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ABSTRACT

It is shown that the Debye length formulation, for plasmas described by kappa distributions, depends on the polytropic index, rather than the parameter that labels and governs these distributions, the kappa index—in contrast to what it was previously derived. As a consequence, the ratio of the Debye length over the plasma oscillation period gives exactly the sound speed, instead of being proportional to the thermal speed; this ratio is generalized to the fast magnetosonic speed when the magnetic Debye length is considered, leading also to the development of the vector Debye length. Finally, as an application, we derive the Debye length values for the solar wind plasma near 1 AU, exhibiting clear distinction between slow and fast wind modes, while we provide insights into the connection between plasma and polytropic processes.

The interest in the generalized Debye length formulation is triggered by numerous applications of kappa distributions in plasmas,[1–3] their natural connection with thermodynamics,[4] and the fundamental importance of the Debye length in basic plasma physics.[5–8] There are four papers exclusively devoted to developing the Debye length modified[5–8] formula for plasmas described by kappa distributions.[1–3] Following similar or different approaches, they arrive at the same analytical result, that is,

$$\lambda_{\rm D} = \sqrt{\frac{\varepsilon_0 k_{\rm B} T_{e,i}}{e^2 n_\infty}\cdot f_\kappa}, \quad \text{with } f_\kappa \equiv \frac{\kappa - \frac{3}{2}}{\kappa - \frac{1}{2}}, \tag{1}$$

where the temperature $T_{e,i}$ refers to the summation of the electron and ion inverse temperature (also called reduced temperature), $T_{e,i}^{-1} = T_i^{-1} + T_e^{-1}$; n_∞ indicates the density at infinity, that is, at a position where the potential is practically zero. The factor f_κ modifies the case of Maxwell–Boltzmann distributions for which $f_\kappa = 1$; it depends on the kappa index κ, the parameter that labels and governs the kappa distribution.

It has been shown[1,9,10] that the kappa index depends on the correlated degrees of freedom d so that

$$\kappa(d) = \kappa_0 + \frac{1}{2}d, \tag{2}$$

where the difference $\kappa(d) - \frac{1}{2}d$ is a quantity independent of d; it has the meaning of the actual kappa index, indicating the stationary state in which the system resides; it is noted by κ_0, as of zero dimensionality kappa. Equation (1) is derived using a 3D kappa distribution; in this case, we have $\kappa = \kappa(3) = \kappa_0 + \frac{3}{2}$, and hence, the factor can be written in an invariant form as

$$f_\kappa = \frac{\kappa_0}{\kappa_0 + 1}. \tag{3}$$

Next, we derive the formula of the Debye length and its kappa dependent factor in a more accurate way, which involves the formulation of phase-space kappa distributions of Hamiltonians with nonzero potential degrees of freedom.[1,11–13]

Density and Temperature positional profiles in plasmas described by kappa distributions—The distribution of the Hamiltonian, $H(\vec{r},\vec{u}) = \varepsilon_{\rm K}(\vec{u}) + \Phi(\vec{r})$, where $\varepsilon_{\rm K}(\vec{u}) = \frac{1}{2}m\cdot u^2$ and $\Phi(\vec{r})$ denote the kinetic and potential energies, respectively, is given by

$$P(\vec{r},\vec{u}\,;\kappa\,,T\,) \propto \left[1 + \frac{1}{\kappa}\cdot\frac{H(\vec{r},\vec{u}) - \langle H\rangle}{k_{\rm B}T}\right]^{-\kappa-1}, \tag{4a}$$

or, equivalently,

$$P(\vec{r},\vec{u}\,;\kappa_0\,,T\,) \propto \left[1 + \frac{1}{\kappa_0}\cdot\frac{\varepsilon_{\rm K}(\vec{u}) + \Phi(\vec{r})}{k_{\rm B}T}\right]^{-\kappa_0-1-\frac{1}{2}d}, \tag{4b}$$

where the degrees of freedom d sum both the kinetic and potential degrees of freedom, which are expressed through the ensemble averages

$$\frac{1}{2}d \equiv \frac{\langle H \rangle}{k_B T} = \frac{1}{2}d_K + \frac{1}{2}d_\Phi, \tag{5a}$$

with

$$\frac{1}{2}d_K = \frac{\langle \varepsilon_K \rangle}{k_B T}, \quad \frac{1}{2}d_\Phi \equiv \frac{\langle \Phi \rangle}{k_B T}. \tag{5b}$$

(Note: In the case where the potential may be negative, we use the cut-off operator,[1–3,11] instead of the simple square brackets: $[x]_+$: $f([x]_+) = x$ if $x > 0$; $f([x]_+) = 0$ if $x \leq 0$. In this analysis, we avoid this notation for simplicity.)

