right)\right] \cdot \frac{\partial f_i(x,v,t)}{\partial \mathbf{v}} = 0 \tag{1}$$

where $f_i(x, v, t)$ is distribution function in six-dimensional phase space, $E(\mathbf{x},t)$ and $B(\mathbf{x},t)$ are the electric and magnetic fields, q_i and m_i are the charge and mass of ith particle, respectively, and i denotes multi-ions, i.e. H^+, He^+ and O^+ ions.

To determine the dispersion relation and damping rate, the loss-cone distribution function [23, 29] is used as

$$F_j\left(\mathbf{v}_\perp, v_{||}\right) = \frac{v_\perp^{2J}}{\pi^{3/2} v_{T\perp i}^{2(J+1)} v_{T||i} J!} \exp\left(-\frac{v_\perp^2}{v_{T\perp i}^2}\right) \exp\left(-\frac{v_{||}^2}{v_{T||i}^2}\right) \tag{2}$$

where J is the loss-cone index and integer which characterises the steepness of loss cone and $v_{T||i}^2 = \frac{2T_{||i}}{m_i}$ and $v_{T\perp i}^2 = (J+1)^{-1}\frac{2T_{\perp i}}{m_i}$ are the parallel and perpendicular components of thermal velocities, respectively, in reference to the uniform external magnetic field. $T_{||i}$ and $T_{\perp i}$ are the parallel and perpendicular components of temperature of ith particle, and m_i is mass of ion. Here, steep loss-cone distribution is considered only for ions with $k_\perp \rho_i$ term and is considered negligible for electrons due to small electron gyroradius.

3. Dispersion relation

The basic components of dispersion relation are used as given by Davidson [33]. Here loss-cone distribution function (Eq. 2) is substituted in derivations. The dispersion tensors are solved similar to Ref. [29]; here we have also considered exponential factor as it regulates oscillatory nature of wave propagation. So, the dispersion tensors for kinetic Alfven wave propagating in x–z plane are obtained which are expanded for multi-ions plasma as

$$D_{xx}(\mathbf{k},\omega) = \left(1 - \frac{\omega_{pH^+}^2}{\Omega_{H^+}^2}\frac{D_1^J(\lambda_{H^+})}{\lambda_{H^+}} - \frac{\omega_{pHe^+}^2}{\Omega_{He^+}^2}\frac{D_1^J(\lambda_{He^+})}{\lambda_{He^+}} - \frac{\omega_{pO^+}^2}{\Omega_{O^+}^2}\frac{D_1^J(\lambda_{O^+})}{\lambda_{O^+}}\right) - \frac{c^2k_{||}^2}{\omega^2} \tag{3}$$

$$D_{xz}(\mathbf{k},\omega) = D_{zx}(\mathbf{k},\omega) = \frac{c^2k_{||}k_{\perp}}{\omega^2} \tag{4}$$

$$D_{zz}(\mathbf{k},\omega) = 1 - \frac{c^2k_{\perp}^2}{\omega^2} + \sum_i \frac{m_i}{m_e}\frac{\omega_{pi}^2}{k_{||}^2v_{T||e}^2}C_0^J(\lambda_i)\left(1 + i\sqrt{\frac{\pi}{2}}\left(\frac{\omega}{k_{||}v_{T||e}}\right)\exp(-\xi_e^2)\right) \tag{5}$$

where $k_{||}$ and $k_{\perp}$ are the parallel and perpendicular wave vector, respectively, ω denotes wave frequency, $\omega_{pi}^2 = \frac{4\pi n_0 q_i^2}{m_i}$ is the square of plasma frequency, c is the velocity of light, $\xi_e^2 = \frac{\omega^2}{k_{||}^2 v_{T||e}^2}$, $\Omega_i = \frac{q_i B_0}{m_i c}$ is the cyclotron frequency of ith ion, $\lambda_i = \frac{1}{2}(J+1)k_{\perp}^2\rho_i^2$

$$D_n^J(\lambda) = \int_0^{\infty} \mathrm{d}v_{\perp}^2\left(1 - \frac{Jv_{T\perp i}^2}{v_{\perp}^2}\right)J_n^2\left(\frac{k_{\perp}v_{\perp}}{\Omega_i}\right)\frac{v_{\perp}^{2J}}{v_{T\perp i}^{2(J+1)}\cdot J!}\exp\left(-\frac{v_{\perp}^2}{v_{T\perp i}^2}\right),$$

$$C_n^J(\lambda) = \int_0^{\infty} 2\pi v_{\perp}J_n^2\left(\frac{k_{\perp}v_{\perp}}{\Omega_i}\right)\cdot\frac{v_{\perp}^{2J}}{\pi v_{T\perp i}^{2(J+1)}\cdot J!}\exp\left(-\frac{v_{\perp}^2}{v_{T\perp i}^2}\right)\mathrm{d}v_{\perp},$$

where n represents harmonic of wave and value of $D_n^J(\lambda)$ and $C_n^J(\lambda)$ at J = 0, 1, 2 is derived using recurrence relation [35].

