Suicune

Grace Jiang
Period 4

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I. OVERVIEW

A) Note To Reader

The instructions for this project included the banning of using "cross sections" in order to calculate the volume of any part of the project.

If all cross sections are banned, then it is purely impossible to calculate the volume of a solid using single-variable calculus. Therefore, I have assumed that the banned "cross sections" merely refers to the use of any elementary cross section formulas that are based on simple cross-sectional shapes with defined area formulas. These shapes include:

- *Regular polygons (Equilateral Triangles, Squares, Pentagons, Hexagons, etc.)*
- *Rectangles*
- *Ellipses*
- *Circles*

Because rotational cross-sections are allowed in order to calculate volumes, they have not been ruled out. In addition, I have not ruled out the juxtaposition of cross sections with other methods in order to calculate a volume. This means that the subtraction of a cross section from a rotationally-determined volume is allowed for the purpose of this project.

In short, as long the volume of any part of this project is not purely determined through the use of cross sections, the calculations will be allowed. In addition, any cross sections used cannot involve the use of any of the elementary shapes listed above.

I have concluded that under stricter guidelines, it is not possible to use single-variable calculus to calculate the volume of this project.

- Grace

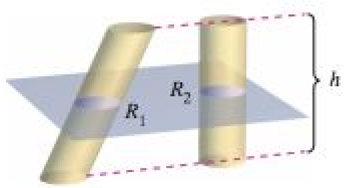
B) Cavalieri's Principle

In mathematics, Cavalieri's Principle states:

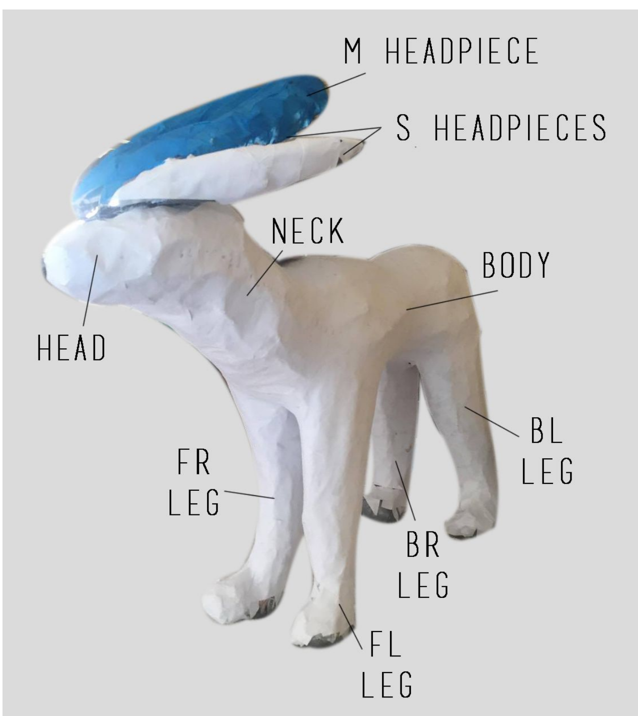
"*Suppose two regions in three-space (solids) are included between two parallel planes. If every plane parallel to these two planes intersects both regions in cross-sections of equal area, then the two regions have equal volumes.*" (Wikipedia)

In other words, when two solids have equal altitudes and all plane sections parallel to their bases and equal distances from their bases have equal areas, then the solids have the same volume.

For example, the following two solids have the same volume.

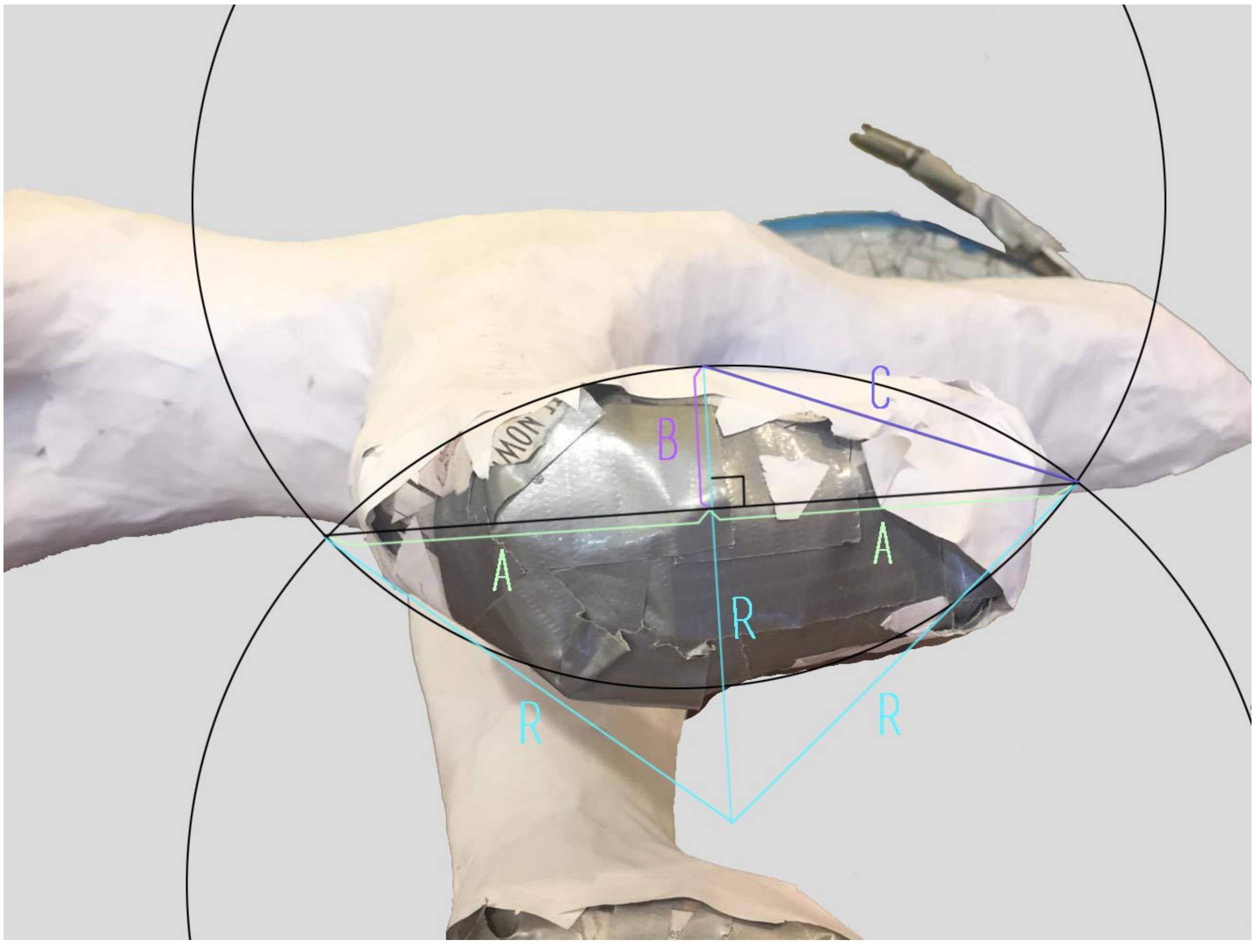


C) Sections of the Project

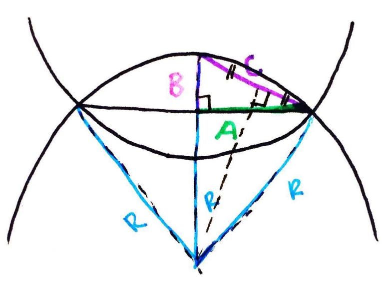


II. LEGS

A) Calculation Method

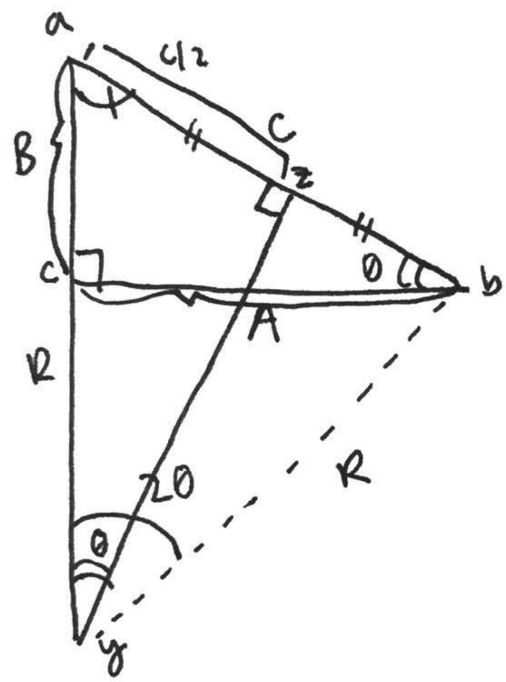


Because elementary cross-sections aren't allowed in this project, I decided to use two circles to approximate the areas to integrate in the legs. The math is as follows:



partial rotation:

$$V = \underbrace{\pi\left(\frac{2\theta}{\pi}\right)\int_{\alpha}^{\beta}\left[R^2\right]dx}_{\text{ROTATION}} - \underbrace{\frac{1}{2}\int_{\alpha}^{\beta}(R-\beta)2A}_{\text{GEOMERY}}$$