If we integrate the phase-space distribution (4b) over the velocity space, we derive the positional kappa distribution $P(\vec{r};\kappa_0, T)$. The density profile $n(\vec{r})$ is proportional to the positional probability distribution, $n(\vec{r}) = n_\infty \cdot P(\vec{r};\kappa_0, T)$, where, again, the normalization density n_∞ refers to a distant position for which the potential is practically zero

$$\begin{aligned} P(\vec{r};\kappa_0, T) &\propto \int_{-\infty}^{+\infty} \left[1 + \frac{1}{\kappa_0} \cdot \frac{\varepsilon_K(\vec{u}) + \Phi(\vec{r})}{k_B T}\right]^{-\kappa_0 - 1 - \frac{1}{2}d} d\vec{u} \\ &\propto \left[1 + \frac{1}{\kappa_0} \cdot \frac{\Phi(\vec{r})}{k_B T}\right]^{-\kappa_0 - 1 - \frac{1}{2}d_\Phi}, \end{aligned} \tag{6a}$$

or

$$n(\vec{r}) = n_\infty \cdot \left[1 + \frac{1}{\kappa_0} \cdot \frac{\Phi(\vec{r})}{k_B T}\right]^{-\kappa_0 - 1 - \frac{1}{2}d_\Phi}. \tag{6b}$$

On the other hand, the temperature profile $T(\vec{r})$ is derived by finding the second statistical moment of the phase-space distribution, that is, by integrating $\varepsilon_K(\vec{u})$ over the velocity space

$$\frac{1}{2}d_K k_B n(\vec{r}) T(\vec{r}) = n_\infty \cdot \int_{-\infty}^{+\infty} P(\vec{r}, \vec{u};\kappa_0, T)\varepsilon_K(\vec{u})\, d\vec{u}, \tag{7a}$$

where we find

$$T(\vec{r}) = T_\infty \cdot \left[1 + \frac{1}{\kappa_0} \cdot \frac{\Phi(\vec{r})}{k_B T}\right]. \tag{7b}$$

The two positional profiles in Eqs. (6b) and (7b) can be connected together to derive the polytropic relationship, by mutually canceling the dependence on the potential energy, $[1 + \Phi(\vec{r})/(\kappa_0 k_B T)]$, i.e.,

$$\frac{n(\vec{r})}{n_\infty} = \left[\frac{T(\vec{r})}{T_\infty}\right]^\nu \quad \text{or} \quad \frac{T(\vec{r})}{T_\infty} = \left[\frac{n(\vec{r})}{n_\infty}\right]^{\gamma - 1}, \tag{8}$$

where the polytropic index ν (or $\gamma \equiv 1 + 1/\nu$) is given in terms of the kappa index by[1,12–14]

$$\nu + 1 + \kappa_0 + \frac{1}{2}d_\Phi = 0, \quad \text{or} \quad \gamma = \frac{\kappa_0 + \frac{1}{2}d_\Phi}{1 + \kappa_0 + \frac{1}{2}d_\Phi}. \tag{9}$$

(Note that as the kappa index tends to infinity, the potential degrees of freedom become less important, while the polytropic index describes an isothermal process, $\gamma = 1$, independent of their finite value.) From Eq. (7b), we find that the globally averaged temperature $\langle T(\vec{r})\rangle = T$ is related to the temperature at zero potential T_∞

$$T_\infty = T \cdot \frac{\kappa_0}{\kappa_0 + \frac{1}{2}d_\Phi} \tag{10a}$$

or using Eq. (9)

$$(\nu + 1)T_\infty + \kappa_0 T = 0. \tag{10b}$$

Derivation of the Debye length formulation—Next, we follow the typical theoretical analysis[1,8,15–18] for deriving the Debye length formula, where the ion n_i and electron n_e densities are described by the kappa distributions. We first introduce the potential $\tilde{\Phi}(\vec{r})$ that differs from the potential energy $\Phi(\vec{r})$: A charge q_e needs potential energy equal to $\Phi(\vec{r}) \equiv q_e \cdot \tilde{\Phi}(\vec{r})$ to be brought from infinity where potential is zero to the position $\vec{r}$; then