The above dispersion tensor components are used with multi-ions effect to derive the frequency of KAW. For deriving frequency expression, Eqs. (3), (4) and (5) are substituted in Eq. (6)

$$\begin{vmatrix} D_{xx} & D_{xz} \\ D_{zx} & D_{zz} \end{vmatrix} = 0 \tag{6}$$

and determinant is solved by considering only the real components,

$$1 + \sum_i \frac{\omega_{pi}^2}{\Omega_i^2}\frac{D_1^J(\lambda_i)}{\lambda_i} - \frac{c^2k_{||}^2}{\omega^2} = \frac{c^2k_{||}^2}{\omega^2}\frac{\Omega_i^2}{\omega_{pi}^2}\frac{k_{\perp}^2v_{T||e}^2}{\Omega_i^2}\frac{m_e}{m_i}\frac{1}{2C_0^J(\lambda_i)} \times \left(1 + \sum_i \frac{\omega_{pi}^2}{\Omega_i^2}\frac{D_1^J(\lambda_i)}{\lambda_i}\right) \tag{7}$$

Equation (7) is solved further and then expanded over i components, i.e. multi-ions, and hence, the frequency expression is obtained as

$$\omega^2 = c^2k_{||}^2\left\{\begin{array}{l} \frac{\Omega_{H^+}^2}{\omega_{pH^+}^2}\left(\frac{\lambda_{H^+}}{D_1^J(\lambda_{H^+})} + \left(\frac{T_{||e}}{T_{\perp H^+}}\right)\frac{2\lambda_{H^+}}{C_0^J(\lambda_{H^+})}\right) \\ + \frac{\Omega_{He^+}^2}{\omega_{pHe^+}^2}\left(\frac{\lambda_{He^+}}{D_1^J(\lambda_{He^+})} + \left(\frac{T_{||e}}{T_{\perp He^+}}\right)\frac{2\lambda_{He^+}}{C_0^J(\lambda_{He^+})}\right) + \\ \frac{\Omega_{O^+}^2}{\omega_{pO^+}^2}\left(\frac{\lambda_{O^+}}{D_1^J(\lambda_{O^+})} + \left(\frac{T_{||e}}{T_{\perp O^+}}\right)\frac{2\lambda_{O^+}}{C_0^J(\lambda_{O^+})}\right) \end{array}\right\} \tag{8}$$

Here, the frequency expression is containing two variables. First is the loss-cone index J, and second is the density of multi-ions. The present study involves the variation of loss-cone index J to examine the effect on kinetic Alfven wave with each gyrating ion $k_{\perp}\rho_i$. The contribution of density of each ion affecting the nature of wave with loss-cone variation will be dealt in next study to understand generation of the frequency in plasma sheet boundary layer.

4. Damping rate

The growth/damping rate of KAW is derived by assuming

$$\omega' \rightarrow \omega + i\gamma \text{ with } \gamma < \omega \tag{9}$$

where collisionless damping rate of KAW is derived by substituting the values of imaginary component of $D_{zz}(\boldsymbol{k},\omega)$ and differential value of real dispersion components from Eqs. (3), (4) (5) and (8), in expression [29]

$$\gamma = -\frac{\mathrm{Im}\ \mathrm{D}(\mathbf{k},\omega)}{\frac{\partial}{\partial\omega}\mathrm{Re}\ D(\mathbf{k},\omega)} \tag{10}$$