$$C = \sqrt{A^2 + B^2}$$

$\Delta abc \sim \Delta ayz$ through AAA

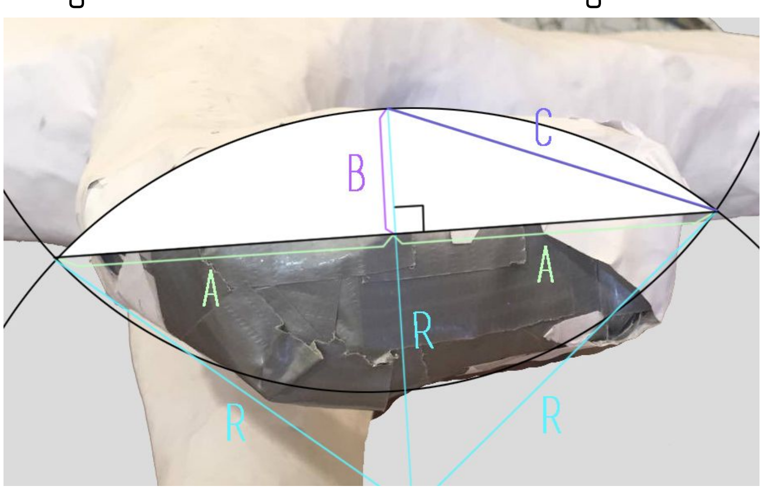
$$\therefore \frac{\frac{C}{2}}{R} = \frac{B}{C}$$

$$R = \frac{C^2}{2B} = \frac{A^2+B^2}{2B}$$

$$\tan\theta = \frac{B}{A}, \quad \theta = \tan^{-1}\frac{B}{A}$$

(Note: Δbyz is also similar to Δayz and Δabc through AAA)

This gives us the volume of the following shaded area:

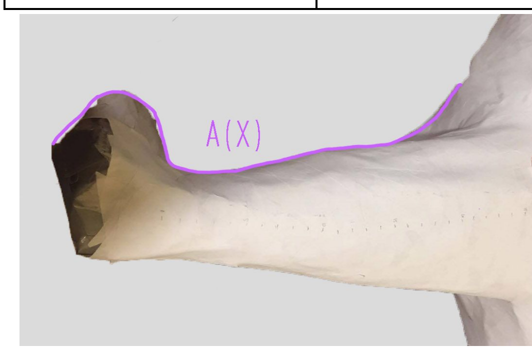


Similarly, we can calculate the lower region using the same method.

Through Cavalieri's principle, when we integrate from the bottom of the leg to the upper area of the leg, the volumes will be consistent no matter how shifted the radii are.

B) Front Left Leg

	a(x)	b(x)	R(x)
Top Section	$1.104202605\cdot10^{-10}x^{10} - 1.708323336\cdot10^{-8}x^{9} + 1.118039276\cdot10^{-6}x^{8} - 4.017581355\cdot10^{-5}x^{7} + 8.590928596\cdot10^{-4}x^{6} - 1.102386196\cdot10^{-2}x^{5} + 8.027717669\cdot10^{-2}x^{4} - 2.752917758\cdot10^{-1}x^{3} + 1.660705637\cdot10^{-1}x^{2} + 2.473793756\cdot10^{-1}x + 6.064406376$ $R^2 = 0.9643021805$	$3.96637299\cdot10^{-11}x^{10} - 6.424097587\cdot10^{-9}x^{9} + 4.426230218\cdot10^{-7}x^{8} - 1.690222314\cdot10^{-5}x^{7} + 3.904052749\cdot10^{-4}x^{6} - 5.580333156\cdot10^{-3}x^{5} + 4.835973808\cdot10^{-2}x^{4} - 2.366701858\cdot10^{-1}x^{3} + 0.554764668x^{2} - 0.397094578x + 2.505210113$ $R^2 = 0.9524194075$	$1.457492984\cdot10^{-10}x^{10} - 2.213041519\cdot10^{-8}x^{9} + 1.412971069\cdot10^{-6}x^{8} - 4.911455096\cdot10^{-5}x^{7} + 1.002344046\cdot10^{-3}x^{6} - 1.197009116\cdot10^{-2}x^{5} + 7.620344067\cdot10^{-2}x^{4} - 1.719350778\cdot10^{-1}x^{3} - 3.543116832\cdot10^{-1}x^{2} + 5.253465893\cdot10^{-1}x + 8.678252417$ $R^2 = 0.9790972072$
Bottom Section	$1.081955586\cdot10^{-10}x^{10} - 1.682401099\cdot10^{-8}x^{9} + 1.10570973\cdot10^{-6}x^{8} - 3.986990612\cdot10^{-5}x^{7} + 8.549739009\cdot10^{-4}x^{6} - 1.099725008\cdot10^{-2}x^{5} + 8.024638658\cdot10^{-2}x^{4} - 0.275550406x^{3} + 0.164976107x^{2} + 2.455509733\cdot10^{-1}x + 5.966579193$ $R^2 = 0.9747248507$	$3.646247882\cdot10^{-11}x^{10} - 5.992337675\cdot10^{-9}x^{9} + 4.181839905\cdot10^{-7}x^{8} - 1.61350586\cdot10^{-5}x^{7} + 3.751779577\cdot10^{-4}x^{6} - 5.364867622\cdot10^{-3}x^{5} + 4.595459873\cdot10^{-2}x^{4} - 2.163332996\cdot10^{-1}x^{3} + 4.503974821\cdot10^{-1}x^{2} - 1.825369289\cdot10^{-1}x + 2.396340453$ $R^2 = 0.9501082551$	$1.432202651\cdot10^{-10}x^{10} - 2.168306493\cdot10^{-8}x^{9} + 1.380255253\cdot10^{-6}x^{8} - 4.781818299\cdot10^{-5}x^{7} + 9.718414012\cdot10^{-4}x^{6} - 1.15324496\cdot10^{-2}x^{5} + 7.243209181\cdot10^{-2}x^{4} - 1.535394271\cdot10^{-1}x^{3} - 3.987363717\cdot10^{-1}x^{2} + 5.629412597\cdot10^{-1}x + 8.675242663$ $R^2 = 0.9818234167$



(side view, A(x) from center of leg)

(front view, B(x) from center of leg)

$$V_{top} = \pi \int_0^{31} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{31}(R(x) - b(x))2a(x)dx$$

$$= 1366.822cm^3$$

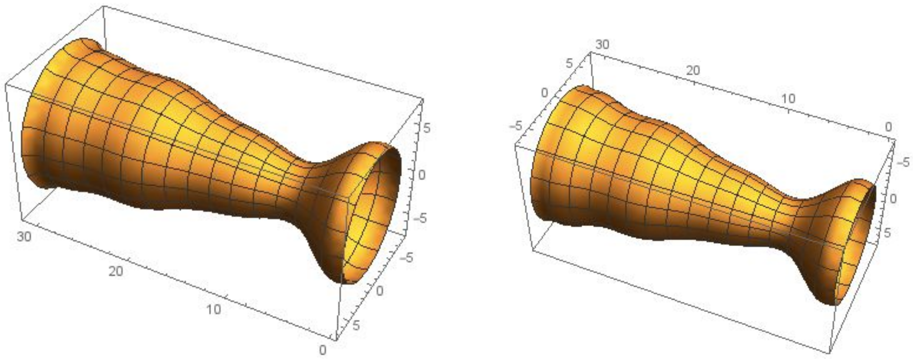
$$V_{bottom} = \pi \int_0^{31} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{31}(R(x) - b(x))2a(x)dx$$

$= 1283.674 cm^3$

$V_{total} = V_{top} + V_{bottom}$

$= 1366.822 cm^3 + 1283.674 cm^3$

$= \mathbf{2650.496\ cm^3}$



(Cavalieri's principle)