$$\begin{aligned} n_i(\vec{r}) &= n_\infty \cdot \left[1 + \frac{e\tilde{\Phi}(\vec{r})}{\kappa_{0i} k_B T_i}\right]^{-\kappa_{0i} - 1 - \frac{1}{2}d_{\Phi_i}}, \\ n_e(\vec{r}) &= n_\infty \cdot \left[1 - \frac{e\tilde{\Phi}(\vec{r})}{\kappa_{0e} k_B T_e}\right]^{-\kappa_{0e} - 1 - \frac{1}{2}d_{\Phi e}}, \end{aligned} \tag{11}$$

we find the charge density

$$\rho_e = -\left(\frac{\kappa_{0i} + 1 + \frac{1}{2}d_{\Phi i}}{\kappa_{0i}} \cdot \frac{1}{T_i} + \frac{\kappa_{0e} + 1 + \frac{1}{2}d_{\Phi e}}{\kappa_{0e}} \cdot \frac{1}{T_e}\right) \cdot \frac{n_\infty e^2}{k_B}\tilde{\Phi}(\vec{r}). \tag{12a}$$

Using Eq. (10a) that connects the temperatures T and T_∞ and Eq. (9) that connects κ_0 and ν indices, we obtain

$$\begin{aligned} \rho_e = &-\left(\frac{\kappa_{0i} + 1 + \frac{1}{2}d_{\Phi i}}{\kappa_{0i} + \frac{1}{2}d_{\Phi i}} \cdot \frac{1}{T_{\infty i}} + \frac{\kappa_{0e} + 1 + \frac{1}{2}d_{\Phi e}}{\kappa_{0e} + \frac{1}{2}d_{\Phi e}} \cdot \frac{1}{T_{\infty e}}\right) \\ &\times \frac{n_\infty e^2}{k_B}\tilde{\Phi}(\vec{r}), \end{aligned} \tag{12b}$$

or

$$\rho_e = -\left(\frac{1}{\gamma_i} \cdot \frac{1}{T_{\infty i}} + \frac{1}{\gamma_e} \cdot \frac{1}{T_{\infty e}}\right) \cdot \frac{n_\infty e^2}{k_B}\tilde{\Phi}(\vec{r}) = -\frac{n_\infty e^2}{\gamma_{e,i} k_B T_{\infty e,i}}\tilde{\Phi}(\vec{r}), \tag{12c}$$

where we defined $T_{\infty e,i}$ and $\gamma_{e,i}$, according to

$$\gamma_{e,i}^{-1} T_{\infty e,i}^{-1} \equiv \gamma_i^{-1} T_{\infty i}^{-1} + \gamma_e^{-1} T_{\infty e}^{-1}, \tag{12d}$$

$$T_{\infty e,i}^{-1} \equiv T_{\infty i}^{-1} + T_{\infty e}^{-1}. \tag{12e}$$

Equation (12e) was set, so that $\gamma_{e,i} = \gamma$, when $\gamma_i = \gamma_e = \gamma$. Hence, if $T_{\infty i} = T_{\infty e} = T_\infty$, then $\gamma_{e,i}^{-1} = (\gamma_i^{-1} + \gamma_e^{-1})/2$. The quantity $T_{\infty e,i}$ is also called reduced temperature; it coincides with the smaller species temperature, if there is a large temperature difference with the rest of the species, i.e., if $T_{\infty e} \ll T_{\infty i}$, then $T_{\infty e,i} \cong T_{\infty e}$. Similarly, $\gamma_{e,i}$ can be called the reduced weighted polytropic index; it coincides with the index of that species that has the significantly smaller temperature: If $T_{\infty e} \ll T_{\infty i}$, then $\gamma_{e,i} \cong \gamma_e$.