On solving, we obtained

$$\gamma = -2\sqrt{\pi}\sum_i \frac{m_e}{m_i}\frac{\omega_{pi}^2}{k_{||}^3 v_{T||e}^3}\frac{C_0^J(\lambda_i)}{c^2k_\perp^2} \times \left(c^2k_{||}^2\left(\frac{\Omega_i^2}{\omega_{pi}^2}\left(\frac{\lambda_i}{D_1^J(\lambda_i)}+\frac{T_{||e}}{T_{\perp i}}\frac{2\lambda_i}{C_0^J(\lambda_i)}\right)\right)-\frac{c^2k_{||}^2}{\frac{\omega_{pi}^2}{\Omega_i^2}\frac{D_1^J(\lambda_i)}{\lambda_i}}\right)^2 \exp(-\xi_e^2) \tag{11}$$

This expression is solved, and we obtained damping rate of KAW as

$$\gamma = -4\sqrt{\pi}k_{||}v_{T||e}\left\{\begin{array}{l}\frac{\Omega_{H^+}^2}{\omega_{pH^+}^2}\left(\frac{m_e}{m_{H^+}}\frac{c^2}{v_{T\perp H^+}^2}\frac{\lambda_{H^+}}{C_0^J(\lambda_{H^+})}\right)\\ +\frac{\Omega_{He^+}^2}{\omega_{pHe^+}^2}\left(\frac{m_e}{m_{He^+}}\frac{c^2}{v_{T\perp He^+}^2}\frac{\lambda_{He^+}}{C_0^J(\lambda_{He^+})}\right)\\ +\frac{\Omega_{O^+}^2}{\omega_{pO^+}^2}\left(\frac{m_e}{m_{O^+}}\frac{c^2}{v_{T\perp O^+}^2}\frac{\lambda_{O^+}}{C_0^J(\lambda_{O^+})}\right)\end{array}\right\}\exp\left(-\frac{\omega^2}{k_{||}^2v_{T||e}^2}\right) \tag{12}$$

So each ion is affecting the damping of wave. Here, again the gyroradius of each ion can be varied with loss-cone index J. Effects of density of ions in plasma on damping of wave may be investigated for each ion. The present study focuses on variation of J along with $k_\perp\rho_i$ of each ion to investigate the damping phenomena of KAW associated with energy transfer, heating and acceleration mechanism of multi-ions. The density variation of multi-ions with loss-cone will be studied further. The exponential term in the Eq. (12) is determining the oscillatory nature of wave frequency of KAW in PSBL.

5. Results and discussion

The discussion of results is done by numerical evaluation of Eqs. (8) and (12). Frequency and damping rate of KAW derived for homogeneous magnetised plasma with multi-ions are studied graphically. The effect of loss-cone index variation on wave is examined by considering gyroradius of H^+, He^+ and O^+ ions. The parameters relevant to PSBL region are used as reported earlier [3, 8]

$B_0 = 4\times10^{-3}$Gauss, $k_{||} = 10^{-10}$ cm^{-1}, $c = 3\times10^{10}$ cm s^{-1}, $T_{||e} = 2$ keV, $T_{\perp i} = 4$ keV, $\Omega_{H^+} = 38.3234$ s^{-1}, $\Omega_{He^+} = 9.5808$ s^{-1}, $\Omega_{O^+} = 2.3952$ s^{-1} $\omega_{pH^+}^2 = 6.1027\times10^5$ s^{-2}, $\omega_{pHe^+}^2 = 1.734\times10^4$ s^{-2}, $\omega_{pO^+}^2 = 3.034\times10^3$ s^{-2}, $v_{T\perp H^+} = 8.76\times10^7$ cm s^{-1}, $v_{T\perp He^+} = 4.38\times10^7$ cm s^{-1}, $v_{T\perp O^+} = 2.19\times10^7$ cm s^{-1}, $v_{T||e} = 2.652\times10^9$ cm s^{-1}, $n_0 = 1$ cm^{-3}

Equations (8) and (12) are evaluated by substituting above values of PSBL to observe nature of frequency ω(s^{-1}) and damping rate γ(s^{-1}) of wave by varying loss-cone index J or by increasing the steepness of loss cone with gyration of each ion, i.e. H^+, He^+ and O^+ ions. Also $k_\perp$ is varied in each calculation. The loss-cone index is varied from $J = 0$ to $J = 2$ where nature of wave is examined at $J = 1$ and $J = 2$ by comparing with Maxwellian plasma at $J = 0$. J value is not increased beyond 2 as the $D_n^J(\lambda_i)$ value becomes negative resulting in imaginary value of frequency. The results are shown in Figs. 1, 2, 3, 4, 5 and 6. The KAW travels from distant PSBL towards auroral ionosphere which has magnetic trapping like converging magnetic field lines, so ions distribution becomes conic. This study shows He^+ and O^+ ions influence wave differently than H^+ ions due to their Larmor precession. Earlier the effect of each ion was considered to be similar to H^+ ions. The frequency of wave is found to vary from 0.1 to 2.9 s^{-1} which is in agreement with observational results [8, 9].