C) Back Left Leg

	a(x)	b(x)	R(x)
Top Section	$4.297183832 \cdot 10^{-11} x^{10} - 7.211034446 \cdot 10^{-9} x^9 + 5.152002819 \cdot 10^{-7} x^8 - 2.036385411 \cdot 10^{-5} x^7 + 4.83472447 \cdot 10^{-4} x^6 - 6.97761431 \cdot 10^{-3} x^5 + 5.844823723 \cdot 10^{-2} x^4 - 0.24491112 x^3 + 2.932049394 \cdot 10^{-1} x^2 + 1.988393088 \cdot 10^{-1} x + 5.873913176$ *$R^2 = 0.9871238156$*	$-3.18273931 \cdot 10^{-12} x^{10} + 4.820415178 \cdot 10^{-10} x^9 - 3.04095078 \cdot 10^{-8} x^8 + 1.036634018 \cdot 10^{-6} x^7 - 2.08940462 \cdot 10^{-5} x^6 + 2.634514894 \cdot 10^{-4} x^5 - 2.317578939 \cdot 10^{-3} x^4 + 1.685389434 \cdot 10^{-2} x^3 - 8.899112651 \cdot 10^{-2} x^2 + 1.607682281 \cdot 10^{-1} x + 3.073539608$ *$R^2 = 0.9907593963$*	$6.511607472 \cdot 10^{-11} x^{10} - 1.085690978 \cdot 10^{-8} x^9 + 7.699116194 \cdot 10^{-7} x^8 - 3.016178259 \cdot 10^{-5} x^7 + 7.082331955 \cdot 10^{-4} x^6 - 1.007360095 \cdot 10^{-2} x^5 + 8.256636277 \cdot 10^{-2} x^4 - 0.331330601 x^3 + 3.181481728 \cdot 10^{-1} x^2 + 4.386707103 \cdot 10^{-1} x + 7.131556423$ *$R^2 = 0.9836795702$*
Bottom Section	$5.510482074 \cdot 10^{-11} x^{10} - 9.102709883 \cdot 10^{-9} x^9 + 6.409267669 \cdot 10^{-7} x^8 - 2.501278333 \cdot 10^{-5} x^7 + 5.880977088 \cdot 10^{-4} x^6 - 8.448913829 \cdot 10^{-3} x^5 + 7.11907355 \cdot 10^{-2} x^4 - 3.091207955 \cdot 10^{-1} x^3 + 4.609221539 \cdot 10^{-1} x^2 + 1.307089345 \cdot 10^{-2} x + 5.769752641$ *$R^2 = 0.9872438659$*	$2.370754131 \cdot 10^{-12} x^{10} - 4.651656832 \cdot 10^{-10} x^9 + 3.803092134 \cdot 10^{-8} x^8 - 1.691537485 \cdot 10^{-6} x^7 + 4.450318104 \cdot 10^{-5} x^6 - 6.984628922 \cdot 10^{-4} x^5 + 6.14133719 \cdot 10^{-3} x^4 - 2.401612703 \cdot 10^{-2} x^3 - 1.586007889 \cdot 10^{-3} x^2 + 0.131416451 x + 2.896514257$ *$R^2 = 0.9868184251$*	$8.439445728 \cdot 10^{-11} x^{10} - 1.380055991 \cdot 10^{-8} x^9 + 9.612178402 \cdot 10^{-7} x^8 - 3.706965773 \cdot 10^{-5} x^7 + 8.599647372 \cdot 10^{-4} x^6 - 1.215893536 \cdot 10^{-2} x^5 + 1.003214223 \cdot 10^{-1} x^4 - 4.208439087 \cdot 10^{-1} x^3 + 5.649308733 \cdot 10^{-1} x^2 + 9.478797902 \cdot 10^{-2} x + 7.189378781$ *$R^2 = 0.985100791$*

$$V_{top} = \pi \int_0^{32} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{32} (R(x) - b(x))2a(x)dx$$

$= 1624.675cm^3$

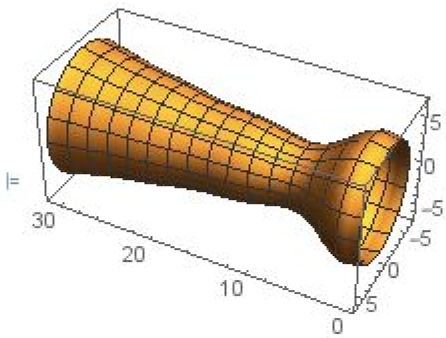
$$V_{bottom} = \pi \int_0^{32} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{32} (R(x) - b(x))2a(x)dx$$

$= 1527.728cm^3$

$V_{total} = V_{top} + V_{bottom}$

$= 1624.675cm^3 + 1527.728cm^3$

$=$ **$3152.403cm^3$**



D) Front Right Leg

	a(x)	b(x)	R(x)
Top Section	$9.084818802\cdot10^{-11} x^{10} - 1.467059968\cdot10^{-8} x^9 + 1.008724089\cdot10^{-6} x^8 - 3.843670999\cdot10^{-5} x^7 + 8.839450952\cdot10^{-4} x^6 - 1.249450565\cdot10^{-2} x^5 + 1.051617981\cdot10^{-1} x^4 - 0.474387771 x^3 + 8.341765897\cdot10^{-1} x^2 + 1.078270916\cdot10^{-1} x + 4.572695653$ $R^2 = 0.9842634193$	$-1.475769745\cdot10^{-11} x^{10} + 2.299497134\cdot10^{-9} x^9 - 1.514245351\cdot10^{-7} x^8 + 5.478538213\cdot10^{-6} x^7 - 1.187149037\cdot10^{-4} x^6 + 1.581002685\cdot10^{-3} x^5 - 1.289403893\cdot10^{-2} x^4 + 6.374595617\cdot10^{-2} x^3 - 1.935960259\cdot10^{-1} x^2 + 3.535110271\cdot10^{-1} x + 2.59866772$ $R^2 = 0.9634062507$	$1.559862206\cdot10^{-10} x^{10} - 2.52177754\cdot10^{-8} x^9 + 1.735999212\cdot10^{-6} x^8 - 6.623367125\cdot10^{-5} x^7 + 1.525525128\cdot10^{-3} x^6 - 2.161233871\cdot10^{-2} x^5 + 1.827359446\cdot10^{-1} x^4 - 8.346226113\cdot10^{-1} x^3 + 1.545843606 x^2 - 9.064167831\cdot10^{-2} x + 5.320127837$ $R^2 = 0.9731876332$
Bottom Section	$8.676050427\cdot10^{-11} x^{10} - 1.387200052\cdot10^{-8} x^9 + 9.443762812\cdot10^{-7} x^8 - 3.562512873\cdot10^{-5} x^7 + 8.10747435\cdot10^{-4} x^6 - 1.132424863\cdot10^{-2} x^5 + 9.376002295\cdot10^{-2}$	$2.022207954\cdot10^{-11} x^{10} - 3.180373673\cdot10^{-9} x^9 + 2.137537818\cdot10^{-7} x^8 - 8.008861698\cdot10^{-6} x^7 + 1.826882108\cdot10^{-4} x^6 - 2.59347176\cdot10^{-3} x^5 + 2.23495244\cdot10^{-2} x^4 -$	$1.228616864\cdot10^{-10} x^{10} - 1.977522419\cdot10^{-8} x^9 + 1.354857092\cdot10^{-6} x^8 - 5.140029288\cdot10^{-5} x^7 + 1.174843139\cdot10^{-3} x^6 - 1.644221717\cdot10^{-2} x^5 + 0.135787172 x^4 -$

	x^4 - 4.094360727·10^{-1} x^3+ 6.364051327·10^{-1} x^2 + 3.672477799·10^{-1} x + 4.369232102 R^2 = 0.983911289	1.071171828·10^{-1} x^3+ 2.227589302·10^{-1} x^2 - 7.706852164·10^{-3} x + 2.499678761 *R^2 = 0.9839523717*	5.850273408·10^{-1} x^3+ 8.512145197·10^{-1} x^2 + 6.803506436·10^{-1} x + 5.074537902 *R^2 = 0.9721293343*

$$V_{top} = \pi \int_0^{31} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{31} (R(x) - b(x))2a(x)dx$$

= 1441.607cm^3

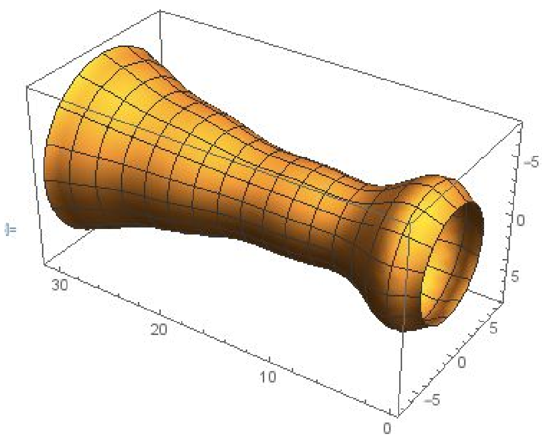
$$V_{bottom} = \pi \int_0^{31} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{31} (R(x) - b(x))2a(x)dx$$

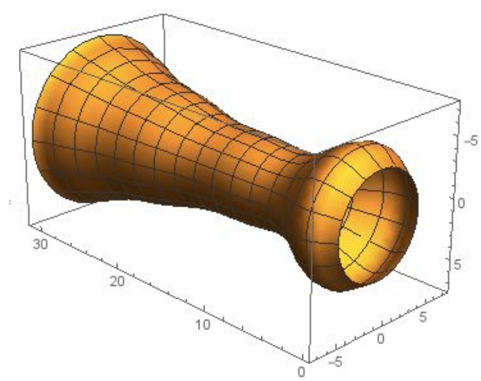
= 1366.655cm^3

$V_{total} = V_{top} + V_{bottom}$

= 1441.607cm^3 + 1366.655cm^3

= **2808.263 cm^3**





E) Back Right Leg

	a(x)	b(x)	R(x)
Top Section	3.276843443·10^{-10} x^{10} - 5.167815514·10^{-8} x^9 + 3.462742693·10^{-6} x^8 - 1.281136237·10^{-4} x^7+ 2.841410334·10^{-3} x^6 - 3.820354212·10^{-2} x^5 + 2.962244464·10^{-1} x^4 - 1.118699784 x^3+ 8.480290249·10^{-1} x^2 +	4.973940487·10^{-11} x^{10} - 7.568601829·10^{-9} x^9 + 4.861253435·10^{-7} x^8 - 1.709403656·10^{-5} x^7+ 3.564653971·10^{-4} x^6 - 4.451418077·10^{-3} x^5 + 3.185647447·10^{-2} x^4 - 1.168391812·10^{-1} x^3+ 1.832507271·10^{-1}	2.134880296·10^{-10} x^{10} - 3.380128913·10^{-8} x^9 + 2.277897129·10^{-6} x^8 - 8.501603565·10^{-5} x^7+ 1.912465998·10^{-3} x^6 - 2.636886062·10^{-2} x^5 + 2.152418552·10^{-1} x^4 - 9.315313837·10^{-1} x^3 + 1.56863688 x^2-