Therefore, Gauss's law gives

$$\nabla^2\tilde{\Phi}(\vec{r}) = -\frac{\rho_e}{\varepsilon_0} = \frac{n_\infty e^2}{\varepsilon_0\gamma_{e,i}k_B T_{\infty e,i}}\cdot\tilde{\Phi}(\vec{r}) = \frac{1}{\lambda_D^2}\cdot\tilde{\Phi}(\vec{r}), \quad (13)$$

where we finally obtain the Debye length form

$$\lambda_D = \sqrt{\frac{\varepsilon_0 k_B T_{\infty e,i}}{e^2 n_\infty}\cdot\gamma_{e,i}}. \quad (14)$$

The modifying factor f_κ of Eq. (1) or Eq. (3) is now generalized to

$$f_\kappa = \left(\frac{\frac{\kappa_{0i}+1+\frac{1}{2}d_{\Phi i}}{\kappa_{0i}+\frac{1}{2}d_{\Phi i}}\cdot T_{\infty i}^{-1} + \frac{\kappa_{0e}+1+\frac{1}{2}d_{\Phi e}}{\kappa_{0e}+\frac{1}{2}d_{\Phi e}}\cdot T_{\infty e}^{-1}}{T_{\infty i}^{-1}+T_{\infty e}^{-1}}\right)^{-1}$$
$$= \left(\frac{\gamma_i^{-1}T_{\infty i}^{-1}+\gamma_e^{-1}T_{\infty e}^{-1}}{T_{\infty i}^{-1}+T_{\infty e}^{-1}}\right)^{-1} = \gamma_{e,i}, \quad (15a)$$

while in the case where $\gamma_{e,i}=\gamma$, we obtain

$$f_\kappa = \frac{\kappa_0+\frac{1}{2}d_\Phi}{\kappa_0+1+\frac{1}{2}d_\Phi} = \gamma. \quad (15b)$$

Hence, the factor f_κ is not kappa dependent but is rather equal to the polytropic index. Note that Eq. (15b) generalizes Eq. (3) by taking into account the potential degrees of freedom, which were ignored in the earliest studies.[5–8] Finally, we recap the considerations of the above developments: (i) weakly interacting plasmas (e.g., space plasmas), and thus, the multiparticle Hamiltonian terms can be ignored; (ii) potential energy depends only on the position and not the velocity; (iii) potential energy must be small compared to the thermal energy for compressible plasma; and (iv) potential energy of electromagnetic interaction is dominant compared to other interactions that may exist among protons and electrons.

Related definitions and formulations—The Debye length and plasma oscillation frequency for a species j are given by

$$\lambda_{Dj} = \sqrt{\frac{\varepsilon_0}{n_\infty e^2}\cdot\gamma_j k_B T_{\infty j}} \quad \text{and} \quad \omega_{pj}^2 = \frac{n_\infty e^2}{\varepsilon_0 m_j}, \quad (16a)$$

(where m_j is the mass of the j species), while the Debye length from both the ion and the electron is smaller because of the inverse summation in Eq. (12d)

$$\lambda_D^{-2} = \frac{n_\infty e^2}{\varepsilon_0\gamma_{e,i}k_B T_{\infty e,i}} = \frac{n_\infty e^2}{\varepsilon_0\gamma_i k_B T_{\infty i}} + \frac{n_\infty e^2}{\varepsilon_0\gamma_e k_B T_{\infty e}} = \lambda_{Di}^{-2}+\lambda_{De}^{-2}. \quad (16b)$$

Instead, the sonic Debye length can be defined by summing $\gamma_i T_{\infty i}+\gamma_e T_{\infty e}$, i.e.,

$$\lambda_{Ds}^2 = \lambda_{Di}^2+\lambda_{De}^2. \quad (16c)$$

(Note: Hereafter, we ignore the subscript notation of "∞" for densities and temperatures, for simplicity. Thermodynamic variables are measured away from the charge perturbation where both are practically stabilized. Hence, T does not mean the average temperature as meant during the derivations, but the temperature at infinity, practically, far from the charge perturbation.)