Figure 1 shows the variation of frequency with $k_\perp\rho_{H^+}$ for $J = 0$, $J = 1$ and $J = 2$. It is clear from the graph that for

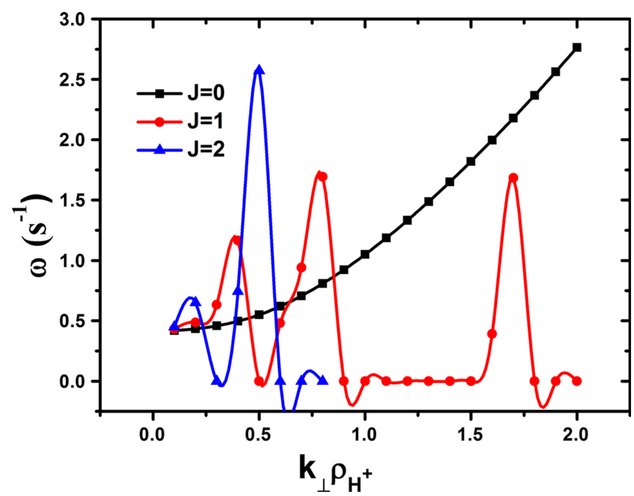

Fig. 1 Variation of wave frequency $\omega(s^{-1})$ with $k_\perp\rho_{H^+}$ for varying loss-cone index $J = 0$, $J = 1$ and $J = 2$

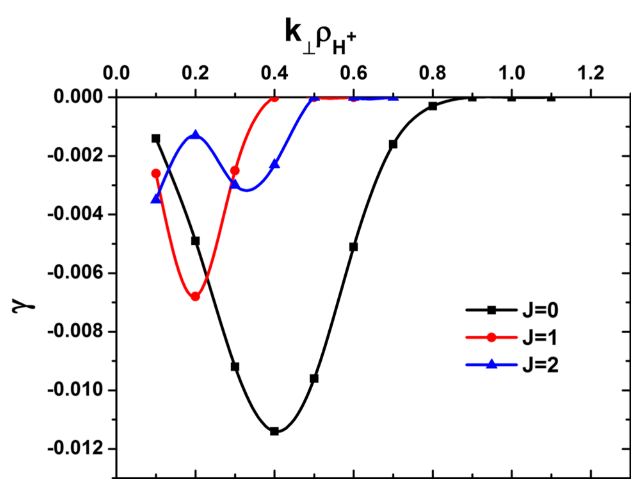

Fig. 2 Variation of damping rate $\gamma(s^{-1})$ with $k_\perp\rho_{H^+}$ for varying loss-cone index $J = 0$, $J = 1$ and $J = 2$

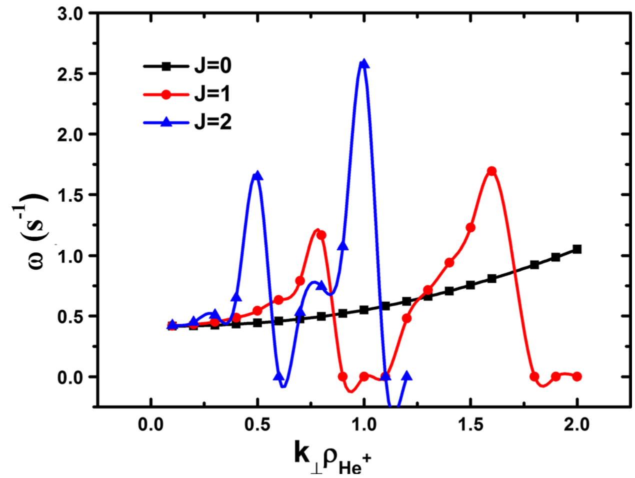

Fig. 3 Variation of wave frequency $\omega(s^{-1})$ with $k_{\perp}\rho_{He^+}$ for varying loss-cone index J = 0, J = 1 and J = 2

