



# Kelvin-Helmholtz Instabilities of Black Hole Jets and Accretion Discs

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cm cm

## Background

Kelvin-Helmholtz Instability:

- An instability occurring at a velocity shear in a single fluid or at the interface of two fluids with differing velocities.

Two Boundary Cases:

- Zero width layer: Immediate jump from one fluid to the next.
- Finite width shear layer: Adds a transition region of mixed fluids between the two fluids.

Applications/Previous Works:

- Supersonic Shear Layer of finite width in space plasmas. [1]
- Counter-Rotating Gas Discs in astrophysical settings implement cylindrical symmetries and utilize a zero width layer. [2]

This Work:

- Black hole jets and accretion disks
- Cylindrical symmetries
- Pressure anisotropy
- Zero and finite width velocity shear layers

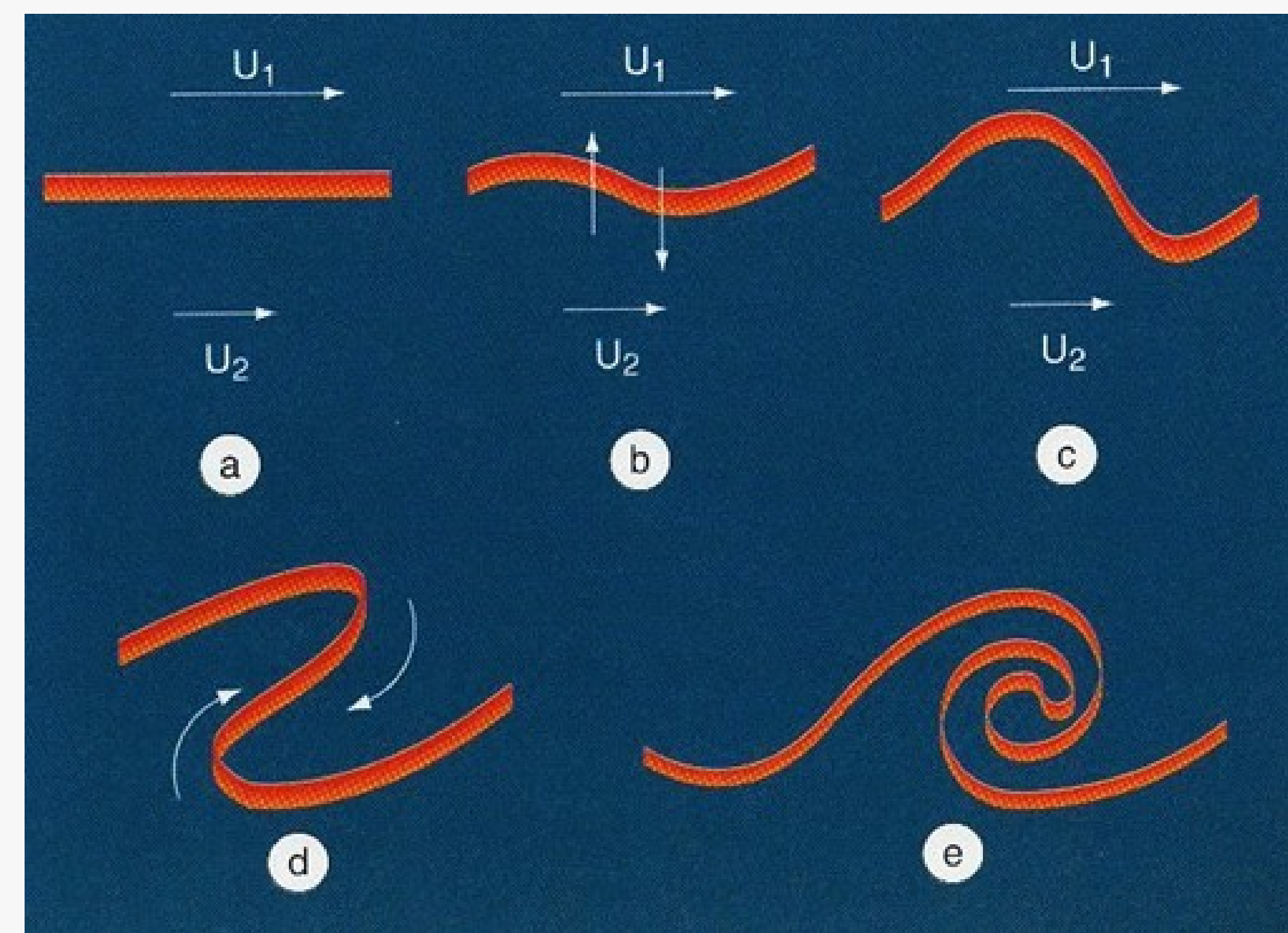


Fig 1. Time evolution of a Kelvin-Helmholtz instability. Such instabilities can be observed in a wide range of natural phenomena, such as cloud formation, ocean waves, Earth's magnetopause, etc. [5]

