Problem 43:

The number, 1406357289, is a 0 to 9 pandigital number because it is made up of each of the digits 0 to 9 in some order, but it also has a rather interesting sub-string divisibility property. Let d1 be the 1st digit, d2 be the 2nd digit, and so on. In this way, we note the following:

d2d3d4=406 is divisible by 2
d3d4d5=063 is divisible by 3
d4d5d6=635 is divisible by 5
d5d6d7=357 is divisible by 7
d6d7d8=572 is divisible by 11
d7d8d9=728 is divisible by 13
d8d9d10=289 is divisible by 17
Find the sum of all 0 to 9 pandigital numbers with this property.

Knowns from problem:

- Substrings are divisible by prime numbers in ascending order (2, 3, 5, 7, 11, 13, 17)
- This is a 10-digit 0 to 9 pandigital with no redundant digits and contains 0 (i.e. cannot use the same number twice within string) (shout out to Sudoku)
- The total number of possible numbers is 3,265,920 (9 * 9!)
- To make it a little more confusing, the digits are labeled from 1 to 10 instead of 0 to 9. I used a table when writing the problem out and recreated it below:

	1	4	0	6	3	5	7	2	8	9
d	1	2	3	4	5	6	7	8	9	10
substring										

A = 2, 3, 4 = {406} = by 2
B = 3, 4, 5 = {063} = by 3
C = 4, 5, 6 = {635} = by 5
D = 5, 6, 7 = {357} = by 7
E = 6, 7, 8 = {572} = by 11
F = 7, 8, 9 = {728} = by 13
G= 8, 9, 10 = {289} = by 17

Endings (d4) = 0, 2, 4, 6, 8
Endings (d5)
Endings (d6) = 0, 5
Endings (d7) =
Endings (d8) =
Endings (d9) =
Endings (d10) =

- From there, we know that d6 can only be 0 or 5; however, it cannot start with 0 because for E to be a multiple of 11, the second and third (d7, d8) would have to be the same digits.
- Because d6 = 5, can take that and search for 3 digit numbers beginning with 5 that are divisible by 11(and don't repeat any numbers) ⇒ yield: 506, 517, 528, 539, 561, 572, 583, and 594. 506 gets removed because the only possible combination it can make later requires using another 5 (065). 517 is also removed because there aren't any multiples of 13 that start with 17.

- Next step is to take on F, divisible by 13. We now have the possible combinations that d7 and d8 could be; therefore, we just find numbers starting with the last two digits of the E possibilities (06, 17, 28, 39, 61, 72, 83, 94). New options: for F(d7,8,9)= 286, 390, 728, 832
- Now to G, divisible by 17. Same process as for F only using the new potential digits: 286, 390, 728, 832. G= 867, 901, 289, 323. 323 doesn't work since it contains redundant 3's so it's out.

Touching in:
E=(6, 7, 8)= 528, 539, 561, 572, 583, 594
F=(7, 8, 9)= 286, 390, 728, 832
G=(8, 9, 10)= 867, 901, 289

- Now to work backwards: D must end with the first two digits of E and be divisible by 7. Because of redundancies, the only combinations that work are 357 and 952.

Chain update:
D=(5, 6, 7)=357, 952
E=(6, 7, 8)= 528, 539, 561, 572, 583, 594
F=(7, 8, 9)= 286, 390, 728, 832
G=(8, 9, 10)= 867, 901, 289

- Now we just need d1, 2, 3, 4: d4 had the options of (0, 2, 4, 6, 8). 2, 8 are used in both possible ending sequences (357289, 952867). Leaving the d4 possibilities as: 0, 4, 6
- d5 must be divisible by 3 and cannot contain 2, 5, 7, 8. Narrowed down further, the d3,4,5 sequence must have an even number in the middle leaving the possibilities as 063, 309, and 603 for B.
- The remaining options for d1,d2 are 1 and 4, so those are broken out between the different combinations to complete the set.

d	1	2	3	4	5	6	7	8	9	10
	1	4	0	6	3	5	7	2	8	9
	1	4	3	0	9	5	2	8	6	7
	1	4	6	0	3	5	7	2	8	9
	4	1	0	6	3	5	7	2	8	9
	4	1	3	0	9	5	2	8	6	7
	4	1	6	0	3	5	7	2	8	9

All that remains is calculating the sum of the combinations (all possible combinations are shown above) which brings the total to 16,695,334,890.