	2.843986044 x + 11.39131815 R^2 = *0.9274154827*	x^2 - 1.199181257·10^{-1} x + 5.756424082 R^2 = *0.9816857235*	3.429596275·10^{-1} x + 7.188315871 R^2 = *0.9339677955*
Bottom Section	1.763249641·10^{-10} x^{10} - 2.770164012·10^{-8} x^9 + 1.844990277·10^{-6} x^8 - 6.759582564·10^{-5} x^7+ 1.474447718·10^{-3} x^6 - 1.921971978·10^{-2} x^5 + 1.391807719·10^{-1} x^4 - 4.190014938·10^{-1} x^3- 4.471470714·10^{-1} x^2 + 3.410404135 x + 5.630493622 R^2 = *0.9186581449*	2.019425116·10^{-11} x^{10} - 3.079936917·10^{-9} x^9 + 1.976103428·10^{-7} x^8 - 6.8967625·10^{-6} x^7+ 1.409613928·10^{-4} x^6 - 1.67993279·10^{-3} x^5 + 1.072870719·10^{-2} x^4 - 2.751406422·10^{-2} x^3- 9.757706895·10^{-3} x^2 + 6.718791183·10^{-2} x + 2.867004221 R^2 = *0.9686897714*	2.914986446·10^{-10} x^{10} - 4.53811657·10^{-8} x^9 + 2.984472419·10^{-6} x^8 - 1.073368777·10^{-4} x^7+ 2.27361732·10^{-3} x^6 - 2.81053525·10^{-2} x^5 + 1.796550959·10^{-1} x^4 - 2.775304038·10^{-1} x^3 - 2.620682009 x^2+ 9.32759246 x + 6.944537963 R^2 = *0.9147351956*

$$V_{top} = \pi \int_0^{31} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{31} (R(x) - b(x))2a(x)dx$$

= 1985.690cm^3

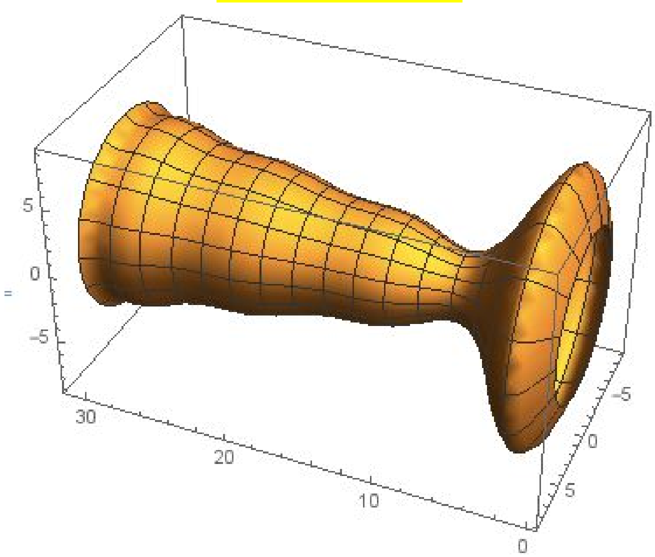
$$V_{bottom} = \pi \int_0^{31} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{31} (R(x) - b(x))2a(x)dx$$

= 1571.406cm^3

V_{total} = V_{top} + V_{bottom}

= 1985.690cm^3 + 1571.406cm^3

= **3557.096cm^3**



III. BODY

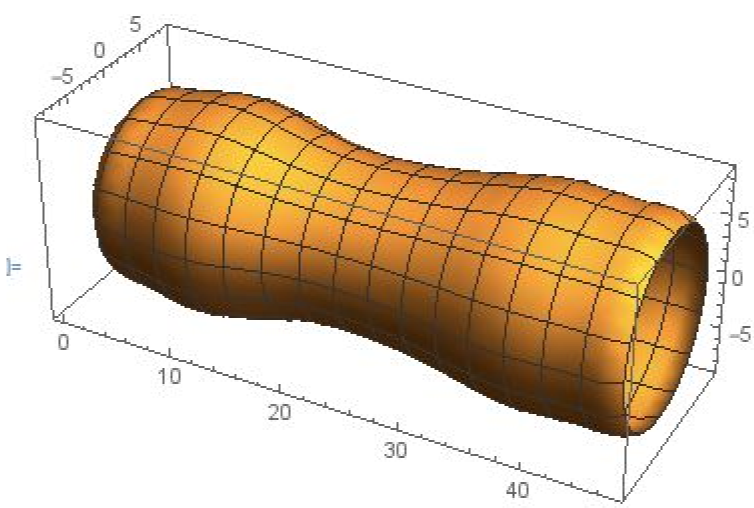
A) Method of Calculation

Suicune's body is circular, so I used regular rotations to calculate its volume. Although the individual slices may be shifted up or down by a little, the volume is the same by Cavalieri's Principle.

B) Calculations

$f(x) = -2.067541679\cdot10^{-12}\,x^{10} + 5.109741701\cdot10^{-10}\,x^{9} - 5.439864173\cdot10^{-8}\,x^{8} + 3.26286222\cdot10^{-6}\,x^{7} - 1.210075817\cdot10^{-4}\,x^{6} + 2.863928057\cdot10^{-3}\,x^{5} - 4.309434118\cdot10^{-2}\,x^{4} + 3.994635481\cdot10^{-1}\,x^{3} - 2.159828153\,x^{2} + 6.281663192\,x + 3.139020171\cdot10^{-1}$

$R^2 = 0.9806965116$



$$V = \pi \int_{0}^{47} f^{2}(x)\,dx$$

$$= \mathbf{9114.294\ cm^3}$$

IV. NECK

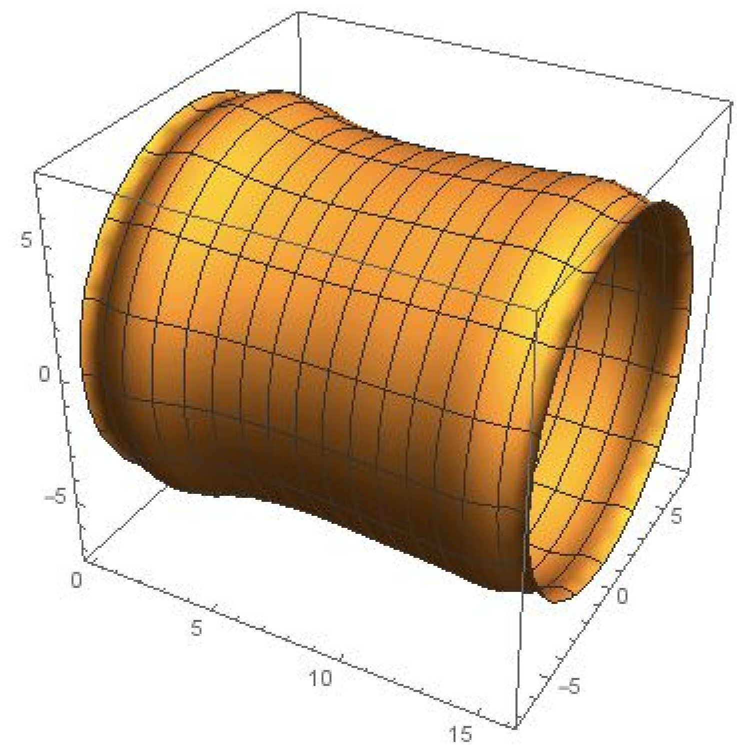
A) Method of Calculation

Suicune's neck is also circular, so I used regular rotations to calculate its volume. Although the individual slices may be shifted up or down by a little, the volume is the same by Cavalieri's Principle.