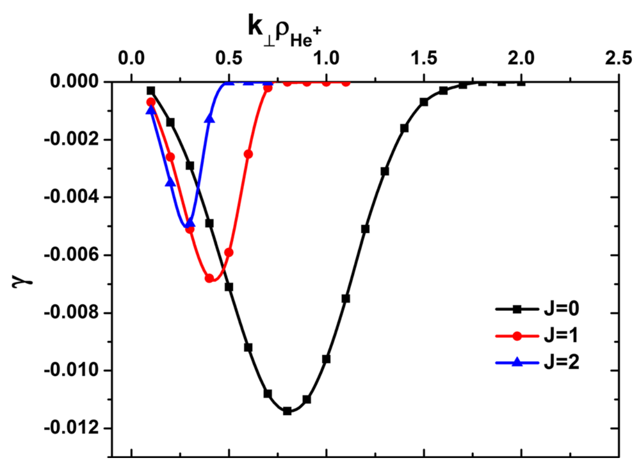

Fig. 4 Variation of damping rate $\gamma(s^{-1})$ with $k_{\perp}\rho_{He^+}$ for varying loss-cone index J = 0, J = 1 and J = 2

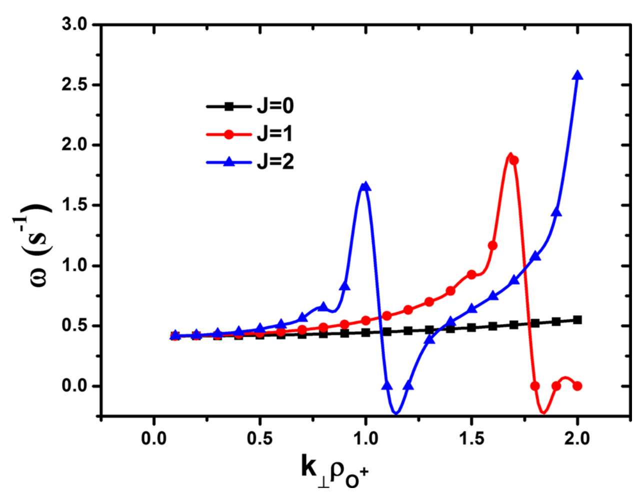

Fig. 5 Variation of wave frequency $\omega(s^{-1})$ with $k_{\perp}\rho_{O^+}$ for varying loss-cone index J = 0, J = 1 and J = 2

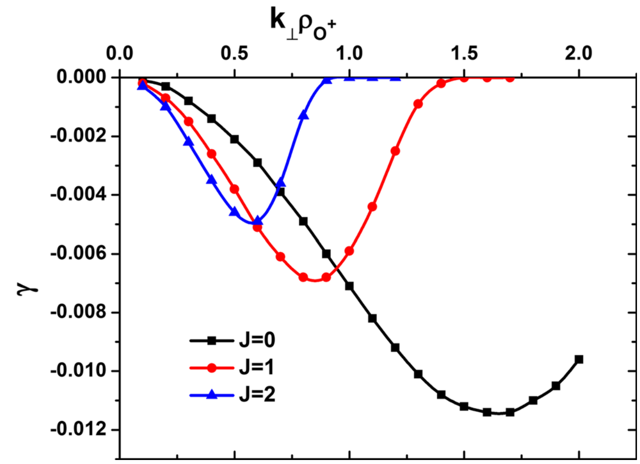

Fig. 6 Variation of damping rate $\gamma(s^{-1})$ with $k_{\perp}\rho_{O^+}$ for varying loss-cone index J = 0, J = 1 and J = 2

J = 0 the wave is showing bi-Maxwellian nature and frequency of wave is increasing linearly. For J = 1, wave frequency is showing oscillatory nature and increasing to attain a peak at particular value of $k_{\perp}\rho_{\mathrm{H}^+}$ and then suddenly drops to zero, but oscillatory characteristic is obtained till $k_{\perp}\rho_{\mathrm{H}^+} \approx 1.7$. For J = 2, the existence of wave is obtained only up to certain value of $k_{\perp}\rho_{\mathrm{H}^+} = 0.6$ with oscillatory frequency in nature. The oscillatory nature of wave frequency is due to the property of modified Bessel function and variation of $C_0^J(\lambda_{\mathrm{H}^+})$ and $D_1^J(\lambda_{\mathrm{H}^+})$. So, the existence of KAW is limiting towards lower $k_{\perp}\rho_{\mathrm{H}^+}$ with the increase in loss-cone index with gyration of hydrogen ion. This may be due to penetration of H^+ ions through loss cone. With the increase in steepness of loss cone, wave is showing its signature at $k_{\perp}\rho_{\mathrm{H}^+} < 1$. It shows that wave exists with perpendicular wavelength higher than the H^+ ion gyroradius.