## Methods

1. Begin with CGL equations for anisotropic plasma flows from [4] assuming a negligible magnetic field in the direction of the jets.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \quad (1)$$

$$\frac{\partial \rho \vec{u}}{\partial t} + \nabla \cdot [\rho \vec{u} \vec{u} + p_{\perp} \mathbf{I} + (p_{\parallel} - p_{\perp}) \hat{z} \hat{z}] = 0 \quad (2)$$

$$\frac{\partial (p_{\parallel} - p_{\perp})}{\partial t} + \nabla \cdot ((p_{\parallel} - p_{\perp}) \vec{u}) + (2p_{\parallel} + p_{\perp}) \hat{z} \cdot \nabla \vec{u} \cdot \hat{z} - p_{\perp} \nabla \cdot \vec{u} = \frac{p_{\perp} - p_{\parallel}}{\tau} \quad (3)$$

$$\frac{\partial E}{\partial t} + \nabla \cdot [\vec{u} (E + p_{\perp}) + \vec{u} \cdot ((p_{\parallel} - p_{\perp}) \hat{z} \hat{z})] = 0 \quad (4)$$

2. Find the steady state of the system by setting the partial derivatives with respect to time to zero.
3. Perturb each variable: density, pressure in both the directions parallel and perpendicular to the magnetic field, and velocity.

$$\delta f = f(r) \exp[i(m\phi + k_z z - \omega t)] \quad (5)$$

4. Plug in perturbed variables to initial equations to linearize them.

$$i\Delta\omega \frac{\delta \rho}{\rho_0} = \frac{\delta u_r}{r} + \delta u'_r + ik_{\phi} \delta u_{\phi} + ik_z \delta u_z \quad (6)$$

$$\delta p'_{\perp} = i\rho_0 \Delta\omega \delta u_r \quad (7)$$

$$k_{\phi} \delta p_{\perp} = \rho_0 \Delta\omega \delta u_{\phi} \quad (8)$$

$$ik_z \delta p_{\parallel} = i\rho(\Delta\omega - k_z u_z) \delta u_z + i\Delta\omega u_z \delta \rho - \rho_0(u_z + u'_z) \delta u_r - \rho_0 u_z \delta u'_r - ik_{\phi} \rho_0 \delta u_{\phi} \quad (9)$$

$$\left(i\Delta\omega - \frac{1}{\tau}\right)(\delta p_{\parallel} - \delta p_{\perp}) = (p_{\parallel} - 2p_{\perp}) \left(\frac{\delta u_r}{r} + \delta u'_r + ik_{\phi} \delta u_{\phi}\right) + ik_z (3p_{\parallel} - p_{\perp}) \delta u_z \quad (10)$$

$$i\Delta\omega \delta p_{\perp} + \frac{i}{2}(\Delta\omega - 2k_z u_z) \delta p_{\parallel} = \left(\frac{(E_0 + p_{\perp})}{r} + \rho_0 u_z u'_z\right) \delta u_r + (E_0 + p_{\perp}) \delta u'_r + ik_{\phi} (E + p_{\perp}) \delta u_{\phi} + i(k_z(E_0 + p_{\parallel}) - \Delta\omega \rho_0 u_z) \delta u_z - \frac{i u_z^2}{2} \Delta\omega \delta \rho \quad (11)$$

5. Integrate over the boundary to find zero width solutions.
6. Solve the finite width layer case outside the boundaries of the velocity shear.
7. Employ numerical shooting method to solve the linearized equations in the shear layer between the boundaries for physical solutions.