Moreover, the magnetic Debye length[19–23] λ_{Dm} is given by substituting the thermal energy (for compressible plasmas) with the magnetic energy in the regular electric Debye length

$$\lambda_D^2 = \frac{\varepsilon_0}{ne^2}\cdot(\gamma k_B T) \to \lambda_{Dm}^2 = \frac{\varepsilon_0}{ne^2}\cdot\left(\frac{B^2}{\mu_0 n}\right) = \left(\frac{B}{\mu_0 nec}\right)^2, \quad (17a)$$

where we used the identity $\varepsilon_0\mu_0 c^2=1$, arriving at the magnetic Debye length

$$\lambda_{Dm} = \frac{B}{\mu_0 nec}. \quad (17b)$$

The magnetic Debye length has the property of measuring the distance of magnetic field screening,[23] similar to the property of measuring the distance of the electric field screening by the electric Debye length. The two screening effects are independent, and the total Debye length can be defined by

$$\lambda_{Dms}^2 = \lambda_{Dm}^2+\lambda_{Ds}^2. \quad (18)$$

In addition, we notice that the electric and magnetic Debye lengths are involved in similar relations, leading to the sound and Alfvén speeds, respectively,

$$\frac{1}{2}m_0\lambda_{Ds}^2\omega_p^2 = \frac{1}{2}m_i V_s^2 \equiv E_s, \quad \frac{1}{2}m_0\lambda_{Dm}^2\omega_p^2 = \frac{1}{2}m_i V_A^2 \equiv E_m \quad (19a)$$

(where m_0 refers to the reduced mass $m_0^{-1}=m_i^{-1}+m_e^{-1}$); the respective relationship for the total Debye length gives the fast magnetosonic speed, $V_{ms}^2 = V_m^2+V_s^2$, i.e.,

$$\frac{1}{2}m_0\lambda_{Dms}^2\omega_p^2 = \frac{1}{2}m_i V_{ms}^2 \equiv E_{ms}. \quad (19b)$$

The common proportionality constant $\tau \equiv \sqrt{1+m_i/m_e}\cdot\omega_p^{-1}$ among the Debye lengths $\lambda_{Da}=\tau\cdot V_a$, with subscripts a: s, m (or A), ms, as well as the common Euclidean L_2-norm in the summation of the sonic and magnetic Debye lengths $\lambda_{Dms}^2=\lambda_{Dm}^2+\lambda_{Ds}^2$ and speeds $V_{ms}^2=V_m^2+V_s^2$, leads to the concept of the vector Debye length, defined by

$$\vec{\lambda}_{Da} \equiv \tau\cdot\vec{V}_a, \quad a: s, m\ (\text{or A}), ms, \quad \text{with } \tau \equiv \sqrt{1+m_i/m_e}\cdot\omega_p^{-1}. \quad (20)$$

Finally, we note that the key point that allows the above connection between Debye lengths and magnetic/sonic velocities is that the factor f_κ is interpreted by the polytropic index, instead of some function of the kappa index; only then, the ratio of the Debye length over the plasma oscillation period gives the sound speed, instead of just being proportional to the thermal speed, as it was previously thought.

Application in the solar wind plasma—As an application, in Fig. 1, we depict (a) the sonic, (b) the magnetic, and (c) the total magnetosonic Debye lengths and (d) the ratio of the ion Debye length over the magnetic Debye length, in the case of the solar wind proton-electron plasma near 1 AU. The values of the Debye lengths are plotted in normalized[24] 2D histograms with respect to the solar wind flow speed V_{sw}. In particular, we use measurements of ~92-s solar wind plasma moments (speed V_{sw}, density n, and temperature T) and the interplanetary magnetic field B, which were taken, respectively, using SWE and MAG instruments onboard Wind S/C,[12,24–27] during the year 1995. We observe that the sonic Debye length increases linearly with the solar wind speed, $\lambda_{Ds}\cong\tau\cdot V_{sw}$, with the slope given by the time scale $\tau\cong 1$ ms (more accurately, 0.975 ± 0.015 ms), that is, the