B) Calculations

$f(x) = 2.360944506 \cdot 10^{-8} x^{10} - 1.915376928 \cdot 10^{-6} x^{9} + 6.690165412 \cdot 10^{-5} x^{8} - 1.316178079 \cdot 10^{-3} x^{7} + 1.600272306 \cdot 10^{-2} x^{6} - 1.238937833 \cdot 10^{-1} x^{5} + 6.038947293 \cdot 10^{-1} x^{4} - 1.745187803 x^{3} + 2.559347317 x^{2} - 1.240429843 x + 6.883992815$

$R^2 = 0.9393802076$



$$V = \pi \int_{0}^{16} f^{2}(x)dx$$

$= \mathbf{2276.942\ cm^3}$

V. HEAD

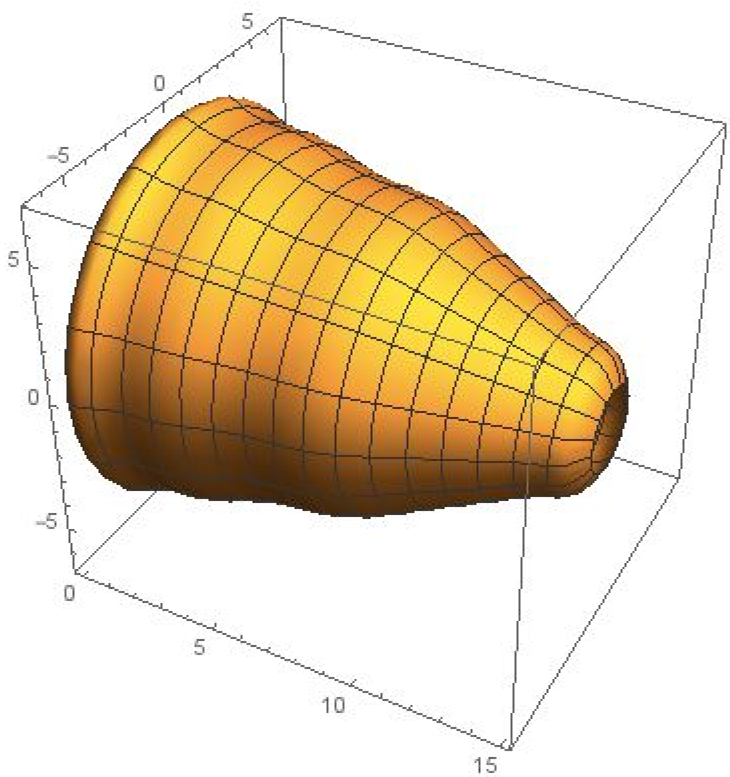
A) Method of Calculation

Suicune's head is also circular, so I used regular rotations to calculate its volume. Although the individual slices may be shifted up or down by a little, the volume is the same by Cavalieri's Principle.

B) Calculations

$f(x) = -7.131006868 \cdot 10^{-8} x^{10} + 5.42198665 \cdot 10^{-6} x^9 - 1.760274779 \cdot 10^{-4} x^8 + 3.17972316 \cdot 10^{-3} x^7 - 3.489797024 \cdot 10^{-2} x^6 + 2.385566777 \cdot 10^{-1} x^5 - 1.001530234 x^4 + 2.451078719 x^3 - 3.137798652 x^2 + 1.466441922 x + 6.472092202$

$R^2 = 0.9992603482$



$$V = \pi \int_0^{15} f^2(x)dx$$

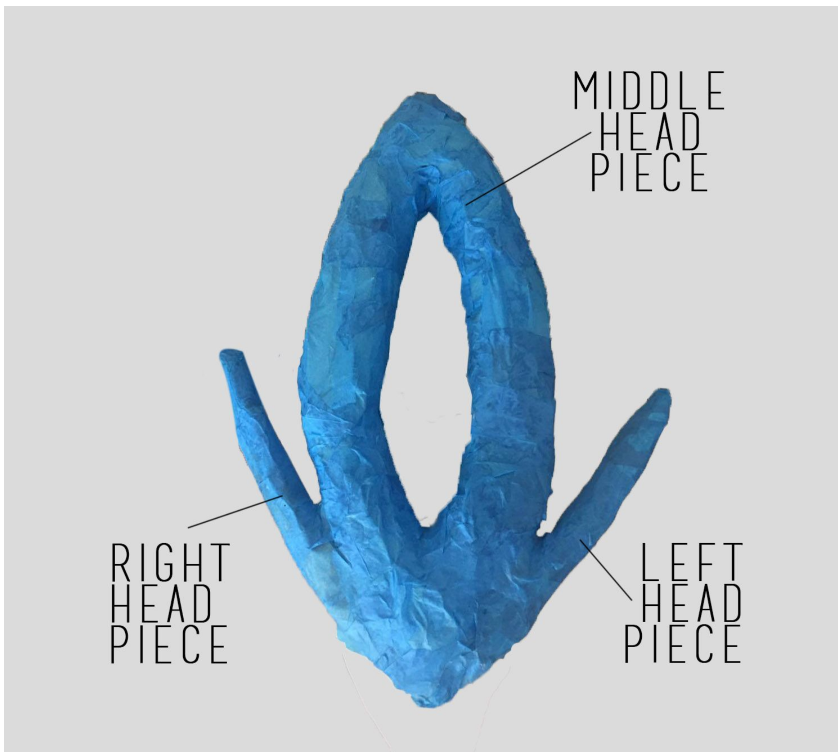
$= \mathbf{1152.858cm^3}$

VI. HEAD PIECES

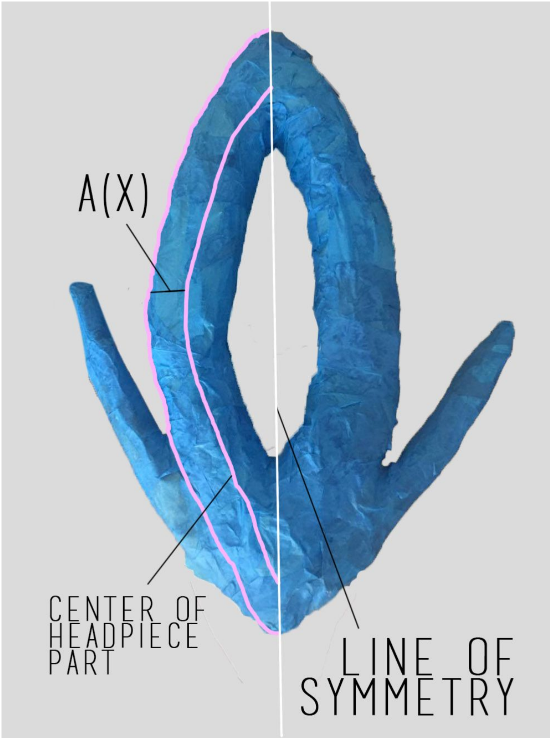
A) Overview & Calculations
For this section of the project, since the headpieces are mostly eye shaped (like two parentheses), I used the same method of calculating volume as I did with the legs.

Cavalieri's Principle accounts for the curvature of the headpieces, especially the middle one.

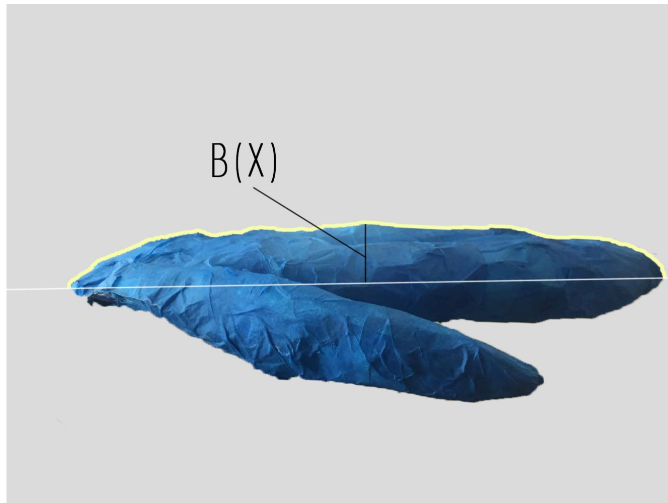
The headpiece is divided into three parts: The middle piece, the right piece, and the left piece.



The following pictures show which function is a(x) and b(x). R(x) is the calculated R value that we later use to integrate for calculating volume.



(top view, A(x) from center of headpiece)



(side view, B(x) from center of headpiece)

B) Middle Headpiece

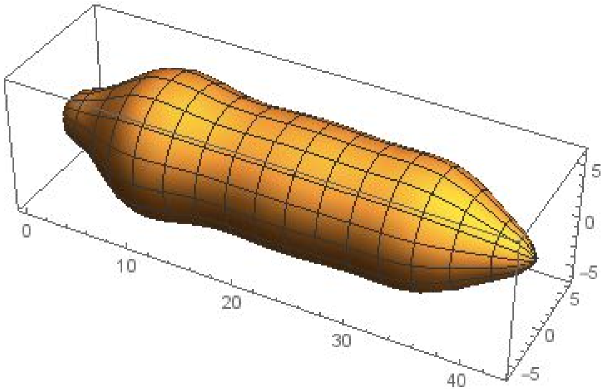
Here we assume that the middle headpiece is symmetrical about its center. Therefore, we apply the "eye-method" (*see Section II*) and multiply the value by 2.