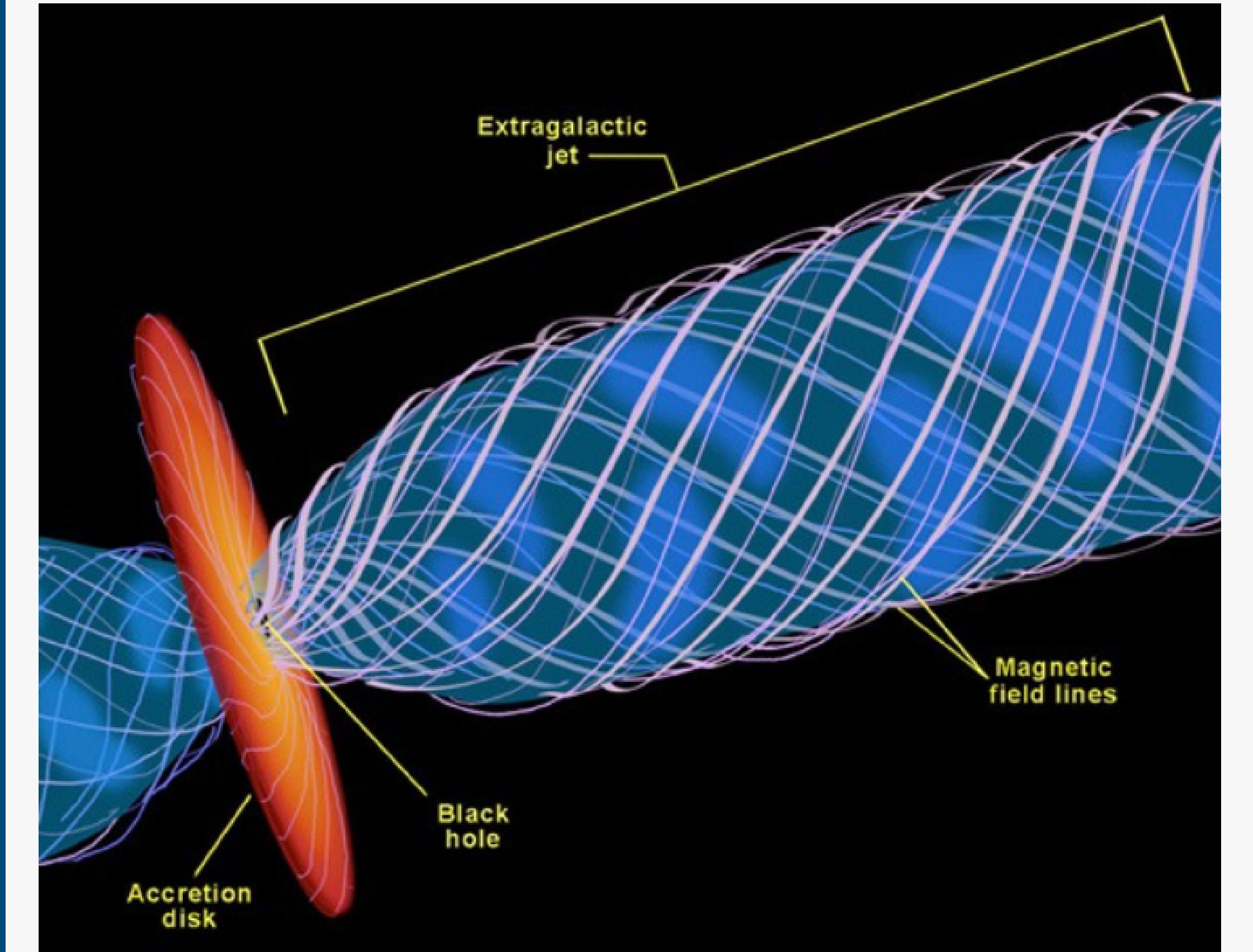


Fig 2. The accretion disc is comprised of spiraling material from nearby stars and astrophysical objects. Jets form due to a spiraling of magnetic fields in the system. These fields pull the material of the accretion disk around the black hole. Then, if the system has enough energy, the material is shot out as jets. [3]

## References

- [1] S. R. Choudhury and R. V. E. Lovelace. On the Kelvin-Helmholtz instabilities of supersonic shear layers. , 283:331–342, August 1984.
- [2] S. Dyda D. Quaach and R. Lovelace. Kelvin-Helmholtz instability of counter-rotating discs . *Monthly Notices of the Royal Astronomical Society*, 446:622–626, 2015.
- [3] Ann Feild. Formation of extragalactic jets from a black hole accretion disk, October 1999.
- [4] Chetan Singh, Deepak Bhoirya, Anshu Yadav, Harish Kumar, and Dinshaw S. Balsara. Chew, Goldberger & Low Equations: Eigensystem Analysis and Applications to One-Dimensional Test Problems. *arXiv e-prints*, page arXiv:2504.05503, April 2025.
- [5] Bahman Zohuri. *Physics of Inertial Confinement Fusion (ICF)*, pages 133–192. Springer International Publishing, Cham, 2017.