Figure 2 shows damping of KAW with gyrating H^+ ion for J = 0, J = 1 and J = 2. It is clear from the graph that the damping rate of KAW is decreasing with the increase in loss-cone index.

The graph with increasing J value shows decreased damping rate. This corresponds to wave frequency in Fig. 1 where large perpendicular wavelength is generated in steep loss cone. Reduced damping suggests that wave is less Landau damped, and hence, it can travel longer distance towards ionosphere by accelerating H^+ ions. The damping rate of KAW is shown to exist only up to $k_{\perp}\rho_{\mathrm{H}^+} = 0.8$, i.e. in $k_{\perp}\rho_{\mathrm{H}^+} < 1$ regime, and no damping is found for $k_{\perp}\rho_{\mathrm{H}^+} > 1$ case. This is in accordance with [1] where they have suggested that wave may not show its existence at higher $k_{\perp}\rho_{\mathrm{H}^+}$.

Figure 3 shows the variation of frequency with $k_{\perp}\rho_{\mathrm{He}^+}$ for J = 0, J = 1 and J = 2. For J = 0, the bi-Maxwellian characteristic is observed, but frequency is increased only till 1 Hz. The existence of wave frequency is showing oscillatory behaviour at higher J values. The frequency is increasing to some peak value and then suddenly approaching zero. The frequency of wave is obtained up to $k_\perp \rho_{\mathrm{He}^+} = 1.6$. The frequency values in Fig. 3 with loss-cone index $J = 1$ are nearly equal to that with $k_\perp \rho_{\mathrm{H}^+}$ in Fig. 1. So, this suggests that wave with nearly similar frequency is generated with gyration of lighter H^+ and He^+ ions at $J = 1$. For $J = 2$, existence of wave frequency is shown till $k_\perp \rho_{\mathrm{He}^+} = 1.0$ with oscillatory nature. For $J = 2$, the frequency at $k_\perp \rho_{\mathrm{He}^+} \approx 0.5$ is less than for $k_\perp \rho_{\mathrm{H}^+}$, and at $k_\perp \rho_{\mathrm{He}^+} \approx 1.0$, the frequency attains a maximum value for helium ion gyration. So for $J = 2$, gyrating He^+ ions generates wave with perpendicular wavelength nearly equal to He^+ gyroradius, and reflecting ions through loss-cone may be responsible for generation of KAW at higher J value compared to gyration of H^+ ions.

Figure 4 shows variation of damping rate of KAW with respect to $k_\perp \rho_{\mathrm{He}^+}$ for conic distribution of He^+ ions in PSBL region corresponding to Fig. 3. The value of damping rate is nearly same for bi-Maxwellian distribution of helium ions and hydrogen ions (Fig. 2), i.e. for $J = 0$. For $J = 0$, damping of wave is observed up to $k_\perp \rho_{\mathrm{He}^+} \approx 1.5$. The damping is reduced for conic distribution of He^+ ions corresponding to higher values of $k_\perp \rho_{\mathrm{He}^+}$ compared to H^+ ion distributions (Fig. 4) signifying less penetration of helium ions as compared to hydrogen ions. Damping of wave is reduced with narrow conic width which may be due to loss cone itself acting as a source of free energy or it may be due to less wave–particle interaction as density of helium ions is less compared to H^+ ions density.

The variation of frequency $\omega(\mathrm{s}^{-1})$ with respect to $k_\perp \rho_{\mathrm{O}^+}$ at $J = 0$, $J = 1$ and $J = 2$ is shown in Fig. 5. Here, for $J = 0$, the frequency is nearly constant showing its bi-Maxwellian nature. For $J = 1$, wave frequency is showing oscillatory nature and existing for complete range of $k_\perp \rho_{\mathrm{O}^+}$ from 0 to 1.6 and no frequency drop is observed in between these values though frequency is zero beyond $k_\perp \rho_{\mathrm{O}^+} \approx 1.6$. The frequency is increasing continuously over this range showing continuous existence of kinetic Alfven wave due to gyrating O^+ ions.