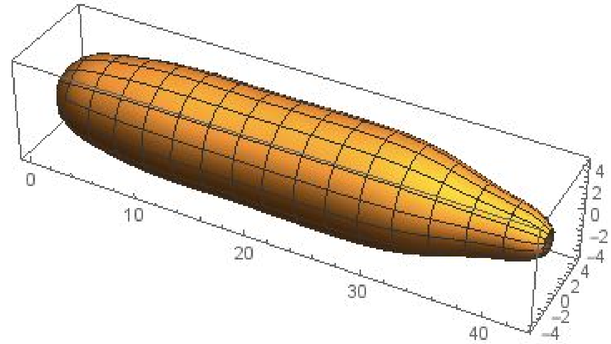
a(x)	b(x)	R(x)
$-5.871420644 \cdot 10^{-12} x^{10} + 1.306722269 \cdot 10^{-9} x^9 - 1.241447236 \cdot 10^{-7} x^8 + 6.562593208 \cdot 10^{-6} x^7 - 2.107738669 \cdot 10^{-4} x^6 + 4.210838113 \cdot 10^{-3} x^5 - 5.138381229 \cdot 10^{-2} x^4 + 3.608065829 \cdot 10^{-1} x^3 - 1.323418475 x^2 + 2.68471244 x + 9.45793819 \cdot 10^{-2}$ $R^2 = 0.9923742169$	$-1.572450098 \cdot 10^{-12} x^{10} + 3.399117674 \cdot 10^{-10} x^9 - 3.172882947 \cdot 10^{-8} x^8 + 1.676619228 \cdot 10^{-6} x^7 - 5.520800278 \cdot 10^{-5} x^6 + 1.173943059 \cdot 10^{-3} x^5 - 1.616003778 \cdot 10^{-2} x^4 + 1.410037885 \cdot 10^{-1} x^3 - 0.753415725 x^2 + 2.413358205 x + 1.765971708 \cdot 10^{-1}$ $R^2 = 0.9800345169$	$-7.114514695 \cdot 10^{-12} x^{10} + 1.592962101 \cdot 10^{-9} x^9 - 1.521697587 \cdot 10^{-7} x^8 + 8.083419362 \cdot 10^{-6} x^7 - 2.607225977 \cdot 10^{-4} x^6 + 5.227076953 \cdot 10^{-3} x^5 - 6.394230592 \cdot 10^{-2} x^4 + 4.489027959 \cdot 10^{-1} x^3 - 1.626734453 x^2 + 3.073252118 x + 5.642295213 \cdot 10^{-2}$ $R^2 = 0.9892123013$

$$V = 2 * [\pi \int_0^{43} \frac{2tan^{-1}(\frac{b(x)}{a(x)})}{\pi} * R^2(x)dx - \frac{1}{2} \int_0^{43} (R(x) - b(x))2a(x)dx]$$

$= \mathbf{2679.593cm^3}$

A(x) *B(x)*





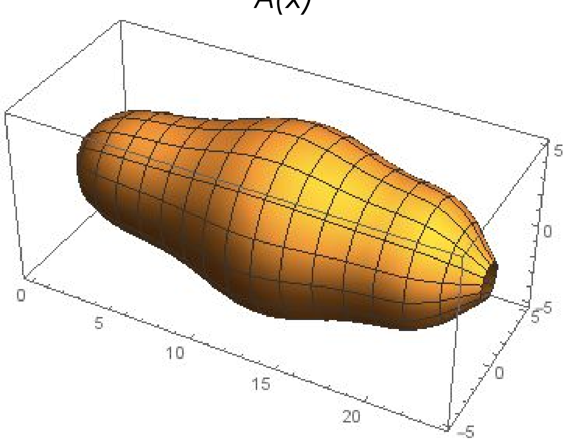
C) Right Headpiece

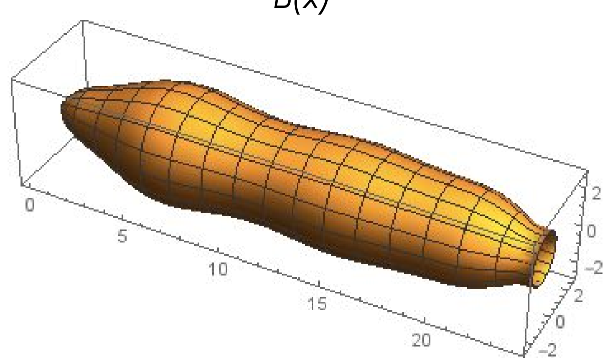
a(x)	b(x)	R(x)
$-1.61988024 \cdot 10^{-11} x^{10} + 5.782304096 \cdot 10^{-9} x^9 - 4.911186061 \cdot 10^{-7} x^8 + 1.930301616 \cdot 10^{-5} x^7 - 4.208718644 \cdot 10^{-4} x^6 + 5.503968611 \cdot 10^{-3} x^5 - 4.558955199 \cdot 10^{-2} x^4 + 2.543033668 \cdot 10^{-1} x^3 - 0.986475277 x^2 + 2.571390717 x + 2.706452637 \cdot 10^{-3}$ $R^2 = 0.9964322178$	$1.541905809 \cdot 10^{-10} x^{10} - 1.222578654 \cdot 10^{-8} x^9 + 2.510107626 \cdot 10^{-7} x^8 + 5.294314388 \cdot 10^{-6} x^7 - 3.417549504 \cdot 10^{-4} x^6 + 6.796846656 \cdot 10^{-3} x^5 - 6.853844585 \cdot 10^{-2} x^4 + 3.686109136 \cdot 10^{-1} x^3 - 1.038824695 x^2 + 1.777725821 x - 7.577227858 \cdot 10^{-3}$ $R^2 = 0.98760855$	$7.627698993 \cdot 10^{-11} x^{10} - 3.218384032 \cdot 10^{-9} x^9 - 1.736098802 \cdot 10^{-7} x^8 + 1.564655457 \cdot 10^{-5} x^7 - 4.765501799 \cdot 10^{-4} x^6 + 7.625979066 \cdot 10^{-3} x^5 - 6.958611996 \cdot 10^{-2} x^4 + 3.628397183 \cdot 10^{-1} x^3 - 1.066166556 x^2 + 1.986221433 x - 8.668638489 \cdot 10^{-3}$ $R^2 = 0.995048854$

$$V = \pi \int_0^{24} \frac{2tan^{-1}\left(\frac{b(x)}{a(x)}\right)}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{24} (R(x) - b(x))2a(x)dx$$

= **649.677cm³**

A(x) *B(x)*





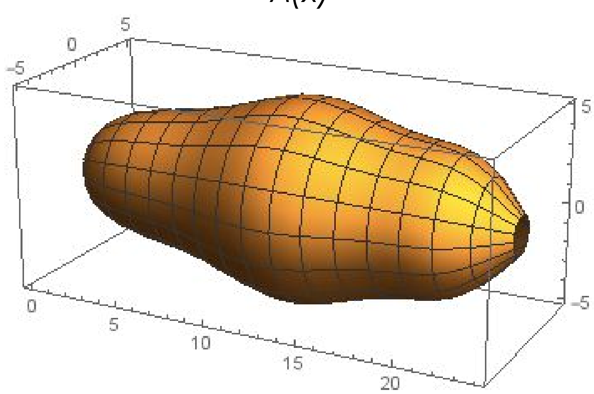
D) Left Headpiece

a(x)	b(x)	R(x)
$-1.234014592 \cdot 10^{-10} x^{10} + 1.8545503 \cdot 10^{-8} x^{9} - 1.140307113 \cdot 10^{-6} x^{8} + 3.775766648 \cdot 10^{-5} x^{7} - 7.429777436 \cdot 10^{-4} x^{6} + 9.070392404 \cdot 10^{-3} x^{5} - 7.045108962 \cdot 10^{-2} x^{4} + 3.573184276 \cdot 10^{-1} x^{3} - 1.203014341 x^{2} + 2.712509652 x - 2.610959738 \cdot 10^{-3}$ $R^2 = 0.9969064464$	$-3.298322607 \cdot 10^{-10} x^{10} + 4.475484037 \cdot 10^{-8} x^{9} - 2.587163715 \cdot 10^{-6} x^{8} + 8.306962387 \cdot 10^{-5} x^{7} - 1.618425853 \cdot 10^{-3} x^{6} + 1.961445969 \cdot 10^{-2} x^{5} - 1.457845507 \cdot 10^{-1} x^{4} + 0.633250957 x^{3} - 1.504700391 x^{2} + 2.095768422 x - 2.806943521 \cdot 10^{-3}$ $R^2 = 0.994175653$	$-2.488819404 \cdot 10^{-10} x^{10} + 3.465865953 \cdot 10^{-8} x^{9} - 2.037419963 \cdot 10^{-6} x^{8} + 6.600532135 \cdot 10^{-5} x^{7} - 1.289727449 \cdot 10^{-3} x^{6} + 1.563611709 \cdot 10^{-2} x^{5} - 1.168166029 \cdot 10^{-1} x^{4} + 5.207646892 \cdot 10^{-1} x^{3} - 1.336533645 x^{2} + 2.157001942 x - 2.982225487 \cdot 10^{-3}$ $R^2 = 0.9969149121$

