Discrete Math Propositional Logic Test

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1. Determine if the statements $P \to (Q \vee R)$ and $(P \to Q) \vee (P \to R)$ are logically equivalent.

   From the following truth table (I cheated and let Python make it for me ;-):

   ```
   $ ttg_cli.py "['P', 'Q', 'R']" -p "['P => (Q or R)', '(P => Q) or (P => R)']" -i False
   ```

P	Q	R	P => (Q or R)	(P => Q) or (P => R)
True	True	True	True	True
True	True	False	True	True
True	False	True	True	True
True	False	False	False	False
False	True	True	True	True
False	True	False	True	True
False	False	True	True	True
False	False	False	True	True

   we see that the two statements have the same truth values, and they are therefore logically equivalent by definition.

2. Prove the following:

   $$\begin{array}{r} P \to Q \\ P \to R \\ \hline \therefore\ P \to (Q \wedge R). \end{array}$$

   From the following truth table:

   ```
   $ ttg_cli.py "['P', 'Q', 'R']" -p "['P => Q', 'P => R', 'P => (Q and R)']" -i False
   ```

P	Q	R	P => Q	P => R	P => (Q and R)
True	True	True	True	True	True
True	True	False	True	False	False
True	False	True	False	True	False
True	False	False	False	False	False
False	True	True	True	True	True
False	True	False	True	True	True
False	False	True	True	True	True
False	False	False	True	True	True

   we can see that the final statement (the conclusion) is true whenever the two hypotheses are true, so the conclusion follows from the hypotheses.

Alternatively, we can establish this using rules of inference.

1. $P \rightarrow Q$	Given
2. $P \rightarrow R$	Given
3. P	conditional proof assumption
4. Q	1, 3, modus ponens
5. R	2, 3, modus ponens
6. $Q \wedge R$	4, 5, conjunction introduction
$\therefore P \rightarrow (Q \wedge R)$	3 and 6

3. Using the statement: *If you read for understanding and work all the exercises you will score well on the test*. Translate it into symbols, then write its **negation**, **converse**, and **contrapositive**. What can you conclude if you don't score well on the test?

   Let R: You read for understanding; W: You work all the exercises; and S: You score well on the test.

   The given statement translates into symbols as:

$$(R \wedge W) \rightarrow S$$

   The negation, converse, and contrapositive statements are as follows:

Negation: $(R \wedge W) \wedge \neg S$

Converse: $S \rightarrow (R \wedge W)$

Contrapositive: $\neg S \rightarrow (\neg R \vee \neg W)$

   Using the contrapositive, which is logically equivalent to the original statement, you can conclude that if you don't score well on the test, then you didn't read for understanding or you didn't work all the exercises.