The frequency of wave with $k_\perp \rho_{\mathrm{O}^+}$ is lower compared to gyration of H^+, and He^+ showing wave generated with gyrating O^+ ions is of large timescale compared to lighter ions. The frequency of wave generated for $J = 2$ is again oscillatory in nature. First frequency drop is observed at around $k_\perp \rho_{\mathrm{O}^+} \approx 1$ showing the absence of wave at this value. A further rise in frequency is observed for $k_\perp \rho_{\mathrm{O}^+} > 1$, but at $k_\perp \rho_{\mathrm{O}^+} \approx 2.0$, it reaches $\approx 2.5\ \mathrm{s}^{-1}$ which is greater than the cyclotron frequency of oxygen ion. Hence, it is observed that beyond $k_\perp \rho_{\mathrm{O}^+} \approx 1.9$ the wave generated due to O^+ ion gyration is not KAW, but it is electromagnetic ion cyclotron wave.

Figure 6 shows damping rate of KAW due to gyrating oxygen O^+ ions for different values of loss-cone index corresponding to wave generation in Fig. 5. It can be clearly seen from the graph that for bi-Maxwellian distribution of O^+ ions, damping rate of wave is not reducing to zero even at $k_\perp \rho_{\mathrm{O}^+} \approx 2$ showing existence of wave. But as observed in Fig. 5 beyond $k_\perp \rho_{\mathrm{O}^+} \approx 1.9$, the wave may become electromagnetic ion cyclotron, so the damping beyond $k_\perp \rho_{\mathrm{O}^+} \approx 1.9$ may correspond to EMIC wave nature.

Hence, the frequency graphs show that KAW is being generated continuously in the presence of heavier ions for broad as well as narrow conic section as heavy ions penetrate least through the loss cone, and due to magnetic trapping of O^+ ions as Larmor radius of O^+ ions is large compared to lighter ions, whereas lighter ions can penetrate through the broad and sharp loss-cone section more easily so the wave generated with gyrating H^+ and He^+ ions resulted frequency drop around zero showing the absence of wave at some $k_\perp \rho_i$ for particular ions. For the conic distribution of O^+ ions, damping is increased for higher values of $k_\perp \rho_{\mathrm{O}^+}$ compared to gyrating lighter ions distribution. With reducing width of loss cone, damping of KAW is decreasing. Density of O^+ ions is very less so very few heavy ions can penetrate through loss cone; hence, reflected gyrating O^+ ions may be responsible for lower damping of KAW over higher values of $k_\perp \rho_{\mathrm{O}^+}$. This also suggests reduced Landau damping and effective energization of O^+ ions. It was suggested that for obliquely propagating electromagnetic waves, the loss-cone index influences wave growth/damping [24], and this has been clearly examined in the present study for KAW in multi-ions plasma.

6. Conclusion

The kinetic Alfven waves with loss-cone distribution of ions in PSBL region are investigated. The effect of increase in steepness is studied on frequency and damping rate of KAW. Conic distribution of multi-ions in plasma sheet plays an important role in distant tail magnetospheric physics through wave–particle interaction; however, we have considered the parameters of plasma sheet boundary layer region in this study. The frequency obtained is in agreement with observationally reported values range 0.1–4 Hz [8] and signifies generation of kinetic Alfven wave. This study indicates that the damping of wave is reduced with increasing steepness of loss-cone suggesting that wave is less Landau damped and can travel long distances towards auroral ionosphere by accelerating H^+, He^+

and O^+ ions. However, the decrease in energy of H^+ and O^+ ions on travelling from PSBL region towards Earth changes differently [36]. This study also presents a mechanism of KAW generation from interaction of multi-ions. Earlier it was limited to proton consideration [3–5]. The study also reveals that heavy ions can penetrate least through the loss cone and hence can be more energised. Sometimes loss cone itself acts as a source of free energy. Here, this study also presents that on increasing steepness, the wave generated with gyrating H^+ ions has perpendicular wavelength greater than ion gyroradius, with gyrating He^+ has perpendicular wavelength equal to ion gyroradius and with gyrating O^+ ions has perpendicular wavelength less than ion gyroradius. This study also fulfils observational evidence that these waves may provide a principal mechanism for heating particles throughout the plasma sheet to keV energies [8]. So, this study provides important mechanism to understand magnetospheric dynamics. The effect of density variation of multi-ions with conic distribution of ions will be focused in next study as density is also an important parameter for understanding dynamic of ions heating, acceleration, energization and energy transport.

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