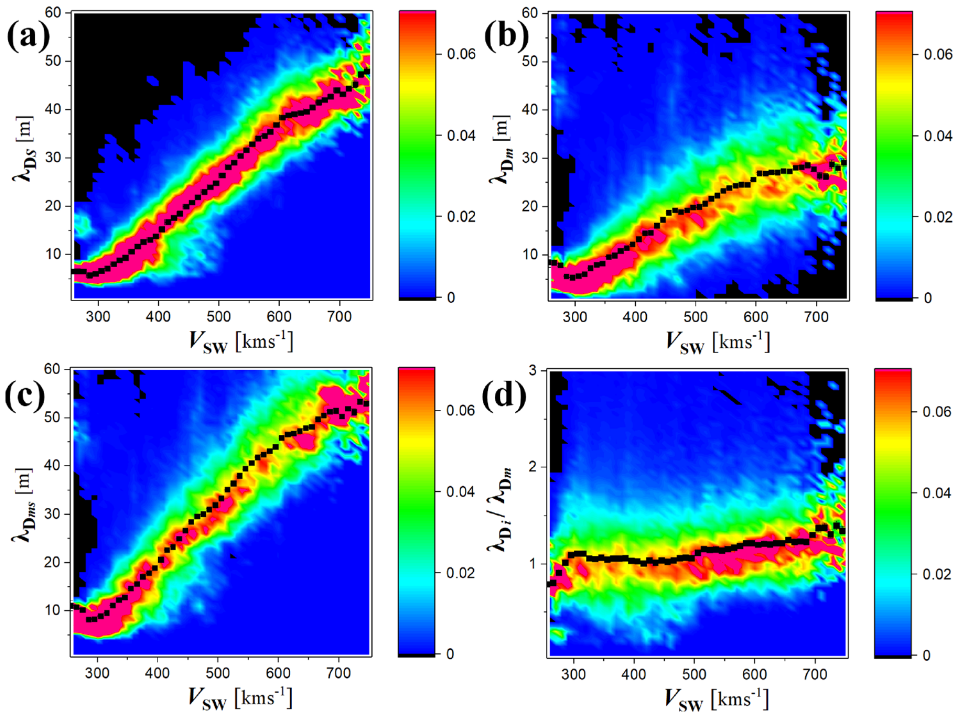

FIG. 1. Normalized 2D histograms of the values of the (a) sonic, (b) magnetic, and (c) magnetosonic Debye lengths and (d) ratio of the ion Debye length over the magnetic Debye length, with respect to the bulk flow speed for the solar wind proton-electron plasma near 1 AU.

time needed for the solar wind flow to cover a distance equal to the sonic Debye length, which is about 25 times the period of the plasma oscillation t_p (for the same dataset, we found $\langle \log(\omega_p\,[\mathrm{rad\,Hz}])\rangle \cong 5.20 \pm 0.14$ or $t_p \cong 40\ \mu s$). The magnetic Debye length appears to have a concave relationship with solar wind speed, clearly departing from linearity above $\sim$550 km s^{-1}, a range corresponding to the fast solar wind mode. The linearity of the sonic Debye length and concavity of the magnetic Debye length are moderate in the magnetosonic Debye length. The ratio of the ion Debye length over the magnetic Debye length, $\lambda_{D\,i}/\lambda_{D\,m}$, has values close to $\sim$1 in the slow solar wind ($V_{sw} < 550$ km s^{-1}), while clearly $>$1 in the fast solar wind ($V_{sw} > 550$ km s^{-1}).

Conclusions and future applications—In summary, it was shown that the general formulation of the Debye length for plasmas described by kappa distributions involves the polytropic index, instead of the kappa index, the parameter characterizing these distributions. The ratio of the Debye length over the plasma oscillation period gives the sound speed, instead of the thermal speed that was previously thought. This ratio is generalized to the fast magnetosonic speed when the magnetic Debye length is considered, while the vector Debye length can also be defined. Finally, as an application, we derive the Debye length values for the solar wind plasma near 1 AU, exhibiting clear distinction between slow and fast wind modes.

The importance of the newly developed formulation of the Debye length is the connection of plasma with polytropic processes (that is, the thermodynamic processes following a polytropic relationship among the plasma thermal observables). For instance, the observation of plasma particles flowing under a quasi-isobaric process (with polytropic index closed to $\gamma \sim 0$) means the existence of small values of the Debye length and a very effective shielding of charge fluctuations, while the Landau damping would be effective in smaller scales than usual, allowing the existence of waves of small wavelengths. When the polytropic index is quite small (smaller than the adiabatic value, $\gamma < 5/3$), then the plasma is endothermic, i.e., a heat-absorbing system. If a process allows for the polytropic index to be continuously decreasing, then an external heating source has to be connected to the system;—e.g., the radial expansion of the solar wind may lead to a slow decrease in the polytropic index, which must be connected to turbulent heating sources, more effective for smaller Debye lengths. In this case, it would certainly be a challenge to find the energy balance among thermal and wave energy and other plasma processes that are possibly affected by the polytropic index variation.

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