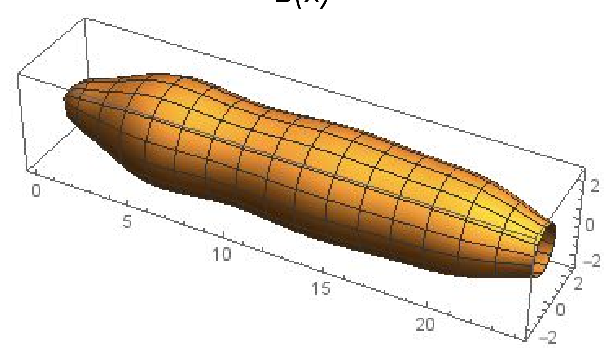
$$V = \pi \int_0^{24} \frac{2\tan^{-1}\left(\frac{b(x)}{a(x)}\right)}{\pi} * R^2(x)dx - \frac{1}{2}\int_0^{24} (R(x) - b(x))2a(x)dx$$

$$= \mathbf{634.077cm^3}$$

A(x)



B(x)



VII. CONCLUSION

A) The total volume of the project is the sum of the individual volumes. Thus,

$V_{total} = V_{front\ left\ leg} + V_{back\ left\ left} + V_{front\ right\ leg} + V_{back\ right\ leg} + V_{body} + V_{neck} + V_{head} + V_{middle\ headpiece} + V_{right\ headpiece} + V_{left\ headpiece}$

$= 2650.496\ cm^3 + 3152.403cm^3 + 2808.263\ cm^3 + 3557.096cm^3 + 9114.294\ cm^3 + 2276.942\ cm^3 + 1152.858cm^3 + 2679.593cm^3 + 649.677cm^3 + 634.077cm^3$

$=$ **$28{,}585.699cm^3$**

In other units…

$$28,585.699\ cm^3 * \frac{1000mm^3}{cm^3} = 28,585,699.129\ mm^3$$
$$28,585.699\ cm^3 * \frac{0.061in^3}{cm^3} = 1743.728\ in^3$$

VIII. RAW DATA

LEGS

Front Left

z (cm)	b_1 (cm)	a_1 (cm)	r_1 (cm)	b_2 (cm)	a_2 (cm)	r_2 (cm)
0	2.5	6.1	8.692	2.4	6	8.7
1	2.5	6.2	8.938	2.5	6.1	8.692
2	2.6	5.9	7.994230769	2.6	5.8	7.769230769
3	2.7	5.3	6.551851852	2.8	5.2	6.228571429
4	2.8	5	5.864285714	2.7	4.8	5.616666667
5	2.5	2.6	2.602	2.3	2.5	2.508695652
6	2.3	2.8	2.854347826	2.2	2.6	2.636363636
7	2.4	2.9	2.952083333	2.2	2.8	2.881818182
8	2.4	3.1	3.202083333	2.2	2.9	3.011363636
9	2.5	3.2	3.298	2.3	3	3.106521739
10	2.5	3.4	3.562	2.4	3.2	3.333333333
11	2.6	3.6	3.792307692	2.5	3.4	3.562
12	2.6	3.7	3.932692308	2.4	3.5	3.752083333
13	2.6	3.9	4.225	2.4	3.8	4.208333333
14	2.6	4.2	4.692307692	2.5	4	4.45
15	2.6	4.5	5.194230769	2.5	4.3	4.948
16	2.7	4.7	5.440740741	2.5	4.6	5.482
17	2.7	5	5.97962963	2.6	4.8	5.730769231
18	2.7	5.2	6.357407407	2.6	5.1	6.301923077
19	2.7	5.3	6.551851852	2.6	5.2	6.5
20	2.9	6.1	7.865517241	2.7	5.9	7.796296296
21	3	6.1	7.701666667	2.8	5.9	7.616071429
22	2.9	6.3	8.293103448	2.8	5.95	7.721875
23	2.7	5.9	7.796296296	2.6	5.85	7.88125
24	2.8	6.1	8.044642857	2.7	5.9	7.796296296
25	2.8	6.1	8.044642857	2.7	6.15	8.354166667
26	2.9	6.3	8.293103448	2.7	6.2	8.468518519
27	2.9	6.6	8.960344828	2.8	6.4	8.714285714
28	3	6.7	8.981666667	2.8	6.55	9.061160714
29	3.3	6.2	7.474242424	3.2	7.3	9.9265625
30	3.4	7.1	9.113235294	3.3	6.9	8.863636364
31	3.5	7.3	9.362857143	3.3	7.2	9.504545455

Back Left

z (cm)	b_1 (cm)	a_1 (cm)	r_1 (cm)	b_2 (cm)	a_2 (cm)	r_2 (cm)
0	3.1	5.9	7.164516129	2.9	5.8	7.25
1	3.1	6.1	7.551612903	3	5.9	7.301666667
2	3.2	6.2	7.60625	3	6.1	7.701666667
3	3.1	5.8	6.975806452	3	5.6	6.726666667
4	3	5.6	6.726666667	3	5.5	6.541666667
5	2.8	4.2	4.55	2.6	3.9	4.225
6	2.8	3.6	3.714285714	2.6	3.5	3.655769231
7	2.7	3.5	3.618518519	2.6	3.3	3.394230769
8	2.7	3.4	3.490740741	2.5	3.2	3.298
9	2.7	3.5	3.618518519	2.5	3.3	3.428
10	2.7	3.8	4.024074074	2.6	3.5	3.655769231
11	2.7	3.7	3.885185185	2.6	3.6	3.792307692
12	2.8	3.8	3.978571429	2.7	3.8	4.024074074
13	2.8	4	4.257142857	2.7	3.9	4.166666667
14	2.9	4.2	4.49137931	2.7	3.9	4.166666667
15	2.9	4.1	4.348275862	2.8	4.1	4.401785714
16	3	4.3	4.581666667	2.9	4.2	4.49137931
17	3.1	4.5	4.816129032	2.9	4.4	4.787931034
18	3.1	4.6	4.962903226	3	4.4	4.726666667
19	3.1	4.6	4.962903226	3	4.6	5.026666667
20	3.2	4.8	5.2	3	4.8	5.34
21	3.2	4.9	5.3515625	3.1	4.8	5.266129032
22	3.2	5.1	5.6640625	3.1	5	5.582258065
23	3.3	5.4	6.068181818	3.2	5.2	5.825
24	3.4	5.5	6.148529412	3.3	5.4	6.068181818
25	3.6	5.7	6.3125	3.4	5.6	6.311764706
26	3.6	5.8	6.472222222	3.5	5.8	6.555714286
27	3.6	5.9	6.634722222	3.5	5.9	6.722857143
28	3.7	6.2	7.044594595	3.6	6.1	6.968055556
29	3.8	6.3	7.122368421	3.7	6.3	7.213513514
30	3.9	6.6	7.534615385	3.9	6.5	7.366666667
31	4	7.1	8.30125	3.9	6.9	8.053846154
32	4.1	7.6	9.093902439	3.9	7.5	9.161538462

Front Right

z (cm)	b_1 (cm)	a_1 (cm)	r_1 (cm)	b_2 (cm)	a_2 (cm)	r_2 (cm)
0	2.6	4.6	5.369230769	2.5	4.4	5.122
1	2.8	5.1	6.044642857	2.6	5	6.107692308
2	2.9	5.5	6.665517241	2.9	5.4	6.477586207

3	2.9	5.8	7.25	2.8	5.7	7.201785714
4	2.8	5.6	7	2.8	5.4	6.607142857
5	3.1	4.1	4.261290323	2.9	3.9	4.072413793
6	2.8	3.7	3.844642857	2.8	3.7	3.844642857
7	2.7	3.6	3.75	2.7	3.4	3.490740741
8	2.8	3.6	3.714285714	2.6	3.4	3.523076923
9	2.8	3.6	3.714285714	2.7	3.4	3.490740741
10	2.9	3.6	3.684482759	2.7	3.5	3.618518519
11	2.7	3.6	3.75	2.8	3.4	3.464285714
12	2.8	3.6	3.714285714	2.8	3.4	3.464285714
13	2.8	3.6	3.714285714	2.8	3.4	3.464285714
14	2.9	3.6	3.684482759	2.7	3.5	3.618518519
15	2.9	3.7	3.810344828	2.8	3.6	3.714285714
16	3	3.8	3.906666667	2.8	3.7	3.844642857
17	3	3.9	4.035	2.8	3.9	4.116071429
18	2.9	4.2	4.49137931	2.8	4	4.257142857
19	2.8	4.3	4.701785714	2.8	4.3	4.701785714
20	2.9	4.5	4.94137931	2.8	4.5	5.016071429
21	2.9	4.6	5.098275862	2.9	4.6	5.098275862
22	3	4.9	5.501666667	3	4.7	5.181666667
23	3.1	5.2	5.911290323	3	4.9	5.501666667
24	3.3	5.3	5.906060606	3.1	5.3	6.080645161
25	3.3	5.6	6.401515152	3.2	5.6	6.5
26	3.5	5.8	6.555714286	3.4	5.7	6.477941176
27	3.6	6.1	6.968055556	3.6	6	6.8
28	3.7	6.4	7.385135135	3.7	6.3	7.213513514
29	3.9	6.9	8.053846154	3.8	6.7	7.806578947
30	3.8	7.3	8.911842105	3.7	7.1	8.662162162
31	3.8	7.6	9.5	3.7	7.4	9.25

Back Right

z (cm)	b_1 (cm)	a_1 (cm)	r_1 (cm)	b_2 (cm)	a_2 (cm)	r_2 (cm)
0	2.9	5.8	7.25	2.9	5.7	7.051724138
1	2.8	5.9	7.616071429	2.8	8.2	13.40714286
2	2.8	5.9	7.616071429	2.9	8.7	14.5
3	2.9	6	7.656896552	2.9	8.1	12.76206897
4	2.9	5.9	7.451724138	2.8	8.1	13.11607143
5	2.7	2.8	2.801851852	2.5	2.7	2.708
6	2.8	2.8	2.8	2.7	2.6	2.601851852
7	3.1	2.8	2.814516129	2.9	2.7	2.706896552

8	3.1	2.9	2.906451613	3.1	2.7	2.725806452
9	3.3	3	3.013636364	3.2	2.9	2.9140625
10	3.3	3.1	3.106060606	3.2	3	3.00625
11	3.3	3.3	3.3	3.3	3.1	3.106060606
12	3.4	3.4	3.4	3.2	3.3	3.3015625
13	3.4	3.5	3.501470588	3.2	3.5	3.5140625
14	3.3	3.8	3.837878788	3.2	3.7	3.7390625
15	3.4	3.9	3.936764706	3.3	3.9	3.954545455
16	3.4	4.2	4.294117647	3.2	4.2	4.35625
17	3.4	4.3	4.419117647	3.3	4.3	4.451515152
18	3.5	4.5	4.642857143	3.3	4.4	4.583333333
19	3.5	4.6	4.772857143	3.3	4.6	4.856060606
20	3.5	4.8	5.041428571	3.3	4.7	4.996969697
21	3.5	5	5.321428571	3.4	4.9	5.230882353
22	3.5	5.1	5.465714286	3.4	5	5.376470588
23	3.5	5.2	5.612857143	3.5	5.2	5.612857143
24	3.6	5.4	5.85	3.5	5.3	5.762857143
25	3.6	5.5	6.001388889	3.6	5.4	5.85
26	3.7	5.7	6.240540541	3.6	5.7	6.3125
27	3.8	5.9	6.480263158	3.7	5.9	6.554054054
28	3.9	6.2	6.878205128	3.8	6	6.636842105
29	4	6.3	6.96125	3.8	6.2	6.957894737
30	4.1	6.5	7.202439024	4.1	6.5	7.202439024
31	4.4	7	7.768181818	4.3	6.8	7.526744186

BODY

z (cm)	circumference (in)	radius (cm)
0	0	0
1	14	5.659549776
2	16	6.468056887
3	18	7.276563998
4	19	7.680817554
5	20	8.085071109
6	21	8.489324665
7	21	8.489324665
8	21	8.489324665
9	21	8.489324665
10	21.5	8.691451442
11	22	8.89357822
12	21	8.489324665

13	21	8.489324665
14	20	8.085071109
15	20	8.085071109
16	20	8.085071109
17	19	7.680817554
18	18	7.276563998
19	18	7.276563998
20	17.5	7.07443722
21	17	6.872310443
22	17	6.872310443
23	17	6.872310443
24	17	6.872310443
25	17	6.872310443
26	17.25	6.973373832
27	17.25	6.973373832
28	17.75	7.175500609
29	18	7.276563998
30	18.5	7.478690776
31	19	7.680817554
32	19.5	7.882944331
33	19.75	7.98400772
34	20.25	8.186134498
35	20.75	8.388261276
36	21	8.489324665
37	21.25	8.590388053
38	21.75	8.792514831
39	21.25	8.590388053
40	21.25	8.590388053
41	21.25	8.590388053
42	21.25	8.590388053
43	21.25	8.590388053
44	21	8.489324665
45	21	8.489324665
46	20.5	8.287197887
47	19.5	7.882944331

NECK

z (cm)	circumference (in)	radius (cm)
0	17	6.872310443
1	17.25	6.973373832

2	17.75	7.175500609
3	17.75	7.175500609
4	16.75	6.771247054
5	16.25	6.569120276
6	16.25	6.569120276
7	16.25	6.569120276
8	16	6.468056887
9	16.25	6.569120276
10	16.25	6.569120276
11	16.75	6.771247054
12	16.5	6.670183665
13	16.75	6.771247054
14	17	6.872310443
15	16.25	6.569120276
16	16.25	6.569120276

HEAD

z (cm)	circumference (in)	radius (cm)
0	16	6.468056887
1	16	6.468056887
2	15.25	6.164866721
3	15.25	6.164866721
4	14.5	5.861676554
5	13.5	5.457422999
6	13.5	5.457422999
7	13	5.255296221
8	12.5	5.053169443
9	11.5	4.648915888
10	8.5	4.244662332
11	7.5	3.739345388
12	6.5	3.234028444
13	56	2.829774888
14	4.95	2.425521333
15	2.88	1.414887444

HEADPIECES

Middle

z (cm)	a (cm)	b (cm)	R (cm)
0	0	0	0

1	2.064606742	2.48	2.099395363
2	2.162921348	2.64	2.206028174
3	2.654494382	3.26	2.710727059
4	2.935393258	3.16	2.943375566
5	4.04494382	4.34	4.054973561
6	4.606741573	4.26	4.620853042
7	4.957865169	4.24	5.018635263
8	5.58988764	4.2	5.81986236
9	6.095505618	4.3	6.470370784
10	6.573033708	4.36	7.134675703
11	6.516853933	4.48	6.979886739
12	6.179775281	4.52	6.484515766
13	5.95505618	4.52	6.182864392
14	5.842696629	4.7	5.981606798
15	5.561797753	4.64	5.653361449
16	5.617977528	4.6	5.730616468
17	5.674157303	4.58	5.804853832
18	5.730337079	4.48	5.904817303
19	5.646067416	4.6	5.765008398
20	5.533707865	4.5	5.652435859
21	5.47752809	4.56	5.569837059
22	5.47752809	4.58	5.565470958
23	5.491	4.46	5.610166031
24	5.53	4.58	5.628526201
25	5.581	4.62	5.68094816
26	5.721	4.68	5.836777885
27	5.681	4.6	5.8080175
28	5.613	4.6	5.724540109
29	5.372	4.58	5.440478603
30	5.198	4.57	5.241149234
31	5.403225806	4.41	5.515073595
32	5.463709677	4.23	5.643619791
33	5.302419355	4.05	5.496068027
34	5.423387097	3.82	5.759885812
35	5.241935484	3.68	5.573408644
36	4.828629032	3.32	5.171394327
37	4.284274194	3.06	4.529183884
38	3.760080645	2.83	3.912916335
39	3.185483871	2.51	3.276375995
40	2.721774194	2.28	2.764573413
41	2.096774194	2.04	2.097564221

42	1.330645161	1.82	1.396433117
43	0.2318548387	0.98	0.5174268705

Right

z (cm)	a (cm)	b (cm)	R (cm)
0	0	0	0
1	1.01	1.812	1.187484547
2	1.43	2.631	1.704116496
3	1.92	3.105	2.146123188
4	2.09	3.275	2.304385496
5	2.43	3.485	2.589687948
6	2.81	3.613	2.89923457
7	2.95	3.512	2.994966401
8	2.63	4.034	2.874325235
9	2.47	4.329	2.869154655
10	2.38	4.539	2.893470037
11	2.37	4.689	2.943444338
12	2.37	5.023	3.070618057
13	2.48	5.312	3.234915663
14	2.66	5.118	3.250246581
15	2.49	4.771	3.03526944
16	2.51	4.573	2.97533665
17	2.47	4.232	2.836805766
18	2.44	4.098	2.775403123
19	2.28	3.821	2.590740775
20	2.19	3.702	2.498771475
21	2.25	3.52	2.479105114
22	1.78	2.941	2.00916032
23	1.43	2.045	1.52247555
24	1.24	1.01	1.266188119

Left

z (cm)	a (cm)	b (cm)	R (cm)
0	0	0	0
1	1.081081081	1.78913738	1.221188746
2	1.441441441	2.587859425	1.695372196
3	1.801801802	3.03514377	2.052388352
4	2.192192192	3.162939297	2.341159632
5	2.372372372	3.38658147	2.524239395

6	2.642642643	3.610223642	2.772304
7	2.612612613	3.610223642	2.750447255
8	2.492492492	3.993610224	2.774612469
9	2.372372372	4.313099042	2.808998099
10	2.282282282	4.536741214	2.842440426
11	2.402402402	4.664536741	2.950929732
12	2.432432432	5.047923323	3.110017269
13	2.552552553	5.239616613	3.241564154
14	2.642642643	5.047923323	3.215687713
15	2.492492492	4.664536741	2.998199327
16	2.402402402	4.47284345	2.881597145
17	2.402402402	4.153354633	2.771481615
18	2.432432432	4.121405751	2.778507418
19	2.372372372	3.801916933	2.641131223
20	2.162162162	3.674121406	2.473259769
21	2.102102102	3.482428115	2.375661216
22	1.891891892	2.939297125	2.07851095
23	1.471471471	2.044728435	1.551830198
24	1.201201201	0.9904153355	1.223631529

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